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Synthesis of an Adaptive State-constrained Control for MIMO Euler-Lagrange Systems

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Abstract: A state-constrained adaptive control synthesis is presented in this paper for multi-input multi-output Euler-Lagrange nonlinear systems associated with structured uncertainties. The controller is synthesized in two steps: (i) an approximated system is constructed to approximate model uncertainties (ii) a novel nonlinear error transformation based control law is designed to ensure the desired reference command tracking. A neural network is used in the approximated system to approximate the model uncertainties, and the weights of the neural network are updated using a stable weight update rule. The proposed controller ensures that the closed-loop states of the system will remain bounded by the user-defined constraints and the steady-state errors will converge asymptotically to a predefined domain. The proposed formulation also gives the flexibility to impose independent constraints on system states and leads to an easily on-board implementable closed-form control solution. The effectiveness of the control design is demonstrated by extensive computer simulations.

Keywords: State-constrained control, Error transformation, Barrier Lyapunov function.

1. INTRODUCTION

Euler-Lagrange (EL) systems are considered as a very important class of nonlinear systems as they represent a wide class of practical systems (Zhao et al., 2018). These systems are often associated with constraints in the form of saturation, performance, or safety requirement. Therefore, to handle constraints on EL systems, various solutions have been proposed in the literature (Tee et al., 2012; Zhang et al., 2018; Li and Li, 2017; He and Dong, 2018; Sun et al., 2018; Zhao et al., 2018; Sachan and Padhi, 2019)

The use of *barrier Lyapunov function* (BLF) in the context of backstepping control design philosophy is one of the ways to enforce system constraints. Utilizing BLF, several output-constrained controllers are proposed for EL systems, for instance, refer the work carried out by Tee et al. (2012); Zhang et al. (2018); Li and Li (2017); He and Dong (2018) and the references therein. State-constrained control solutions can also be derived using BLF based control design. A state-constrained backstepping control law is proposed by He et al. (2016) using BLF and Moore-Penrose inverse. Other BLF based control solutions are proposed by Sun et al. (2018); Zhao et al. (2018), where constraints are imposed on the norm of states. Since constraints cannot be imposed on each component of the system states, this limits the control applications. Additionally, since backstepping philosophy is used in the core of above control formulations, these control solutions suffer from several other limitations, such as, a complex intermediate stability term and its time derivatives are required for control implementation, which may be difficult to obtain for large dimensional systems. Moreover, before implementation, an offline feasibility analysis is also required for these controllers.

To mitigate the above limitations, a new robust adaptive control formulation is proposed in this paper for EL systems. The controller is synthesized in two steps. In the first step, a *neural network* (NN) based novel approximated system dynamics is constructed to approximate the model uncertainties. The weights of the NN are updated by a Lyapunov stable weight update learning rule. Since structured uncertainties are considered in this work, the basis functions for the NN are selected by the combination of system states. In the second step, the approximated system is transformed using a newly proposed error transformation and a controller is designed to ensure the asymptotic stability of the closed-loop system. It should be noted that using this error transformation, the limitations of backstepping control design are avoided. It is proven that the closed-loop states of the system will remain bounded by the imposed state constraints and asymptotically converge to a user-defined bounded region. The performance and effectiveness of the proposed controller are demonstrated by computer simulations.

It can be mentioned here that using a similar type of error transformation, controllers are designed by Sachan and Padhi (2018); Sachan and Padhi (2019). However, only output constraints can be imposed using these controllers. Furthermore, several other error transformation based adaptive control methods are available in the literature (Bechlioulis and Rovithakis, 2008; Wang et al., 2016; Liu et al., 2017; Arabi et al., 2019). However, using these controllers, only closed-loop error of the system can be enforced within the prescribed bounds. Therefore, the major difference between the proposed and existing error transformations is that the proposed transformation can enforce state constraints, whereas the existing transformations cannot handle state constraints.

The major contributions and salient features of the proposed control design can be summarized as follows: (i) states will remain bounded by the user-defined constraints; (ii) the errors of the system converge asymptotically to a predefined domain; (iii) the formulation of the controller is simple and reduce to output-constrained control and dynamic inversion control by just changing the nature of the control gain matrices; (iv) unlike backstepping design, the proposed control law results in a simple closed-form solution, which is easy for onboard implementations and does not require any feasibility analysis.

Notations: In this paper, vectors are represented by $\boldsymbol{z}$ (small and bold) and their i^{th} components are denoted by z_i. The matrices are represented by $\boldsymbol{Z}$ (capital and bold). The symbol for exponential is represented by $\mathbf{e}$.

2. PROBLEM FORMULATION

Consider the dynamics of EL systems as follows

$$\begin{aligned}\dot{\boldsymbol{x}}_{\boldsymbol{1}} &= \boldsymbol{x}_{\boldsymbol{2}}\\ \dot{\boldsymbol{x}}_{\boldsymbol{2}} &= \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{d}(\boldsymbol{x}) \end{aligned} \tag{1}$$

where the kinematic and dynamics state variables $\boldsymbol{x}_{\boldsymbol{1}}$ and $\boldsymbol{x}_{\boldsymbol{2}}$ are defined as $\boldsymbol{x}_{\boldsymbol{1}} \triangleq [x_{11},\ x_{12},\ \dots,\ x_{1n}]^T \in \Re^n$, and $\boldsymbol{x}_{\boldsymbol{2}} \triangleq [x_{21},\ x_{22},\ \dots,\ x_{2n}]^T \in \Re^n$, respectively. The system output and control input are represented by $\boldsymbol{y} = \boldsymbol{x}_{\boldsymbol{1}}$ and $\boldsymbol{u} \in \Re^n$, respectively. Total states are denoted by $\boldsymbol{x} = \left[\boldsymbol{x}_{\boldsymbol{1}}^{\boldsymbol{T}}, \boldsymbol{x}_{\boldsymbol{2}}^{\boldsymbol{T}}\right]^{\boldsymbol{T}}$ and assumed to be measurable. The term $\boldsymbol{d}(\boldsymbol{x})$ represents the additive structured uncertainties. The component of vector function $\boldsymbol{f}(\boldsymbol{x}) \in \Re^n$, and matrix $\boldsymbol{G}(\boldsymbol{x}) \in \Re^{n\times n}$ are the known part of system dynamics and assumed to be smooth and continuous. Moreover, $\boldsymbol{G}(\boldsymbol{x})$ is assumed to be invertible for all time.

The objective of the proposed control design is to ensure that the output error $(y_i - y_i{}^*), \forall i$ converges asymptotically to a user defined set Ω_ρ, where $\Omega_\rho \subset (-\rho_i, \rho_i)$. The controller should also ensure the boundedness of all closed-loop signals without violation of imposed state constraints in presence of structured uncertainties $\boldsymbol{d}(\boldsymbol{x})$, where the imposed constraints on the components of kinematic $\boldsymbol{x}_{\boldsymbol{1}}$ and dynamic $\boldsymbol{x}_{\boldsymbol{2}}$ states are defined as $\underline{\kappa}_i < x_{1i} < \overline{\kappa}_i$ and $\underline{\zeta}_i < x_{2i} < \overline{\zeta}_i$, respectively where $i = 1, 2, \dots, n$.

It should be mentioned that the reference output $\boldsymbol{y}^*$ and its derivatives are assumed to be smooth and bounded, viz. $\underline{d}_{p_i} \le y_i{}^* \le \overline{d}_{p_i}$ and $\underline{d}_{d_i} \le \dot{y}_i^* \le \overline{d}_{d_i}, \forall i$, where $\underline{d}_{p_i}, \overline{d}_{p_i}, \underline{d}_{d_i}, \overline{d}_{d_i}$ are constants. Moreover, the bounds on $\boldsymbol{y}^*$ and $\dot{\boldsymbol{y}}^*$ are assumed to be bounded by the imposed state bounds, i.e., $\underline{\kappa}_i < \underline{d}_{p_i} \le y_i{}^* \le \overline{d}_{p_i} < \overline{\kappa}_i$ and $\underline{\zeta}_i < \underline{d}_{d_i} \le \dot{y}_i^* \le \overline{d}_{d_i} < \overline{\zeta}_i$, respectively $\forall i$.

To approximate model uncertainties, a separate NN is used in each dynamic state channel to approximate model uncertainties. The uncertainties of each channel can be defined as $\boldsymbol{d}(\boldsymbol{x}) = [d_1(\boldsymbol{x}), \dots, d_n(\boldsymbol{x})]^T$. The structure of the model uncertainties is assumed as $d_i(\boldsymbol{x}) = \boldsymbol{w}_{\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{\phi}_{\boldsymbol{i}}(\boldsymbol{x})$, where $\boldsymbol{w}_{\boldsymbol{i}}$ represent ideal weights and $\phi_i(\boldsymbol{x})$ represents known basis functions. The output of the NN corresponds to $d_i(\boldsymbol{x})$ is calculated as $\hat{d}_i(\boldsymbol{x}) = \hat{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{\phi}_{\boldsymbol{i}}(\boldsymbol{x})$, where $\hat{\boldsymbol{w}}_{\boldsymbol{i}}$ are the approximated weights of the NN. The basis function $\phi_i(\boldsymbol{x})$ is constructed from system states and weights of NN are updated by a weight update rule (given in (12)). Noticed that since structured uncertainties are assumed, the NN approximation error is zero.

3. ADAPTIVE CONTROL FORMULATION

The proposed control formulation is inspired by model-following neuro-adaptive control formulation, developed by Padhi et al. (2007). The controller is synthesized in two steps. In the first steps, the model uncertainties are estimated using the approximated system dynamics and in the second step, an asymptotically stable controller is design using the online estimation of uncertainties. The synthesis of overall controller along with approximated system is explained in the following subsections.

3.1 Approximated System Dynamics

A novel approximated system dynamics to approximate model uncertainties is constructed as

$$\begin{aligned}\dot{\boldsymbol{x}}_{\boldsymbol{a_1}} &= \boldsymbol{x}_{\boldsymbol{a_2}}\\ \dot{\boldsymbol{x}}_{\boldsymbol{a_2}} &= \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} + \hat{\boldsymbol{d}}(\boldsymbol{x}) + \boldsymbol{K}_{\boldsymbol{p_a}}\boldsymbol{e}_{\boldsymbol{a}} + \boldsymbol{K}_{\boldsymbol{d_a}}\dot{\boldsymbol{e}}_{\boldsymbol{a}} \end{aligned} \tag{2}$$

where $\boldsymbol{x}_{\boldsymbol{a_1}}$ and $\boldsymbol{x}_{\boldsymbol{a_2}}$ are the state variables for the approximated system. The error variables $\boldsymbol{e}_{\boldsymbol{a}}$ and $\dot{\boldsymbol{e}}_{\boldsymbol{a}}$ are defined as $\boldsymbol{e}_{\boldsymbol{a}} = \boldsymbol{x}_{\boldsymbol{1}} - \boldsymbol{x}_{\boldsymbol{a_1}}$ and $\dot{\boldsymbol{e}}_{\boldsymbol{a}} = \boldsymbol{x}_{\boldsymbol{2}} - \boldsymbol{x}_{\boldsymbol{a_2}}$. The matrices $\boldsymbol{K}_{\boldsymbol{d_a}}$ and $\boldsymbol{K}_{\boldsymbol{p_a}}$ are symmetric and diagonal, defined as

$$\boldsymbol{K}_{\boldsymbol{d_a}} = \text{diag}\left[\frac{b_{d_1}}{\left(\sigma_1^2 - \dot{e}_{a_1}^2\right)}, \dots, \frac{b_{d_n}}{\left(\sigma_n^2 - \dot{e}_{a_n}^2\right)}\right] \tag{3}$$

$$\boldsymbol{K}_{\boldsymbol{p_a}} = \text{diag}\left[\frac{b_{p_1}\left(\sigma_1^2 - \dot{e}_{a_1}^2\right)}{\left(\rho_1^2 - e_{a_1}^2\right)}, \dots, \frac{b_{p_n}\left(\sigma_n^2 - \dot{e}_{a_n}^2\right)}{\left(\rho_n^2 - e_{a_n}^2\right)}\right] \tag{4}$$

where, b_{d_i} and b_{p_i} are positive constants. The variables σ_i and ρ_i are user defined constants, serve as the bounds on steady-state error of the overall system. The term $\hat{\boldsymbol{d}}(\boldsymbol{x})$ is the approximation of model uncertainties, obtained as $\hat{d}_i(\boldsymbol{x}) = \hat{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{\phi}_{\boldsymbol{i}}(\boldsymbol{x}), \forall i$.

3.2 Approximation of Model Uncertainties

The approximation error is defined as

$$\boldsymbol{e}_{\boldsymbol{a}} = \boldsymbol{x}_{\boldsymbol{1}} - \boldsymbol{x}_{\boldsymbol{a_1}} \tag{5}$$

The double time derivative of the error in (5) is

$$\ddot{\boldsymbol{e}}_{\boldsymbol{a}} = \dot{\boldsymbol{x}}_{\boldsymbol{2}} - \dot{\boldsymbol{x}}_{\boldsymbol{a_2}} \tag{6}$$

By substituting $\dot{\boldsymbol{x}}_{\boldsymbol{2}}$ and $\dot{\boldsymbol{x}}_{\boldsymbol{a_2}}$ from (1) and (2), $\ddot{\boldsymbol{e}}_{\boldsymbol{a}}$ is

$$\ddot{\boldsymbol{e}}_{\boldsymbol{a}} = \boldsymbol{d}(\boldsymbol{x}) - \hat{\boldsymbol{d}}(\boldsymbol{x}) - \boldsymbol{K}_{\boldsymbol{p_a}}\boldsymbol{e}_{\boldsymbol{a}} - \boldsymbol{K}_{\boldsymbol{d_a}}\dot{\boldsymbol{e}}_{\boldsymbol{a}} \tag{7}$$

Since $\boldsymbol{K}_{\boldsymbol{p_a}}$ and $\boldsymbol{K}_{\boldsymbol{d_a}}$ are diagonal matrices, the i^{th} component of error dynamics (7) can be expressed as

$$\ddot{e}_{a_i} = d_i(\boldsymbol{x}) - \hat{d}_i(\boldsymbol{x}) - \frac{b_{p_i}\left(\sigma_i^2 - \dot{e}_{a_i}^2\right)e_{a_i}}{\left(\rho_i^2 - e_{a_i}^2\right)} - \frac{b_{d_i}\dot{e}_{a_i}}{\left(\sigma_i^2 - \dot{e}_{a_i}^2\right)} \tag{8}$$

where, initial conditions $e_a(0) = 0$ and $\dot{e}_a(0) = 0$. Using the structures of $d_i(\boldsymbol{x}) = \boldsymbol{w}_{\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{\phi}_{\boldsymbol{i}}(\boldsymbol{x})$ and $\hat{d}_i(\boldsymbol{x}) = \hat{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{\phi}_{\boldsymbol{i}}(\boldsymbol{x})$, following is obtained

$$\ddot{e}_{a_i} = \tilde{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{\phi_i(x)} - \frac{b_{p_i}\left(\sigma_i^2 - \dot{e}_{a_i}^2\right) e_{a_i}}{\left(\rho_i^2 - e_{a_i}^2\right)} - \frac{b_{d_i}\dot{e}_{a_i}}{\left(\sigma_i^2 - \dot{e}_{a_i}^2\right)} \quad (9)$$

where, $\tilde{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{T}}$ is defined as $\tilde{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{T}} = \boldsymbol{w}_{\boldsymbol{i}}^{\boldsymbol{T}} - \hat{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{T}}$.

To analyze the stability of the error dynamics (9), the following BLF is selected

$$L_i = \frac{b_{p_i}b_{d_i}}{2}\log\left(\frac{\rho_i^2}{\rho_i^2 - e_{a_i}^2}\right) + \frac{b_{d_i}}{2}\log\left(\frac{\sigma_i^2}{\sigma_i^2 - \dot{e}_{a_i}^2}\right) + \frac{\tilde{\boldsymbol{w}}_i^T\tilde{\boldsymbol{w}}_i}{2\theta_i}$$

where, $\theta_i > 0$ is known as learning rate constant. All other terms are already defined in (3) and (4). The time derivative of the BLF is obtained as

$$\dot{L}_i = \frac{b_{p_i}b_{d_i}e_{a_i}\dot{e}_{a_i}}{\left(\rho_i^2 - e_{a_i}^2\right)} + \frac{b_{d_i}\dot{e}_{a_i}\ddot{e}_{a_i}}{\left(\sigma_i^2 - \dot{e}_{a_i}^2\right)} - \frac{\tilde{\boldsymbol{w}}_i^T\dot{\hat{\boldsymbol{w}}}_i}{\theta_i} \quad (10)$$

Using (9), the derivative of the Lyapunov function in (10) is modified as

$$\dot{L}_i = \left(\frac{-b_{d_i}\dot{e}_{a_i}}{\left(\sigma_i^2 - \dot{e}_{a_i}^2\right)}\right)^2 + \tilde{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{T}}\left(\frac{\boldsymbol{\phi_i(x)}b_{d_i}\dot{e}_{a_i}}{\left(\sigma_i^2 - \dot{e}_{a_i}^2\right)} - \frac{\dot{\hat{\boldsymbol{w}}}_i}{\theta_i}\right) \quad (11)$$

By selecting weight learning rule as

$$\dot{\hat{\boldsymbol{w}}}_i = \frac{b_{d_i}\theta_i\dot{e}_{a_i}\boldsymbol{\phi_i(x)}}{\left(\sigma_i^2 - \dot{e}_{a_i}^2\right)}, \quad (12)$$

the Lyapunov derivative in (11) is obtained as

$$\dot{L}_i = -\left(\frac{b_{d_i}\dot{e}_{a_i}}{\left(\sigma_i^2 - \dot{e}_{a_i}^2\right)}\right)^2 \quad (13)$$

which means $\dot{L}_i \leq 0$. This implies that $L_i(t) \leq L_0, \forall t$, where L_0 is the value of L_i at $t = 0$. Furthermore, in the selected BLF, each term is positive definite and hence, the following is true for individual term

$$\frac{b_{p_i}b_{d_i}}{2}\log\left(\frac{\rho_i^2}{\left(\rho_i^2 - e_{a_i}^2\right)}\right) \leq L_0 \quad (14)$$

By taking the exponential on both sides and followed by algebra, (14) can be written as

$$|e_{a_i}| \leq \rho_i\sqrt{1 - \mathbf{e}^{-\left(2L_0/\left(b_{p_i}b_{d_i}\right)\right)}} \quad (15)$$

where $\mathbf{e}$ represents exponential. Since L_0 is always positive except at equilibrium point zero, the term $\mathbf{e}^{-\left(2L_0/\left(b_{p_i}b_{d_i}\right)\right)}$ is bounded in the domain $(0, 1)$. Therefore, the errors e_{a_i} will remain bounded as $|e_{a_i}| < \rho_i, \forall i$. Similar analysis can be performed for $\dot{e}_{a_i}$ and the bounds on $\dot{e}_{a_i}$ can be shown as $|\dot{e}_{a_i}| < \sigma_i, \forall i$.

Since $\dot{L}_i \leq 0$, the closed-loop signals e_{a_i}, $\dot{e}_{a_i}$, and $\tilde{\boldsymbol{w}}_{\boldsymbol{i}}$ are bounded. Moreover, since the basis functions $\boldsymbol{\phi_i(x)}$ are bounded in nature, by analyzing (9), $\ddot{e}_{a_i}$ is also bounded. Therefore, all closed-loop signals are bounded and stable in the sense of Lyapunov.

We can further analyze the nature of $\dot{e}_{a_i}$ by calculating the time derivative of $\dot{L}_i$ in (13) as

$$\ddot{L}_i = \frac{-2b_{d_i}\dot{e}_{a_i}}{\left(\sigma_i^2 - \dot{e}_{a_i}^2\right)^3}\left[b_{d_i}\ddot{e}_{a_i}\left(\sigma_i^2 - \dot{e}_{a_i}^2\right) + 2b_{d_i}\dot{e}_{a_i}^2\ddot{e}_{a_i}\right]$$

Since each term of $\ddot{L}_i$ is bounded, using Barbalat's Lemma (Sachan and Padhi, 2018), $\dot{L}_i \to 0$, i.e., $\dot{e}_{a_i} \to 0$ as $t \to \infty$.

The above analysis can be summarized in the form of following theorem

Theorem 1. Consider the systems (1) and (2) and if the model uncertainties are approximated as $\hat{d}_i(\boldsymbol{x}) = \hat{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{\phi_i(x)}$ and the weight update rule is chosen as $\dot{\hat{\boldsymbol{w}}}_i = \frac{b_{d_i}\theta_i\dot{e}_{a_i}\boldsymbol{\phi_i(x)}}{\left(\sigma_i^2 - \dot{e}_{a_i}^2\right)}$, then following properties hold

i) The error e_{a_i} and its derivative $\dot{e}_{a_i}$ will remain bounded in compact sets Ω_ρ and Ω_σ, respectively, where $\Omega_\rho \subset (-\rho_i, \rho_i)$ and $\Omega_\sigma \subset (-\sigma_i, \sigma_i)$
ii) The error derivative will be asymptotically stable, i.e., $\dot{e}_{a_i} \to 0$ as $t \to \infty$.

After the estimation the model uncertainties, the next step is to design an asymptotically stable controller for the approximated system, which is presented as follows.

3.3 Tracking Control Design

The tracking error between approximated system (2) and reference output $\boldsymbol{y^*}$ is defined as

$$\boldsymbol{e_r} = \boldsymbol{x_{a_1}} - \boldsymbol{y^*} \quad (16)$$

The time derivatives of (16) can be calculated as

$$\ddot{\boldsymbol{e}}_{\boldsymbol{r}} = \dot{\boldsymbol{x}}_{\boldsymbol{a_2}} - \ddot{\boldsymbol{y}}^* \quad (17)$$

By substituting $\dot{\boldsymbol{x}}_{\boldsymbol{a_2}}$ from (2), (17) can be written as

$$\ddot{\boldsymbol{e}}_{\boldsymbol{r}} = \boldsymbol{f(x)} + \boldsymbol{Gu} + \hat{\boldsymbol{d}}(\boldsymbol{x}) + \boldsymbol{K_{p_a}e_a} + \boldsymbol{K_{d_a}}\dot{\boldsymbol{e}}_{\boldsymbol{a}} - \ddot{\boldsymbol{y}}^* \quad (18)$$

To stabilize the error dynamics (18), a closed-form adaptive control law is proposed as

$$\boldsymbol{u} = [\boldsymbol{G(x)}]^{-1}\begin{pmatrix} -\boldsymbol{f(x)} - \boldsymbol{K_{pr}e_r} - \boldsymbol{K_{dr}}\dot{\boldsymbol{e}}_{\boldsymbol{r}} \\ -\boldsymbol{K_{pa}e_a} - \boldsymbol{K_{da}}\dot{\boldsymbol{e}}_{\boldsymbol{a}} - \hat{\boldsymbol{d}}(\boldsymbol{x}) + \ddot{\boldsymbol{y}}^* \end{pmatrix} \quad (19)$$

where, $\boldsymbol{G(x)}$ is known and assumed to be invertible. The matrices $\boldsymbol{K_{dr}}$ and $\boldsymbol{K_{pr}}$ are symmetric and diagonal matrices. The matrix $\boldsymbol{K_{dr}}$ is defined as

$$\boldsymbol{K_{dr}} = \text{diag}\left[k_{dr_1},\ k_{dr_2}, \ldots,\ k_{dr_n}\right] \quad (20)$$

where the i^{th} component of matrix $\boldsymbol{K_{dr}}$ is

$$k_{dr_i} = \left[\frac{\gamma_i c_{d_i}}{\left(\overline{\zeta}_{\dot{e}_i}^2 - \dot{e}_{r_i}^2\right)} + \left(\underline{\zeta}_{\dot{e}_i} / \overline{\zeta}_{\dot{e}_i}\right)^2 \frac{(1-\gamma_i)\, c_{d_i}}{\left(\underline{\zeta}_{\dot{e}_i}^2 - \dot{e}_{r_i}^2\right)}\right]$$

with, $\gamma_i(\dot{e}_{r_i}) = \begin{cases} 1, & \text{if } 0 < \dot{e}_{r_i} < \overline{\zeta}_{\dot{e}_i} \\ 0, & \text{if } -\underline{\zeta}_{\dot{e}_i} < \dot{e}_{r_i} \leq 0 \end{cases}$

where constant $c_{d_i} > 0, \forall i$. Constants $\overline{\zeta}_{\dot{e}_i}$ and $\underline{\zeta}_{\dot{e}_i}$ represent upper and lower bounds on error $\dot{e}_{r_i}$, obtained as

$$\overline{\zeta}_{\dot{e}_i} = \overline{\zeta}_i - \overline{d}_{d_i} - \sigma_i \quad (21)$$

$$\underline{\zeta}_{\dot{e}_i} = \underline{d}_{d_i} - \underline{\zeta}_i - \sigma_i \quad (22)$$

where, $\overline{\zeta}_i$ and $\underline{\zeta}_i$ are imposed state constraints on dynamic states x_{2i}, known constant σ_i is defined in (3), and $\overline{d}_{d_i}$ and

$\underline{d}_{d_i}$ are the known bounds on the derivative of reference signal. The matrix $\boldsymbol{K_{pr}}$ is defined as

$$\boldsymbol{K_{pr}} = \text{diag}\left[k_{pr_1},\ k_{pr_2}, \ldots,\ k_{pr_n}\right] \tag{23}$$

where, the i^{th} component of matrix is given as

$$k_{pr_i} = \left[\frac{\beta_i c_{p_i}}{\left(\overline{\kappa}_{e_i}^2 - e_{r_i}^2\right)} + \frac{(1-\beta_i)\,\underline{\kappa}_{e_i}^2 c_{p_i}}{\overline{\kappa}_{e_i}^2\left(\underline{\kappa}_{e_i}^2 - e_{r_i}^2\right)}\right]\left(\frac{1}{k_{dr_i}}\right)$$

$$\text{with,} \quad \beta_i(e_{r_i}) = \begin{cases} 1, & \text{if } 0 < e_{r_i} < \overline{\kappa}_{e_i} \\ 0, & \text{if } -\underline{\kappa}_{e_i} < e_{r_i} \leq 0 \end{cases}$$

where $c_{p_i} > 0, \forall i$ are constants. The constants $\overline{\kappa}_{e_i}$ and $\underline{\kappa}_{e_i}$ are upper and lower bounds on e_{r_i}, calculated as

$$\overline{\kappa}_{e_i} = \overline{\kappa}_i - \overline{d}_{p_i} - \rho_i \tag{24}$$

$$\underline{\kappa}_{e_i} = \underline{d}_{p_i} - \underline{\kappa}_i - \rho_i \tag{25}$$

where, $\overline{\kappa}_i$ and $\underline{\kappa}_i$ are imposed state constraints on kinematic states x_{1i}, known constant ρ_i is defined in (4), and $\overline{d}_{p_i}$ and $\underline{d}_{p_i}$ are the bounds on reference signal.

Note that, in the expressions of proposed controller gain matrices k_{dr_i} and k_{pr_i}, terms $\left(\underline{\zeta}_{\dot{e}_i}\ /\ \overline{\zeta}_{\dot{e}_i}\right)^2$ and $\left(\underline{\kappa}_{e_i}\ /\ \overline{\kappa}_{e_i}\right)^2$ are added to prevent a very sharp change in the control histories around equilibrium point.

Further, by substituting the control expression (19) in (18), the closed-loop error dynamics is obtained as

$$\boldsymbol{\ddot{e}_r} = -\boldsymbol{K_{pr}e_r} - \boldsymbol{K_{dr}\dot{e}_r} \tag{26}$$

Since the matrices $\boldsymbol{K_{pr}e_r}$ and $\boldsymbol{K_{dr}\dot{e}_r}$ are diagonal in nature, the i^{th} component of closed-loop error dynamic can be written as

$$\ddot{e}_{r_i} = -k_{pr_i}e_{r_i} - k_{dr_i}\dot{e}_{r_i} \tag{27}$$

To analyze the stability of (27), the following asymmetric logarithm BLF (Sachan and Padhi, 2018) is chosen

$$\begin{aligned} V_i = &\frac{c_{p_i}}{2}\left[\beta_i \log\left(\frac{\overline{\kappa}_{e_i}^2}{\left(\overline{\kappa}_{e_i}^2 - e_{r_i}^2\right)}\right)\right] \\ &+ \frac{c_{p_i}}{2}\left[\frac{(1-\beta_i)\,\underline{\kappa}_{e_i}^2}{\overline{\kappa}_{e_i}}^2 \log\left(\frac{\underline{\kappa}_{e_i}^2}{\left(\underline{\kappa}_{e_i}^2 - e_{r_i}^2\right)}\right)\right] \\ &+ \frac{c_{d_i}}{2}\left[\gamma_i \log\left(\frac{\overline{\zeta}_{\dot{e}_i}^2}{\left(\overline{\zeta}_{\dot{e}_i}^2 - \dot{e}_{r_i}^2\right)}\right)\right] \\ &+ \frac{c_{d_i}}{2}\left[\frac{(1-\gamma_i)\,\underline{\zeta}_{\dot{e}_i}^2}{\overline{\zeta}_{\dot{e}_i}^2} \log\left(\frac{\underline{\zeta}_{\dot{e}_i}^2}{\left(\underline{\zeta}_{\dot{e}_i}^2 - \dot{e}_{r_i}^2\right)}\right)\right] \end{aligned} \tag{28}$$

where all variables used in (28) are already defined. The time derivative of (28) can be obtained as

$$\begin{aligned} \dot{V}_i = &c_{p_i}\left[\frac{\beta_i e_{r_i}\dot{e}_{r_i}}{\left(\overline{\kappa}_{e_i}^2 - e_{r_i}^2\right)} + \frac{(1-\beta_i)\,\underline{\kappa}_{e_i}^2 e_{r_i}\dot{e}_{r_i}}{\overline{\kappa}_{e_i}^2\left(\underline{\kappa}_{e_i}^2 - e_{r_i}^2\right)}\right] \\ &+ c_{d_i}\left[\frac{\gamma_i \dot{e}_{r_i}\ddot{e}_{r_i}}{\left(\overline{\zeta}_{\dot{e}_i}^2 - \dot{e}_{r_i}^2\right)} + \frac{(1-\gamma_i)\,\underline{\zeta}_{\dot{e}_i}^2 \dot{e}_{r_i}\ddot{e}_{r_i}}{\overline{\zeta}_{\dot{e}_i}^2\left(\underline{\zeta}_{\dot{e}_i}^2 - \dot{e}_{r_i}^2\right)}\right] \end{aligned}$$

Using the definitions of k_{pr_i} and k_{dr_i} from (23) and (20), $\dot{V}_i$ is expressed as

$$\dot{V}_i = k_{pr_i}k_{dr_i}e_{r_i}\dot{e}_{r_i} + k_{dr_i}\dot{e}_{r_i}\ddot{e}_{r_i} \tag{29}$$

By substituting the value of $\ddot{e}_{r_i}$ from (27), following simplified expression of $\dot{V}_i$ is obtained

$$\dot{V}_i = -k_{dr_i}^2\dot{e}_{r_i}^2 \tag{30}$$

Since $\dot{V}_i \leq 0$, the asymptotic stability of the error dynamics in (26) cannot be concluded. However, $\dot{V}_i = 0$ only when $\dot{e}_{r_i} = 0, \forall i$ for all time, which implies that $\ddot{e}_{r_i} = 0, \forall i$. By using (27), when $\dot{e}_{r_i} = \ddot{e}_{r_i} = 0$, one can conclude that $e_{r_i} = 0, \forall i$ (provided $e_{r_i} \in \left(-\underline{\kappa}_{e_i}, \overline{\kappa}_{e_i}\right)$ and $\dot{e}_{r_i} \in \left(-\underline{\zeta}_{\dot{e}_i}, \overline{\zeta}_{\dot{e}_i}\right)$ satisfy, which is proven next). Thus, the equilibrium point is the only invariant set and hence, using LaSalle's Theorem (Sachan and Padhi, 2018), the asymptotic stability of the closed loop error dynamics can be concluded. Furthermore, as $\dot{V}_i \leq 0$, it can be concluded that all closed-loop signals will remain bounded. Moreover, the upper bound on selected Lyapunov is obtained as $V_i \leq V_0, \forall t$, which is (from (28))

$$\begin{aligned} &\frac{c_{p_i}}{2}\left[\beta_i \log\left(\frac{\overline{\kappa}_{e_i}^2}{\left(\overline{\kappa}_{e_i}^2 - e_{r_i}^2\right)}\right)\right] \\ &+ \frac{c_{p_i}}{2}\left[\frac{(1-\beta_i)\,\underline{\kappa}_{e_i}^2}{\overline{\kappa}_{e_i}^2} \log\left(\frac{\underline{\kappa}_{e_i}^2}{\left(\underline{\kappa}_{e_i}^2 - e_{r_i}^2\right)}\right)\right] \\ &+ \frac{c_{d_i}}{2}\left[\gamma_i \log\left(\frac{\overline{\zeta}_{\dot{e}_i}^2}{\left(\overline{\zeta}_{\dot{e}_i}^2 - \dot{e}_{r_i}^2\right)}\right)\right] \\ &+ \frac{c_{d_i}}{2}\left[\frac{(1-\gamma_i)\,\underline{\zeta}_{\dot{e}_i}^2}{\overline{\zeta}_{\dot{e}_i}^2} \log\left(\frac{\underline{\zeta}_{\dot{e}_i}^2}{\left(\underline{\zeta}_{\dot{e}_i}^2 - \dot{e}_{r_i}^2\right)}\right)\right] \leq V_0 \end{aligned}$$

where V_0 is the value of Lyapunov function at $t = 0$. Since the left hand side terms are positive definite, the individual term is upper bounded by V_0 (can be followed from Sachan and Padhi (2018)). Furthermore, by solving and taking the exponent on both sides, the following inequality is obtained after simplification (similar to the procedure followed after (14))

$$-\underline{\kappa}_{e_i} < e_{r_i} < \overline{\kappa}_{e_i} \tag{31}$$

Similar analysis can be carried out for $\dot{e}_{r_i}$ and the bounds can be obtained as $-\underline{\zeta}_{\dot{e}_i} < \dot{e}_{r_i} < \overline{\zeta}_{\dot{e}_i}$. Thus, the error e_{r_i} will remain bounded in a compact set Ω_κ and its derivative $\dot{e}_{r_i}$ will remain bounded in a compact set Ω_ζ, where $\Omega_\kappa \subset \left(-\underline{\kappa}_{e_i}, \overline{\kappa}_{e_i}\right)$, $\Omega_\zeta \subset \left(-\underline{\zeta}_{\dot{e}_i}, \overline{\zeta}_{\dot{e}_i}\right) \forall i$.

The above analysis can be summarized as follows.

Theorem 2. Consider the systems (2) under the controller (19), and if the initial conditions are chosen as $e_{r_i}(0) \in \Omega_\kappa$, $\dot{e}_{r_i}(0) \in \Omega_\zeta$, then following properties hold

i) The closed-loop error dynamics is asymptotically stable, i.e., $e_{r_i} \to 0$ as $t \to \infty$
ii) The error e_{r_i} and its derivative $\dot{e}_{r_i}$ will remain bounded in compact sets Ω_κ and Ω_ζ, respectively where $\Omega_\kappa \subset \left(-\underline{\kappa}_{e_i}, \overline{\kappa}_{e_i}\right)$, $\Omega_\zeta \subset \left(-\underline{\zeta}_{\dot{e}_i}, \overline{\zeta}_{\dot{e}_i}\right) \forall i$.

Remark 1: In the current formulation, the output-constrained control can be achieved by selecting nonzero diagonal elements of the matrix $\boldsymbol{K_{dr}}$ as positive constants. Similarly, the proposed formulation is reduced to dynamic inversion formulation by selecting nonzero diagonal elements of the matrices $\boldsymbol{K_{pr}}$, $\boldsymbol{K_{dr}}$ as positive constants.

In Theorems 1 and 2, it is shown that all closed loop signal will be bounded in both steps of the control design. However, the behavior of the overall closed-loop actual system is not analyzed yet. Thus, the closed-loop response of the actual system is analyzed in the following theorem.

Theorem 3. Consider the actual systems (1) under the controller (19) with the weight update learning rule in (12), and if the initial conditions are chosen as $e_{r_i}(0) \in (-\underline{\kappa}_{e_i}, \overline{\kappa}_{e_i})$, $\dot{e}_{r_i}(0) \in \left(-\underline{\zeta}_{\dot{e}_i}, \overline{\zeta}_{\dot{e}_i}\right) \forall i$, then following properties hold

i) The overall error $(e_i = y_i - y_i^*)$ will asymptotically converge to a user-defined compact set Ω_ρ, where $\Omega_\rho \in (-\rho_i, \rho_i)$ and $\rho_i > 0$ is a user-defined constant.
ii) Closed-loop states of the system will remain bounded as $\underline{\kappa}_i < x_{1_i} < \overline{\kappa}_i$ and $\underline{\zeta}_i < x_{2_i} < \overline{\zeta}_i, \forall t \geq 0, \forall i$.

Proof.

i) The overall error of the actual system is defined as $e_i = x_{1_i} - y_i^*$. By adding and subtracting $x_{a_{1_i}}$, the error e_i is modified as $e_i = e_{a_i} + e_{r_i}$. From Theorem 1, $e_{a_i} \in \Omega_\rho$ and from Theorem 2, e_{r_i} is asymptotically stable, i.e., $e_{r_i} \to 0$ as $t \to \infty$. Thus, e_i will asymptotically converge to the compact set Ω_ρ.
ii) From Theorem 1 and 2, the errors e_{r_i} and e_{a_i} are shown to be bounded as $-\underline{\kappa}_{e_i} < e_{r_i} < \overline{\kappa}_{e_i}$ and $|e_{a_i}| < \rho_i, \forall i$, respectively. Thus, following is true

$$-\underline{\kappa}_{e_i} - \rho_i < e_{r_i} + e_{a_i} < \overline{\kappa}_{e_i} + \rho_i \tag{32}$$

By substituting $e_{r_i} = x_{a_{1_i}} - y_i^*$, $e_{a_i} = x_{1_i} - x_{a_{1_i}}$ and bounds of y_i^*, (32) is modified as

$$-\underline{\kappa}_{e_i} - \rho_i + \underline{d}_{p_i} < x_{1_i} < \overline{\kappa}_{e_i} + \rho_i + \overline{d}_{p_i} \tag{33}$$

Using the definitions of $\overline{\kappa}_i$ and $\underline{\kappa}_i$ from (24) and (25), inequality (33) can be expressed as

$$\underline{\kappa}_i < x_{1_i} < \overline{\kappa}_i \tag{34}$$

Similarly from the bounds on $\dot{e}_{r_i}$ and $\dot{e}_{a_i}$, the same analysis can be performed and the bounds on x_{2_i} can be obtained as

$$\underline{\zeta}_i < x_{2_i} < \overline{\zeta}_i \tag{35}$$

Thus, the closed-loop states of the actual system will remain bounded by the imposed state constraints. This completes the proof.

4. SIMULATION RESULTS

To demonstrate the capability of the proposed control design, consider the following nonlinear system as

$$\ddot{x}_1 = f_1(x_1, x_2) + \left(1 + x_1^2\right) u_1 + u_2 + d_1 \tag{36}$$

$$\ddot{x}_2 = f_2(x_1, x_2) + \left(1 + \dot{x}_1^2\right) u_2 + d_2 \tag{37}$$

where, $f_1(x_1, x_2) = x_1\dot{x}_2 + x_2^2 + c_1$ and $f_2(x_1, x_2) = x_2\dot{x}_2 + c_2$, and $c_1 \in \Re$, $c_2 \in \Re$ are constants. The reference signals, without loss of generality, are assumed as zero. The outputs of the system are selected as $y_1 = x_1$ and $y_2 = x_2$. Since the proposed *state-constrained* (SC) control formulation can be easily converted into *output-constrained* (OC) control, the results are presented for both scenarios. In simulations, the asymmetric lower and upper constraints on the system output (x_1 and x_2) are chosen as -0.2 and 0.56, respectively. Similarly, the lower and upper constraints on dynamic states ($\dot{x}_1$ and $\dot{x}_2$) are chosen as -0.7 and 1, respectively. The constraints on e_{a_i}, $\dot{e}_{a_i}$ are chosen as ± 0.05, which means that the overall system error will asymptotically converge to the compact set Ω_ρ, where $\Omega_\rho \in (-0.05, 0.05)$. The model parametric uncertainties are assumed as $d_1 = 11.8\dot{x}_2\dot{x}_1 + 12.5\dot{x}_1$ and $d_2 = -13x_2 + 11.5x_1^2 + 21.7\dot{x}_2^2$. Since the model uncertainties are structured in nature and the basis functions ϕ_1 and ϕ_2 are chosen as $\phi_1 = [\dot{x}_2\dot{x}_1, \ \dot{x}_1]^T$ and $\phi_2 = [x_2, \ x_1^2, \ \dot{x}_2^2]^T$, respectively.

The initial conditions for the simulations are selected as $x_1(0) = 0.51$, $\dot{x}_1(0) = 0.6$, $x_2(0) = 0.45$, and $\dot{x}_2(0) = 0.65$. Moreover, for weight update, $\hat{\mathbf{w}}_i(0), \forall i$ are chosen as zero. The learning rate θ_i is selected as 10 and all other constant controller gains are taken as unity. The constants c_1 and c_2 in the system dynamics (36) and (37) are chosen as $c_1 = 0.2$ and $c_2 = 0.4$, respectively.

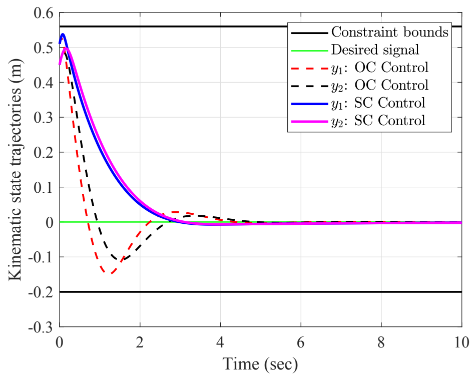

Fig. 1. Kinematic state histories obtained using output and state constrained adaptive control.

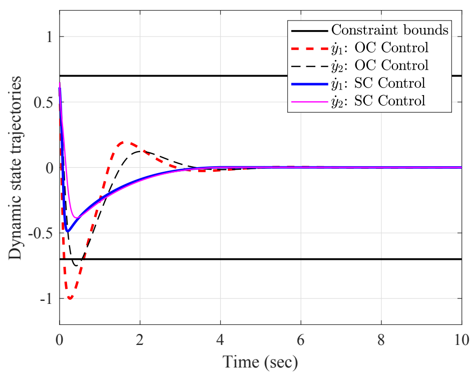

Fig. 2. Dynamic state histories obtained using output and state constrained adaptive control.

The kinematic and dynamic state trajectories of the system are shown in Figs 1 and 2, respectively where both the OC controller and SC controller prevent the violation of

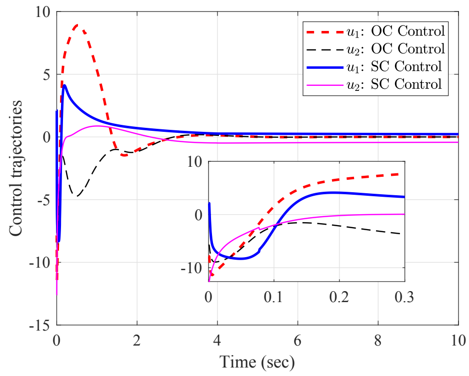

Fig. 3. Control histories obtained using output and state constrained adaptive control.

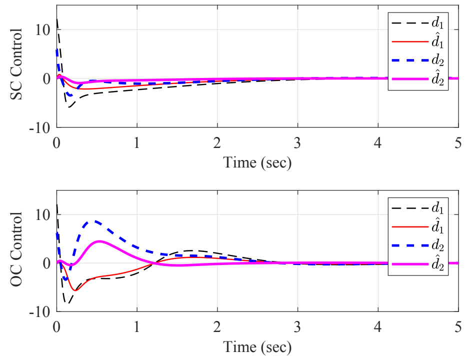

Fig. 4. Uncertainty learning history obtained using output and state constrained adaptive control.

imposed constraints. Moreover, the desired reference signal tracking is ensured by both controllers, which is shown in Theorem 3. Additionally, it can be observed that the transients are negligible during uncertainty learning. This happens due to the constraint on the states of the learning error dynamics in (7). The control history are presented in Fig 3, where all profiles are smooth and continuous. The uncertainty learning trajectories are shown in Fig 4, which shows that the unknown model uncertainties are approximated well using neural network.

5. CONCLUSION

A robust adaptive state-constrained control technique has been proposed in this paper for practically relevant Euler-Lagrange systems. The control technique has been formulated using an asymptotically stable nonlinear error dynamics, which enforces the user-defined constraints on the system states. A neural network based adaptive law has been used for the estimation of system modeling uncertainties. The stability of the closed-loop system has been shown through the Lyapunov stability theory. It has been pointed out that the proposed controller can overcome several limitations of the existing BLF based backstepping control designs and leads to an easily implementable control solution. The effectiveness of the control design has been demonstrated through extensive simulations. This work can be further upgraded by considering unstructured model uncertainties.

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