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Digital Identification and Control of Multivariable Plants Using Markov Parameters

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Abstract: The algorithm of the digital adaptive controller is offered in this article for control systems with identification of uncertain multivariable continuous-time objects. The adaptive controller includes unit of digital identification and unit of digital control. The algorithm of the identification unit allows to define the transfer matrix of the uncertain continuous-time plant. Unlike traditional methods of identification, the offered analytical algorithm uses Markov parameters not of the uncertain continuous-time plant, but virtual discrete-like plant. Markov parameters are determined by data of I/O of the uncertain continuous-time plant. The algorithm of the control unit is developed with use of control of two types: decomposition control and digital control on the output and impacts. The order and parameters of the control unit are determined by the method of the analytical design of digital control systems. Normalized standard transfer functions are used to create the transfer functions of the adaptive system according to required values of astatism order, settling time and overshot. The effectiveness of the proposed algorithm of adaptive controller is illustrated by a numerical example. The proposed approach can be applied to create control systems for agricultural, food, mining, and other industries.

Keywords: uncertain multivariable continuous-time plant, identification, Markov parameters, adaptive control, design, digital adaptive controller, standard transfer function, astatism, settling time, overshoot.

1. INTRODUCTION

Requirements to the quality of control systems increase and their area of use expands continuously. Moreover, plant models are often absent. Therefore, the problem of control of uncertain, continuous-time multivariable (MIMO) plants is considered in many papers. To solve this problem, adaptive systems are usually used. Adaptive control systems are designed without using a model of an uncertain plant often. Adaptive MIMO systems are designed based on learning control in Radac et al. (2015), Xu et al. (2014). An adaptive controller is designed according to minimum of a certain functional by Zhu and Hou (2015), and Zhu et al. (2016) use the backstepping method to design adaptive feedbacks of an output. Practically, adaptive design methods assume that the order of an uncertain plant is known and constant.

Adaptive systems with identification of models are more efficient. They can be used when the order and parameters of a plant change in steps at some points in time. Constancy intervals of the order and the parameters can be short Ning et al. (2019) or rather long Xu et al. (2014). To create adaptive systems of this type, digital algorithms for identification of the plant model and for the control design are needed.

The frequency identification method based on the Wiener-Hopf equation is one of the earliest. This method is statistical; therefore, the identification process has long duration. Currently, for identification, harmonious signals are used more often. A model of an object is created on the basis of sinusoidal test signals Gerasimov et al. (2018), Ahmed et al. (2009). In Voevoda et al. (2019) this problem is solved on the basis of the amplitude-frequency characteristic of a closed system. However, this approach can be implemented if changing parameters and ranges of their changes are known a priori. That is a significant drawback of this approach.

One of the earliest non-frequency identification methods of B. Ho was represented in Kalman et al. (1971). This method was developed based on the Markov parameters of dynamical systems Chen (1999) for the identification of discrete plants with the use of pulse transient functions. But these functions are almost impossible to obtain; therefore, the B. Ho's method is not applied practically.

Identification is often carried out based on the optimal filtering theory. Model coefficients and boundaries of their constancy are determined by solving an optimization problem with constraints in Aida-zade and Ragimov (2017). Parameters of an uncertain plant model are determined by integrating the highest derivative variables, which are measured with some noise, in Shao et al. (2017). In Genov et al. (2017), the state of a complex system is identified under ambiguity measurement conditions.

At present, the LQG method is the most common method for the analytical control systems design Chen (1999), Ghaffar and Richardson (2015). However, this method is difficult to apply when the plant order is uncertain, therefore, adaptive control systems for uncertain plants are often designed using PI or PID controllers, see He and Wang (2006).

The paper is devoted to the development of an algorithm of a digital adaptive control with identification of uncertain multivariable continuous-time (UC MIMO) plants. The algorithm of the identification has been developed using the Markov parameters of a virtual discrete-like plant (DLP). The Markov parameters are calculated based on continuous-time plant reaction to a step test action. The algorithm of the adaptive control is designed using decomposition, digital control on an output and impacts, and standard normalized transfer functions. The offered controller can be implemented using parallel digital processing of experimental data by multiprocessor controllers, see Levin (2015), Minaev (2016).

This paper is structured into seven sections. In the sections 2 and 3 the problem statement and the virtual discrete-like plant are introduced. In the section 4 the connection of the Markov parameters of any dynamic plant with its transfer function and the algorithm of the identification unit following from them are considered. The design problem of adaptive control is formulated in the section 5. The algorithm as its solution is offered in the section 6. Finally, in the section 7 a numerical example shows the performance of the proposed algorithms.

2. PROBLEM STATEMENT

We assume that an uncertain continuous-time plant is described by the equations

$$\dot{w} = Pw + Qu\,,\ \ y = Kw + Du\,, \qquad (1)$$

where $w \in R^n$ is the state vector; $u \in R^m$ is the vector of controls; $y \in R^q$ is the vector of output variables of the plant; P, Q, K, D are numeric matrices of the corresponding dimensions. The order n of the plant and the parameters of its model (1) are unknown a priori. They can suddenly change and then remain unchanged for quite a long time. At any time UC MIMO plant (1) is full, i.e. it has only a controllable and observable path between the input and the output of the plant Stengel (1994), Gaiduk (1998); but its order is $n < n_{\max}$, where $n_{\max}$ is a priory known value. All the output variables $y_i = y_i(t)$, $i = 1, 2, \ldots q$ are measured.

The task consists in the development of the digital identification algorithm to define the current model (1) and the control algorithm of the adaptive digital controller (ADC) based on the model (1). This controller must be realizable and to provide a required astatism order to reference and external disturbances; the settling time and overshoot should have values less than required, Raven (1983), Gaiduk (2012).

3. VIRTUAL DISCRETE-LIKE PLANT

Let T_{spi} be the sampling period of the variables $y_i=y_i(t)$ at the identification, then $y_k^j = y^j(kT_{spi}) = [y_{1k}^j \;\; y_{2k}^j \;\; \ldots \;\; y_{qk}^j]^T$ is the vector of discrete values of the deviations of the output variables caused by a test action $u_j(t)=[0\ldots0 \;\; u_{j0} \;\; 0\ldots0]$ at $w(0)=0$; $u_{j0}=const$, $j=[1,\ m]$. The identification algorithm is based on virtual discrete plants which have the sampling period T_{spi}. These plants are described by the equations:

$$\tilde{w}_{\nu,k+1} = \tilde{P}_\nu \tilde{w}_{\nu k} + \tilde{Q}_\nu \tilde{u}_k\,, \quad \tilde{y}_k = \tilde{K}_\nu \tilde{w}_{\nu k} + \tilde{D}\tilde{u}_k\,. \qquad (2)$$

Here $\tilde{w}_{\nu k} = [\tilde{w}_{1k} \;\; \tilde{w}_{2k} \;\; \ldots \;\; \tilde{w}_{\nu k}]^T$ and $\tilde{y}_k = [\tilde{y}_{1k} \;\; \tilde{y}_{2k} \;\; \ldots \;\; \tilde{y}_{qk}]^T$ are the deviations vectors of the state and output variables; $\tilde{u}_k = [\tilde{u}_{1k} \;\; \tilde{u}_{2k} \;\; \ldots \;\; \tilde{u}_{mk}]^T$ are the discrete controls, $k = 0, 1, 2, \ldots$; $\tilde{P}_\nu$, $\tilde{Q}_\nu$, $\tilde{K}_\nu$, $\tilde{D}$ are the numerical matrices of the plants (2), $\nu = [1, \ldots n_{\max}]$, see: Gaiduk and Plaksienko (2016).

Let $\tilde{y}_k^j = [\tilde{y}_{1k}^j \;\; \tilde{y}_{2k}^j \;\; \ldots \;\; \tilde{y}_{qk}^j]^T$ be the deviations of the output variables of the plant (2) caused by the control $\tilde{u}_k = [0\ldots0 \;\; \tilde{u}_{jk} \;\; 0\ldots0]^T$ at $\tilde{w}_{\nu 0} = 0$ and

$$\tilde{u}_{jk} = u_{j0}\,, \ \ \tilde{y}_k^j = y_k^j\,, \ \ j = [1, \ldots m]\,, \ \ k = 0, 1, 2, \ldots. \qquad (3)$$

Definition 1. The virtual plants (2) are called discrete-like plants (DLP) to the continuous plant (1) if the conditions (3) are fulfilled.

The test action $u_j(t)= u_{j0}1(t)$ can be represented as a sequence of rectangular pulses having the duration T_{spi} and amplitude u_{j0}. Therefore, the transfer functions (TF) $\Theta_{y_i u_j}(s)$ of the channels $u_j \to y_i$ of the plant (1) are connected with the TF $\tilde{\Theta}_{\tilde{y}_i \tilde{u}_j}(z, T_{spi}, \nu)$ of the channels $\tilde{u}_j \to \tilde{y}_i$ of the DLP (2) with $\nu=n$ by the special Z_T-transform:

$$\Theta_{y_i u_j}(s) = Z_T^{-1}\left\{\tilde{\Theta}_{\tilde{y}_i \tilde{u}_j}(z, T_{sp}, \tilde{n}_{ij})\right\},$$

$$\tilde{\Theta}_{\tilde{y}_i \tilde{u}_j}(z, T_{sp}, \tilde{n}_{ij}) = Z_T\left\{\Theta_{y_i u_j}(s)\right\}, \quad n_{ij} = \tilde{n}_{ij}\,, \qquad (4)$$

where $\tilde{n}_{ij}, n_{ij}$ are the degrees of the denominators of the TF $\tilde{\Theta}_{\tilde{y}_i \tilde{u}_j}(z, T_{spi}, \tilde{n}_{ij})$ and $\Theta_{y_i u_j}(s)$ respectively, $j=1,2,\ldots,m$, $i=1,2,\ldots,q$. Evidently, the matrices $\tilde{P}_\nu$, $\tilde{Q}_\nu$, $\tilde{K}_\nu$ depend on T_{spi}.

The transforms (4) are carried simply, see Moore (2013). However, if the continuous-time plant (1) has "left" poles, then the possibility of its identification based on the DLP (2) is lost if T_{spi} is too large. In this regard, the following definition is introduced. Let $\tilde{\eta}_{ij}^{\nu,0}$ be the free coefficient of the denominator of the TF $\tilde{\Theta}_{\tilde{y}_i \tilde{u}_j}(z, T_{spi}, \nu)$ of the system (2) and $\tilde{\Delta}_0$ be the computational error of the controller used.

Definition 2. If the sampling period $T_{spi}=T^{\circ}_{spi}$, and the TF $\tilde{\Theta}_{\tilde{y}_i \tilde{u}_j}(z, T^{\circ}_{spi}, \nu)$ is such that

$$\tilde{\eta}_{ij}^{\nu,0} \gg \tilde{\Delta}_0\,, \ \ \nu = [1, \ldots n_{\max}]\,, \qquad (5)$$

then the DLP (2) is correct. If $T_{spi} > T^{\circ}_{spi}$ and $\tilde{\eta}_{ij}^{\nu,0} \ll \tilde{\Delta}_0$, then the DLP (2) is incorrect.

4. MARKOV PARAMETERS AND TRANSFER FUNCTIONS

The Markov parameters $\tilde{m}_{ij}^{\varsigma}$ of each channel $\tilde{u}_j \to \tilde{y}_i$ of the DLP (2) with the order v are defined by the expressions:

$$\tilde{m}_{ij}^{0} = \tilde{d}_{ij}\,,\ \tilde{m}_{ij}^{\varsigma} = \tilde{K}_{\nu i}\tilde{P}_{\nu}^{\varsigma-1}\tilde{Q}_{\nu}^{j}\,,\quad \varsigma = 1, 2, 3, \ldots\,, \tag{6}$$

where $\tilde{d}_{ij}$ is the element of the matrix $\tilde{D}$; $\tilde{K}_{\nu i}$ is the i-row of the matrix $\tilde{K}_{\nu}$, $i=\overline{1,q}$; $\tilde{Q}_{\nu}^{j}$ is the j-column of the matrix $\tilde{Q}_{\nu}$, $j=\overline{1,m}$, see: Chen (1999), Gaiduk and Plaksienko (2016). The values $\tilde{y}_{i,k}^{j}$ of the DLP (2) are associated with the values $\tilde{u}_{jk}$ by the expressions:

$$\tilde{y}_{i0}^{j} = \tilde{K}_{\nu i}\tilde{w}_0 + d_{ij}\tilde{u}_{j0}\,,$$

$$\tilde{y}_{ik}^{j} = \tilde{K}_{\nu i}\tilde{P}_{\nu}^{k}\tilde{w}_{\nu 0} + \sum_{\varsigma=0}^{k-1}\tilde{K}_{\nu i}\tilde{P}_{\nu}^{\varsigma-1}Q_{\nu}^{j}\tilde{u}_{j\varsigma} + \tilde{d}_{ij}\tilde{u}_{jk}\,,\quad k=1,2,3,\ldots. \tag{7}$$

From the expressions (6) and (7) it follows that if $\tilde{w}_0 = 0$, and the condition (3) is satisfied, then the Markov parameters can be calculated by the formulas:

$$\tilde{m}_{ij}^{0} = y_{i0}^{j} / u_{j0}\,,\quad \tilde{m}_{ij}^{\varsigma} = y_{i\varsigma}^{j}u_{j0}^{-1} + \sum_{\rho=0}^{\varsigma-1}\tilde{m}_{ij}^{\rho}\,,\ \varsigma=1,2,3,\ldots. \tag{8}$$

As shown above, the virtual DLP (2) includes the systems of the order $v=1,2,\ldots n_{\max}$. Evidently, all these systems have the same Markov parameters $\tilde{m}_{ij}^{\varsigma}$ (8). The transfer function of the channel $\tilde{u}_j \to \tilde{y}_i$ of the system (2) with the order v is

$$\tilde{\Theta}_{\tilde{y}_i\tilde{u}_j}(z,\nu) = \tilde{P}^{-1}(z,\nu)\left\{\tilde{K}_{\nu i}\,\mathrm{Adj}(zI-\tilde{P}_{\nu})\tilde{Q}_{\nu}^{j} + \tilde{d}_{ij}\tilde{P}(z,\nu)\right\}, \tag{9}$$

$$\tilde{P}(z,\nu) = \det(zI-\tilde{P}_{\nu}) = \tilde{\eta}_{\nu,\nu}z^{\nu} + \tilde{\eta}_{\nu,\nu-1}z^{\nu-1} + \ldots + \tilde{\eta}_{\nu,1}z + \tilde{\eta}_{\nu,0}\,, \tag{10}$$

where $\tilde{\eta}_{\nu,\nu} = 1$; $\mathrm{Adj}(zI-\tilde{P}_{\nu})$ is the adjoint $v\times v$-matrix, which can be represented as follows, Gantmakher (1988, p. 88):

$$A\mathrm{dj}(zI-\tilde{P}_{\nu}) = \tilde{\eta}_{\nu,\nu}Iz^{\nu-1} + (\tilde{\eta}_{\nu,\nu}\tilde{P}_{\nu} + \tilde{\eta}_{\nu,\nu-1}I)z^{\nu-2} + \ldots$$
$$\ldots + (\tilde{\eta}_{\nu}\tilde{P}_{\nu}^{\nu-1} + \tilde{\eta}_{\nu,\nu-1}\tilde{P}_{\nu}^{\nu-2} + \ldots + \tilde{\eta}_{\nu,1}I)\,. \tag{11}$$

According to (10), (11) the coefficients of TF $\tilde{\Theta}_{\tilde{y}_i\tilde{u}_j}(z,T_{spi},\nu)$ (9) depend on $\tilde{m}_{ij}^{\varsigma}$ (8) and T_{spi}. This TF can be found from the input–output equations of the DLP (2) in the form:

$$\tilde{\Theta}_{\tilde{y}_i\tilde{u}_j}(z,T_{spi},\nu) = \frac{\tilde{\gamma}_{ij}^{\nu,0} + \tilde{\gamma}_{ij}^{\nu,1}z + \ldots + \tilde{\gamma}_{ij}^{\nu,\nu}z^{\nu}}{\tilde{\eta}_{ij}^{\nu,0} + \tilde{\eta}_{ij}^{\nu,1}z + \ldots + \tilde{\eta}_{ij}^{\nu,\nu-1}z^{\nu-1} + z^{\nu}}\,. \tag{12}$$

Two systems of algebraic equations relate the coefficients of the transfer functions (9) – (11), (12) and the parameters $\tilde{m}_{ij}^{\varsigma}$, Gaiduk and Plaksienko (2016). The first system allows us to estimate the degree $\hat{n}_{ij}$ of the denominator of the TF (12) with the corresponding TF $\Theta_{y_iu_j}(s)$ according to (4):

$$\hat{n}_{ij} = \max\left\{\nu\,\middle|\,\tilde{\delta}_{ij}^{\beta\nu} \neq 0\right\},\ \delta_{ij}^{\beta\nu} = \det\tilde{M}_{ij}^{\beta\nu}\,, \tag{13}$$

$$\tilde{M}_{ij}^{\beta\nu} = \begin{bmatrix} \tilde{m}_{ij}^{\beta+1} & \tilde{m}_{ij}^{\beta+2} & \ldots & \tilde{m}_{ij}^{\beta+\nu} \\ \tilde{m}_{ij}^{\beta+2} & \tilde{m}_{ij}^{\beta+3} & \ldots & \tilde{m}_{ij}^{\beta+\nu+1} \\ \vdots & \vdots & .\cdot\cdot & \vdots \\ \tilde{m}_{ij}^{\beta+\nu} & \tilde{m}_{ij}^{\beta+\nu+1} & \ldots & \tilde{m}_{ij}^{\beta+2\nu-1} \end{bmatrix},\ \nu=1,2,3,\ldots. \tag{14}$$

Here β is an arbitrary integer from the interval $[0, \beta_{\max}]$. Then we can calculate the coefficients $\tilde{\eta}_{ij}^{\nu,\rho}$, $\rho=[0,1,\ldots\hat{n}-1]$ of the TF (12) with $\nu=\hat{n}_{ij}$ using the formulas:

$$\tilde{\eta}_{ij}^{\hat{n}_{ij}} = -\left(\tilde{M}_{ij}^{\beta\hat{n}_{ij}}\right)^{-1}\tilde{\Upsilon}_{ij}^{\beta\hat{n}_{ij}}\,, \tag{15}$$

$$\tilde{\eta}_{ij}^{\hat{n}_{ij}} = \left[\tilde{\eta}_{ij}^{\hat{n}_{ij},0}\quad \tilde{\eta}_{ij}^{\hat{n}_{ij},1}\quad \ldots\quad \tilde{\eta}_{ij}^{\hat{n}_{ij},\hat{n}_{ij}-1}\right]^{T},$$

$$\tilde{\Upsilon}_{ij}^{\beta\hat{n}_{ij}} = \left[\tilde{m}_{ij}^{\beta+\hat{n}_{ij}+1}\quad \tilde{m}_{ij}^{\beta+\hat{n}_{ij}+2}\quad \ldots\quad \tilde{m}_{ij}^{\beta+\hat{n}_{ij}+\hat{n}_{ij}}\right]^{T}. \tag{16}$$

If the condition (5) is satisfied, the coefficients $\tilde{\gamma}_{ij}^{\nu,\rho}$, $\rho=[0,1,\ldots\hat{n}_{ij}]$ of the TF (12) with $\nu=\hat{n}_{ij}$ can be calculated as

$$\tilde{\gamma}_{ij}^{\hat{n}_{ij},\rho} = \tilde{m}_{ij}^{\hat{n}_{ij}-\rho} + \sum\nolimits_{\mu=0}^{\hat{n}_{ij}-1-\rho}\tilde{m}_{ij}^{\mu}\tilde{\eta}_{ij}^{\rho+\mu}\,,$$

$$\rho=[0,1,\ldots\hat{n}_{ij}-1]\,,\qquad \tilde{\gamma}_{ij}^{\hat{n}_{ij},\hat{n}_{ij}} = \tilde{m}_{ij}^{0}\,. \tag{17}$$

Application of the Z_T-transform (4) to (12) with the extension zoh yields the TF $\Theta_{y_iu_j}(s) = H_{ij}(s)/P_{ij}(s)$. The estimate $\hat{n}$ of the plant (1) order is determined by the degree of the estimate of its characteristic polynomial:

$$\hat{P}(s) = \mathrm{l.c.m.}\{P_{ij}(s)\}\,,\quad \hat{n} = \deg\hat{P}(s)\,. \tag{18}$$

The transfer matrix estimate of the plant (1) is determined using the expression

$$\hat{\Theta}_{yu}(s) = \hat{P}^{-1}(s)\begin{bmatrix} H_{11}(s)\hat{Q}_{11}(s) & \ldots & H_{1m}(s)\hat{Q}_{1m}(s) \\ \vdots & \ddots & \vdots \\ H_{q1}(s)\hat{Q}_{q1}(s) & \ldots & H_{qm}(s)\hat{Q}_{qm}(s) \end{bmatrix}, \tag{19}$$

where $\hat{Q}_{y_iu_j}(s) = \hat{P}(s)/P_{ij}(s)$.

The identification algorithm using the expressions above allows you to identify UC MIMO continuous plants. Information processing, when separate channels are identified, can be performed in parallel. It is advisable to implement the identification algorithm based on the expressions (3) - (19) using computing tools oriented on parallel information processing, see: Levin (2015), Minaev (2016). The proposed DLP and expressions (4), (5) ensure the high efficiency of this identification method using the Markov parameters.

5. ADAPTIVE CONTROL DESIGN

The adaptive control system with identification of the UC MIMO plant (1) with $m = q$ has been designed using the decomposition control, which forms q independent channels (plants) channels with single output, proposed in Gaiduk (2012). In this case, the control units of ADC of the system for each this plant are created considering the influence of other controls u_j as disturbances. We consider analytical design of the control unit algorithm on the base of control on output and impacts, see Gaiduk and Plaksienko (2016).

Let T_{spc} be the sampling period of the digital control unit and T_{spc}>>T_{spi}. The matrix $\bar{\Theta}_{yu}(z)$ is the result of the direct Z_T-transform (4) with $T = T_{spc}$ of the q×q-transfer matrix $\hat{\Theta}_{yu}(s)$ (19). Application of the transformation, which corresponds to the decomposition control, to the transfer matrix $\bar{\Theta}_{yu}(z)$ gives q independent plants. Suppose each such plant is described by the equation:

$$P(z)y(z) = H(z)u(z) + F(z)f(z) \tag{20}$$

where $y(z)$, $u(z)$ and $f(z)$ are z-images of the controlled variable, the control and the external non-measurable disturbance; $P(z)$, $H(z)$, $F(z)$ are some polynomials. The coefficients of the polynomials in (20) have known values; and $P(z)$ and $H(z)$ do not have common factor. The digital control unit of ADC is described by the equation:

$$\Delta(z)u(z) = \Gamma(z)r(z) - \Lambda(z)y(z) \tag{21}$$

where $r(z)$ is the z-image of the reference signal $r = r(t)$ of control system of plant (20), see Gaiduk and Plaksienko (2018); $\Delta(z)$, $\Gamma(z)$, $\Lambda(z)$ are required polynomials whose degrees are such that

$$\mu_{dc} \geq \mu_{dc}^* \tag{22}$$

$$\mu_{dc} = \min\{r_{dc} - \gamma,\ r_{dc} - l,\ 0\},$$

$$r_{dc} = \deg \Delta(z)\ ,\ \gamma = \deg \Gamma(z)\ ,\ l = \deg \Lambda(z)\ .$$

Usually $\mu_{dc}^* = 1$ or $\mu_{dc}^* = \tau_{del} / T_{spc}$ where τ_{del} is a time delay. The system (20), (21) must have the order of astatism $\zeta_r^* = 1$ to the reference $r = r(t)$ and $\zeta_r^* = 1$ to the disturbance $f = f(t)$; settling time less than t_{st}^* s; overshoot less than σ^* % and the roots stability margin β^*<1.

6. DESIGN PROBLEM SOLUTION

Let the condition on stability of the designed system be

$$\beta_{sys} = \left(1 - \max_{\rho} \left| z_\rho \right| \right) \geq \beta^*\ , \tag{23}$$

where z_ρ are the roots of the characteristic polynomial $D(z)$ of the system (20), (21). This condition defines a region Ω on the complex z-plane, i.e. an area of permissible location of the roots of the polynomial $D(z)$. The polynomial $P(z)$ from (20) is factorized as $P(z) = P_\Omega(z) P_{\bar{\Omega}}(z)$, where the polynomial $P_\Omega(z)$ includes all the roots of the polynomial $P(z)$ which satisfy condition (23), and $P_{\bar{\Omega}}(z) = P(z) / P_\Omega(z)$. Suppose, in (20) $H(z)=h_\vartheta$ or if $H(z) = h_\vartheta \prod_{\rho=1}^{\vartheta} (z - z_\rho^H)$ then the roots z_ρ^H satisfy to the condition (23), i.e. $H(z)=H_\Omega(z)$. In this case, the polynomial $D(z) = P_\Omega(z) H_0(z) \tilde{D}(z)$, where $H_0(z) = h_\vartheta^{-1} H(z)$, and $\tilde{D}(z) = \bar{\delta}_0 + \bar{\delta}_1 z + \ldots + \bar{\delta}_{n_{\tilde{D}}} z^{n_{\tilde{D}}}$ is some polynomial.

In (20), let $P(z) = (z-1)^{n_u} P_1(z)$ and $F(z) = (z-1)^{n_f} F_1(z)$, where $P_1(1)$≠0, $F_1(1)$≠0. In this case, the system (20), (21) will have a required order of astatism, if

$$\bar{\nu} = \max\{\zeta_r^* - n_u;\ \zeta_f^* - n_f;\ 0\}\ , \tag{24}$$

$$\Delta(z) = H_0(z)(z-1)^{\bar{\nu}} \tilde{\Delta}(z)\ ,\ \Lambda(z) = P_\Omega(z) \tilde{\Lambda}(z)\ , \tag{25}$$

where $\tilde{\Delta}(z)$ and $\tilde{\Lambda}(z)$ are some polynomials with degrees:

$$\tilde{r} = r_{ac} - \vartheta - \bar{\nu}\ ,\ \tilde{l} = r_{ac} - \mu_{ac} - n_\Omega\ ;\ n_\Omega = \deg P_\Omega(z)\ .$$

The coefficients of the polynomials

$$\tilde{\Delta}(z) = \sum_{\rho=0}^{\tilde{r}} \theta_\rho z^\rho\ ,\ \tilde{\Lambda}(z) = \sum_{\rho=0}^{\tilde{l}} \lambda_\rho z^\rho \tag{26}$$

are determined by the solution of the next algebraic system:

$$\begin{bmatrix} h_\vartheta & 0 & 0 & \eta_0 & 0 & \ldots \\ 0 & h_\vartheta & 0 & \eta_1 & \eta_0 & \ldots \\ 0 & 0 & \ddots & \vdots & \eta_1 & \ddots \\ \vdots & 0 & \ddots & \eta_{\breve{n}} & \vdots & \ddots \\ 0 & \vdots & \ddots & 0 & \eta_{\breve{n}} & \ddots \\ 0 & 0 & \ldots & 0 & 0 & \ddots \end{bmatrix} \cdot \begin{bmatrix} \lambda_0 \\ \vdots \\ \lambda_{\tilde{l}} \\ \theta_0 \\ \vdots \\ \theta_{\tilde{r}} \end{bmatrix} = \begin{bmatrix} \bar{\delta}_0 \\ \bar{\delta}_1 \\ \vdots \\ \vdots \\ \bar{\delta}_{n_{\tilde{D}}-1} \\ \bar{\delta}_{n_{\tilde{D}}} \end{bmatrix}. \tag{27}$$

The matrix of system (27) has $\tilde{l}+1$ columns composed by the coefficient h_ϑ and $\tilde{r}+1$ columns composed by the coefficients η_ρ of the $\breve{P}(z) = (z-1)^{\bar{\nu}} P_{\bar{\Omega}}(z) = \eta_{\breve{n}} z^{\breve{n}} + \ldots + \eta_0$. The degrees of the polynomials $\tilde{D}(z)$ and (25), (26) are calculated as follows

$$r_{dc} = n_P + \bar{\nu} + \mu_{dc}^* - 1\ ,\quad \tilde{r} = n_P + \mu_{dc}^* - \vartheta - 1\ ,\quad \breve{n} = n_{\bar{\Omega}} + \bar{\nu}\ ,$$

$$\tilde{l} = n_{\bar{\Omega}} + \bar{\nu} - 1\ ,\quad n_{\tilde{D}} = n_P + n_{\bar{\Omega}} + \mu_{dc}^* + \bar{\nu} - \vartheta - 1\ , \tag{28}$$

where n_p=deg$P(z)$. The formulas (28) follow from solvability conditions of the system (27), the conditions (23), (24), and the equality $D(z) = P_\Omega(z) H_0(z) \tilde{D}(z)$.

The coefficients $\bar{\delta}_i$ of the polynomial $\tilde{D}(z)$ are assigned using the standard normalized transfer functions (SNTF) $\Theta_{sn}(s)$ of continuous systems, see Gaiduk (2012, p. 344-346). For this purpose, the coefficients of the required SNTF are stored in the ADC microprocessor memory. As you know, the functions $\bar{\Theta}(z)$, which are the result of the conversion $\bar{\Theta}(z) = Z_T\{\Theta(s)\}$, usually have a relative degree $\mu_{\bar{\Theta}}$ equal to zero or one. Therefore, the denominator degree of the TF $\Theta_{sn}(s)$ is taken equal to $n_\Theta = n_{\tilde{D}} - \mu_{ad}$, where $\mu_{ad} = \mu_{cp} + \mu_{dc}^* - \mu_{\bar{\Theta}}$; $\mu_{cp} = \deg P(z) - \deg H(z)$. The coefficients Δ_ρ and the settling time $t_{st,tab}$ of the suitable SNTF are selected from the microprocessor memory by using the values $\zeta_g^* = 1$, n_{tab}=n_Θ and σ<σ* %. Then the coefficients of the denominator of the sought TF $\Theta_{yr}(s)$ are calculated using the formula:

$$\delta_\rho = \Delta_\rho \omega_r^{n_\Theta - \rho}\ ,\quad \rho = [n_\Theta,\ 0]\ , \tag{29}$$

where $\omega_r = t_{tab} / (t_{st}^* - T_{dc})$.

If $\zeta_r^* = 1$, then $\Theta_{yr}(s) = \delta_0 / (s^{n_\Theta} + \delta_{n_\Theta - 1} s^{n_\Theta - 1} + \ldots + \delta_1 s + \delta_0)$. Its Z_T-transform with $T=T_{spc}$ yields $\bar{\Theta}_{yr}(z) = \zeta_0(z)/\zeta(z)$. The condition $\zeta(z) = \zeta_\Omega(z)$ must be met. The coefficients of the polynomials $\tilde{D}(z) = z^{\mu_{ad}}\zeta(z) = \bar{\delta}_{n_{\tilde{D}}} z^{n_{\tilde{D}}} + \ldots + \bar{\delta}_0$ and $\breve{P}(z) = (z-1)^{\bar{\nu}} P_{\bar{\Omega}}(z) = \eta_{\bar{n}} z^{\bar{n}} + \ldots + \eta_1 z + \eta_0$ are calculated. The coefficients $\bar{\delta}_\rho$, η_ρ and also h_ϑ are substituted into the system (27); its solution yields to the polynomials: $\tilde{\Delta}(z)$, $\tilde{\Lambda}(z)$ (26). The polynomials $\Delta(z)$ and $\Lambda(z)$ are calculated using (25); the polynomial $\Gamma(z) = h_\vartheta^{-1} P_\Omega(z)\zeta_0(z)$. Substituting these polynomials in (21), we find the equation of the control unit of the desired ADC. The calculation results are correct, if the polynomials $D_1(z) = H_0(z)P_\Omega(z)\tilde{D}(z)$ and $P(z)\Delta(z)+H(z)\Lambda(z)$ have similar coefficients Gaiduk, and Plaksienko (2018).

If the polynomial $\Delta(z) = (z-1)^{\bar{\nu}}\Delta_1(z)$, then we take the equations of the control unit (21) as

$$\Delta_1(z)\tilde{\xi}(z) = \Gamma(z)r(z) - \Lambda(z)y(z), \quad (z-1)^{\bar{\nu}} u(z) = \tilde{\xi}(z), \quad (30)$$

where $\tilde{\xi}(z)$ is a z-image of an additional variable $\tilde{\xi}_k$. We have formed the algorithm for calculating the values u_k, k = 0,1,2, ... by converting the equations (30) to the original variables u_k, $\tilde{\xi}_{k-\bar{\nu}}$, $g_{k-\bar{\nu}}$, $y_{k-\bar{\nu}}$. Expressions (20) – (30) allow to calculate parameters of the digital control unit (21) for each channel of the UC MIMO (1) automatically. The desired connections between the channels are provided by the method presented in Gaiduk (1998).

7. EXAMPLE

Assume, that the order of an uncertain plant is not more than $n_{\max}=4$; the plant has one output and two inputs: control u and disturbance $f=f(t)$. If $\beta_{\max}=2$, $T_{spi}=1$ s, then $N_m=2n_{\max}+\beta_{\max}=10$; the duration of each test $t_{tes}=N_m T_{spi}=10$ s; test action $u_{10}=u_{20}=3\cdot 1(t)$; $\tilde{\Delta}_0 = 0.01$; number $\xi=10$. The identification algorithm of ADC includes the following steps:
Step 1. First $u_1(t)$, and then $u_2(t)$ operates on the plant in steady state 3-4 times. The output variable is measured over t_{tes} with the sampling period $T_{spi}=1$ s. The averaged values $y_1^1(kT_{spi}) = y_{1k}^1$, $y_1^2(kT_{spi}) = y_{1k}^2$ and the Markov parameters, calculated with using (8), are listed in Table 1.

Step 2. Let $\tilde{m}_{11}^{\nu} = \tilde{m}_{11}^{k}$, then using (14) with $\beta=0$, $v=1,2,3,4$ we find: $\det\tilde{M}_{11}^{01} = 0.0672$, $\det\tilde{M}_{11}^{02} = 0.0027$, $\det\tilde{M}_{11}^{03} = -2.8868\cdot 10^{-5}$, $\det\tilde{M}_{11}^{04} = -8.6722\cdot 10^{-9} \approx 0$; further using (13) we find $\hat{n}_{11} = 3$.

Step 3. The vector $\tilde{\eta}_{11}^{3} = [-3.4072\cdot 10^{-10} \;\; 1.2214 \;\; -2.2214]^T$ is calculated using (15), (16) with $i=j=1$, $\hat{n}_{11}=3$, $\beta=0$. Since $3.4072\cdot 10^{-10} << 0.01$ the condition (5) is not satisfied, i.e. DLP (2) is incorrect at T_{spi} = 1s. Therefore, we go to step 7. **Step 7**. New sampling period and duration of the test are calculated: $T_{spi}=T_{spi}/\xi$ s; $t_{tes}=0.1\cdot 10=1$ s; new values of the test actions $u_{10}=15$, $u_{20}=0.5$ are selected, and then we go to step 1 again.

Table 1. Digital data with T_{spi} = 1 s

k	y_{1k}^1	$\tilde{m}_{11}^k$	y_{1k}^2	$\tilde{m}_{12}^k$
0	0	0	4.2000	1.4
1	0.2017	0.0672	5.2091	0.3364
2	0.3784	0.0589	5.8091	0,2000
3	0.6547	0.0921	6.4091	0.2000
4	1.0525	0.1326	7.0091	0,2000
5	1.5988	0.1821	7.6091	0.2000
6	2.3264	0.2425	8.2091	0.2000
7	3.2755	0.3164	8.8091	0.2000
8	4.4951	0.4065	9.4091	0.2000
9	6.0452	0.5167	10.209	0.2000
10	7.9988	0.6512	10.609	0.2000

Step 1. Repeating the actions described above gives the new rounded values listed in Table 2. Then we go to step 2 again.

Table 2. Digital data with T_{spi} = 0.1 s

k	y_{1k}^1	$100\cdot\tilde{m}_{11}^k$	y_{1k}^2	$\tilde{m}_{12}^k$
0	0	0	0.7000	1.4
1	0.5045	3.3632	0.7706	0.1413
2	0.5918	0.5821	0.7873	0.0334
3	0.6360	0.2946	0.7981	0.0215
4	0.6785	0.2837	0.8082	0.0202
5	0.7241	0.1821	0.8182	0.0200
6	0.7733	0.2425	0.8282	0.0200
7	0.8262	0.3164	0.8382	0.0200
8	0.8830	0.4065	0.8482	0.0200
9	0.9436	0.5167	0.8582	0.0200
10	1.0083	0.6512	0.0200	0.0200

Step 2. The determinants $\det\tilde{M}_{11}^{02} = 6.518\cdot 10^{-5}$, $\det\tilde{M}_{11}^{01} = 0.034$, $\det\tilde{M}_{11}^{03} = -8.561\cdot 10^{-10}$, and $\det\tilde{M}_{11}^{04} = -2.767\cdot 10^{-26}$ are calculated using (14) with $\tilde{m}_{11}^{\nu} = \tilde{m}_{11}^{k}$, $\beta=0$, $v=1,2,3,4$; the order $\hat{n}_{11}=3$ is determined using (13). Then we go to step 3. **Step 3**. The vector $\tilde{\eta}_{11}^{3}$=[−0.113 1.244 −2.131] is calculated using (15), (16) with $i=j=1$, $\hat{n}_{11}=3$, $\beta=0$. The value $\left|\tilde{\eta}_{11}^{3,0}\right| = 0.113 > \tilde{\Delta}_0 = 0,01$, i.e. condition (5) is satisfied, and DLP (2) is correct with $T_{spi}=0.1$ s; therefore, we can go to next step. **Step 4**. The factors $\tilde{\gamma}_{11}^{3,0} = 0.0324$, $\tilde{\gamma}_{11}^{3,1} = -0.0658$, $\tilde{\gamma}_{11}^{3,2} = 0.0336$, $\tilde{\gamma}_{11}^{3,3} = 0$ are calculated using (17). Then the transfer function

$$\tilde{\Theta}_{11}(z,0.1;3) = \frac{0.0336z^2 - 0.0658z + 0.0324}{z^3 - 2.131z^2 + 1.244z - 0.113}$$

can be written using (12) and we go to step 5. **Step 5**. Z_T^{-1}-transform (4) of the TF $\tilde{\Theta}_{11}(z,0.1;3)$ with $T_{spi}=0.1$ yields to

$$\hat{\Theta}_{11}(s) = \frac{0.8s^2 + 0.3s + 0.4}{s^3 + 21.8s^2 - 4.4s + 3.54\cdot 10^{-10}}, \quad (31)$$

and we go to step 6. **Step 6**. Performing step 2 – step 5 with $i=1$, $j=2$, dates of the Table 2 and T_{spi} = 0.1 s gives

$$\hat{\Theta}_{12}(s) = \frac{1.4s^2 - 27.3s - 7.8}{s^2 + 22s + 3.908 \cdot 10^{-13}}; \qquad (32)$$

we go to step 8. **Step 8**. The estimates $\hat{P}(s)$, $\hat{n}$, $\hat{Q}_{12}(s)$ and $\hat{\Theta}_{yu}(s)$ are determined by the formulas (18), (19) with $\hat{\eta}_{3,0} = \hat{\eta}_{2,0} = 0$ in (31) and (32): $\hat{P}(s) = s^3 + 21.8s^2 - 4.4s$, $\hat{n} = 3$, $\hat{Q}_{12}(s) = s - 0.2$,

$$\hat{\Theta}_{yu}(s) = \begin{bmatrix} \dfrac{0.8s^2 + 0.3s + 0.4}{s^3 + 21.8s^2 - 4.4s} \\ \dfrac{1.4s^3 - 27.31s^2 - 2.394s + 1.56}{s^3 + 21.8s^2 - 4.4s} \end{bmatrix}^T. \qquad (33)$$

Then we go to **Step 9**. Let, the control unit (21) should have sampling period T_{spc}=0.6 s and μ^*_{dc} = 1 . The control system should have $\zeta^*_r = \zeta^*_f = 1$, $\beta^* \geq 0.11$; $t^*_{st} \leq 5.6$ s; $\sigma^* \leq 5$ %; the reference $r(t)$ and output $y(t)$ are measured. The algorithm of the control unit of the ADC includes the next steps:

Step 9. The Z_T-transform (4) of the matrix $\hat{\Theta}_{yu}(s)$ (33) with T_{spc} = 0.6 s, see Moore, (2013), gives:

$$\Theta_{yu}(z) = \begin{bmatrix} \dfrac{0.05155z^2 - 0.08458z + 0.03998}{z^3 - 2.128z^2 + 1.128z - 2.087 \cdot 10^{-6}} \\ \dfrac{1.4z^3 - 5.804z^2 + 7.376z - 2.946}{z^3 - 2.128z^2 + 1.128z - 2.087 \cdot 10^{-6}} \end{bmatrix}^T. \qquad (34)$$

In (34) the factor 2.087·10-6 is replaced by zero, and the polynomials of the equation (20) are written:

$$P(z) = z^3 - 2.128z^2 + 1.128z,$$
$$H(z) = 0.05155(z^2 - 1.6407z + 0.7756),$$
$$F(z) = 1.4z^3 - 5.804z^2 + 7.376z - 2.946. \qquad (35)$$

Step 10. In this case: $|z^H_{1,2}| = 0.8807$; ϑ=2, h_ϑ=0.0516; $z^P_1 = 0$, $z^P_2 = 1.128$; $z^P_3 = 1.0$; n_p=3, n_u=1, $F(1)\neq0$, n_f=0; μ_{cp}=3-2=1; condition (23) is satisfied, since 1-0.8807≥0.11; factorization of the polynomial $P(z)$ gives: $P_\Omega(z)=z$, $P_{\bar{\Omega}}(z) = z^2 - 2.128z + 1.128$, n_Ω=1, $n_{\bar{\Omega}} = 2$; number $\bar{\nu} = \max\{1-1; 1-0; 0\} = 1$. Calculations by formulas (28) give: r_{dc}=4, $\tilde{r} = 1$, $\breve{n} = 3$, $\tilde{l} = 2$ and $n_{\tilde{D}} = 4$. Let $\mu_{\bar{\Theta}} = 1$, then μ_{ad}=1+1-1=1, n_Θ=4-1=3. **Step 11**. The coefficients Δ_ρ and time $t_{st,tab}$ are selected from the memory of the controller using values: $\zeta^*_r = 1$, n_{tab}=n_Θ=3 and σ=0<5%; Δ_0=1, Δ_1=3, Δ_2=3, Δ_3=1, $t_{st,tab}$=6.31 s. The coefficients ω_r=6.31/(5.6-0.6)≈1.262 and δ_ρ are calculated using (29): δ_3=1, δ_2=3.786, δ_1=4.7779; δ_0=2.0099. These coefficients lead to transfer function: $\Theta(s)$=2.01/(s^3+3.786s^2+4.778s+2.01). Z_T-transform of this $\Theta(s)$ with T_{spc} = 0.6 gives the polynomials: $\zeta_0(z)$=0.0415z^2+0.095z+0.0133 and $\zeta(z)$=z^3-1.407z^2+0.66z-0.103. The roots of the polynomial $\zeta(z)$ satisfy to the condition (23). Therefore, in this case $\tilde{D}(z) = z\zeta(z) = z^4 - 1.407z^3 + 0.66z^2 - 0.103z$. **Step 12**. System (27) is composed using the coefficients of the polynomial $\breve{P}(z) = z^3 - 3.128z^2 + 3.256z - 1.128$ and the numbers h_ϑ=0.0516, $\tilde{l} = 2$ and $\tilde{r} = 1$. Solution of this system and expressions (25), (26) yield to the polynomials: $\tilde{\Lambda}(z) = 54.0688z^2 - 88.8233z + 37.6594$; $\tilde{\Delta}(z) = z + 1.7211$;

$$\Delta(z) = (z-1)(z^3 + 0.0804\,z^2 - 2.0482\,z + 1.3349);$$
$$\Lambda(z) = 54.0688z^3 - 88.8233z^2 + 37.6594z;$$
$$\Gamma(z) = 0.8045z^3 + 1.8423z^2 + 0.2582z. \qquad (36)$$

Step 13. Polynomials: $D_1(z)$=z^7-3.048z^6+3.744z^5-2.277z^4+ 0.681z^3-0.08z^2, $D_2(z)$=z^7-3.048z^6+3.744z^5-2.277z^4+ 0.681z^3-0.08z^2-0.0001z. The parameters of the control unit are calculated correctly, since the coefficients of the polynomials $D_1(z)$ and $D_2(z)$ are close to each other. **Step 14**. The equations of this control unit are written according to (30) considering $\bar{\nu} = 1$ and the polynomials (36):

$$(z-1)u(z) = \xi(z),$$
$$(z^3 - 0.0804\,z^2 - 2.0482\,z + 1.3349)\xi(z) =$$
$$= (0.8045z^3 + 1.8423z^2 + 0.2582z)r(z) -$$
$$-(54.0688z^3 - 88.8233z^2 + 37.6594z)y(z). \qquad (37)$$

Step 15. The control unit algorithm of the digital controller is formed by converting the equations (37) to the origin

$$\xi_{k-1} = 0.0804\xi_{k-2} + 2.048\xi_{k-3} - 1.335\xi_{k-4} + 0.805r_{k-1} +$$
$$+1.842r_{k-2} + 0.258r_{k-3} - 54.069y_{k-1} +$$
$$+88.823y_{k-2} - 37.659y_{k-3}; \quad u_k = u_{k-1} + \xi_{k-1}, \qquad (38)$$

where k = 0, 1, 2, ….; $\xi_i = 0$, r_i=0, y_i=0, u_i=0, if i<0.

The rounded values of the coefficients are given above to be short. Let's note that the adaptive digital controller will correctly work if not less than 13-15 digits are considered in the given above expressions of its algorithm. In Fig. 1 shows the response to a step input $r(t)$ =1(t) of the designed adaptive control system (33), (38).

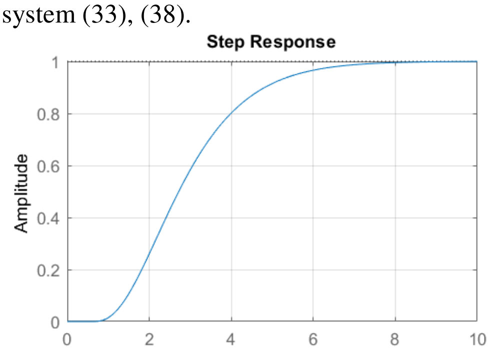

Fig. 1. System response to the input $r(t)$ = 1(t)

Obviously, the astatism orders, the overshoot and the settling time of the system are not worse than the required values. The expressions (6) – (30) allow to analytically find the ADC algorithm for each channel of the UC MIMO plant (1) if a part of the SNTF is stored in the memory of the controller.

8. CONCLUSIONS

The offered algorithm of digital controllers for systems of adaptive control with the identification of uncertain plants can be implemented with minimal prior information. This algorithm can be applied only to fully plants, the order and parameters of which jump quite rarely. The digital controllers provide required values of the quality criteria of a closed adaptive control system. The problem of design of adaptive control systems for uncertain plants is currently relevant, since it is increasingly necessary to control complex plants whose mathematical models are a priori complex or impossible to find.

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