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String Stability of Homogenous Vehicle Platoons Based on Cooperative Extended State Observers ⋆

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Abstract: We study platoon control of homogeneous vehicles with linear third-order longitudinal dynamics with the constant time headway (CTH) policy. In order to ensure string stability with a small time headway, a distributed control law based on extended state observers is proposed. The controller of each follower vehicle only depends on its own velocity, acceleration, inter-vehicle distance and velocity difference with respect to its immediate predecessor. First, a dynamic model based on velocity differences between adjacent vehicles is established. Then cooperative extended state observers are designed to estimate the acceleration differences between adjacent vehicles, based on which distributed cooperative controllers are designed. By analyzing the transfer function of inter-vehicle distances errors, the sufficient conditions to ensure string stability are presented. It is shown that for any given positive time headway, the parameters of distributed cooperative state observers and controllers can be properly designed so that the inter-vehicle distance errors are not amplified during the backward propagation along the platoon. The effectiveness of the cooperative control law is demonstrated by simulations.

Keywords: Vehicle platoon, Constant time headway, Extended state observer, String stability.

1. INTRODUCTION

Vehicle platoon has potential to improve road utilization rate and reduce fuel consumption (Hedrick et al., 1994; Zheng et al., 2016), therefore, it has been a hot research topic since the 1980s. String stability is an important aspect of vehicle platoon control, which means that the inter-vehicle distance errors are not amplified during the backward propagation along the platoon. It is well known that the spacing policy, which specifies the expected values of inter-vehicle distances, greatly affects string stability of the vehicle platoon system. The constant spacing policy (Guo & Yue, 2011; Dunbar & Caveney, 2012; Zheng et al., 2016) and the constant time headway policy (Naus et al., 2010; Darbha et al., 2017; Ploeg et al., 2014; Klinge & Middleton, 2009) are frequently used in the literature. The constant spacing policy means that the expected inter-vehicle distance is fixed, which leads to high road utilization rate. The constant time headway spacing policy refers to that the expected inter-vehicle distance is proportional to the vehicle velocity, which limits the achievable traffic flow density, but agrees with the driving characteristics of drivers. Rajamani and Zhu (2002) and Naus et al. (2010) revealed that a controller only relies on information obtained by on-board sensors can guarantee string stability with the constant time headway spacing policy if sufficiently large time headways are selected. However, it is well known that large time headways lead to low road utilization rate.

It is meaningful to study how to guarantee string stability of a vehicle platoon system with small time headways. Rajamani and Zhu (2002) designed a control law which takes advantage of the accelerations of preceding vehicles to reduce the lower bound of the time headway required. Zhou and Peng (2004) analyzed the requirements for the time headway of several kinds of control laws, among which given proper control parameters, the sliding mode control law by using the accelerations of preceding vehicles can ensure string stability for any given positve time headway. Naus et al. (2010) studied the platoon control of heterogeneous vehicles, taking advantage of the accelerations of preceding vehicles to design the feedforward controller, where both theoretical analysis and experiment showed the importance of the accelerations of preceding vehicles to string stability. Darbha et al. (2017) indicated the benefit of using the accelerations of the preceding vehicles to the reduction of the time headway. Al-Jhayyish and Schmidt (2018) analyzed the effect of feedforward strategies with diverse information, such as the accelerations and the control inputs of preceding vehicles, on the stability of heterogeneous vehicle platoon systems, which also verified that the feedforward of the accelerations of preceding

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vehicles benefits string stability. The above literatures all assumed that the accelerations of the preceding vehicle can be obtained by the wireless communication network accurately, nevertheless, accurate communication doesn't exist in practical applications. Wireless communications will bring about problems concerning time-delays (Di Bernardo et al., 2015), packet losses (Jia & Ngoduy, 2016), quantization errors (Guo & Yue, 2011). Therefore, the design of a cooperative control law with a small time headway, which is independent of the inter-vehicle wireless communication network, is of especially significance for practical applications. Ploeg et al. (2015) and Wen and Guo (2019) proposed methods to estimate the acceleration differences between adjacent vehicles, respectively. The control laws in Ploeg et al. (2015) and Wen and Guo (2019) are indeed independent of wireless communication networks, however, there are no quantitative relationship between string stability and control parameters. Moreover, the time headways in Ploeg et al. (2015) and Wen and Guo (2019) cannot be arbitrarily small.

Motived by the above, we design a distributed cooperative control law for each follower vehicle only using its own velocity, acceleration, inter-vehicle distance and velocity difference with respect to its immediate predecessor, all of which can be measured by on-board sensors. We consider the third-order linear vehicle dynamics and adopt the constant time headway spacing policy. Firstly, we formulate the models of the velocity differences between adjacent vehicles, based on which we design distributed cooperative extended state observers to estimate the acceleration differences between adjacent vehicles. Then based on the estimate of the acceleration differences between adjacent vehicles, we design distributed cooperative control laws for follower vehicles. We give the range of control parameters to guarantee string stability. We show that for any given positive time headway, one can design the control parameters properly to guarantee string stability. It should be point out that our method for estimating the acceleration differences between adjacent vehicles is based on distributed cooperative extended state observers, which is totally different from those in Ploeg et al. (2015) and Wen and Guo (2019), and we give an explicit range of the control parameters quantitatively related to the system parameters to ensure string stability for any given positive time headway.

In Section 2, we present the vehicle platoon model and the control objective. In Section 3, firstly, we formulate the dynamic models of velocity differences between every adjacent vehicles. Then, we design distributed cooperative extended state observers to estimate the acceleration differences between adjacent vehicles. Finally, we propose distributed cooperative control laws for follower vehicles. In Section 4, we give the sufficient conditions to ensure string stability. In Section 5, we demonstrate the proposed control law via simulations. We give some conclusions in Section 6.

2. PROBLEM FORMULATION

2.1 Vehicle platoon model

Suppose a homogeneous vehicle platoon contains $N+1$ vehicles. Each vehicle is considered as a particle. The position, velocity and acceleration of the ith vehicle at time t are denoted by $p_i(t)$, $v_i(t)$ and $a_i(t)$, $i=0,1,...,N$, respectively. We specify that $i=0$ indicates the leader vehicle and $i=1,2,...,N$ indicate the follower vehicles.

We consider each vehicle with the following third-order linear longitudinal dynamics which is commonly used in the literature (Rajamani & Zhu, 2002, Zheng et al., 2016, Wen & Guo, 2019).

$$\begin{cases} \dot{p}_i(t) = v_i(t), \\ \dot{v}_i(t) = a_i(t), & i=0,1,...,N, \\ \dot{a}_i(t) = -\dfrac{1}{\tau}a_i(t) + \dfrac{1}{\tau}u_i(t), \end{cases} \tag{1}$$

where the constant τ is the inertial delay of vehicle longitudinal dynamics, $u_0(t)$ is the control input of the leader vehicle and $u_i(t)$ is the control input of the ith follower vehicle, which needs to be designed, $i=1,2,...,N$. Here, as a preliminary study, we consider homogenous vehicle dynamics, where the inertial delay is assumed to be the same for each vehicle.

The constant time headway spacing policy is adopted. Denote the inter-vehicle distance error

$$e_i(t) = p_{i-1}(t) - p_i(t) - r - hv_i(t),\ i=1,2,...,N, \tag{2}$$

where the constants r and h are the standstill distance and the time headway, respectively.

2.2 Control objective

The control objective is to design $u_i(t), i=1,2,\ldots,N$, so that string stability is satisfied, *i.e.*

$$\sup_{\omega\in\mathbb{R}} \left| \frac{\mathscr{E}_i(j\omega)}{\mathscr{E}_{i-1}(j\omega)} \right| \le 1,\ i=2,...,N,$$

where $\mathscr{E}_i(s)$ is the Laplace transform of $e_i(t)$.

3. COOPERATIVE CONTROL LAW BASED ON EXTENDED STATE OBSERVERS

We suppose that each follower vehicle relies on the on-broad sensors to measure its own velocity, acceleration, the inter-vehicle distance and velocity difference with respect to its immediate predecessor. It is well known that using the accelerations of the preceding vehicles is beneficial to string stability (Rajamani & Zhu, 2002; Zhou & Peng, 2004; Naus et al., 2010). Due to the limitation of sensors, the accelerations of the preceding vehicles cannot obtained by the sensors of follower vehicles, so we need to design observers to estimate them. The concept of extended state observer (ESO) was first put forward by Han (1995), whose core idea is to expand the unmodeled dynamics into new state and by a new state equation, an observer is designed, so all the states can be estimated by the observer. The extended state observer proposed by Han (1995) is nonlinear, which are difficult in parameter tuning and stability analysis. In order to overcome the above problems, Gao (2003) put forward linear extended state observers, which simplify the parameter tuning. Linear extended state observer are also beneficial for stability analysis. In this paper, linear extended state observers are designed to estimate the acceleration differences between adjacent vehicles.

From (1), the models of the velocity difference between adjacent vehicles are given by

$$\begin{cases} \dot{v}_{d,i}(t) = a_{d,i}(t), \\ \dot{a}_{d,i}(t) = q_i(t) - \frac{1}{\tau}u_i(t), \qquad i = 1,2,...,N, \\ \dot{q}_i(t) = w_i(t), \end{cases} \tag{3}$$

where

$$\begin{aligned} v_{d,i}(t) =& v_{i-1}(t) - v_i(t), \\ a_{d,i}(t) =& a_{i-1}(t) - a_i(t), \\ q_i(t) =& \frac{-a_{i-1}(t) + u_{i-1}(t) + a_i(t)}{\tau}, \\ w_i(t) =& \frac{a_{i-1}(t) - u_{i-1}(t) - a_i(t) + u_i(t) + \tau \dot{u}_{i-1}(t)}{\tau^2}. \end{aligned} \tag{4}$$

Here, $q_i(t)$ is the unmodeled dynamics, which contains the control input and the acceleration of $i-1$th vehicle that cannot be measured directly by the ith vehicle. Linear extended state observers for follower vehicles are given by,

$$\begin{cases} \dot{z}_{1,i}(t) = z_{2,i}(t) + \beta_1(v_{d,i}(t) - z_{1,i}(t)), \\ \dot{z}_{2,i}(t) = z_{3,i}(t) + \beta_2(v_{d,i}(t) - z_{1,i}(t)) - \frac{1}{\tau}u_i(t), \\ \dot{z}_{3,i}(t) = \beta_3(v_{d,i}(t) - z_{1,i}(t)), \\ \qquad i = 1,2,\ldots,N, \end{cases} \tag{5}$$

where $z_{1,i}(t)$, $z_{2,i}(t)$, $z_{3,i}(t)$ are the estimate of $a_{d,i}(t)$, $v_{d,i}(t)$, $q_i(t)$, respectively. The constants $\beta_1 > 0$, $\beta_2 > 0$ and $\beta_3 > 0$ are the observer gains to be designed.

The output $z_{2,i}(t)$ of the ESO (5) is the estimate of the acceleration difference between adjacent vehicles. By the $z_{2,i}(t)$ and $a_i(t)$, the estimate of the acceleration of $i-1$th vehicle $z_{2,i}(t) + a_i(t)$ can be obtained. Combining with the inter-vehicle distance error, the acceleration of the ith vehicle and the velocity difference between adjacent vehicles, the control law of ith vehicle is designed as

$$\begin{aligned} u_i(t) =& k_p e_i(t) + k_v(v_{d,i}(t) - h a_i(t)) \\ &+ k_a(z_{2,i}(t) + a_i(t)), \ i = 1,2,...,N, \end{aligned} \tag{6}$$

where $k_p > 0$, $k_v > 0$, $k_a > 0$ are the control parameters to be designed. The controller (6) consists of two parts. The first part $k_p e_i(t) + k_v(v_{d,i}(t) - h a_i(t))$ is the feedback item, which consists of the inter-vehicle distance error between the adjacent vehicles and its differential; the second part $k_a(z_{2,i}(t) + a_i(t))$ is the feedforward item, which consists of the estimate of the acceleration of the $i-1$th vehicle. It is worth mentioning that the design of the extended state observer (5) and the control law (6) only use the information obtained by on-board sensors of follower vehicles.

4. STRING STABILITY ANALYSIS

In this section, we give an explicit range of the control parameters quantitatively to ensure string stability of the vehicle platoon system.

Theorem 1. Consider the system (1) under the control law (5) and (6). Let $k_p = \mu_p k$, $k_v = \mu_v k$, $k_a = \mu_a k$, $\beta_1 = 3\omega_o$, $\beta_2 = 3\omega_o^2$, $\beta_3 = \omega_o^3$, where k, μ_p, μ_v, μ_a, ω_o are positive parameters to be designed. For any given $h > 0$, if $\mu_a > 0$, $\mu_p > 0$,

$$\mu_v > \max\left\{\frac{\sqrt{3}\mu_a}{h}, \frac{(\tau - 2h)\mu_p}{2}\right\}, \tag{7}$$

$$\omega_o > \max\left\{\theta_\mu, \theta_\lambda, \frac{16\mu_a}{3\tau h^2 \mu_p}\right\}, \tag{8}$$

$$k \geq \max\left\{\theta_1, \theta_2, \theta_3, \theta_4, \frac{\gamma_5}{\alpha_5}\right\}, \tag{9}$$

where

$$\theta_\mu = \begin{cases} \sqrt{\frac{\lambda_4}{h^2\mu_v^2 - \mu_a^2}}, & \text{if } \lambda_4 \geq 0, \\ 0, & \text{if } \lambda_4 < 0, \end{cases} \tag{10}$$

$$\theta_\lambda = \begin{cases} \frac{-\lambda_2 + \sqrt{\lambda_2^2 - 4\lambda_1\lambda_3}}{2\lambda_1}, & \text{if } \lambda_2^2 - 4\lambda_1\lambda_3 \geq 0, \\ 0, & \text{if } \lambda_2^2 - 4\lambda_1\lambda_3 < 0, \end{cases} \tag{11}$$

$$\theta_i = \begin{cases} \frac{-\gamma_i + \sqrt{\gamma_i^2 - 4\alpha_i\rho_i}}{2\alpha_i}, & \text{if } \gamma_i^2 - 4\alpha_i\rho_i \geq 0, \\ 0, & \text{if } \gamma_i^2 - 4\alpha_i\rho_i < 0, \\ & i = 1,2,3,4, \end{cases} \tag{12}$$

and

$$\begin{aligned} \lambda_1 =& 3h^2\mu_v^2 - 9\mu_a^2, \\ \lambda_2 =& \frac{16(h-\tau)\mu_a\mu_v}{\tau}, \\ \lambda_3 =& \frac{9}{\tau^2}\mu_a^2 + \frac{12h\mu_p + 12\mu_v - 6\mu_p\tau}{\tau}\mu_a + 3h^2\mu_p^2, \\ \lambda_4 =& \frac{(12\mu_a\tau - 6h\mu_a - 6\mu_a)\mu_p}{\tau}, \\ \alpha_1 =& h^2\mu_v^2, \\ \gamma_1 =& 2(h-\tau)\mu_v - 2h\tau\mu_p - 2\mu_a, \\ \rho_1 =& 3\tau^2\omega_0^2 + 1, \\ \alpha_2 =& 3h^2\mu_v^2\omega_o^2 + \frac{\mu_a^2}{\tau^2} + \frac{2h\mu_a\mu_p + 2\mu_a\mu_v}{\tau} + h^2\mu_p^2, \\ \gamma_2 =& [6(h-\tau)\mu_v - 12\mu_a - 6h\tau\mu_p]\omega_o^2 - 2\mu_p, \\ \rho_2 =& 3\tau^2\omega_0^4 + 3\omega_o^2, \\ \alpha_3 =& \lambda_1\omega_o^4 + \lambda_2\omega_o^3 + \lambda_3\omega_o^2, \\ \gamma_3 =& 6(h\mu_v + \mu_a - \tau\mu_v - h\tau\mu_p)\omega_o^4 + \frac{16\mu_a}{\tau}\omega_o^3 - 6\mu_p\omega_o^2, \\ \rho_3 =& \tau^2\omega_o^6 + 3\omega_o^4, \\ \alpha_4 =& [(h^2\mu_v^2 - \mu_a^2)\omega_o^2 - \lambda_4]\omega_o^4 + (3h^2\mu_p^2\omega_o - \frac{16\mu_a\mu_p}{\tau})\omega_o^3, \\ \gamma_4 =& 2(h\mu_v - \tau\mu_v - h\tau\mu_p)\omega_o^6 - 6\mu_p\omega_o^4, \\ \rho_4 =& \omega_o^6, \\ \alpha_5 =& (h^2\mu_p^2 + 2\mu_a\mu_p)\omega_o^6, \\ \gamma_5 =& 2\mu_p\omega_o^6, \end{aligned}$$

then $\sup_{\omega \in \mathbb{R}} \left|\frac{\mathscr{E}_i(j\omega)}{\mathscr{E}_{i-1}(j\omega)}\right| \leq 1$.

Proof. By (2) and (4), we get

$$a_i(t) = \frac{v_{d,i}(t) - \dot{e}_i(t)}{h}. \tag{13}$$

Taking the Laplace transform of (13), we have

$$\mathscr{A}_i(s) = \frac{\mathscr{V}_{d,i}(s) - s\mathscr{E}_i(s)}{h}, \tag{14}$$

where $\mathscr{A}_i(s)$ and $\mathscr{V}_{d,i}(s)$ are the Laplace transform of $a_i(t)$, $v_{d,i}(t)$, respectively.

From (1), we know

$$u_i(t) = \tau \dot{a}_i(t) + a_i(t). \tag{15}$$

This together with (13) leads to

$$u_i(t) = \tau \frac{\dot{v}_{d,i}(t) - \ddot{e}_i(t)}{h} + \frac{v_{d,i}(t) - \dot{e}_i(t)}{h}. \tag{16}$$

Taking the Laplace transform of (16), we get

$$\mathscr{U}_i(s) = \tau \frac{s\mathscr{V}_{d,i}(s) - s^2\mathscr{E}_i(s)}{h} + \frac{\mathscr{V}_{d,i}(s) - s\mathscr{E}_i(s)}{h}, \tag{17}$$

where $\mathscr{U}_i(s)$ is the Laplace transform of $u_i(t)$. Taking the Laplace transform of (5), we have

$$\begin{cases} s\mathscr{Z}_{1,i}(s) = \mathscr{Z}_{2,i}(s) + \beta_1(\mathscr{V}_{d,i}(s) - \mathscr{Z}_{1,i}(s)), \\ s\mathscr{Z}_{2,i}(s) = \mathscr{Z}_{3,i}(s) + \beta_2(\mathscr{V}_{d,i}(s) - \mathscr{Z}_{1,i}(s)) - \dfrac{1}{\tau}\mathscr{U}_i(s), \\ s\mathscr{Z}_{3,i}(s) = \beta_3(\mathscr{V}_{d,i}(s) - \mathscr{Z}_{1,i}(s)), \end{cases} \tag{18}$$

where $\mathscr{Z}_{1,i}(s)$, $\mathscr{Z}_{2,i}(s)$ and $\mathscr{Z}_{3,i}(s)$ are the Laplace transform of $z_{1,i}(t)$, $z_{2,i}(t)$ and $z_{3,i}(t)$, respectively. Substituting (17) into (18), we obtain

$$\begin{aligned} \mathscr{Z}_{2,i}(s) = & \frac{-s^3 + \left(\frac{h\beta_2\tau - \beta_1\tau - 1}{\tau}\right)s^2 + \left(\frac{h\beta_3\tau - \beta_1}{\tau}\right)s}{h\left(s^3 + \beta_1 s^2 + \beta_2 s + \beta_3\right)}\mathscr{V}_{d,i}(s) \\ & + \frac{s^4 + \left(\frac{\beta_1\tau+1}{\tau}\right)s^3 + \frac{\beta_1}{\tau}s^2}{h(s^3 + \beta_1 s^2 + \beta_2 s + \beta_3)}\mathscr{E}_i(s). \end{aligned} \tag{19}$$

By (6), (13) and (15), we get

$$\begin{aligned} \tau\dot{a}_i(t) + a_i(t) = & k_p e_i(t) + k_v \dot{e}_i(t) \\ & + k_a(z_{2,i}(t) + a_i(t)). \end{aligned} \tag{20}$$

Taking the Laplace transform of (20), we have

$$\begin{aligned} \tau s\mathscr{A}_i(s) + \mathscr{A}_i(s) = & k_p\mathscr{E}_i(s) + k_v s\mathscr{E}_i(s) + k_a\mathscr{A}_i(s) \\ & + k_a\mathscr{Z}_{2,i}(s). \end{aligned} \tag{21}$$

Denote $H(s) = \frac{\mathscr{V}_{d,i}(s)}{\mathscr{E}_i(s)}$. By (14), (19) and (21), we get

$$H(s) = \frac{n_5 s^5 + n_4 s^4 + n_3 s^3 + n_2 s^2 + n_1 s + n_0}{d_4 s^4 + d_3 s^3 + d_2 s^2 + d_1 s + d_0}, \tag{22}$$

where

$$\begin{aligned} n_0 &= -k_p h\beta_3, \\ n_1 &= -k_p h\beta_2 - (1 - k_a + k_v h)\beta_3, \\ n_2 &= -(k_p h + \frac{k_a}{\tau})\beta_1 - (1 - k_a + k_v h)\beta_2 - \tau\beta_3, \\ n_3 &= -k_p h - \frac{k_a}{\tau} - (1 + k_v h)\beta_1 - \tau\beta_2, \\ n_4 &= -1 - k_v h - \tau\beta_1, \\ n_5 &= -\tau, \\ d_0 &= (k_a - 1)\beta_3, \\ d_1 &= -\frac{k_a\beta_1}{\tau} - (1 - k_a)\beta_2 - (\tau - k_a h)\beta_3, \\ d_2 &= -\beta_1 - (\tau - k_a h)\beta_2 - \frac{k_a}{\tau}, \\ d_3 &= -\tau\beta_1 - 1, \\ d_4 &= -\tau. \end{aligned}$$

By (2) and (4), we get

$$\dot{e}_{i-1}(t) - \dot{e}_i(t) = v_{d,i-1}(t) - v_{d,i}(t) - h\dot{v}_{d,i}(t). \tag{23}$$

Denote $G_e(s) = \frac{\mathscr{E}_i(s)}{\mathscr{E}_{i-1}(s)}$. Taking the Laplace transform of (23), we have

$$s\mathscr{E}_{i-1}(t) - s\mathscr{E}_i(t) = \mathscr{V}_{d,i-1}(t) - \mathscr{V}_{d,i}(t) - hs\mathscr{V}_{d,i}(t).$$

This together with $\mathscr{V}_{d,i}(s) = H(s)\mathscr{E}_i(s)$ leads to

$$G_e(s) = \frac{s - H(s)}{s - (hs + 1)H(s)}.$$

This together with (22) leads to

$$\begin{aligned} G_e(s) = & (\overline{n}_4 s^4 + \overline{n}_3 s^3 + \overline{n}_2 s^2 + \overline{n}_1 s + \overline{n}_0)/(\overline{d}_6 s^6 \\ & + \overline{d}_5 s^5 + \overline{d}_4 s^4 + \overline{d}_3 s^3 + \overline{d}_2 s^2 + \overline{d}_1 s + \overline{d}_0), \end{aligned} \tag{24}$$

where

$$\begin{aligned} \overline{n}_0 &= k_p\beta_3, \\ \overline{n}_1 &= k_p\beta_2 + k_v\beta_3, \\ \overline{n}_2 &= k_p\beta_1 + k_v\beta_2 + k_a\beta_3, \\ \overline{n}_3 &= k_v\beta_1 + k_a\beta_2 + k_p, \\ \overline{n}_4 &= k_v, \\ \overline{d}_0 &= k_p\beta_3, \\ \overline{d}_1 &= k_p\beta_2 + (k_p h + k_v)\beta_3, \\ \overline{d}_2 &= k_p\beta_1 + (k_p h + k_v)\beta_2 + (1 + k_v h)\beta_3, \\ \overline{d}_3 &= (k_p h + \frac{k_a}{\tau} + k_v)\beta_1 + (1 + k_v h)\beta_2 + \tau\beta_3 + k_p, \\ \overline{d}_4 &= (1 + k_v h)\beta_1 + \tau\beta_2 + k_p h + \frac{k_a}{\tau} + k_v, \\ \overline{d}_5 &= \tau\beta_1 + k_v h + 1, \\ \overline{d}_6 &= \tau. \end{aligned}$$

Substituting $s = j\omega$ into (24), we get

$$G_e(j\omega) = \frac{x_n(\omega) + y_n(\omega)j}{x_d(\omega) + y_d(\omega)j}, \tag{25}$$

where $x_n(\omega) = \overline{n}_0 - \overline{n}_2\omega^2 + \overline{n}_4\omega^4$, $y_n(\omega) = \overline{n}_1\omega - \overline{n}_3\omega^3$, $x_d(\omega) = \overline{d}_0 - \overline{d}_2\omega^2 + \overline{d}_4\omega^4 - \overline{d}_6\omega^6$, $y_d(\omega) = \overline{d}_1\omega - \overline{d}_3\omega^3 + \overline{d}_5\omega^5$.

By $\mu_p > 0$ and $\mu_a > 0$, we know $\alpha_5 > 0$ and $\gamma_5 > 0$. From (9), we obtain $k \geq \frac{\gamma_5}{\alpha_5}$. This together with $\alpha_5 > 0$ and $\gamma_5 > 0$ leads to

$$\alpha_5 k^2 - \gamma_5 k \geq 0. \tag{26}$$

From (7) and $\mu_a > 0$, we know $\mu_v > \frac{\mu_a}{h}$. By (8), we know $\omega_o > \theta_\mu$. This together with $\mu_v > \frac{\mu_a}{h}$ leads to $[(h^2\mu_v^2 - \mu_a^2)\omega_o^2 - \lambda_4]\omega_o^4 > 0$. By (8), we know $\omega_o > \frac{16\mu_a}{3\tau h^2\mu_p}$. This together with $\mu_p > 0$ and $\mu_a > 0$ leads to $(3h^2\mu_p^2\omega_o - \frac{16\mu_a\mu_p}{\tau})\omega_o^3 > 0$. So we get $\alpha_4 > 0$. From (9), we know $k \geq \theta_4$. This together with $\alpha_4 > 0$ and $\rho_4 > 0$ leads to

$$\alpha_4 k^2 + \gamma_4 k + \rho_4 \geq 0. \tag{27}$$

By (7), we know $\mu_v > \frac{\sqrt{3}\mu_a}{h}$. This together with $\mu_a > 0$ leads to $\lambda_1 > 0$. By (7), we get $\mu_v > \frac{(\tau - 2h)\mu_p}{2}$. This leads to $12h\mu_p + 12\mu_v - 6\mu_p\tau > 0$. This together with $\mu_a > 0$ leads to $\lambda_3 > 0$. By (8), we know $\omega_o > \theta_\lambda$. This together with $\lambda_1 > 0$ and $\lambda_3 > 0$ leads to $\alpha_3 > 0$. From (9), we know $k \geq \theta_3$. This together with $\alpha_3 > 0$ and $\rho_3 > 0$ leads to

$$\alpha_3 k^2 + \gamma_3 k + \rho_3 \geq 0. \tag{28}$$

By (7) and $\mu_a > 0$, we know $\mu_v > 0$. This together with $\mu_p > 0$ and $\mu_a > 0$ leads to $\alpha_2 > 0$. From (9), we obtain $k \geq \theta_2$. This together with $\alpha_2 > 0$ and $\rho_2 > 0$ leads to

$$\alpha_2 k^2 + \gamma_2 k + \rho_2 \geq 0. \tag{29}$$

From (9), we obtain $k \geq \theta_1$. This together with $\alpha_1 > 0$ and $\rho_1 > 0$ leads to

$$\alpha_1 k^2 + \gamma_1 k + \rho_1 \geq 0. \tag{30}$$

By (26)-(30), we know

$$\begin{aligned}&(\alpha_5 k^2 - \gamma_5 k)\omega^2 + (\alpha_4 k^2 + \gamma_4 k + \rho_4)\omega^4 \\ &+ (\alpha_3 k^2 + \gamma_3 k + \rho_3)\omega^6 + (\alpha_2 k^2 + \gamma_2 k + \rho_2)\omega^8 \\ &+ (\alpha_1 k^2 + \gamma_1 k + \rho_1)\omega^{10} + \tau^2\omega^{12} \geq 0, \ \forall\, \omega \in \mathbb{R}.\end{aligned} \tag{31}$$

Through calculation, we get

$$\begin{cases} \alpha_5 k^2 - \gamma_5 k = 2\overline{n}_0\overline{n}_2 + \overline{d}_1^2 - 2\overline{d}_0\overline{d}_2 - \overline{n}_1^2, \\ \alpha_4 k^2 + \gamma_4 k + \rho_4 = 2\overline{n}_1\overline{n}_3 + 2\overline{d}_0\overline{d}_4 + \overline{d}_2^2 - \overline{n}_2^2 \\ \qquad -2\overline{n}_0\overline{n}_4 - 2\overline{d}_1\overline{d}_3, \\ \alpha_3 k^2 + \gamma_3 k + \rho_3 = 2\overline{n}_2\overline{n}_4 + 2\overline{d}_1\overline{d}_5 + \overline{d}_3^2 - \overline{n}_3^2 \\ \qquad -2\overline{d}_0\overline{d}_6 - 2\overline{d}_2\overline{d}_4, \\ \alpha_2 k^2 + \gamma_2 k + \rho_2 = \overline{d}_4^2 + 2\overline{d}_2\overline{d}_6 - \overline{n}_4^2 - 2\overline{d}_3\overline{d}_5, \\ \alpha_1 k^2 + \gamma_1 k + \rho_1 = \overline{d}_5^2 - 2\overline{d}_4\overline{d}_6. \end{cases}$$

This together with (31) leads to

$$\begin{aligned}&(2\overline{n}_0\overline{n}_2 + \overline{d}_1^2 - 2\overline{d}_0\overline{d}_2 - \overline{n}_1^2)\omega^2 \\ &+ (2\overline{n}_1\overline{n}_3 + 2\overline{d}_0\overline{d}_4 + \overline{d}_2^2 - \overline{n}_2^2 - 2\overline{n}_0\overline{n}_4 - 2\overline{d}_1\overline{d}_3)\omega^4 \\ &+ (2\overline{n}_2\overline{n}_4 + 2\overline{d}_1\overline{d}_5 + \overline{d}_3^2 - \overline{n}_3^2 - 2\overline{d}_0\overline{d}_6 - 2\overline{d}_2\overline{d}_4)\omega^6 \\ &+ (\overline{d}_4^2 + 2\overline{d}_2\overline{d}_6 - \overline{n}_4^2 - 2\overline{d}_3\overline{d}_5)\omega^8 \\ &+ (\overline{d}_5^2 - 2\overline{d}_4\overline{d}_6)\omega^{10} + \overline{d}_6^2\omega^{12} \geq 0, \ \forall\, \omega \in \mathbb{R}.\end{aligned} \tag{32}$$

Through calculation, we know

$$\begin{aligned}&x_d^2(\omega) + y_d^2(\omega) - x_n^2(\omega) - y_n^2(\omega) \\ &= \overline{d}_0^2 - \overline{n}_0^2 + (2\overline{n}_0\overline{n}_2 + \overline{d}_1^2 - 2\overline{d}_0\overline{d}_2 - \overline{n}_1^2)\omega^2 \\ &\quad + (2\overline{n}_1\overline{n}_3 + 2\overline{d}_0\overline{d}_4 + \overline{d}_2^2 - \overline{n}_2^2 - 2\overline{n}_0\overline{n}_4 - 2\overline{d}_1\overline{d}_3)\omega^4 \\ &\quad + (2\overline{n}_2\overline{n}_4 + 2\overline{d}_1\overline{d}_5 + \overline{d}_3^2 - \overline{n}_3^2 - 2\overline{d}_0\overline{d}_6 - 2\overline{d}_2\overline{d}_4)\omega^6 \\ &\quad + (\overline{d}_4^2 + 2\overline{d}_2\overline{d}_6 - \overline{n}_4^2 - 2\overline{d}_3\overline{d}_5)\omega^8 \\ &\quad + (\overline{d}_5^2 - 2\overline{d}_4\overline{d}_6)\omega^{10} + \overline{d}_6^2\omega^{12}.\end{aligned}$$

This together with $\overline{d}_0 = \overline{n}_0$ and (32) leads to

$$x_d^2(\omega) + y_d^2(\omega) - x_n^2(\omega) - y_n^2(\omega) \geq 0, \ \forall\, \omega \in \mathbb{R}. \tag{33}$$

By (33), we know

$$\frac{x_n^2(\omega) + y_n^2(\omega)}{x_d^2(\omega) + y_d^2(\omega)} \leq 1, \ \forall\, \omega \in \mathbb{R}. \tag{34}$$

Through calculation, we obtain

$$\left|\frac{x_n(\omega) + y_n(\omega)j}{x_d(\omega) + y_d(\omega)j}\right| = \frac{\sqrt{x_n^2(\omega) + y_n^2(\omega)}}{\sqrt{x_d^2(\omega) + y_d^2(\omega)}}.$$

This together with (34) leads to

$$\left|\frac{x_n(\omega) + y_n(\omega)j}{x_d(\omega) + y_d(\omega)j}\right| \leq 1, \ \forall\, \omega \in \mathbb{R}. \tag{35}$$

From (25) and (35), we know $|G_e(j\omega)| \leq 1$ for any $\omega \in \mathbb{R}$. That is $\left|\frac{\mathscr{E}_i(j\omega)}{\mathscr{E}_{i-1}(j\omega)}\right| \leq 1$ for any $\omega \in \mathbb{R}$. ■

Remark 1. From Theorem 1, we know that for any given positive time headway, string stability can be guaranteed by properly designed control parameters. Note that in Zhou and Peng (2004) and Rajamani and Zhu (2002), string stability can also be ensured for any positive time headway, however the wireless communication networks are needed. Compared with Zhou and Peng (2004) and Rajamani and Zhu (2002), the control law (5) and (6) only makes use of the information obtained by on-board sensors.

5. NUMERICAL SIMULATIONS

Suppose that there are 1 leader vehicle and 9 follower vehicles in the platoon. The initial velocities are given by $v_i(0) = 30\,m/s$, $i = 0, 1, ..., 9$. The initial accelerations are given by $a_i(0) = 0\,m/s^2$, $i = 0, 1, ..., 9$. The initial position are taken as $p_0(0) = 108\,m$, $p_1(0) = 96\,m$, $p_2(0) = 84\,m$, $p_3(0) = 72\,m$, $p_4(0) = 60\,m$, $p_5(0) = 48\,m$, $p_6(0) = 36\,m$, $p_7(0) = 24\,m$, $p_8(0) = 12\,m$, $p_9(0) = 0\,m$.

We assume $\tau = 0.25$ and $r = 3\,m$. The $u_0(t)$ is given by

$$u_0(t) = \begin{cases} -1, & 0 < t \leq 4, \\ 0, & 4 < t \leq 10, \\ 0.5, & 10 < t \leq 16, \\ 0, & t > 16. \end{cases}$$

Let $h = 0.3$. By Theorem 1, we choose $\mu_p = 0.008$, $\mu_v = 0.05$, $\mu_a = 0.0015$, through (10), (11) and (12), we get $\theta_1 = 0$, $\theta_2 = 0$, $\theta_3 = 0$, $\theta_4 = 0$, $\frac{\gamma_5}{\alpha_5} = 537.6$, $\theta_\lambda = 0$, $\theta_\mu = 0$, $\frac{16\mu_a}{3\tau h^2 \mu_p} = 44.4$. From (8) and (9), we choose $\omega_o = 50$, $k = 800$, thus, we get $k_p = 6.4$, $k_v = 40$, $k_a = 1.2$, $\beta_1 = 150$, $\beta_2 = 7500$ and $\beta_3 = 3.75 \times 10^5$. In practical applications, the velocity differences between adjacent vehicles $v_{d,i}(t)$, $i = 1, 2, ..., N$, measured by on-board sensors are usually corrupted by random noises. In the numerical simulations, we implement the sampled-data version of the control law (5) and (6) with $v_{d,i}(k\sigma)$ replaced by $v_{d,i}(k\sigma) + \delta_i(k\sigma)$, where $\sigma = 0.005s$ is the sampling period and $\{\delta_i(k\sigma), k = 0, 1, ...\}$ is a sequence of random variables with the Gaussian distribution $N(0, 10^{-4})$.

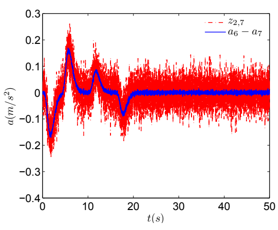

Fig. 1. (a)

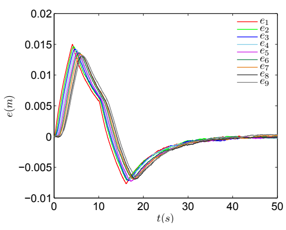

Fig. 1. (b)

Fig. 1. Vehicle platoon under control law (5) and (6) with $h = 0.3$, $k_p = 6.4$, $k_v = 40$, $k_a = 1.2$, $\beta_1 = 150$, $\beta_2 = 7500$ and $\beta_3 = 3.75 \times 10^5$. (a) The actual and the estimated acceleration differences between the 6th and 7th follower vehicle. (b) The evolution of inter-vehicle distance errors.

The actual and the estimated acceleration differences between the 6th and 7th follower vehicle are shown in Fig. 1.(a). The evolution of inter-vehicle distance errors are shown in Fig. 1.(b). From Fig. 1, it is shown that although the output of ESO $z_{2,i}(t)$, $i = 1, 2, ..., N$, are corrupted by random noises, the inter-vehicle distance errors can still converge to a small neighborhood of zero and they are not amplified in the backward propagation along the platoon.

In fact, the result of Theorem 1 is conservative, so the parameters of the ESO, β_1, β_2, β_3 can be selected to be more smaller. For example, we choose $\omega_o = 15$, then we get $\beta_1 = 45$, $\beta_2 = 675$, $\beta_3 = 3375$. The actual and the estimated acceleration differences between the 6th and 7th vehicle and the evolution of inter-vehicle distance errors are shown in Fig. 2.(a) and Fig. 2.(b), respectively.

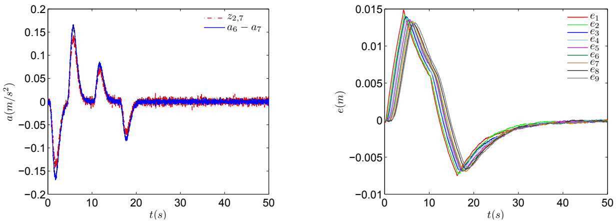

Fig. 2. (a) Fig. 2. (b)

Fig. 2. Vehicle platoon under control law (5) and (6) with $h = 0.3$,$k_p = 6.4$, $k_v = 40$, $k_a = 1.2$, $\beta_1 = 45$, $\beta_2 = 675$ and $\beta_3 = 3375$. (a) The actual and the estimated acceleration differences between the 6th and 7th follower vehicle. (b) The evolution of inter-vehicle distance errors.

From Fig. 1.(a) and Fig. 2.(a), it can be seen that the noise amplification of the ESO can be suppressed by selecting the smaller β_1, β_2 and β_3. It can be seen from Fig. 2.(b) that the string stability is still guaranteed with smaller β_1, β_2 and β_3.

6. CONCLUSION

In this paper, we have proposed distributed cooperative control laws based on extended state observers for a homogeneous vehicle platoon with the constant time headway spacing policy, which are independent of wireless communication networks. Firstly, we have designed distributed cooperative extended state observers to estimate the acceleration differences between adjacent vehicles. Then we have proposed the control law for each follower vehicle based on its own velocity, acceleration, inter-vehicle distance, velocity difference and estimated acceleration difference between adjacent vehicles. The information required by the control law in this paper can be obtained by on-board sensors. We have given the sufficient conditions to ensure string stability. It is shown that for any given positive time headway, string stability can be guaranteed by properly designed control parameters.

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