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A Distributed, Rolling-Horizon Demand Side Management Algorithm under Wind Power Uncertainty

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Abstract: In this paper, we consider a smart grid where users behave selfishly, aiming at minimizing cost in the presence of uncertain wind power availability. We adopt a demand side management (DSM) model, where active users (so-called prosumers) have both private generation and local storage availability. These prosumers participate to the DSM strategy by updating their energy schedule, seeking to minimize their local cost, given their local preferences and the global grid constraints. The energy price is defined as a function of the aggregate load and the wind power availability. We model the resulting problem as a non-cooperative Nash game and propose a semi-decentralized algorithm to compute an equilibrium. To cope with the uncertainty in the wind power, we adopt a rolling-horizon approach, and in addition we use a stochastic optimization technique. We generate several wind power production scenarios from a defined probability density function (PDF), determining an approximate stochastic cost function. Simulations results on a real dataset show that the proposed approach generates lower individual costs compared to a standard expected value approach.

Keywords: Demand side management, Smart grid, Sample average approximation, Stochastic optimization

1. INTRODUCTION

Renewable energy sources (RES) are becoming significant for power generation around the world; these sources will develop even faster in the next years, becoming an essential part of the world energy supply. Indeed, the contemporary energy network comprehends a significant deficit in coordination between demand and supply. In fact, most of the time, suppliers have to run costly ancillary plants in order to satisfy irregular peak demand. Moreover, the unpredictable and intermittent nature of RES dramatically increases the planning complexity, making a considerable market penetration and integration of these sources difficult.

Given these challenges, smart grids are envisioned to facilitate the transition towards a user-oriented system that can ensure high efficiency, security, and quality for the energy distribution [Gao et al. (2014)]. Demand side management (DSM), together with distributed generation (DG) and distributed storage (DS), are recognized as very efficient solutions to plan and adjust the grid operations, mitigating the issues related to the uncertainty in the system.

Let us discuss the scientific literature on DSM for smart grids. Significant studies have been conducted on DSM programs and on their role in balancing the demand and supply side. In Atzeni et al. (2012) the flow of information between users is combined with a pricing strategy to encourage the users' optimal strategies. This model, however, considers only a day ahead optimization problem while not including RES on the supply side. Carli and Dotoli (2019) present a deterministic decentralized control strategy for the scheduling of electrical energy activities of a microgrid. In this model, users are connected to a distributor and exchange renewable energy produced by distributed resources.

The uncertainty in renewable energy production makes it hard to choose the optimal scheduling strategy. Estrella et al. (2019) propose a non-cooperative model that includes a wind power source. In their model, all the possible uncertain variables are assumed to be deterministic, and a shrinking-horizon approach is used to handle their variations.

Typical DSM methods to include uncertainty in the wind power availability are based on the assumption that wind power prediction errors follow a Gaussian distribution. This approach, however, exhibits some issues due to the

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nonlinearity introduced in the wind power generation process.

The Weibull and Rayleigh distribution models are often used in the literature to characterize the wind speed historically. In Biswas et al. (2017), a Weibull distribution is used to describe the wind speed, and a nonlinear function is employed to transform it into a power probability density function for the generated power. In Aghajani et al. (2017), the use of incentive-based payment as a price offer package is suggested; the integration of wind power is made using a historical curve, and results are validated with the wind speed forecast. In Kun et al. (2018) and Ko et al. (2015) a normal distribution is used to take into account the wind speed variability; moreover, the wind speed uncertainty is partially evaluated in Iizaka et al. (2014). In Pazouki et al. (2014), a Monte Carlo simulation is employed to generate a scenario tree to predict wind power availability. Lastly, a stochastic planning approach based on a Monte Carlo method is proposed in Afshar and Shokri Gazafroudi (2007) to model the uncertain wind behaviour.

Differently from the aforementioned literature, in this paper we design a rolling-horizon DSM model that comprises a central unit that serves several users: the central unit relies on traditional energy procurement but also owns a wind power production facility. We assume that each user is selfish, namely, he selects his strategy attempting to minimize his total expense thanks to DG and DS devices. In turn, the central unit updates the energy price based on the aggregate load and the wind power production. However, decisions have to be made by users considering their local preference and the grid constraint. Given the selfish nature of users, we model the problem as a Nash game, and we compute its equilibrium via a preconditioned forward-backward algorithm [Belgioioso and Grammatico (2018)].

To take into account the wind speed uncertainty, we propose a stochastic-optimization approach. Specifically, we model the wind speed uncertainty with a Gaussian distribution, whose parameters are based on the prediction horizon and the forecasted wind speed. Then, we generate a PDF for the wind power by adopting an approximated nonlinear relation between wind speed and power. We then perform several samplings from the resulting PDF and use the sample average approximation (SAA) to generate an averaged cost function. This leads to an optimization problem that considers several different scenarios. Finally, employing real data, we investigate the advantage achieved by a single user that adopts a stochastic approach with respect to the other competitors achieving a long term strategic advantage.

2. SMART GRID MODEL

Let us consider as a starting base the smart grid model presented in Atzeni et al. (2012); this model comprehends a day-ahead optimization process between a group of active users regulated by a central unit. Estrella et al. (2019) further improve the model, including a RES source and implementing a shrinking-horizon approach on a daily base, with a time slot of one hour. On the purpose of a long-time simulation, this approach is modified in the sequel, considering a receding-horizon strategy where the control and the prediction horizon have a fixed length H and are moving forward (e.g., straddling two days eventually).

2.1 Demand-side model

We consider the model presented in Atzeni et al. (2012) where a group of demand-side users $\mathcal{D}$ are divided into two subsets of passive $\mathcal{P}$ and $\mathcal{N}$ active users or prosumers, where $\mathcal{D} = \mathcal{P} \cup \mathcal{N}$, with cardinality D, P and N respectively. Each user has a per-slot energy consumption $e_i(h)$, for $h \in \mathcal{H} = \{1, ..., H\}$, where H is the control horizon. However, passive users are merely energy consumers and follow the traditional demand side approach. Instead, active users $i \in \mathcal{N}$ participate in the grid optimization process. We assume that each active user is connected bidirectionally to the power grid, and to a communication infrastructure that enables two-way communication between the user's smart meter and the central unit. The set of possible strategies depends on the equipment owned by active users; they can be either a dispatchable energy producer $i \in \mathcal{G}$ with a per-slot energy generation $g_i(h)$, or an energy storage user $i \in \mathcal{S}$ with per-slot energy storage $s_i(h)$. Note that in general $\mathcal{G} \cap \mathcal{S} \neq \emptyset$ and $\mathcal{G} \cup \mathcal{S} = \mathcal{N}$. The per-slot load profile $l_i(h)$ is hence:

$$l_i(h) = \begin{cases} e_i(h) & \text{if } i \in \mathcal{P} \\ e_i(h) - g_i(h) + s_i(h) & \text{if } i \in \mathcal{N} \end{cases} \tag{1}$$

which expresses the energy flow between the grid and user $i \in \mathcal{D}$. For every user $i \in \mathcal{D}$ let us then define the energy consumption scheduling vector $\mathbf{e}_i = (e_i(h))_{h=1}^H$ and the energy load scheduling vector $\boldsymbol{l}_i = (l_i(h))_{h=1}^H$.

2.2 Energy generation model

Here we consider the group of dispatchable energy producers (e.g., internal combustion engines or gas turbines) that can either produce energy to satisfy their energy demand, charge their battery, or sell to the grid during the peak time slots when the price is higher. These energy prosumers are subject to variable production costs and are interested in optimizing their energy achievable production strategies to obtain the highest advantage. We define an energy production cost function $W_i(g_i(h))$, comprehending the variable production costs for producing the energy $g_i(h)$ at time slot h. We assume a first constraint in the generation capacity; besides a non-negative minimum, a maximum per-slot energy generation can take into account the devices' technological restrictions:

$$0 \leq g_i(h) \leq g_i^{\max}(h), \quad \forall h \in \mathcal{H}, \forall i \in \mathcal{N}\,. \tag{2}$$

Furthermore, we include an additional constraint on the maximum daily production to prevent the device's overuse. Since the control horizon can straddle on two days, it is useful to define $\mathcal{H}_1$ as the set that comprehends the remaining time slots of the first day, and consequently $\mathcal{H}_2$. Naming $\bar{h}$ the last time slot of the first day, we can write more formally:

$$\mathcal{H}_1 = \left[1, \bar{h}\right] \cap \mathbb{Z} \tag{3}$$

$$\mathcal{H}_2 = \left[\bar{h} + 1, H\right] \cap \mathbb{Z} \tag{4}$$

$$\sum_{h \in \mathcal{H}_1} g_i(h) \leq \psi_i^{\text{left}}, \quad \forall i \in \mathcal{G} \tag{5}$$

$$\sum_{h\in\mathcal{H}_2} g_i(h) \le \psi_i^{\max}, \quad \forall i \in \mathcal{G} \tag{6}$$

where ψ_i^{left} is the remaining generable energy through the time slot in the first day, whose value depends on the previously implemented strategy. Let us define for each producer $i \in \mathcal{G}$ an energy production scheduling vector $\mathbf{g}_i = (g_i(h))_{h=1}^H$ and a set of feasible strategies:

$$\mathbf{\Omega}_{\mathbf{g}_i} = \left\{\mathbf{g}_i \in \mathbb{R}^H_{\ge 0} \mid (2),(5),(6) \text{ hold}\right\}, \quad \forall i \in \mathcal{G}. \tag{7}$$

2.3 Energy storage model

We assume that the storage devices used by the prosumers are lithium-ion batteries, characterized by charging efficiency, discharging efficiency, leakage rate, capacity, and maximum charging rate, as modeled next. For each user $i \in \mathcal{S}$, it is useful to define: the charging and discharging efficiencies $0 < \beta_i^{(+)} \le 1$ and $\beta_i^{(-)} \ge 1$, the leakage rate $0 < \alpha_i \le 1$ and the maximum hourly charging rate $s_i^{\max}$. The battery charge level is a function of the previous charge level decreased by the leakage rate and incremented or reduced by the hourly energy storage profile altered through the charging and discharging efficiency. As in Atzeni et al. (2012), it is possible to define it as:

$$q_i(h) = \alpha_i q_i(h-1) + \beta_i^\top s_i(h), \quad \forall i \in \mathcal{S} \tag{8}$$

where $s_i(h) = [s_i^{(+)}(h), s_i^{(-)}(h)]^\top$ are respectively the battery charging and discharging profile at time slot h, and $\beta_i = [\beta_i^{(+)}, -\beta_i^{(-)}]^\top$ the charging and discharging inefficiency. The battery charge level should be comprised in $[0, \zeta_i]$, where ζ_i is the battery capacity; however, it is convenient to include a minimum charge level $q_i^{\min}$ for each time slot to prevent damage on the devices:

$$q_i^{\min} - \alpha_i q_i(h-1) \le \beta_i^\top s_i(h) \le \zeta_i - \alpha_i q_i(h-1), \quad \forall i \in \mathcal{S}. \tag{9}$$

Furthermore, the maximum charging and discharging rate should be respected:

$$-s_i^{\max} \le \beta_i^\top s_i(h) \le s_i^{\max}, \quad \forall h \in \mathcal{H}, \forall i \in \mathcal{S}\,. \tag{10}$$

As for the distributed generation, it is possible to define for each producer $i \in \mathcal{S}$ an energy storage scheduling vector $\mathbf{s}_i = ((s_i^{(+)}(h))_{h=1}^H, (s_i^{(-)}(h))_{h=1}^H)$ and a set of feasible strategies:

$$\mathbf{\Omega}_{\mathbf{s}_i} = \left\{\mathbf{s}_i \in \mathbb{R}^{2H}_{\ge 0} \mid (9),(10) \text{ hold}\right\}, \quad \forall i \in \mathcal{S}. \tag{11}$$

2.4 Energy cost and pricing market

Let us denote by $\omega(h)$ the power generated at time slot h by the renewable source connected with the grid and $L(h)$ the total aggregate grid load for time slot h. We consider a function C_h to model the cost per unit of energy as:

$$C_h(L(h) - \omega(h)) = K_h(L(h) - \omega(h)), \quad \forall h \in H \tag{12}$$

where $K_h > 0$ are time varying price coefficients. A global constraint is introduced on the aggregate per-slot energy load:

$$L_{\min} \le L(h) \le L_{\max}, \quad \forall h \in H \tag{13}$$

where the upper bound is the maximum aggregate load that the grid can afford before a blackout occurs and the lower bound prevents the turning off of some primary power plants. We remark that (13) is a constraint that couples together all the prosumers' decisions. It is important to stress that in this model we assume that the renewable energy production influences only the energy price and not the global grid constraints.

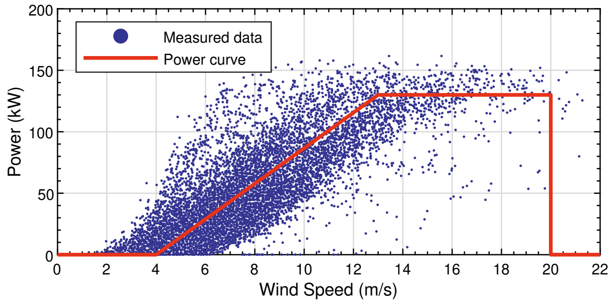

Fig. 1. Power curve for a wind turbine with: $v_{\text{in}} = 4$ m/s, $v_{\text{rated}} = 13$ m/s, $v_{\text{out}} = 20$ m/s and $\omega_{\text{rated}} = 130$kW.

3. WIND POWER CHARACTERIZATION

In this paper we consider the availability of wind power generation for the smart grid, under uncertainty on the wind speed, assuming that the relation between the power and the wind speed is nonlinear [Aghajani et al. (2017)].

More in detail, the relation between wind speed and the generated power in a conventional wind turbine is a function of many factors. In the turbine, if the wind speed is lower than a so-called cut-in wind speed value v_{in}, the wind cannot overcome the mechanical friction in the system. After this threshold, the output power increases with the wind speed, following the Betz law until the so-called rated wind speed v_{rated}. The turbines are equipped with a braking system, that after this value, keeps the output equal to the turbine rated power ω_{rated}. Once the wind speed exceeds a so-called safety cut-off value v_{out}, the turbine stops producing energy and is secured by completely stopping the rotor. An approximation for this relation can be thus defined as follows:

$$\omega(v) = \begin{cases} \omega_{\text{rated}} \dfrac{v - v_{\text{in}}}{v_{\text{rated}} - v_{\text{in}}} & \text{if} \quad v\in[v_{\text{in}}, v_{\text{rated}}] \\ 0 & \text{if} \quad v\in[0,v_{\text{in}}]\cup[v_{\text{out}},\infty) \\ \omega_{\text{rated}} & \text{if} \quad v\in[v_{\text{rated}}, v_{\text{out}}]. \end{cases} \tag{14}$$

Specifically, we assume that the wind speed is a random variable with normal distribution [Kun et al. (2018)], where μ is the forecasted speed, and σ is the standard deviation that we assign at the wind speed forecast. Due to the turbine performance curve, the probability that $\omega = 0$ can be calculated considering the cumulative probability that $v < v_{\text{in}}$ and that $v > v_{\text{out}}$, is:

$$\mathbb{P}[\omega = 0] = F_v(v_{\text{in}}) + (1 - F_v(v_{\text{out}})). \tag{15}$$

Moreover, the probability that $\omega = \omega_{\text{rated}}$ can be calculated considering the cumulative probability that $v_{\text{rated}} \le v \le v_{\text{out}}$:

$$\mathbb{P}[\omega = \omega_{\text{rated}}] = F_v(v_{\text{out}}) - F_v(v_{\text{rated}}). \tag{16}$$

In the interval $v_{\text{in}} \le v \le v_{\text{rated}}$ the probability density function of the generated power ω can be obtained by a one to one mapping with the wind speed [Schinazi (2012)]. By denoting $A = \left(\frac{v_{\text{rated}}}{v_{\text{in}}}\right) - 1$ and $C = \frac{v_{\text{in}}}{\omega_{\text{rated}}}$, we have:

$$f(\omega) = \frac{C}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\left(\left(1 + \frac{A\,\omega}{\omega_{\text{rated}}}\right) v_{\text{in}} - \mu\right)^2}{2\sigma^2}\right). \tag{17}$$

Summarizing, the PDF for the generated power, given a forecasted wind speed value μ and a standard deviation

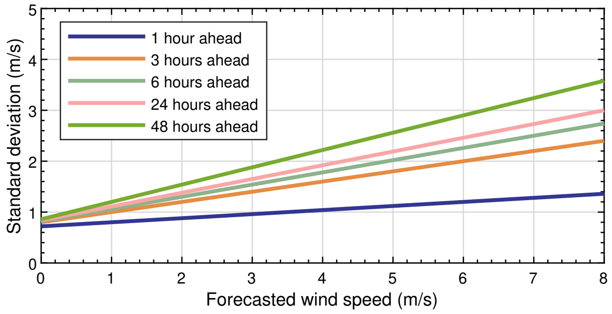

Fig. 2. Correlation between the standard deviation of the forecast error, the wind speed value, and the forecast horizon [Iizaka et al. (2014)].

σ, is defined in (18). The proposed approach is useful in particular when the predicted wind speed values are extreme; here, the expected value is not sufficient to describe the real situations. As shown in prior research, the uncertainty is strongly related to the forecasted wind speed value, and naturally, to the forecasting horizon [Iizaka et al. (2014)]. In Fig. 2 we report these relations estimated through historical data.

4. NON-COOPERATIVE AGGREGATIVE GAME WITH UNCERTAINTY

4.1 Aggregative game formulation

Assuming that users act selfishly, we can model the optimization problem as a non-cooperative game where every user chooses its generation and storage strategy, with the aim of reducing his own cost function, given the current wind forecast and the aggregate load. Thus, for every active user $i \in \mathcal{N}$ let us define the strategy vector at the generic time slot h as $x_i(h) = (g_i(h), s_i(h))^\top$, the strategy scheduling vector $\boldsymbol{x}_i = (\boldsymbol{g}_i, \boldsymbol{s}_i)^\top$ and a local feasible strategy set that takes into account the user preferences:

$$\Omega_{\boldsymbol{x}_i} = \left\{ \boldsymbol{x}_i \in \mathbb{R}^{3H} \mid \mathbf{g}_i \in \Omega_{\boldsymbol{g}_i}, \mathbf{s}_i \in \Omega_{\boldsymbol{s}_i} \right\}, \quad \forall i \in \mathcal{N}. \quad (19)$$

By using the pricing model in (12), the cost function of user $i \in \mathcal{N}$ over the time horizon is defined as:

$$J_i(x_i, \boldsymbol{x}_{-i}, \boldsymbol{\omega}) = \sum_{h=1}^{H} K_h(\boldsymbol{l}_{-i}(h) + e_i(h) + \delta^\top x_i(h) - \omega(h)) \cdot \\ \cdot (e_i(h) + \delta^\top x_i(h)) + \sum_{h=1}^{H} W_i(\delta_g^T x_i(h)). \quad (20)$$

where $l_{-i}(h) = \sum_{i \in \mathcal{D} \setminus \{i\}} l_i(h)$, $\delta = (-1, 1, -1)^\top$ and $\delta_g = (1, 0, 0)^\top$.

While each user acts selfishly to satisfy the local preferences, the global grid constraint in (13) should be respected. Let us define $L_P(h) = \sum_{i \in \mathcal{P}} l_i(h)$, $b = [L_{\max} - L_P(1), ..., L_{\max} - L_P(H), L_P(1) - L_{\min}, ..., L_P(H) - L_{\min}]^\top$, $\Delta = [-I_H, I_H, -I_H]$ and $A = [A_1, ..., A_N] = \mathbf{1}_N \otimes \Delta$, where A_i represents how the ith user is involved in the coupling constraint. The collective global feasible set $\mathcal{X}$ can be defined as the intersection of the collection of the local feasible sets of all the prosumers $\boldsymbol{\Omega} = \prod_{i=1}^{N} \Omega_{\boldsymbol{x}_i}$ and the coupling constraint as follows:

$$\mathcal{X} = \boldsymbol{\Omega} \cap \left\{ \boldsymbol{y} \in \mathbb{R}^{3HN} \mid A\boldsymbol{y} - b \le \mathbf{0}_{2H} \right\}. \quad (21)$$

Overall N inter-dependent optimization problems are then obtained as in the following:

$$\forall i \in \mathcal{N}: \begin{cases} \underset{x_i \in \mathbb{R}^{3H}}{\operatorname{argmin}} \, J(x_i, \boldsymbol{x}_{-i}, \boldsymbol{\omega}) \\ \text{ s.t. } (x_i, \boldsymbol{x}_{-i}) \in \mathcal{X}. \end{cases} \quad (22)$$

This formulation defines a generalized Nash equilibrium (GNE) problem, which we can denote in compact form as $\mathcal{G} = (\mathcal{X}, \mathbf{J})$, with $\mathcal{X}$ as in (21) being the collective global feasible set and $\mathbf{J} = (J_i(x_i, \boldsymbol{x}_{-i}, \boldsymbol{\omega}))_{i=1}^{|\mathcal{N}|}$ being the cost function.

In our model, a central unit broadcasts the wind speed forecast; however, each user can choose how to handle the uncertainty related to this variable. The easiest way to approach the problem is to consider the power availability as a deterministic variable by employing the expected value for the wind speed forecast $v(h)$. Using (14), it is possible to calculate the expected power production $\bar{\omega}(h)$. Defining $\bar{\boldsymbol{\omega}} = (\bar{\omega}(h))_{h=1}^{H}$ the power expected production vector, we can write the $i \in \mathcal{N}$ active user's cost function in case of the expected value approach:

$$\bar{J}_i(x_i, \boldsymbol{x}_{-i}, \bar{\boldsymbol{\omega}}) = \sum_{h=1}^{H} K_h(\boldsymbol{l}_{-i}(h) + e_i(h) + \delta^T x_i(h) - \bar{\omega}(h)) \cdot \\ \cdot (e_i(h) + \delta^T x_i(h)) + \sum_{h=1}^{H} W_i(\delta_g^T x_i(h)). \quad (23)$$

Furthermore, as in Shapiro (2008), we can assume the cost function of the ith user $J_i(x_i, \boldsymbol{x}_{-i}, \boldsymbol{\omega})$ is a stochastic variable itself, with expected value:

$$\mathbb{J}_i(x_i, \boldsymbol{x}_{-i}) = \mathbb{E}_\omega[J_i(x_i, \boldsymbol{x}_{-i})]. \quad (24)$$

While calculating (24) is a major challenge, we can use the Sample Average Approximation (SAA) approach presented in Kim et al. (2015) to obtain an approximation of the "true" problem by using M independent and identically distributed (IID) random samples of the stochastic variable, $\omega^1, ..., \omega^M$, from a prior known probability den-

$$f(\omega) = \begin{cases} \left(\dfrac{1}{\sqrt{2\pi\sigma^2}} \left(\displaystyle\int_{-\infty}^{v_{\text{in}}} \exp\left(-\frac{(v-\mu)^2}{2\sigma^2} \right) dv + \int_{v_{\text{out}}}^{+\infty} \exp\left(-\frac{(v-\mu)^2}{2\sigma^2} \right) dv \right) \right) \delta(\omega) & \text{if} \quad \omega = 0 \\ \dfrac{C}{\sqrt{2\pi\sigma^2}} \exp\left(-\dfrac{\left(\left(1 + \frac{A\,\omega}{\omega_{\text{r}}}\right) v_{\text{in}} - \mu \right)^2}{2\sigma^2} \right) & \text{if} \quad \omega = \omega_{\text{r}} \left(\dfrac{v - v_{\text{in}}}{v_{\text{r}} - v_{\text{in}}} \right) \\ \left(\dfrac{1}{\sqrt{2\pi\sigma^2}} \displaystyle\int_{v_{\text{r}}}^{v_{\text{out}}} \exp\left(-\frac{(v-\mu)^2}{2\sigma^2} \right) dv \right) \delta(\omega) & \text{if} \quad \omega = \omega_{\text{r}} \\ 0 & \text{if} \quad \omega > \omega_{\text{out}} \end{cases} \quad (18)$$

Algorithm 1 Preconditioned forward backward (pFB)

1: **for** $i = 1, ..., N$ **do**
2: $\quad x_i^+ = \mathrm{proj}_{\Omega_{x_i}}\left[x_i - \gamma\left(\nabla_{\boldsymbol{x}_i} J_i(x_i, \boldsymbol{x}_{-i}, \boldsymbol{\omega}) + A_i^\top \lambda\right)\right]$
3: **end for**
4: $\lambda^+ = \mathrm{proj}_{\mathbb{R}^{2H}_{\geq 0}}\left[\lambda + \gamma\left(2A\boldsymbol{x}^+ - A\boldsymbol{x} - b\right)\right]$

sity function. In our case, the samplings can be obtained through the wind power PDF in (18). Consequently, the averaged problem has a cost function:

$$\hat{J}_i^M(x_i, \boldsymbol{x}_{-i}, \boldsymbol{\omega}^M) = \frac{1}{M}\sum_{m=1}^{M} J_i^m(x_i, \boldsymbol{x}_{-i}, \omega^m)\,. \tag{25}$$

4.2 Semi-decentralized equilibrium computation

To compute the general Nash equilibrium of the aggregated game $\mathscr{G} = (\mathcal{X}, \mathbf{f})$, here we propose to use the preconditioned forward backward (pFB) algorithm presented by Belgioioso and Grammatico (2018).

Let us divide the active users set $\mathcal{N}$ into $\mathcal{N}_D$ and $\mathcal{N}_S$, where $\mathcal{N} = \mathcal{N}_D \cup \mathcal{N}_S$ and $\mathcal{N}_D \cap \mathcal{N}_S = \emptyset$. The first set comprehends the users that employ the deterministic cost function; these users rely on the expected value given by the central unit. Conversely, the second set contains the user utilizing the SAA cost function (25): these users believe that the implementation of a stochastic strategy would lead to an economic benefit.

The algorithm has a semi-decentralized structure: the central unit broadcasts the price coefficients $\boldsymbol{K} = (K_h)_{h=1}^H$, the wind speed forecast $\boldsymbol{v} = (v(h))_{h=1}^H$, the incentive signal λ and aggregate load L. At each iteration, every user attempts to reduce his cost function, given the current value for the grid coefficient, the current wind forecast, and the aggregative load on the grid, taking a gradient step of length γ, projected into the feasible local set. The central unit then updates at every step the incentive λ based on the expected coupling constraint violation.

4.3 Rolling-horizon DSM algorithm

Next, we embed the pFB algorithm into a rolling horizon implementation. At each time slot, a new aggregated game is defined with the new wind speed forecast and the updated feasible set. Users are divided into those who employ the expected value and those who use the stochastic approximation methodology. The latter users are generating a PDF based on the forecast. Consequently, they are obtaining M IID samplings from the PDF for every time slot of the control horizon and defining an averaged cost function. The optimization problem is solved at every time slot for the whole control horizon $\mathcal{H} = \{1, ..., H\}$; however, only the first step of the solution is implemented. Lastly, the feasible global set $\mathcal{X}^+$ is updated considering the implemented strategy.

5. NUMERICAL SIMULATIONS

In this section, we provide some numerical results that illustrate the performance of the proposed approach. We evaluate the savings obtained employing the proposed stochastic approach with respect to the standard expected

Algorithm 2 Rolling-horizon pFB algorithm

1: Central unit broadcast $\boldsymbol{v}=(v(h))_{h=1}^H$ and $\boldsymbol{K}=(K_h)_{h=1}^H$
2: **for** $i = 1, ..., N$ **do**
3: $\quad$ **if** $i \in \mathcal{N}_{\mathcal{D}}$ **then**
4: $\quad\quad$ Calculate $\bar{\boldsymbol{\omega}} = (\bar{\omega}(h))_{h=1}^H$ with (14)
5: $\quad\quad$ Set $\bar{J}_i(x_i, \boldsymbol{x}_{-i}, \bar{\boldsymbol{\omega}})$ in (23)
6: $\quad$ **else if** $i \in \mathcal{N}_{\mathcal{S}}$ **then**
7: $\quad\quad$ Generate a wind power PDF $\forall h \in \mathcal{H}$
8: $\quad\quad$ Generate M IID sampling from (18), $\forall h \in \mathcal{H}$
9: $\quad\quad$ Set $\hat{J}_i^M(x_i, \boldsymbol{x}_{-i}, \boldsymbol{\omega}^M)$ in (25)
10: $\quad$ **end if**
11: **end for**
12: Consider the new game $\mathscr{G} = (\mathcal{X}, \mathbf{J})$
13: Compute GNE of $\mathscr{G}$, $\boldsymbol{x}^*$, with Algorithm 1
14: Apply $x^*\{1\}$ and discard $x^*\{2, ..., k + H\}$
15: Update the collective feasible set $\mathcal{X}^+$

Table 1. Recap for the different strategies adopted by the active users in the simulations.

Code	Test user strategy	Other users'
Sim00	Expected	29 Expected
Sim01	Prescient	29 Expected
Sim02	Stochastic	29 Expected
Sim03	Stochastic	14 Stochastic & 15 Expected
Sim04	Stochastic	29 Stochastic

value approach. We use real wind speed and the generated power data obtained from an experimental wind farm [sotaventogalicia.com]. We collect the wind speed forecasts data from a weather provider through an API that every hour registers the forecasts for the next 24 hours [openweathermap.org]. To each forecast, we assign a standard deviation value, statistically estimated by employing historical forecasts, and measured values. This, as in the literature, is a function of the forecast wind speed value and the forecast horizon.

We test the approach using a smart grid with 100 users: 70 passive and 30 active ones. In Tab. 1 we show a recap for the different strategies adopted by the active users in the simulations. We consider a scheduling horizon of 30 days and prediction horizon of 24 hours. The prosumers have a consumption curve with a daily average of $\sum e_i(h) = 14$ kWh, with a peak during the evening hours. The cost coefficient K_h is 0.20 euro/kWh for the daily hours (from 8:00 to 24:00) and 0.15 euro/kWh at night (00:00 to 08:00), and the global constraints are proportional with the maximum and minimum aggregate load. The generation cost is assumed linear with a coefficient $\eta_i = 0.04$ euro/kWh, the maximum hourly generation is $g_i^{\max}(h) = 0.4$ kWh and the maximum daily generation $\psi_i^{\max} = 20 g_i^{\max}(h)$. The storage devices are: a lithium-ion battery, with a leakage rate $\alpha = 0.90$, charging and discharging inefficiency equal to $\beta_i^{(+)} = 0.99$ and $\beta_i^{(-)} = 1.01$, $c_i = 4$kW the battery capacity, $s_i^{\max} = 0.5c_i$ KWh the maximum charging rate and $c_i^{\text{initial}} = q_i^{\min} = 0.25c_i$. We assume that all active users own one generation and one storage device and that all the data related to these devices are equal.

We analyze the results from the point of view of a single active user, named "test user". We calculate a reference cost for this test user with Sim00, where all active users employ the expected value strategy.

In Fig. 3(a), we employ a Boxplot graph to show the savings that the test user obtains adopting a different strategy to handle the uncertainty in wind power. In particular, Sim01 shows the maximum savings that the test user can reach when perfect knowledge of the future is available; this is an upper bound for any stochastic technique. In Sim02, we show how much this user can save on average, adopting the proposed stochastic strategy when all its competitors employ the standard expected value strategy.

In Fig. 3(b), we show the savings that the test user obtains employing the stochastic strategy with respect to the reference cost. In particular, we show how the solution changes when also other players adopt the proposed stochastic approach. Simulations show that when other players employ a stochastic strategy (Sim03 and Sim04), the savings that the test user gets decreases.

Furthermore, we calculate the total grid cost (so-called welfare) when all the users employ a stochastic strategy in Sim04. The simulations show that the total grid cost decreases (approximately 5.5% less) with respect to the case when all the active users employ the standard expected value strategy in Sim00.

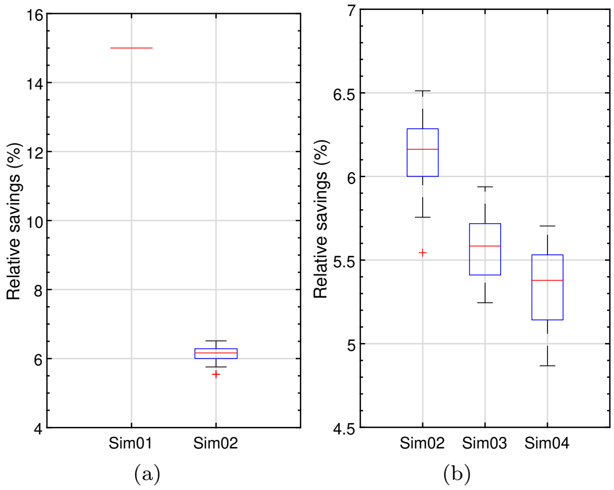

Fig. 3. Boxplot of the test user savings with respect to the expected value strategy (a) in presence of other stochastic users (b).

6. CONCLUSION

In this work, we introduce an approach for demand side management of smart grids with distributed generation and storage that allows considering uncertainty in the wind speed forecast and reducing the individual cost of prosumers, taking into account different wind power scenarios. The approximation results show that, in a realistic situation, the use of the proposed approach increases the advantage for individual users. The approach is able to reduce the individual cost and improve the grid welfare, also in the case when all the users implement the stochastic strategy. Our future works will focus on designing a more precise approach to perform a stochastic optimization and we will consider the renewable energy source in the global grid constraint by employing penalty functions or chance constraints techniques.

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