Sampled-Data Observers for Delay Systems

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Abstract: This paper studies the problem of designing sampled-data observers and observer-based, sampled-data, output feedback stabilizers for systems with both discrete and distributed, state and output time-delays. The obtained results can be applied to time delay systems of strict-feedback structure. The proposed design approach consists in exploiting an existing observer, which features robust exponential convergence of the error when continuous-time output measurements are available. The observer is then modified, mainly by adding an inter-sample output predictor, to compensate for the effect of data-sampling. Using Lyapunov stability tools and small-gain analysis, we show that robust exponential stability of the error is preserved, provided the sampling period is not too large.

Keywords: nonlinear observers, sampled-data observers, delay systems, inter-sample predictor.

1. INTRODUCTION

With the growing penetration of network technology in control systems, the compensation of the effects of time-delay has become a major issue in control theory; see Fridman (2014), Karafyllis & Krstic (2013), Karafyllis et al. (2016), Karafyllis & Krstic (2017), Michiels & Niculescu (2014). A great deal of interest has recently been paid to the problem of designing state observers for linear and nonlinear systems with measurement delays. The dominant design approach consists in starting with the design of an exponentially convergent observer for the delay-free system, which is described by Ordinary Differential Equations (ODEs), and modifying it mainly by adding predictors: static predictors in Karafyllis & Krstic (2017) or dynamic (chain) predictors in Ahmed-Ali et al. (2012), Ahmed-Ali et al. (2013), Besancon et al. (2007), Cacace et al. (2010), Cacace et al. (2014), Germani et al. (2002), Kazantzis & Wright (2005). In parallel to this research activity, which takes into account the time-delay explicitly in the model, a separate activity, based on Partial Differential Equations (PDEs) has been initiated in Krstic (2009). This consists in modeling time-delays by means of first-order hyperbolic PDEs, leading to a representation of the delayed system in the form of an ODE-PDE cascade; see also the recent work Ahmed-Ali et al. (2018), where a PDE-based chain-observer is constructed).

Most existing results on observer design for delayed systems have been established assuming the measurement delay to be of discrete nature. So far, only a few studies have investigated the case of distributed measurement time-delays. The PDE-based observer developed in Bekiaris-Liberis & Krstic (2011) and the recent observer developed in Ammeh et al. (2019) are notable exceptions.

The nowadays-digital implementation of observers entails sampling in time of all system signals needed by the observer. Consequently, the design of sampled-data observers is a major issue. Sampled-data observers for ODE systems can be classified in four main categories:

1) observers where data-sampling is accounted for through a standard Zero-Order-Hold (ZOH) sampling of the output estimation error; see for example Ahmed-Ali et al. (2016), Raff et al. (2008),
2) observers designed on approximate discrete-time models; see Arcak & Nesic (2004), Biyik & Arcak (2006),
3) continuous-discrete time observers where correction terms are employed at the sampling times; see for instance Nadri et al. (2012), and
4) sampled-data observers, where the time-varying delay effect (caused by output sampling) is compensated by using inter-sample output predictors; see Karafyllis & Kravaris (2009).

The use of inter-sample output predictors was extended to systems with asynchronous measurements in Ling & Kravaris (2019) and systems described by parabolic PDEs in Karafyllis et al. (2019).

The combination of time-delay and data-sampling effects necessarily makes the problem of observer design more complex. Indeed, not only data-sampling introduce a time-varying delay but it also entails information lost. The case of discrete measurement delays, in conjunction with data sampling, has been investigated in Ahmed-Ali et al. (2013), Ahmed-Ali et al. (2016), Karafyllis & Krstic (2013), Karafyllis & Krstic (2017), Raff et al. (2008). Results on observer-based output feedback stabilization of delay systems with sampled measurements have been recently given in Di Ferdinando & Pepe (2019), Pepe & Fridman (2017) (but see also Pepe (2014)).

In the present work, we extend for the first time the use of inter-sample predictors to the case of time-delay systems with state and output (discrete and/or distributed) delays. Moreover, we provide observer-based output feedback stabilization results for delay systems with sampled measurements under appropriate assumptions. More specifically, we consider time-delay systems of the form:

$$\begin{aligned} &\dot{x} = f(x_t, u, d) \\ &y = h(x_t) + \xi \\ &(x, u, d) \in \Re^n \times U \times D, y, \xi \in \Re^k \end{aligned} \quad (1)$$

where $U \subseteq \Re^m$, $D \subseteq \Re^q$ are convex sets with $0 \in U$, $0 \in D$, $f : C^0([-r,0];\Re^n) \times U \times D \to \Re^n$, $h : C^0([-r,0];\Re^n) \to \Re^k$ are continuous mappings with $f(0,0,0) = 0$, $h(0) = 0$. The input u is assumed to be available, but the inputs d, ξ are unknown and represent possible modeling errors and measurement noise, respectively. The proposed sampled-data observer design approach consists in starting with an existing observer, which features *robust exponential convergence* when continuous-time output measurements are available (see Definition 2.1 for the precise meaning of the phrase "robust exponential convergence"). The available observer, based on continuous-time measurements, is then modified, by adding an inter-sample output predictor. Using Lyapunov stability tools and small gain analysis, we show that the robust exponential stability feature is preserved, provided that the sampling period is sufficiently small (Theorem 2.2). The sampled-data observer can be used in a straightforward way for the design of observer-based output feedback stabilizers (Corollary 2.4) under certain assumptions.

Sampled-data observers are also provided for uncertain, triangular, globally Lipschitz delay systems of the form

$$\begin{aligned} &\dot{x}_1(t) = f_1(x_{1,t}, u(t)) + x_2(t) + d_1(t) \\ &\vdots \\ &\dot{x}_{n-1}(t) = f_{n-1}(x_{1,t}, ..., x_{n-1,t}, u(t)) + x_n(t) + d_{n-1}(t) \\ &\dot{x}_n(t) = f_n(x_{1,t}, ..., x_{n,t}, u(t)) + d_n(t) \\ &y(t) = x_1(t) \end{aligned} \quad (2)$$

where $x(t) = (x_1(t),...,x_n(t)) \in \Re^n$ is the state, $u(t) \in \Re^m$ is a known input, $d(t) = (d_1(t),...,d_n(t)) \in \Re^n$ is the vector of disturbances and $f_i : C^0([-r,0];\Re^i) \times \Re^m \to \Re$ ($i = 1,...,n$) with $f_i(0) = 0$ ($i = 1,...,n$) are globally Lipschitz functionals with $r > 0$ being the maximum delay. Again, by using the inter-sample predictor design, we are in a position to design sampled-data observers for (2), no matter how large the maximum delay $r > 0$ is (Theorem 3.1). The observer design is based on the high-gain observer design for ODEs, proposed in Gauthier & Kupka (2001).

It should be noted that in all cases the results are global. Moreover, we are in a position to consider uncertain sampling schedules (i.e., the sampling times are not a priori known) and guarantee robustness with respect to measurement noise. Finally, in the absence of measurement noise and unknown disturbances, exponential convergence of the observer error is achieved. The obtained results in all cases are proved by means of a combined use of Lyapunov-Krasovskii functionals and a small-gain analysis.

Due to lack of space, all proofs are omitted and can be found in Ahmed-Ali et al. (2019).

<u>*Notation.*</u>

* By $\Re_+$ we denote the set of non-negative real numbers. Let $S \subseteq \Re^n$ be an open set and let $A \subseteq \Re^n$ be a set that satisfies $S \subseteq A \subseteq cl(S)$. By $C^0(A;\Omega)$, we denote the continuous functions on A, which take values in $\Omega \subseteq \Re^m$.
* For a vector $x \in \Re^n$ we denote by $|x|$ its usual Euclidean norm and by x^T its transpose. By $|A| := \sup\{ |Ax| ; x \in \Re^n, |x| = 1 \}$ we denote the induced norm of a matrix $A \in \Re^{m \times n}$ and I denotes the identity matrix. By $B = diag(b_1,...,b_n)$ we denote the diagonal matrix $B \in \Re^{n \times n}$ with $b_1,...,b_n$ in its diagonal. For $x \in C^0([-r,0];\Re^n)$ we define $\|x\| := \max_{\theta \in [-r,0]} (|x(\theta)|)$.
* Let $x : [a-r, b) \to \Re^n$ be a continuous mapping with $b > a > -\infty$ and $r > 0$. By x_t we denote the "r-history" of x at time $t \in [a,b)$, i.e., $(x_t)(\theta) := x(t+\theta)$; $\theta \in [-r,0]$. Notice that $x_t \in C^0([-r,0];\Re^n)$.
* By K we denote the set of increasing, continuous functions $\rho : \Re_+ \to \Re_+$ with $\rho(0) = 0$. We say that a function $\rho \in K$ is of class K_∞ if $\lim_{s \to +\infty} \rho(s) = +\infty$.
* Let $D \subseteq \Re^l$ be a non-empty set and $I \subseteq \Re_+$ an interval. By $L^\infty_{loc}(I;D)$ we denote the class of Lebesgue measurable and locally bounded mappings $d : I \to D$. Notice that by $\sup\{ |d(\tau)| : \tau \in I \}$ we do not mean the essential supremum of d on I but the actual supremum of d on I.

2. ASSUMPTIONS AND MAIN RESULT

In the present work we study systems of the form (1) under the following assumptions:

(H1) *The mappings* $f : C^0([-r,0];\Re^n) \times U \times D \to \Re^n$, $h : C^0([-r,0];\Re^n) \to \Re^k$ *are continuous and satisfy the following properties: (i)* $f(0,0,0) = 0$, $h(0) = 0$, *(ii) for every bounded* $\Omega \subset C^0([-r,0];\Re^n) \times U \times D$ *the image set* $f(\Omega) \subset \Re^n$ *is bounded, (iii) for every bounded* $\Omega \subset C^0([-r,0];\Re^n)$ *the image set* $h(\Omega) \subset \Re^k$ *is bounded, and (iv) for every bounded* $S \subset C^0([-r,0];\Re^n) \times U \times D$, *there exists a constant* $L_S \geq 0$ *such that*

$$\begin{aligned} &\left(x(0) - \bar{x}(0)\right)^T \left(f(x,u,d) - f(\bar{x},u,d)\right) \leq L_S \|x - \bar{x}\|^2 \\ &\forall (x,u,d) \in S, \forall (\bar{x},u,d) \in S \end{aligned}$$

(H2) *System (1) is Forward Complete, i.e., for every* $x_0 \in C^0([-r,0];\Re^n)$ *and for every* $u \in L^\infty_{loc}(\Re_+;U)$, $d \in L^\infty_{loc}(\Re_+;D)$ *the solution of (1) with initial condition* x_0, *corresponding to inputs* u *and* d *exists for all* $t \geq 0$.

Assumption (H1) is a standard assumption for time-delay systems that guarantees existence and uniqueness of solutions for system (1), i.e. guarantees that for every $x_0 \in C^0([-r,0];\Re^n)$ and for every $u \in L^\infty_{loc}(\Re_+;U)$,

$d \in L^{\infty}_{loc}(\Re_+;D)$ there exists $t_{\max} \in (0,+\infty]$ and a unique continuous mapping $x:[-r,t_{\max}) \to \Re^n$ which is absolutely continuous on $[0,t_{\max})$ and satisfies $x(\theta)=x_0(\theta)$ for $\theta \in [-r,0]$ and (1) for $t \in [0,t_{\max})$ a.e.. This mapping $x:[-r,t_{\max}) \to \Re^n$ is the solution of (1) with initial condition x_0, corresponding to inputs u,d. Assumption (H2) guarantees that $t_{\max}=+\infty$ for every $x_0 \in C^0([-r,0];\Re^n)$ and for every $u \in L^{\infty}_{loc}(\Re_+;U)$, $d \in L^{\infty}_{loc}(\Re_+;D)$.

The following assumption plays a crucial role in what follows.

(H3) *There exists a continuous mapping* $R: C^0([-r,0];\Re^n)\times U \times D \to \Re^k$ *with the following property: for every* $x_0 \in C^0([-r,0];\Re^n)$, $u \in L^{\infty}_{loc}(\Re_+;U)$, $d \in L^{\infty}_{loc}(\Re_+;D)$ *the unique solution* x *of (1) with initial condition* x_0, *corresponding to inputs* u,d, *satisfies for* $t \ge 0$ *a.e. the following equation:*

$$\frac{d}{dt}\left(h(x_t)\right) = R(x_t,u(t),d(t)) \tag{3}$$

Moreover, there exists a constant $L \ge 0$ *and a function* $\kappa \in K$ *such that the following inequality holds for all* $x,\bar{x} \in C^0([-r,0];\Re^n)$, $(u,d) \in U \times D$:

$$\left|R(\bar{x},u,0)-R(x,u,d)\right| \le L\left\|\bar{x}-x\right\| + \kappa\left(|d|\right) \tag{4}$$

Assumption (H3) requires that the derivative of the output of system (1) exists and is expressed by the globally Lipschitz (with respect to x) mapping R. Not every nonlinear time-delay system satisfies (H3). Nevertheless, the class of systems satisfying (H3) is wide, including many systems of practical interest e.g. (2).

The notion of the Robust Exponential Observer (REO) for system (1) is crucial to the development of the main results of the present work and it is given in the following definition.

Definition 2.1 (Robust Exponential Observer): *Consider the following system*

$$\dot{z} = F(z_t,y,u),\ z \in \Re^l$$
$$\hat{x}_t = \Psi(z_t),\ \hat{x} \in \Re^n \tag{5}$$

where $F: C^0([-r,0];\Re^l)\times \Re^k \times U \to \Re^l$, $\Psi : C^0([-r,0];\Re^l) \to C^0([-r,0];\Re^n)$ *are continuous mappings with* $F(0,0,0)=0$, $\Psi(0)=0$. *Suppose that the mapping* F *is such that, for every bounded* $\Omega \subset C^0([-r,0];\Re^l)\times\Re^k\times U$ *the image set* $F(\Omega)\subset \Re^l$ *is bounded and there exists a constant* $L_\Omega \ge 0$ *such that*

$$\left(z(0)-\bar{z}(0)\right)^T\left(F(z,y,u)-F(\bar{z},y,u)\right) \le L_\Omega \left\|z-\bar{z}\right\|^2$$
$$\forall (z,y,u)\in\Omega,\ \forall(\bar{z},y,u)\in\Omega$$

System (5) is called a ***Robust Exponential Observer (REO)*** *for system (1), if there exist constants* $\gamma,\sigma>0$ *and functions* $a,g \in K_\infty$ *such that for every* $u \in L^{\infty}_{loc}(\Re_+;U)$, $(x_0,z_0)\in C^0([-r,0];\Re^n)\times C^0([-r,0];\Re^l)$, $d \in L^{\infty}_{loc}(\Re_+;D)$, $\xi \in L^{\infty}_{loc}(\Re_+;\Re^k)$ *the solution* $(x(t),z(t))$ *of*

$$\dot{x} = f(x_t,u,d)$$
$$\dot{z} = F(z_t,h(x_t)+\xi,u)$$
$$\hat{x}_t = \Psi(z_t) \tag{6}$$

with initial condition (x_0,z_0), *corresponding to inputs* u,d,ξ, *exists for all* $t\ge 0$ *and satisfies the following estimate for all* $t \ge 0$:

$$\left\|\hat{x}_t - x_t\right\| \le \exp(-\sigma t)a\left(\left\|x_0\right\|+\left\|z_0\right\|\right)$$
$$+\gamma \sup_{0\le s\le t}\left(\left|\xi(s)\right|\exp(-\sigma(t-s))\right) + \sup_{0\le s\le t}\left(g(|d(s)|)\right) \tag{7}$$

At this point, it should be noticed that the way that the inputs d and ξ enter the Input-to-Output (IOS) Stability estimate (7) is different. While the input d comes in estimate (7) through a (possibly) nonlinear gain function, the input ξ appears in estimate (7) with a linear gain and with a fading memory effect (see Karafyllis & Jiang (2011)). This difference is important and allows, in what follows, the construction of sampled-data observers.

Besides the fact that Definition 2.1 introduces the notion of REO for systems with state delays, there are important differences between the notion of a REO in Definition 2.1 and other similar notions that were used in the literature for systems described by ODEs; see Ahmed-Ali et al. (2013), Karafyllis & Kravaris (2009):

1) In Definition 2.1, the effect of disturbances is explicitly taken into account (see the term $\sup_{0\le s\le t} g(|d(s)|)$ in estimate (7)), while in other similar notions in the literature no disturbances are assumed to act on the system.

2) In Definition 2.1, the IOS estimate (7) is assumed to hold uniformly for inputs $u \in L^{\infty}_{loc}(\Re_+;U)$, while in other notions in the literature either there is no control input u or the sup-norm of u appears in the corresponding observer error estimate. This difference is important when the observer is to be used in conjunction with a state feedback control law for the dynamic stabilization of the system.

We are now in a position to state our main result.

Theorem 2.2 (Sampled-Data Observer Design): *Consider system (1) under (H1), (H2), (H3) and suppose that system (5) is a REO for system (1). Moreover, suppose that for every bounded* $S \subset C^0([-r,0];\Re^n)\times U$, *there exists a constant* $L_S \ge 0$ *such that*

$$\left|R(\Psi(z),u,0)-R(\Psi(\bar{z}),u,0)\right| \le L_S\left\|z-\bar{z}\right\|,$$
$$\forall (z,u)\in S,\ \forall(\bar{z},u)\in S \tag{8}$$

Let $\delta>0$ *and* $\omega \in (0,\sigma]$ *be constants that satisfy*

$$\delta < \frac{1}{\omega}\ln\left(1+\frac{\omega}{\gamma L}\right) \tag{9}$$

Then for every sampling sequence $\{\tau_i\}_{i=0}^{\infty}$ *with* $\tau_0 = 0$, $\lim(\tau_i)=+\infty$, $0<\tau_{i+1}-\tau_i \le \delta$ *for* $i=0,1,\ldots$, *for every* $(x_0,z_0)\in C^0([-r,0];\Re^n)\times C^0([-r,0];\Re^l)$ *and* $u \in L^{\infty}_{loc}(\Re_+;U)$, $d\in L^{\infty}_{loc}(\Re_+;D)$, $\xi \in L^{\infty}_{loc}(\Re_+;\Re^k)$, *the solution* $(x(t),z(t),w(t))$ *of (1) with*

$$\dot{z}(t) = F(z_t, w(t), u(t)),\ t \ge 0$$
$$\dot{w}(t) = R(\Psi(z_t), u(t), 0),\ t \in [\tau_i, \tau_{i+1})$$
$$\hat{x}_t = \Psi(z_t),\ t \ge 0 \quad (10)$$
$$w(\tau_i) = h(x_{\tau_i}) + \xi(\tau_i) \quad (11)$$

initial condition (x_0, z_0)*, corresponding to inputs* u, d, ξ*, exists for all* $t \ge 0$ *and satisfies the following estimate*

$$\|\hat{x}_t - x_t\| \le (1-B)^{-1}\exp(-\omega t)a\left(\|x_0\| + \|z_0\|\right) + \sup_{0\le s\le t}\left(\tilde{g}(|d(s)|)\right)$$
$$+(1-B)^{-1}\gamma\exp(\omega\delta)\sup_{0\le s\le t}\left(|\xi(s)|\exp(-\omega(t-s))\right) \quad (12)$$

where $\tilde{g}(s) := (1-B)^{-1}(g(s) + \gamma\delta\kappa(s))$ *and* $B := \gamma L\left(\exp(\omega\delta) - 1\right)/\omega < 1$.

Remark 2.3: (a) The observer (10), (11) is simply the REO (5) with the unavailable output signal replaced by the signal produced by the inter-sample output predictor

$$\dot{w}(t) = R(\Psi(z_t), u(t), 0),\ t \in [\tau_i, \tau_{i+1})$$
$$w(\tau_i) = h(x_{\tau_i}) + \xi(\tau_i)$$

(b) Notice that (12) guarantees the IOS property for the output map $Y = \hat{x}_t - x_t$ from the inputs d, ξ, i.e. from the inputs expressing the effect of modeling errors and measurement noise, respectively. However, a comparison of (7) and (12) shows that the input gains are higher for the sampled-data observer (10), (11) than the continuous-time REO (5). It is clear that sampling makes the observer more sensitive to modeling errors and measurement noise.

(c) The sampled-data observer (10), (11) is a hybrid observer with delays. For each sampling sequence $\{\tau_i\}_{i=0}^{\infty}$ with $\tau_0 = 0$, $\lim(\tau_i) = +\infty$, $0 < \tau_{i+1} - \tau_i \le \delta$ for $i = 0, 1, \ldots$, for each $(x_0, z_0) \in C^0([-r,0];\Re^n) \times C^0([-r,0];\Re^l)$ and $u \in L^\infty_{loc}(\Re_+;U)$, $d \in L^\infty_{loc}(\Re_+;D)$, $\xi \in L^\infty_{loc}(\Re_+;\Re^k)$, the solution $(x(t), z(t), w(t))$ of (1) with (10), (11) with initial condition (x_0, z_0), corresponding to inputs u, d, ξ, is produced by the following algorithm:

Step $i \ge 0$:

1) Given x_{τ_i}, calculate x_t for $t \in (\tau_i, \tau_{i+1}]$ from (1) and calculate $w(\tau_i) = h(x_{\tau_i}) + \xi(\tau_i)$,

2) Given z_{τ_i} and $w(\tau_i)$, calculate z_t for $t \in (\tau_i, \tau_{i+1}]$ and $w(t)$ for $t \in (\tau_i, \tau_{i+1})$ as the solution of the system $\dot{z}(t) = F(z_t, w(t), u(t))$ and $\dot{w}(t) = R(\Psi(z_t), u(t), 0)$,

3) Compute the output trajectory $\hat{x}_t$, for $t \in (\tau_i, \tau_{i+1}]$ using the equation $\hat{x}_t = \Psi(z_t)$

(d) Despite the hybrid nature of the observer (10)-(11), the trajectory of the estimated state features continuity. The proof of Theorem 2.2 is based on a small-gain argument. It is therefore expected that the observer error estimate (12) and the upper bound for the diameter of the sampling sequence $\delta > 0$ given by (9) are conservative. However, formulas (9) and (12) are useful because they indicate which parameters affect the performance of the observer and (qualitatively) how the upper bound for the diameter of the sampling sequence depends on the parameters of the system.

(e) Since the mapping $(0,\sigma] \ni \omega \to \frac{1}{\omega}\ln\left(1 + \frac{\omega}{\gamma L}\right)$ is decreasing with $\lim_{\omega\to 0^+}\left(\frac{1}{\omega}\ln\left(1 + \frac{\omega}{\gamma L}\right)\right) = \frac{1}{\gamma L}$, it is clear from (9) that: (i) Theorem 2.2 requires sampling sequences with diameter $\delta > 0$ being less than $1/\gamma L$, and (ii) the smaller the diameter $\delta > 0$ of the sampling sequence is, the larger the constant $\omega > 0$ is, i.e., convergence is faster for a smaller diameter $\delta > 0$ of the sampling sequence in the absence of modeling and measurement errors (recall (12)).

(f) In general, the constants γ and L depend on the value of the maximum delay r. Therefore, inequality (9) provides a useful relation between the diameter of the sampling sequence δ and the delay r.

For the design of observed-based, output feedback we need a stabilizability assumption.

(H4) *The equalities* $f(0,0,0) = 0$ *and* $U = \Re^m$ *hold. Moreover there exist a function* $\bar{\kappa} \in K$*, constants* $\sigma, M > 0$ *and a functional* $\tilde{k} : C^0([-r,0];\Re^n) \to \Re^m$ *with* $\tilde{k}(0) = 0$ *and a constant* $\bar{L} > 0$ *such that the inequalities*

$$\left|\tilde{k}(\bar{x}) - \tilde{k}(x)\right| + \left|f(\bar{x}, \bar{u}, 0) - f(x, u, 0)\right| \le \bar{L}\|\bar{x} - x\| + \bar{L}|\bar{u} - u|$$
$$\left|f(x,u,d) - f(x,u,0)\right| \le \bar{\kappa}\left(|d|\right)$$

hold for $x, \bar{x} \in C^0([-r,0];\Re^n)$, $\bar{u}, u \in \Re^m$, $d \in D$ *and such that for every* $x_0 \in C^0([-r,0];\Re^n)$*, the solution* $x(t)$ *of*

$$\dot{x}(t) = f(x_t, u(t), 0)$$
$$u(t) = \tilde{k}\left(x_t\right) \quad (13)$$

with initial condition x_0 *exists for all* $t \ge 0$ *and satisfies the following estimate*

$$\|x_t\| \le M\exp(-\sigma t)\|x_0\|,\ \forall t \ge 0 \quad (14)$$

When Assumption (H4) holds then we obtain the following stabilization result.

Corollary 2.4 (Global Stabilization by Means of Observer-Based Sampled-Data Output Feedback): *Consider system (1) under (H1), (H2), (H3), (H4) and suppose that system (5) is a REO for system (1). Moreover, suppose that for every bounded* $S \subset C^0([-r,0];\Re^n) \times U$*, there exists a constant* $L_S \ge 0$ *such that (8) holds. Then there exist constants* $\bar{\delta}, \omega, \hat{\gamma} > 0$ *and functions* $\hat{g}, \hat{a} \in K$ *such that for every sampling sequence* $\{\tau_i\}_{i=0}^{\infty}$ *with* $\tau_0 = 0$, $\lim(\tau_i) = +\infty$, $0 < \tau_{i+1} - \tau_i \le \bar{\delta}$ *for* $i = 0, 1, \ldots$*, for every* $(x_0, z_0) \in C^0([-r,0];\Re^n) \times C^0([-r,0];\Re^l)$, $\xi \in L^\infty_{loc}(\Re_+;\Re^k)$, $d \in L^\infty_{loc}(\Re_+;D)$*, the solution* $x(t)$ *of (1) with (10), (11) and*

$$u(t) = \tilde{k}\left(\hat{x}_{\tau_i}\right), t \in [\tau_i, \tau_{i+1}) \quad (15)$$

initial condition (x_0, z_0)*, corresponding to inputs* d, ξ*, exists for all* $t \ge 0$ *and satisfies the following estimate*

$$\|x_t\| + \|\hat{x}_t\| \le \exp(-\omega t)\hat{a}\left(\|x_0\| + \|z_0\| \right) + \hat{\gamma}\sup_{0\le s\le t}\left(|\xi(s)|\right) + \sup_{0\le s\le t}\left(\hat{g}(|d(s)|)\right) \quad (16)$$

Moreover, if $a \in K_\infty$ (the function involved in (7)) is linear then $\hat{a}$ is linear too.

3. TRIANGULAR GLOBALLY LIPSCHITZ DELAY SYSTEMS

In this section we consider systems of the form (2), where we assume that there exists a constant $L \ge 0$ such that the following inequalities hold for $i = 1,...,n$:

$$\left| f_i(x_1,...,x_i,u) - f_i(z_1,...,z_i,u) \right| \le \tilde{L}\sum_{j=1}^{i}\left\| x_j - z_j \right\|,$$

$$\text{for all } (x_1,...,x_i),(z_1,...,z_i) \in C^0([-r,0];\Re^i),\ u \in \Re^m \quad (17)$$

Notice that systems of the form (2) satisfying (17) are Forward Complete and satisfy Assumptions (H1), (H2) with $U = \Re^m$, $D = \Re^n$. More specifically, it can be shown that for every $x_0 \in C^0([-r,0];\Re^n)$, $u \in L^\infty_{loc}(\Re_+;\Re^m)$, $d \in L^\infty_{loc}(\Re_+;\Re^n)$, the solution $x(t)$ of (2) with initial condition x_0, corresponding to u,d, exists for all $t \ge 0$ and satisfies the estimate

$$\|x_t\| \le \exp(n\tilde{L}t)\left(\|x_0\| + t\sup_{0\le s\le t}\left(|d(s)|\right) + t\sup_{0\le s\le t}\left(\sum_{i=1}^{n}\left| f_i(0,u(s)) \right| \right) \right) \quad (18)$$

It is also straightforward to check that (H3) holds as well with $k = 1$, $L = \tilde{L} + 1$, $\kappa(s) = s$ for $s \ge 0$ and $R(x,u,d) := f_1(x_1,u) + d_1$ for $x \in C^0([-r,0];\Re^n)$, $(u,d) \in \Re^m \times \Re^n$.

Define the matrix $A = \{a_{i,j} : i = 1,..,n,\ j = 1,...,n\} \in \Re^{n\times n}$ by the relations

$$a_{i,i+1} = 1 \text{ for } i = 1,...,n-1 \text{ and } a_{i,j} = 0 \text{ if otherwise} \quad (19)$$

and the vector

$$c := (1,0,...,0)^T \in \Re^n \quad (20)$$

Since the pair of matrices (A,c) is observable, there exists $K = (K_1,...,K_n)^T \in \Re^n$ so that the matrix $(A + Kc^T)$ is Hurwitz. Using Theorem 2.2, we are in a position to prove the following result.

Theorem 3.1: *There exist constants $\delta,\omega > 0$, $\theta,Q_1,Q_2,Q_3 \ge 1$ such that for every sampling sequence $\{\tau_i\}_{i=0}^{\infty}$ with $\tau_0 = 0$, $\lim(\tau_i) = +\infty$, $0 < \tau_{i+1} - \tau_i \le \delta$ for $i = 0,1,...$, for every $(x_0,z_0) \in C^0([-r,0];\Re^n)\times C^0([-r,0];\Re^n)$, $u \in L^\infty_{loc}(\Re_+;\Re^m)$, $d \in L^\infty_{loc}(\Re_+;\Re^n)$, $\xi \in L^\infty_{loc}(\Re_+;\Re)$, the solution $(x(t),z(t),w(t))$ of (2) with*

$$\begin{aligned}
&\dot{z}_1(t) = f_i(z_{1,t},u(t)) + z_2(t) + \theta K_1(c^T z(t) - w(t))\\
&\vdots\\
&\dot{z}_{n-1}(t) = f_i(z_{1,t},...,z_{n-1,t},u(t)) + z_n(t) + \theta^{n-1}K_{n-1}(c^T z(t) - w(t))\\
&\dot{z}_n(t) = f_n(z_{1,t},...,z_{n,t},u(t)) + \theta^n K_n(c^T z(t) - w(t))\\
&\dot{w}(t) = f_1(z_{1,t},u(t)) + z_2(t),\ t \in [\tau_i,\tau_{i+1})\\
&\hat{x}_t = z_t
\end{aligned} \quad (21)$$

$$w(\tau_i) = x_1(\tau_i) + \xi(\tau_i) \quad (22)$$

initial condition (x_0,z_0), corresponding to inputs u,d,ξ, exists for all $t \ge 0$ and satisfies the following estimate

$$\|\hat{x}_t - x_t\| \le \exp(-\omega t)Q_1\left(\|x_0\| + \|z_0\| \right) + Q_2\sup_{0\le s\le t}\left(|\xi(s)|\exp(-\omega(t-s))\right) + Q_3\sup_{0\le s\le t}\left(|d(s)|\right) \quad (23)$$

The proof of Theorem 3.1 is based on a combined Lyapunov analysis together with small-gain arguments. The observer (21), (22) is constructed by the combination of a high-gain observer with an inter-sample predictor.

4. AN EXAMPLE WITH DISTRIBUTED OUTPUT DELAY

In Ahmed-Ali et al. (2019) it is shown that a chemical reactor model is equivalent to the following system with distributed state and output delays:

$$\dot{x}(t) = \theta(x(t)) - (\mu+1)x(t) + u(t) + \mu\zeta\int_0^1\int_{t-rl}^{t} x(s)\exp\left(-\zeta(t-s)\right)dsdl \quad (24)$$

$$y(t) = \zeta\int_{t-r}^{t}\exp\left(\zeta(s-t)\right)x(s)ds \quad (25)$$

where $\mu,\zeta > 0$ are constants and $\theta : \Re \to \Re$ is a globally Lipschitz function. Making use of Theorem 2.2, we can prove the following result for the reactor model (24), (25).

Theorem 4.1: *There exist constants $k_1,k_2,\delta,P,M,\omega > 0$ such that for every sampling sequence $\{\tau_i\}_{i=0}^{\infty}$ with $\tau_0 = 0$, $\lim(\tau_i) = +\infty$, $0 < \tau_{i+1} - \tau_i \le \delta$ for $i = 0,1,...$, for every $z_0 \in C^0([-r,0];\Re^2)$, $x_0 \in C^0([-r,0];\Re)$, $\xi,u \in L^\infty_{loc}(\Re_+;\Re)$, the solution $(x(t),z(t),w(t))$ of (24), (25) with*

$$\begin{aligned}
&\dot{z}_1(t) = \zeta z_2(t) - \zeta z_2(t-r)\exp\left(-\zeta r\right) - \zeta z_1(t) - k_1\left(z_1(t) - w(t)\right)\\
&\dot{z}_2(t) = \theta(z_2(t)) - (\mu+1)z_2(t) + u(t) - k_2\left(z_1(t) - w(t)\right)\\
&\qquad + \mu\zeta\int_0^1\int_{t-rl}^{t} z_2(s)\exp\left(-\zeta(t-s)\right)dsdl\\
&\dot{w}(t) = \zeta z_2(t) - \zeta z_2(t-r)\exp\left(-\zeta r\right) - \zeta z_1(t),\quad t \in [\tau_i,\tau_{i+1})\\
&\hat{x}_t = z_{2,t}
\end{aligned} \quad (24)$$

$$w_j(\tau_i) = \xi(\tau_i) + y(\tau_i) \quad (25)$$

initial condition (x_0,z_0), corresponding to u,ξ, exists for all $t \ge 0$ and satisfies

$$|x(t) - \hat{x}(t)| \le \exp(-\omega t)M\left(\|x_0\| + \|z_0\| \right)$$

$$+P\sup_{0\le s\le t}\left(|\xi(s)|\exp(-\omega(t-s))\right) \tag{26}$$

The proof of Theorem 3.4 is constructive and is given next. It should be noticed that there is *no restriction* in the delay $r>0$.

Proof: By virtue of Theorem 2.2, it suffices to prove that for appropriate selection of the constants $k_1,k_2\in\Re$, the system

$$\begin{aligned}
&\dot z_1(t)=\zeta z_2(t)-\zeta z_2(t-r)\exp(-\zeta r)-\zeta z_1(t)-k_1\left(z_1(t)-y(t)\right)\\
&\dot z_2(t)=\theta(z_2(t))-(\mu+1)z_2(t)+u(t)-k_2\left(z_1(t)-y(t)\right)\\
&\qquad+\mu\zeta\int_0^1\int_{t-rl}^{t} z_2(s)\exp\left(-\zeta(t-s)\right)dsdl\\
&\hat x_t=z_{2,t}
\end{aligned}\tag{27}$$

is a REO for system (24), (25). In order to prove this, we consider the functional

$$\begin{aligned}
V(t)=&\frac{R}{2}\left(z_1(t)-\zeta\int_{t-r}^{t}\exp\left(-\zeta(t-s)\right)x(s)ds\right)^2\\
&+Q\int_{t-r}^{t}\left(z_2(s)-x(s)\right)^2\exp\left(-\zeta(t-s)\right)ds\\
&+\frac{1}{2}\left(z_2(t)-x(t)-bz_1(t)+b\zeta\int_{t-r}^{t}\exp\left(-\zeta(t-s)\right)x(s)ds\right)^2
\end{aligned}\tag{28}$$

where $R,b,Q>0$ are constants to be selected. For every $\xi,u\in L^\infty_{loc}(\Re_+;\Re)$, the time derivative of $V(t)$ along the trajectories of (24) with $y(t)=\zeta\int_{t-r}^{t}\exp\left(-\zeta(t-s)\right)x(s)ds+\xi(t)$ is given by

$$\begin{aligned}
\dot V(t)=&-\left((k_1+\zeta)R-R\zeta b-Qb^2\right)E_1^2(t)\\
&-(\mu+1+b\zeta-Q)E_2^2(t)-\zeta Q\int_{t-r}^{t}\left(z_2(s)-x(s)\right)^2\exp\left(-\zeta(t-s)\right)ds\\
&+\left(R\zeta+2bQ+b(k_1+\zeta)-k_2-(\mu+1+b\zeta)b\right)E_1(t)E_2(t)\\
&+\left(bE_2(t)-RE_1(t)\right)\zeta\exp(-\zeta r)\left(z_2(t-r)-x(t-r)\right)\\
&-Q\left(z_2(t-r)-x(t-r)\right)^2\exp(-\zeta r)\\
&+\left(\theta(z_2(t))-\theta(x(t))\right)E_2(t)+\left(\left(k_2-bk_1\right)E_2(t)+Rk_1E_1(t)\right)\xi(t)\\
&+\mu\zeta E_2(t)\int_0^1\int_{t-rl}^{t}\exp\left(-\zeta(t-s)\right)\left(z_2(s)-x(s)\right)dsdl
\end{aligned}\tag{29}$$

for $t\ge 0$ a.e., where $E_2(t):=z_2(t)-x(t)-bE_1(t)$ and $E_1(t):=z_1(t)-\zeta\int_{t-r}^{t}\exp\left(-\zeta(t-s)\right)x(s)ds$. Since $\theta:\Re\to\Re$ is a globally Lipschitz function, there exists a constant $\Phi>0$ such that

$$\left|\theta(z_2(t))-\theta(x(t))\right|\le\Phi\left|z_2(t)-x(t)\right|\le\Phi\left|E_2(t)\right|+\Phi b\left|E_1(t)\right| \tag{30}$$

In the above inequality we have used the triangle inequality and the definition $E_2(t):=z_2(t)-x(t)-bE_1(t)$. Consequently, by using the (Young) inequalities

$$\begin{aligned}
&\Phi b\left|E_1(t)\right|\left|E_2(t)\right|\le\Phi E_2^2(t)+\Phi b^2E_1^2(t)/4\\
&E_2(t)\left(z_2(t-r)-x(t-r)\right)\le E_2^2(t)/2+\left(z_2(t-r)-x(t-r)\right)^2/2\\
&\left|E_1(t)\right|\left|z_2(t-r)-x(t-r)\right|\le E_1^2(t)/2+\left(z_2(t-r)-x(t-r)\right)^2/2\\
&\left|k_2-bk_1\right|\left|E_2(t)\right|\left|\xi(t)\right|\le E_2^2(t)+\left|k_2-bk_1\right|^2\xi^2(t)/4\\
&\left|k_1\right|\left|E_1(t)\right|\left|\xi(t)\right|\le E_1^2(t)+k_1^2\xi^2(t)/4
\end{aligned}$$

we obtain from (29), (30) the following inequality for $t\ge 0$ a.e.:

$$\begin{aligned}
\dot V(t)\le&-\left(\mu+b\zeta-Q-2\Phi-b\zeta\exp(-\zeta r)/2\right)E_2^2(t)\\
&-\left((k_1+\zeta)R-R\zeta b-Qb^2-\Phi\frac{b^2}{4}-\frac{1}{2}R\zeta\exp(-\zeta r)-R\right)E_1^2(t)\\
&+\left(R\zeta+2bQ+b(k_1+\zeta)-k_2-(\mu+1+b\zeta)b\right)E_1(t)E_2(t)\\
&+\left((b+R)\zeta/2-Q\right)\exp(-\zeta r)\left(z_2(t-r)-x(t-r)\right)^2\\
&-\zeta Q\int_{t-r}^{t}\left(z_2(s)-x(s)\right)^2\exp\left(-\zeta(t-s)\right)ds\\
&+\mu\zeta E_2(t)\int_0^1\int_{t-rl}^{t}\exp\left(-\zeta(t-s)\right)\left(z_2(s)-x(s)\right)dsdl\\
&+\left(\left|k_2-bk_1\right|^2+Rk_1^2\right)\xi^2(t)/4
\end{aligned}\tag{31}$$

Using the Cauchy-Schwarz inequality (twice), we bound the double integral appearing in the right hand side of (31) as follows:

$$\begin{aligned}
&\left|\int_0^1\int_{t-rl}^{t}\exp\left(-\zeta(t-s)\right)\left(z_2(s)-x(s)\right)dsdl\right|\\
&\le\left(\int_0^1\left(\int_{t-rl}^{t}\exp\left(-\zeta(t-s)\right)\left|z_2(s)-x(s)\right|ds\right)^2dl\right)^{1/2}\\
&\le\left(\int_0^1\left(\int_{t-r}^{t}\exp\left(-\zeta(t-s)\right)\left|z_2(s)-x(s)\right|ds\right)^2dl\right)^{1/2}\\
&\le\left(\frac{1-\exp(-\zeta r)}{\zeta}\int_{t-r}^{t}\exp\left(-\zeta(t-s)\right)\left|z_2(s)-x(s)\right|^2ds\right)^{1/2}
\end{aligned}$$

Using the above inequality in conjunction with estimate (31), we obtain the following inequality for $t\ge 0$ a.e.:

$$\begin{aligned}
\dot V(t)\le&-\left(\mu+b\zeta-Q-2\Phi-b\zeta\exp(-\zeta r)/2\right)E_2^2(t)\\
&-\left((k_1+\zeta)R-R\zeta b-Qb^2-\Phi\frac{b^2}{4}-\frac{1}{2}R\zeta\exp(-\zeta r)-R\right)E_1^2(t)\\
&+\left(R\zeta+2bQ+b(k_1+\zeta)-k_2-(\mu+1+b\zeta)b\right)E_1(t)E_2(t)\\
&+\left((b+R)\zeta/2-Q\right)\exp(-\zeta r)\left(z_2(t-r)-x(t-r)\right)^2\\
&-\zeta Q\int_{t-r}^{t}\left(z_2(s)-x(s)\right)^2\exp\left(-\zeta(t-s)\right)ds\\
&+\mu\zeta\left|E_2(t)\right|\left(\frac{1-\exp(-\zeta r)}{\zeta}\int_{t-r}^{t}\left(z_2(s)-x(s)\right)^2\exp\left(-\zeta(t-s)\right)ds\right)^{1/2}\\
&+\left(\left|k_2-bk_1\right|^2+Rk_1^2\right)\xi^2(t)/4
\end{aligned}\tag{32}$$

Finally, using the Young inequality

$$|E_2(t)|\left(\frac{1-\exp(-\zeta r)}{\zeta}\int_{t-r}^{t}(z_2(s)-x(s))^2\exp(-\zeta(t-s))ds\right)^{1/2}$$
$$\leq\frac{1}{2}E_2^2(t)+\frac{1-\exp(-\zeta r)}{2\zeta}\int_{t-r}^{t}(z_2(s)-x(s))^2\exp(-\zeta(t-s))ds$$

in conjunction with estimate (32), we obtain the following inequality for $t\geq 0$ a.e.:

$$\dot V(t)\leq-\left(\mu+b\zeta-Q-2\Phi-\zeta\left(b\exp(-\zeta r)+\mu\right)/2\right)E_2^2(t)$$
$$-\left((k_1+\zeta)R-R\zeta b-Qb^2-\Phi\frac{b^2}{4}-\frac{1}{2}R\zeta\exp(-\zeta r)-R\right)E_1^2(t)$$
$$+\left(R\zeta+2bQ+b(k_1+\zeta)-k_2-(\mu+1+b\zeta)b\right)E_1(t)E_2(t)$$
$$-\left(Q-(b+R)\zeta/2\right)\exp(-\zeta r)\left(z_2(t-r)-x(t-r)\right)^2$$
$$-\left(\zeta Q-\mu\frac{1-\exp(-\zeta r)}{2}\right)\int_{t-r}^{t}(z_2(s)-x(s))^2\exp(-\zeta(t-s))ds$$
$$+\left(|k_2-bk_1|^2+Rk_1^2\right)\xi^2(t)/4 \quad (33)$$

By selecting

$$R=\frac{2\mu(1-\exp(-\zeta r))}{\zeta^2},\ b=\frac{4\Phi+(\mu+R+1)\zeta}{\zeta\left(1-\exp(-\zeta r)\right)} \quad (34)$$
$$Q=(b+R)\zeta/2,$$
$$k_2=R\zeta+b(b+R)\zeta+b(k_1+\zeta)-(\mu+1+b\zeta)b \quad (35)$$
$$k_1=\zeta b+\left(2(b+R)b^2\zeta+\Phi b^2\right)/(4R)+\zeta\exp(-\zeta r)/2+1 \quad (36)$$

we obtain from (33) the following inequality for $t\geq 0$ a.e.:

$$\dot V(t)\leq-\zeta V(t)/2+\left(|k_2-bk_1|^2+Rk_1^2\right)\xi^2(t)/4 \quad (37)$$

Applying Lemma 2.12 in Karafyllis & Jiang (2011) in conjunction with (37), we get for all $t\geq 0$:

$$V(t)\leq\exp(-\zeta t/2)V(0)$$
$$+\frac{1}{4}\left(|k_2-bk_1|^2+Rk_1^2\right)\int_0^t\exp(-\zeta(t-s)/2)\xi^2(s)ds \quad (38)$$

Notice that the quadratic form $S(x)=Rx_1^2/2+(x_2-bx_1)^2/2$ on $\Re^2$ is positive definite. Consequently, there exists $K_1>0$ such that $K_1x_2^2\leq S(x)$ for all $x\in\Re^2$. Using this fact in conjunction with (28) and bounding the integral in the right hand side of (38) in the following way for any $\sigma\in(0,\zeta/4)$

$$\int_0^t\exp(-\zeta(t-s)/2)|\xi(s)|^2ds$$
$$\leq\int_0^t\exp(-\zeta(t-s)/2)\exp(-2\sigma s)ds\sup_{0\leq s\leq t}\left(|\xi(s)|^2\exp(2\sigma s)\right)$$
$$\leq\frac{2\exp(-2\sigma t)}{\zeta-4\sigma}\sup_{0\leq s\leq t}\left(|\xi(s)|^2\exp(2\sigma s)\right)$$

we obtain from (38) for all $t\geq 0$:

$$(z_2(t)-x(t))^2\leq K_1^{-1}\exp(-2\sigma t)V(0)$$
$$+\frac{|k_2-bk_1|^2+Rk_1^2}{2K_1(\zeta-4\sigma)}\sup_{0\leq s\leq t}\left(\xi^2(s)\exp(-2\sigma(t-s))\right) \quad (39)$$

Definition (28) implies that there exists a constant $K_2\geq K_1$ (independent of z_t,x_t) such that $K_2\left(\|x_t\|^2+\|z_t\|^2\right)\geq V(t)$. Therefore, we obtain from (39) for all $t\geq 0$:

$$|z_2(t)-x(t)|\leq\sqrt{K_2/K_1}\exp(-\sigma t)\left(\|x_0\|+\|z_0\|\right)$$
$$+\sqrt{\frac{|k_2-bk_1|^2+Rk_1^2}{2K_1(\zeta-4\sigma)}}\sup_{0\leq s\leq t}\left(|\xi(s)|\exp(-\sigma(t-s))\right) \quad (40)$$

Notice that due to the fact that $K_2\geq K_1$, inequality (40) actually holds for all $t\geq -r$. Consequently, we obtain from (40) for all $t\geq -r$:

$$\sup_{-r\leq s\leq t}\left(|z_2(s)-x(s)|\exp(\sigma s)\right)\leq\sqrt{K_2/K_1}\left(\|x_0\|+\|z_0\|\right)$$
$$+\sqrt{\frac{|k_2-bk_1|^2+Rk_1^2}{2K_1(\zeta-4\sigma)}}\sup_{0\leq s\leq t}\left(|\xi(s)|\exp(\sigma s)\right) \quad (41)$$

Using the fact that $\sup_{t-r\leq s\leq t}\left(|z_2(s)-x(s)|\exp(\sigma s)\right)\geq\exp(\sigma(t-r))\|z_{2,t}-x_t\|$, we obtain from (41) for all $t\geq 0$:

$$\|z_{2,t}-x_t\|\leq\sqrt{K_2/K_1}\exp(-\sigma(t-r))\left(\|x_0\|+\|z_0\|\right)$$
$$+\sqrt{\frac{|k_2-bk_1|^2+Rk_1^2}{2K_1(\zeta-4\sigma)}}\exp(\sigma r)\sup_{0\leq s\leq t}\left(|\xi(s)|\exp(-\sigma(t-s))\right) \quad (42)$$

Estimate (42) holds for all $\xi,u\in L_{loc}^{\infty}(\Re_+;\Re)$ and for all $\sigma\in(0,\zeta/4)$ and shows that (27) is a REO for system (5.16), (5.17). The proof is complete. $\lhd$

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