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An application of RLS to the online parameter identification of three-phase AC mesh grids

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Abstract: This paper focuses on parameter identification of mesh distribution grids. In order to tune secondary level controllers, accurate micrograds models, in terms of topology and impedances values, are needed. Furthermore, an efficient energy management strategy relays on the correct estimation of the load behavior considering only measurable signals. We apply the well known Recursive Least Squares (RLS) methodology to identify the complex admittances of the lines between, and loads at, nodes where voltage and current measurements are available. The proposed algorithm has little communication requirements as the signals can be sampled at low frequencies compared with the grid standard. The relatively low computation load of the algorithm makes it suitable for on-line implementation on real time. The main characteristics of the methodology are illustrated with help of simulation examples.

Keywords: Microgrids, Mesh-grids, RLS, Least Squares

1. INTRODUCTION

On-line parameter estimation based on Recursive Least Squares (RLS, Hayes (1996); Haykin (2014)), is a powerful tool for control of any kind of systems. Electric transmission and distribution systems in general are good examples of large Multiple-Input-Multiple-Output (MIMO) systems where hidden or non-measurable quantities are present. In particular, islanded microgrids are local distribution circuits that connect dispersed energy sources and loads in a relatively small geographic region, to take advantage of the distributed nature of renewable energy sources (RES). For the correct operation of this strategy, a three-level hierarchical control scheme has been proposed in several publications, *e.g.* Guerrero et al. (2013); Cheng et al. (2018). At the secondary level, typically an entire islanded microgrid is seen as a MIMO plant where some control strategy is enforced, for example, Schiffer et al. (2016); Han et al. (2016); Simpson-Porco et al. (2015).

In this context, parameter estimation is an alternative to direct measurement techniques to determine the values of the line and load impedances that describe the dynamic behavior of the a grid. This is not new to transmission systems, as several publications, *e.g.* Du and Liao (2012); Li et al. (2017); Zhou et al. (2014), deal with estimation in isolated transmission lines based on the measurements of electric signals. In the case of more complex distribution circuits, other publications focus on topology estimation of medium or low voltage distribution grids, Park et al. (2018); Liao et al. (2019), or of smart grids, Xu et al. (2013); Weckx et al. (2012). Furthermore, standard textbooks as Grainger and Stevenson (1994) apply Least Squares methods to estimate hidden states. Other examples of states estimation include Dobbe et al. (2016); Gelagaev et al. (2008) for general transmission grids, and Rana and Li (2015); Rana et al. (2018); Pereira Barbeiro et al. (2014) specifically applied to smart and microgrids.

In this paper we focus on the impedances estimation of mesh grids in general. Based on RLS, we formally develop an algorithm that can estimate the parameters of the grid at a low sample rate, defining a model that can be used for control design, on-line observation, or future operation prediction porpoises. The proposed methodology does not assume a known structure of the grid and can identify both, the topology of the grid and the line or load parameters.

After this introduction, Section 2 describes the microgrid as a secondary level control plant, first by giving insights about the electric description of the grid, to propose a model which is suitable for parameters identification based on network equations. The following Section 3 presents the main result of the paper, which is the formal deduction of a recursive algorithm to estimate the admittances of the lines and the loads based only on the voltage and current measurements at predefined nodes. Also, some modifications on the original idea are presented in order to make the estimation more efficiently when additional information over the network is known. Section 4 presents some numeric examples to illustrate the main features of the proposed methodology before the conclusion section.

Through this paper, matrix $\boldsymbol{A}'$ is the transpose of $\boldsymbol{A}$. The identity matrix and the null matrix are respectively denoted by $\boldsymbol{I}$ and $\boldsymbol{0}$. A column vector of ones is denoted as $\boldsymbol{1}$, and a vector with zeros in every position except in the i-th row where its value is one, is denoted as $\boldsymbol{s}_i \in \mathbb{R}^N$. A (block) element in position (i, j) of a matrix $\boldsymbol{A}$ is denoted $[\boldsymbol{A}]_{ij}$.

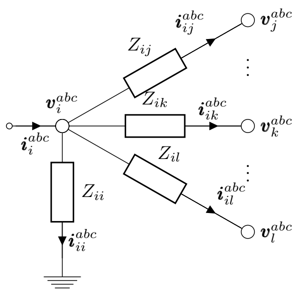

Fig. 1. Voltage and current at node i of a mesh grid with neighbors $\mathcal{N}_i = \{j, ..., k, ..., l\}$.

2. MESH GRID MODEL

2.1 Problem Formulation

We consider that an electric mesh grid can be represented by an undirected Graph with additional quantities associated.

Definition 1 (Mesh Grid)**.** A mesh grid is a tuple

$$\begin{aligned} M = (\mathcal{G} = (\mathcal{V}, \mathcal{E}), \\ \{\boldsymbol{v}_i^{abc}(t)\}_{i\in\mathcal{V}}, \{\boldsymbol{i}_i^{abc}(t)\}_{i\in\mathcal{V}}, \\ \{R_{ij}\}_{\{i,j\}\in\mathcal{E}\cup\{\{i,i\}\}_{i\in\mathcal{V}}}, \\ \{L_{ij}\}_{\{i,j\}\in\mathcal{E}\cup\{\{i,i\}\}_{i\in\mathcal{V}}}\big), \end{aligned}$$

where

- $\mathcal{G}$: an undirected graph with
  - $\mathcal{V}$ a set of N vertices or nodes where voltage amplitude is measured.
  - $\mathcal{E}$ a set of unordered edges between nodes in such a way that if $\{i, j\} = \{j, i\} \in \mathcal{E}$, with $i \neq j \in \mathcal{V}$, then there is an electric three-phase line between the i-th and j-th node of the mesh grid where current can be transmitted.
- $\boldsymbol{v}_i^{abc}$: three-phase balanced sinusoidal voltage measured at node $i \in \mathcal{V}$.
- $\boldsymbol{i}_i^{abc}$: three-phase net current injected at the i-th node in $\mathcal{V}$. That is, the addition of the currents injected by each known source and the subtraction of the currents of each known load connected to the node.
- R_{ij} and L_{ij}: balanced series resistance and inductance associated to the line between node $i \in \mathcal{V}$ and $j \in \mathcal{V}$ if $i \neq j$, or to the unknown load at node $i \in \mathcal{V}$ to ground if $i = j$.

The nominal operation frequency will be denoted $f > 0$ in $[Hz]$. The multiplication $\omega = 2\pi f$ is the nominal operation angular frequency. Note that, because the edges of the mesh grid are not oriented, we have that $R_{ij} = R_{ji}$ and $L_{ij} = L_{ji}$. In phasor notation, with $\hat{\imath} = \sqrt{-1}$ the imaginary unit, the complex impedance associated to an edge or a load is $Z_{ij} = R_{ij} + \hat{\imath}\omega L_{ij}$.

It is also useful to define the line currents $\boldsymbol{i}_{ij}^{abc}$ from node $i \in \mathcal{V}$ to node $j \in \mathcal{V}$ with $i \neq j$, and the load currents $\boldsymbol{i}_{ii}^{abc}$ from each node $i \in \mathcal{V}$ to ground. With this concepts, a circuital representation of a node of a mesh grid can be seen in Figure 1.

Problem 1 (Parameter Identification)**.** Given a set of measurements of the node voltages $\{\boldsymbol{v}_i^{abc}(t)\}_{i\in\mathcal{V}}$, and the net injected currents $\{\boldsymbol{i}_i^{abc}(t)\}_{i\in\mathcal{V}}$ at different time instants $t \in S = \{t_0, t_1, \dots t_T, \dots\}$, recursively determine the value of the parameters of the lines and passive loads, $\{R_{ij}\}_{\{i,j\}\in\mathcal{E}\cup\{\{i,i\}\}_{i\in\mathcal{V}}}$ and $\{L_{ij}\}_{\{i,j\}\in\mathcal{E}\cup\{\{i,i\}\}_{i\in\mathcal{V}}}$.

Assumption 1 (Stationary State at sample Instants)**.** At sample instants, we assume that the voltages and currents are sinusoidal three-phase quantities with frequency identical to the nominal value, constant amplitude and constant phase angle. In this way, with help of the Park transformation Kundur (1994), we can calculate the constant dq equivalent of any abc signal: $\boldsymbol{x}^{dq} := \boldsymbol{T}_{dq-abc}\boldsymbol{x}^{abc}$, with

$$\boldsymbol{T}_{dq-abc}(t) = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\omega t) & \cos(\omega t - 2\pi/3) & \cos(\omega t - 4\pi/3) \\ \sin(\omega t) & \sin(\omega t - 2\pi/3) & \sin(\omega t - 4\pi/3) \end{bmatrix}.$$

Assumption 2 (Access to distributed Measurements)**.** We assume that the processor where the calculations are performed to estimate the parameters has access to the distributed measurements of voltages and currents. This implies that this information needs to be communicated from possibly long distances to the processing unit. Nevertheless, if the sample instants $t \in S = \{t_0, t_1, \dots t_T, \dots\}$ are relatively distant from each other, and the data has a time stamp, delays or other communication issues can be ignored.

It a common practice to model the lines and loads not by their impedance $Z_{ij} = R_{ij} + \hat{\imath}\omega L_{ij}$, but by their admittance $Y_{ij} = Z_{ij}^{-1} = g_{ij} + \hat{\imath} b_{ij}$, where the conductance and susceptance are respectively defined by

$$g_{ij} = \text{real}\{Y_{ij}\} = \frac{R_{ij}}{R_{ij}^2 + \omega^2 L_{ij}^2},$$

$$b_{ij} = \text{imag}\{Y_{ij}\} = \frac{\omega L_{ij}}{R_{ij}^2 + \omega^2 L_{ij}^2}.$$

Note that the relationship between impedance and admittance is invertible,

$$R_{ij} = \frac{g_{ij}}{g_{ij}^2 + b_{ij}^2}, \qquad \omega L_{ij} = \frac{b_{ij}}{g_{ij}^2 + b_{ij}^2},$$

and therefore, if we can estimate the values of the conductances g_{ij} and the susceptances b_{ij}, we can indirectly find the resistances and inductances that describe the mesh grid.

2.2 Circuital model of a Mesh Grid

From a Kirchhoff circuital analysis of Figure 1, considering Assumption 1, we obtain for the currents that

$$\boldsymbol{i}_i^{dq} = \boldsymbol{i}_{ii}^{dq} + \sum_{j\in\mathcal{N}_i}^{N} \boldsymbol{i}_{ij}^{dq}.$$

For the loads we have

$$\boldsymbol{v}_i^{dq} = [R_{ii}\boldsymbol{I} - L_{ii}\omega\boldsymbol{J}]\,\boldsymbol{i}_{ii}^{dq},$$

where

$$\boldsymbol{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Similarly, at each RL transmission line $\{i, j\} \in \mathcal{E}$:

$$\boldsymbol{v}_i^{dq} - \boldsymbol{v}_j^{dq} = [R_{ij}\boldsymbol{I} - L_{ij}\omega\boldsymbol{J}]\,\boldsymbol{i}_{ij}^{dq}.$$

Assumption 2 allows to have access to the dq representation of every abc signal by simply multiplying the measurements vector by the Park transformation evaluated at the corresponding instant. Combining the currents balance equation with the load and lines equations, the current injected at the i-th node can be written as

$$\boldsymbol{i}_i^{dq} = [g_{ii}\boldsymbol{I} + b_{ii}\boldsymbol{J}]\,\boldsymbol{v}_i^{dq} + \sum_{j\in\mathcal{N}_i} [g_{ij}\boldsymbol{I} + b_{ij}\boldsymbol{J}]\left(\boldsymbol{v}_i^{dq} - \boldsymbol{v}_j^{dq}\right) \quad (1)$$

From here, an expression that describes the behavior of the entire set of nodes can be written as a matrix equation. For this effect we define vectors

$$\boldsymbol{i}^d := \operatorname{col}\{i_i^d\}_{i=1}^N \in \mathbb{R}^N,\;\; \boldsymbol{i}^q := \operatorname{col}\{i_i^q\}_{i=1}^N \in \mathbb{R}^N,$$
$$\boldsymbol{v}^d := \operatorname{col}\{v_i^d\}_{i=1}^N \in \mathbb{R}^N,\;\; \boldsymbol{v}^q := \operatorname{col}\{v_i^q\}_{i=1}^N \in \mathbb{R}^N,$$

and matrices

$$\boldsymbol{G} = \operatorname{diag}\{g_{ii}\}_{i=1}^N,\;\; \boldsymbol{B} = \operatorname{diag}\{b_{ii}\}_{i=1}^N,\;\; \boldsymbol{D} = D'(\mathcal{G}^o)$$
$$\boldsymbol{W}_g = \operatorname{diag}\{g_{i_kj_k}\}_{k=1}^{|\mathcal{E}|},\;\; \boldsymbol{W}_b = \operatorname{diag}\{b_{i_kj_k}\}_{k=1}^{|\mathcal{E}|},$$

where the incidence matrix $D(\mathcal{G}^o)$ of a directed graph $\mathcal{G}^o$ is derived by giving arbitrary orientations to the edges of $\mathcal{G}$. To do so, we need to identify the $|\mathcal{E}| \le N(N-1)/2$ edges $e_k = (i_k, j_k) \in \mathcal{E}$ through an index $k \in \{1, 2, \dots, |\mathcal{E}|\}$ and define an orientation for each one of them. In that case, one can define the incidence matrix $D(\mathcal{G}^o) \in \mathbb{R}^{N\times|\mathcal{E}|}$ in such a way that its elements are either $[D(\mathcal{G}^o)]_{ik} = -1$ if the edge e_k has its origin in the i-th node, $[D(\mathcal{G}^o)]_{ik} = 1$ if the i-th node is the destination of e_k, or $[D(\mathcal{G}^o)]_{ik} = 0$ otherwise.

Using this notation, separating (1) in its d and q components and rearranging, we find that a model for the entire mesh grid is given by

$$\begin{bmatrix}\boldsymbol{i}^d\\ \boldsymbol{i}^q\end{bmatrix} = \begin{bmatrix}\boldsymbol{G}+\boldsymbol{D}'\boldsymbol{W}_g\boldsymbol{D} & -\boldsymbol{B}-\boldsymbol{D}'\boldsymbol{W}_b\boldsymbol{D}\\ \boldsymbol{B}+\boldsymbol{D}'\boldsymbol{W}_b\boldsymbol{D} & \boldsymbol{G}+\boldsymbol{D}'\boldsymbol{W}_g\boldsymbol{D}\end{bmatrix}\begin{bmatrix}\boldsymbol{v}^d\\ \boldsymbol{v}^q\end{bmatrix}. \quad (2)$$

Note that this model is not dynamic as the present value of the currents depend only on the present value of the voltages. The expression is linear with respect to the voltages $\boldsymbol{v}^d$ and $\boldsymbol{v}^q$, but also with respect to its parameters. This can be used to write the last equation in a form more convenient for parameter identification.

Remark 1 (Non-linear model). An equivalent quadratic and trigonometric representation of the grid can be proposed in terms of active and reactive power instead of dq currents, and voltages magnitude and phase angles instead of dq components. Even though many measurement devices work with these quantities, it is a matter of algebraic calculation to obtain the equivalent dq currents and voltages. We prefer the representation (2) because it is linear on the variables. However, both representations are linear with respect to the admittance parameters, and therefore an equivalent methodology can be proposed when power measurements must be used.

Observation 1. *The product between a diagonal matrix* $\boldsymbol{M} \in \mathbb{R}^{n\times n}$ *and a vector* $\boldsymbol{u} \in \mathbb{R}^n$ *can be written as* $\boldsymbol{Mu} = \boldsymbol{Um}$, *where* $\boldsymbol{U} = \operatorname{diag}\{[\boldsymbol{u}]_i\}_{i=1}^n$ *and* $\boldsymbol{m} = \operatorname{col}\{[\boldsymbol{M}]_{ii}\}_{i=1}^n$.

Using this property, because matrices $\boldsymbol{G}$, $\boldsymbol{B}$, $\boldsymbol{W}_g$, and $\boldsymbol{W}_b$ are all diagonal, then we can write,

$$\frac{1}{S}\begin{bmatrix}\boldsymbol{i}^d\\ \boldsymbol{i}^q\end{bmatrix} = \frac{1}{S}\begin{bmatrix}\boldsymbol{V}_d & \boldsymbol{D}'\Delta\boldsymbol{V}_d & -\boldsymbol{V}_q & -\boldsymbol{D}'\Delta\boldsymbol{V}_q\\ \boldsymbol{V}_q & \boldsymbol{D}'\Delta\boldsymbol{V}_q & \boldsymbol{V}_d & \boldsymbol{D}'\Delta\boldsymbol{V}_d\end{bmatrix}\begin{bmatrix}\boldsymbol{\theta}_g^v\\ \boldsymbol{\theta}_g^e\\ \boldsymbol{\theta}_b^v\\ \boldsymbol{\theta}_b^e\end{bmatrix},$$

where,

$$\boldsymbol{V}_d = \operatorname{diag}\{v_i^d\}_{i=1}^N,\quad \Delta\boldsymbol{V}_d = \operatorname{diag}\{[\boldsymbol{D}\boldsymbol{v}^d]_k\}_{k=1}^{|\mathcal{E}|},$$
$$\boldsymbol{V}_q = \operatorname{diag}\{v_i^q\}_{i=1}^N,\quad \Delta\boldsymbol{V}_q = \operatorname{diag}\{[\boldsymbol{D}\boldsymbol{v}^q]_k\}_{k=1}^{|\mathcal{E}|},$$
$$\boldsymbol{\theta}_g^v = \operatorname{col}\{g_{ii}\}_{i=1}^N,\quad \boldsymbol{\theta}_g^e = \operatorname{col}\{g_{i_kj_k}\}_{k=1}^{|\mathcal{E}|},$$
$$\boldsymbol{\theta}_b^v = \operatorname{col}\{b_{ii}\}_{i=1}^N,\quad \boldsymbol{\theta}_b^e = \operatorname{col}\{b_{i_kj_k}\}_{k=1}^{|\mathcal{E}|},$$

and $S > 0$ is an arbitrary scalar thought as a way to normalize the data for parameter identification. Indeed, numeric errors can be expected if, for example, the voltage signals are too large when compared with the current signals.. If all nodes share a common current or power rating, or a nominal voltage, a practical choice for $S > 0$ would be this value.

In an even more compact way we have that a model for the mesh grid is given by:

Definition 2 (Mesh Grid Model)**.**

$$\boldsymbol{i}^{dq} = \boldsymbol{V}^{dq}\boldsymbol{\theta}, \quad (3)$$

with,

$$\boldsymbol{i}^{dq} = {}^1/_S \cdot \operatorname{col}\left\{\boldsymbol{i}^d, \boldsymbol{i}^q\right\} \in \mathbb{R}^{2N},$$
$$\boldsymbol{\theta} = \operatorname{col}\{\boldsymbol{\theta}_g^v, \boldsymbol{\theta}_g^e, \boldsymbol{\theta}_b^v, \boldsymbol{\theta}_b^e\} \in \mathbb{R}^{2(N+|\mathcal{E}|)},$$
$$\boldsymbol{V}^{dq} = \frac{1}{S}\begin{bmatrix}\boldsymbol{V}_d & \boldsymbol{D}'\Delta\boldsymbol{V}_d & -\boldsymbol{V}_q & -\boldsymbol{D}'\Delta\boldsymbol{V}_q\\ \boldsymbol{V}_q & \boldsymbol{D}'\Delta\boldsymbol{V}_q & \boldsymbol{V}_d & \boldsymbol{D}'\Delta\boldsymbol{V}_d\end{bmatrix} \in \mathbb{R}^{2N\times 2(N+|\mathcal{E}|)}.$$

Note that, although not explicitly indicated, vector $\boldsymbol{i}^{dq}$ and matrix $\boldsymbol{V}^{dq}$ depend on time, while the parameters are constant.

3. PARAMETER IDENTIFICATION

3.1 Identification through Least Squares (LS)

Define an error vector

$$\boldsymbol{e}(t) = \boldsymbol{i}^{dq}(t) - \boldsymbol{V}^{dq}(t)\hat{\boldsymbol{\theta}}_T \in \mathbb{R}^{2N}$$

where $\hat{\boldsymbol{\theta}}_T \in \mathbb{R}^{2(N+|\mathcal{E}|)}$ is an estimation of the real parameters in model (3). Note that contrary to the most common Least Squares setup (See for example Hayes (1996); Haykin (2014)), the error defined here is a vector and not a scalar and the standard LS estimation method needs to be modified.

If we consider the first $T+1$ time instants in $S = \{t_0, t_1, \dots, t_T, \dots\}$, we can also define an aggregated error vector in the following way:

$$\boldsymbol{e}_T = \begin{bmatrix}\boldsymbol{e}(t_0)\\ \boldsymbol{e}(t_1)\\ \vdots\\ \boldsymbol{e}(t_T)\end{bmatrix} = \begin{bmatrix}\boldsymbol{i}^{dq}(t_0)\\ \boldsymbol{i}^{dq}(t_1)\\ \vdots\\ \boldsymbol{i}^{dq}(t_T)\end{bmatrix} - \begin{bmatrix}\boldsymbol{V}^{dq}(t_0)\\ \boldsymbol{V}^{dq}(t_1)\\ \vdots\\ \boldsymbol{V}^{dq}(t_T)\end{bmatrix}\hat{\boldsymbol{\theta}}_T = \boldsymbol{i}_T^{dq} - \boldsymbol{V}_T^{dq}\hat{\boldsymbol{\theta}}_T,$$

where the vector $\boldsymbol{i}_T^{dq} \in \mathbb{R}^{2N\cdot(T+1)}$ and the matrix $\boldsymbol{V}_T^{dq} \in \mathbb{R}^{2N\cdot(T+1)\times 2(N+|\mathcal{E}|)}$ are implicitly defined.

With a forgetting factor $0 < \lambda < 1$ we can define a quadratic functional,

$$J(\hat{\boldsymbol{\theta}}_T) = \sum_{i=0}^{T}(\lambda)^{T-i}\boldsymbol{e}'(t_i)\boldsymbol{e}(t_i) = \boldsymbol{e}_T'\left(\boldsymbol{L}_T \otimes \boldsymbol{I}_{2N}\right)\boldsymbol{e}_T, \quad (4)$$

which implicitly depends on the parameters estimations through the error vector and where $\otimes$ stands for the Kronecker Product and,

$$\boldsymbol{L}_T := \operatorname{diag}\{\lambda^{T-i}\}_{i=0}^T.$$

When $\lambda = 1$, the functional represents the accumulated quadratic error between the model and the real values of currents and voltages in the last $T-1$ observations. For smaller forgetting factors, the accumulation is weighted so that the last observation is more important than the previous ones. Minimizing this functional in terms of $\hat{\boldsymbol{\theta}}_T$, a procedure can be proposed to estimate the real parameters of the grid.

Theorem 2 (Least Squares). *The positive functional (4) is minimum for*

$$\hat{\boldsymbol{\theta}}_T = \left[(\boldsymbol{V}_T^{dq})'(\boldsymbol{L}_T \otimes \boldsymbol{I})\boldsymbol{V}_T^{dq}\right]^{-1} (\boldsymbol{V}_T^{dq})'(\boldsymbol{L}_T \otimes \boldsymbol{I})\boldsymbol{i}_T^{dq} \quad (5)$$

Proof. At the minimum, the divergence of the functional (4) with respect to the parameters vector must be zero. This standard procedure leads directly to (5). □

3.2 Recursive Parameter identification

It is desired to find a recursive method to identify the parameters.

Theorem 3 (Recursive Least Squares). *Giving an initial estimations for the parameters, $\hat{\boldsymbol{\theta}}_{T-1} \in \mathbb{R}^{2N(T+1)}$, and an initial covariance matrix $\boldsymbol{P}_{T-1} \in \mathbb{R}^{2(N+|\mathcal{E}|)\times 2(N+|\mathcal{E}|)}$, the positive functional (4) is minimum for*

$$\hat{\boldsymbol{\theta}}_T = \hat{\boldsymbol{\theta}}_{T-1} + \frac{1}{\lambda}\boldsymbol{P}_{T-1}(\boldsymbol{V}^{dq}(t_T))'\boldsymbol{e}(t_T), \quad (6)$$

where,

$$\boldsymbol{P}_T = \frac{1}{\lambda}\boldsymbol{P}_{T-1} - \frac{1}{\lambda^2}\boldsymbol{P}_{T-1}(\boldsymbol{V}^{dq}(t_T))'\boldsymbol{\Gamma}_T\boldsymbol{V}^{dq}(t_T)\boldsymbol{P}_{T-1}, \quad (7)$$

and

$$\boldsymbol{\Gamma}_T := \left[\boldsymbol{I} + \frac{1}{\lambda}\boldsymbol{V}^{dq}(t_T)\boldsymbol{P}_{T-1}(\boldsymbol{V}^{dq}(t_T))'\right]^{-1} \in \mathbb{R}^{2N\times 2N}.$$

Proof. First, note the following decompositions:

$$\boldsymbol{V}_T^{dq} = \begin{bmatrix}\boldsymbol{V}_{T-1}^{dq}\\ \boldsymbol{V}^{dq}(t_T)\end{bmatrix}, \boldsymbol{i}_T^{dq} = \begin{bmatrix}\boldsymbol{i}_{T-1}^{dq}\\ \boldsymbol{i}^{dq}(t_T)\end{bmatrix}, \boldsymbol{L}_T\otimes\boldsymbol{I} = \begin{bmatrix}\lambda\boldsymbol{L}_{T-1}\otimes\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}$$

Equation (5) can be equivalently written as $\boldsymbol{R}_T\hat{\boldsymbol{\theta}}_T = \boldsymbol{g}_T$, where

$$\begin{aligned}\boldsymbol{R}_T &:= (\boldsymbol{V}_T^{dq})'(\boldsymbol{L}_T \otimes \boldsymbol{I})\boldsymbol{V}_T^{dq}\\ &= \lambda(\boldsymbol{V}_{T-1}^{dq})'(\boldsymbol{L}_{T-1} \otimes \boldsymbol{I})\boldsymbol{V}_{T-1}^{dq} + (\boldsymbol{V}^{dq}(t_T))'\boldsymbol{V}^{dq}(t_T)\\ &= \lambda\boldsymbol{R}_{T-1} + (\boldsymbol{V}^{dq}(t_T))'\boldsymbol{V}^{dq}(t_T) \quad (8)\end{aligned}$$

and

$$\begin{aligned}\boldsymbol{g}_T &:= (\boldsymbol{V}_T^{dq})'(\boldsymbol{L}_T \otimes \boldsymbol{I})\boldsymbol{i}_T^{dq}\\ &= \lambda(\boldsymbol{V}_{T-1}^{dq})'(\boldsymbol{L}_{T-1} \otimes \boldsymbol{I})\boldsymbol{i}_{T-1}^{dq} + (\boldsymbol{V}^{dq}(t_T))'\boldsymbol{i}^{dq}(t_T)\\ &= \lambda\boldsymbol{R}_{T-1}\hat{\boldsymbol{\theta}}_{T-1} + (\boldsymbol{V}^{dq}(t_T))'\boldsymbol{i}^{dq}(t_T)\end{aligned}$$

Developing this expression based in the noted decompositions and the error definition, defining the covariance matrix $\boldsymbol{P}_T := \boldsymbol{R}_T^{-1}$, we come to the recursive expression (6).

Because we already have the recursive expression (8), then

$$\boldsymbol{P}_T = \left[\lambda\boldsymbol{R}_{T-1} + (\boldsymbol{V}^{dq}(t_T))'\boldsymbol{V}^{dq}(t_T)\right]^{-1},$$

can be expressed recursively as in equation (7) directly applying the Matrix Inversion Lemma stated bellow. □

Lemma 4 (Matrix Inversion Lemma). *For matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$, such that the multiplications and the inverses used bellow are well defined, then*

$$(\boldsymbol{A}+\boldsymbol{BCD})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{B}\left(\boldsymbol{C}^{-1} + \boldsymbol{D}\boldsymbol{A}^{-1}\boldsymbol{B}\right)^{-1}\boldsymbol{D}\boldsymbol{A}^{-1}.$$

Proof. The lemma, also known as the Woodbury matrix identity, Sherman-Morrison-Woodbury formula or just Woodbury formula can be easily found in several textbooks (*e.g*Golub and Loan (2013); Horn and Johnson (2013)) or on the Internet. □

Remark 2 (Inverse of $\boldsymbol{\Gamma}$). In the procedure proposed by Theorem 3, we still need to calculate the inverse of matrix $\boldsymbol{F}_T = \boldsymbol{I} + \frac{1}{\lambda}\boldsymbol{V}^{dq}(t_T)\boldsymbol{P}_{T-1}(\boldsymbol{V}^{dq}(t_T))' = \boldsymbol{\Gamma}_T^{-1} \in \mathbb{R}^{2N\times 2N}$. The dimension of this matrix only depends on the number of nodes and not the number of edges, which is not the case when directly computing the inverse of $\boldsymbol{R}_T \in \mathbb{R}^{2(N+|\mathcal{E}|)\times 2(N+|\mathcal{E}|)}$ in Theorem 2.

In the standard recursive least squares set up, the error is a scalar, which finally makes matrix $\boldsymbol{F}_T$ also to be a scalar easy to invert. In our case this matrix inverse might not even exist and therefore we need to check for existence in order to estimate parameters. One way of doing this is through the condition number (*e.g* Golub and Loan (2013); Horn and Johnson (2013)) of the matrix $\kappa(\boldsymbol{F}_T) = \|\boldsymbol{F}_T^{-1}\|\|\boldsymbol{F}_T\|$, defined for some matrix norm $\|\cdot\|$. With the Euclidean norm, we have that $\kappa(\boldsymbol{F}_T) = \|\boldsymbol{F}_T^{-1}\|_2\|\boldsymbol{F}_T\|_2 = \sigma_{max}(\boldsymbol{F}_T)/\sigma_{min}(\boldsymbol{F}_T)$ is easy to calculate with $\sigma_{max}(\boldsymbol{F}_T)$ and $\sigma_{min}(\boldsymbol{F}_T)$, respectively, the largest and smallest singular values of the matrix.

If the condition number of a matrix is infinite, then the matrix is singular. In principle, any positive value of the condition number characterizes an invertible matrix. However, if the condition number is too large ($\kappa(\boldsymbol{F}_T) \gg 1$), the matrix is ill conditioned (or close to be singular) and the computation of its inverse is prone to large numeric errors.

3.3 Immediate extensions

A priory knowledge on some of the parameters that describe the mesh grid might ease the identification. For example, some parameters might be significantly larger than the values of other parameters, or some parameters might already be known through direct measurement or previous identifications. In this cases, the model in (3) needs to be modified in order to properly include this information.

Define the set of the indexes of all parameters as

$$P = \{1, 2, \ldots, 2(N+|\mathcal{E}|)\}$$

and a partition of this set into a set of known parameters,

$$P_K = \left\{p \in P | [\boldsymbol{\theta}]_p = [\hat{\boldsymbol{\theta}}_T]_p\right\},$$

and a set of unknown parameters such that $P_U \cup P_K = P$ and $P_U \cap P_K = \{\ \}$. Additionally, consider that each unknown parameters is weighted by a constant $w_p > 0$ in order to adjust the units.

In this way,

Table 1. Line and Load Nominal Parameters in Examples 1 and 2.

	i	j	$R_{ij}[\Omega]$	$L_{ij}[mH]$	$g_{ij}[S]$	$b_{ij}[S]$
Loads	1	1	180.00	130.00	0.0053	0.0012
	2	2	280.00	260.00	0.0033	0.0010
	3	3	243.00	084.00	0.0041	0.0004
	4	4	201.00	100.00	0.0049	0.0008
Lines	1	2	0.5000	1.1000	1.3535	0.9354
	1	4	0.9000	1.4000	0.8969	0.4383
	2	3	0.1000	0.7600	1.4924	3.5632
	3	4	0.1000	1.5000	0.4309	2.0306

$$\hat{\boldsymbol{\theta}}_T = \sum_{p\in P_K} \boldsymbol{s}_p[\boldsymbol{\theta}]_p + \sum_{p\in P_U} \boldsymbol{s}_p w_p[\hat{\boldsymbol{\theta}}_T]_p$$
$$= \boldsymbol{S}_K\boldsymbol{\theta}^K + \boldsymbol{S}_U\boldsymbol{W}_U\hat{\boldsymbol{\theta}}_T^U.$$

Where $\boldsymbol{s}_p \in \mathbb{R}^{2(N+|\mathcal{E}|)}$ is a vector with zeros in every entry but the p-th entry where its value is one; and $\boldsymbol{S}_K = \text{row}\left\{\boldsymbol{s}_p\right\}_{p\in P_K}$, $\boldsymbol{S}_U = \text{row}\left\{\boldsymbol{s}_p\right\}_{p\in P_U}$, $\boldsymbol{W}_U = \text{diag}\left\{w_p\right\}_{p\in P_U}$, $\boldsymbol{\theta}^K = \text{col}\left\{[\boldsymbol{\theta}]_p\right\}_{p\in P_K}$ is a vector with all known parameters, and $\boldsymbol{\theta}_T^U = \text{col}\left\{[\hat{\boldsymbol{\theta}}_T]_p\right\}_{p\in P_U}$ is a vector with unknown parameters to be estimated.

Replacing the last expression in the definition of the error leads to

$$\boldsymbol{e}(t_T) = \boldsymbol{i}^{dq}(t_T) - \boldsymbol{V}^{dq}(t_T)\hat{\boldsymbol{\theta}}_T$$
$$= \left(\boldsymbol{i}^{dq}(t_T) - \boldsymbol{V}^{dq}(t_T)\boldsymbol{S}_K\boldsymbol{\theta}^K\right) - \left(\boldsymbol{V}^{dq}(t_T)\boldsymbol{S}_U\boldsymbol{W}_U\right)\hat{\boldsymbol{\theta}}_T^U$$
$$= \boldsymbol{i}_U^{dq}(t_T) - \boldsymbol{V}_U^{dq}(t_T)\hat{\boldsymbol{\theta}}_T^U,$$

where $\boldsymbol{i}_U^{dq}(t_T) \in \mathbb{R}^{2N}$ and $\boldsymbol{V}_U^{dq}(t_T) \in \mathbb{R}^{2N\times|P_U|}$ are implicitly defined. To adapt Theorem 3 is straight forward.

4. NUMERIC EXAMPLES

Example 1 (Four nodes microgrid). Consider a microgrid with $N = 4$ nodes and $|\mathcal{E}| = 4 < N(N-1)/2 = 6$ lines. The parameters of the microgrid are shown in Table 1. Note that two lines do not exist, and therefore the associated admittances are identically zero. The behavior of the microgrid is simulated with nominal values $V = 220[V_{RMS}]$ and $f = 50[Hz]$. The original data used for estimation is available upon request to the author.

We will assume no knowledge about the microgrid, that is, all possible $N(N-1)/2 = 6$ lines are assumed present, implying $P = \{1,\dots,20\}$ and $P_K = \{\ \}$. Sampling the data every $\Delta T := t_T - t_{T-1} = 0.1[s]$, with $\boldsymbol{\theta}_0 = \boldsymbol{0}$, $\boldsymbol{P}_0 = \boldsymbol{I}$, $S = \sqrt{3/2}V$, $\bar{\kappa} = 1.5$ and $\lambda = 0.95$, the estimation results can be seen in Figure 2 for the non-zero admittances and in Figure 3 for the parameters associated to the un-existing lines which should have a value zero.

Note that even though the estimation is achieved, the algorithm intents to find the value of the zero parameters making large errors during a long period, as can be seen in Figure 3 a). Furthermore, at the end of the simulation time, the value at which these parameters are estimated is numerically comparable with the real values of the load admittances, see Figure 3 b). This issues can be addressed by considering additional information about the circuit in order to improve the identification performance. ■

Example 2 (Estimation with known parameters). Consider the exact same data than in Example 1. However, now we assume that the graph that describes

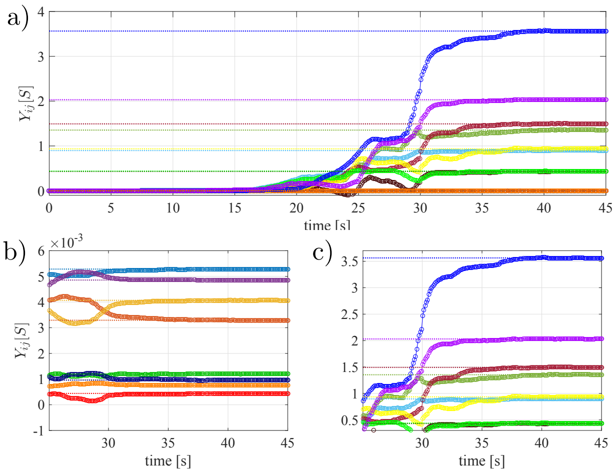

Fig. 2. Identification results for four nodes microgrid with sampling period $t_T - t_{T-1} = 0.1[s]$, $\bar{\kappa} = 1.5$ and $\lambda = 0.95$ in Example 1. a) Positive Admittances identification, b) Detail of load admittances, and c) Detail of line admittances.

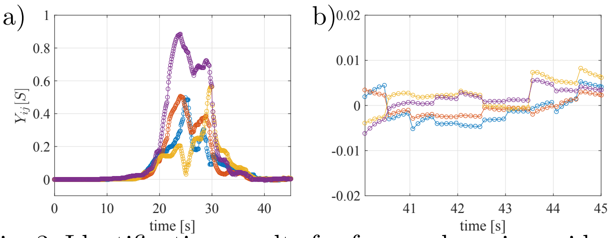

Fig. 3. Identification results for four nodes microgrid with sampling period $t_T - t_{T-1} = 0.1[s]$, $\bar{\kappa} = 1.5$ and $\lambda = 0.95$ in Example 1. a) Zero admittances identification, and b) Detail of zero admittances.

the microgrid is known, so that $\boldsymbol{D} = D'(\mathcal{G}^o)$, $P = \{1,\dots,2(N+|\mathcal{E}|) = 2(4+4) = 16\}$, and we do not need to estimate the parameters associated to the un-existing lines. Furthermore, we will assume that we know the parameters of the loads under the second, third, and fourth node. That is, $P_K = \{2,3,4,10,11,12\}$ and $P_U = \{1,5,6,7,8,9,13,14,15,16\}$. In this way, we only need to estimate ten parameters instead of the twenty parameters from the previous example. Additionally, we know that the load admittances are around a thousand times smaller than the line admittances, therefore we choose $w_p = 1$, $\forall p \in P_U\backslash\{1,9\}$ and $w_1 = w_9 = 0.001$.

With $\Delta T := t_T - t_{T-1} = 0.1[s]$, $\boldsymbol{\theta}_0 = \boldsymbol{0}$, $\boldsymbol{P}_0 = \boldsymbol{I}$, $S = \sqrt{3/2}V$, $\bar{\kappa} = 1.5$ and $\lambda = 0.95$, the same as in Example 1, the estimation results can be seen in Figure 4 with the same color pattern. It is clear at simple sight, that the estimation is carried out faster than before. Note also that the introduction of the weights in matrix $\boldsymbol{W}_U \neq \boldsymbol{I}$, makes the parameters of the first load comparable in magnitude to those of the lines. ■

Example 3 (Estimation of time varying loads). Consider the same microgrid, but this time assume that the values of the line impedances are already known. The unknown loads begin at their nominal values used in the previous examples, but they abruptly change up to a 20% during the simulation time. Figure 5 shows the result of the estimation process including the dynamic behavior of the load. ■

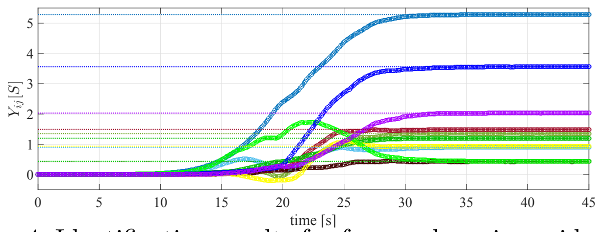

Fig. 4. Identification results for four nodes microgrid with sampling period $t_T - t_{T-1} = 0.1[s]$, $\bar{\kappa} = 1.5$ and $\lambda = 0.95$ in Example 2.

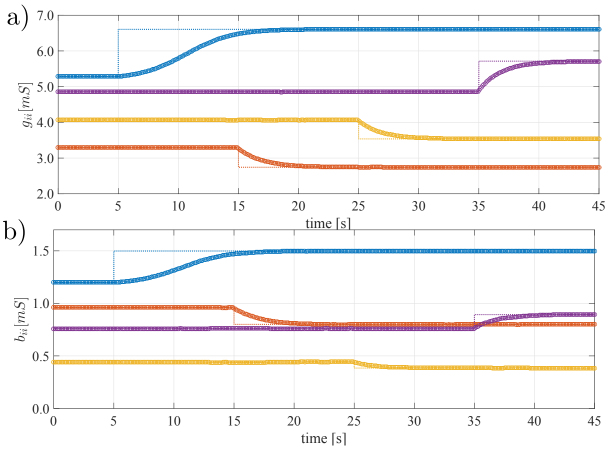

Fig. 5. Identification results for four nodes microgrid with sampling period $t_T - t_{T-1} = 0.1[s]$, $\bar{\kappa} = 1.5$ and $\lambda = 0.95$ in Example 3. a) Conductance of loads, and b) Susceptance of loads.

5. CONCLUSION

In this paper we have applied the RLS methodology to the estimation of the line and load parameters of a three phase mesh grid using only voltage and current measurements at predefined nodes. The resulting proposed recursive algorithm is suitable for on-line operation. The algorithm determines the admittances values, also identifying zero-admittance lines, and therefore the topology of the grid. Because the sample rate at which the parameter estimation is defined by the user, there are no large communication restrictions and the algorithm can be run at a centralized unit.

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