Coalitional MPC with predicted topology transitions [⋆]

E. Masero [*], **J.M. Maestre** [*], **M. Francisco** [**], **E.F. Camacho** [*]

* *Systems Engineering and Automation Department, University of Seville, Seville, Spain, (e-mail: evamasero@us.es (E. Masero), pepemaestre@us.es (J.M. Maestre), efcamacho@us.es (E.F. Camacho))*
** *Computing and Automation Department, University of Salamanca, Salamanca, Spain, (e-mail: mfs@usal.es)*

Abstract: This paper proposes a novel hierarchical coalitional MPC technique, where transitions to the best communication topology are considered over the prediction horizon. For this reason, a new variable, called transition horizon, is added to the optimization problem to compute the optimal instant to introduce a new topology. Consequently, local controllers can anticipate topology transitions and adapt their trajectories whilst optimizing their local interests. Furthermore, stability guarantees in the closed-loop control of each coalition are provided. The benefits of this control method are shown via a simulated non-linear eight-coupled tanks plant.

Keywords: Model predictive control; Multi-agent systems; Networked control; LMI; Stability.

1. INTRODUCTION

Model predictive control (MPC) is an optimization-control method that uses a model to predict the system response over a given time horizon. At each time instant, a cost function is minimized subject to constraints to obtain the best future control actions. Additionally, MPC allows dealing with delays, dead times, and multi-variable systems. Due to these advantages, MPC has become one of the most extended control techniques in both academia and industry (Camacho and Bordons, 2007).

In some large-scale processes, the application of MPC is impossible due to issues such as computation burden and the incapacity to find a centralized model for the overall system. Consequently, a distributed approach is needed. The origin of non-centralized control methods dates back in time, e.g., Kulikowski (1970) discusses the optimization of a large-scale problem, which is divided into smaller sub-problems. Nevertheless, distributed model predictive control (DMPC) techniques have bloomed from the 90s due to massive improvements in information and communication technologies. From then on, DMPC has been applied to large-scale systems such as road traffic networks (De Oliveira and Camponogara, 2010), power grids (Alejandro et al., 2014), gas networks (Arnold et al., 2010), irrigation canals (Negenborn et al., 2009).

The key idea of DMPC is to divide the overall system into subsystems governed by local controllers or agents that have their own control goals. Agents have only partial

information about the global system and communicate to enhance performance. There are numerous DMPC schemes proposed in the literature; see (Scattolini, 2009; Maestre et al., 2014b) for more details.

Lately, the evolution of distributed control methods has led to schemes where the cooperation burden is adapted in real-time. Depending on the coupling between control tasks, the formation of coalitions, i.e., groups of controllers that communicate and cooperate to optimize their control actions, is promoted dynamically. Indeed, should there be a strong coupling between subsystems, controllers will tend to increase their degree of cooperation. Otherwise, a fully decentralized approach will be used. For example, an online cluster formation scheme based on active constraints is presented by Trodden and Richards (2009), and Núñez et al. (2013) show hierarchical controllers that are adapted dynamically to operational conditions. Furthermore, a top-down architecture where a supervisory controller decides which is the best topology is described by Núñez et al. (2015); Fele et al. (2014, 2018). Moreover, Maestre et al. (2014a, 2011) propose that communication links with a poor contribution to overall system performance should be disabled.

This work also presents a hierarchical coalitional MPC scheme where information is exchanged between two control layers: a lower layer where local controllers optimize their control and state sequences, and an upper control layer that evaluates the information received from agents and chooses the best communication topology to achieve a global objective that includes performance and cooperation costs. Once the new topology chosen by the upper layer is sent to the agents, they must implement it. Nevertheless, the instant and how the topology is implemented influence the results. Unlike previous works (Núñez et al.,

2015; Fele et al., 2018; Maestre et al., 2011) where the new topology was selected and implemented by local controllers at the same time instant, here the moment to switch the topology over the prediction horizon becomes a variable of the optimization problem, the so-called transition horizon N_t. As a result, agents can anticipate a topology change over the prediction horizon, which takes place at time $k + N_t$, providing lower control and cooperation costs and smoother system evolution.

To illustrate the benefits of this approach, an eight-coupled tanks benchmark, which is an extension of the quadruple-tank process (Johansson, 2000), has been designed. The quadruple-tank process has been used as a benchmark, e.g., to study the effects of multi-variable dead times (Shneiderman and Palmor, 2010), and to analyze different distributed MPC techniques (Alvarado et al., 2011).

The paper is organized as follows. Section 2 introduces the problem formulation; Section 3 defines the control goal and describes the algorithm proposed; Section 4 provides stability properties and the controller design procedure; and Section 5 details the eight-coupled tanks plant, the three control methods evaluated, and the results obtained from the simulation of the plant. Finally, conclusions are given in Section 6.

2. PROBLEM FORMULATION

Consider a system divided into $\mathcal{N} = \{1, 2, \ldots, N\}$ coupled subsystems, whose discrete-time dynamics are modeled as

$$x_i(k + 1) = A_{ii}x_i(k) + B_{ii}u_i(k) + w_i(k),$$
$$w_i(k) = \sum_{j \in \mathcal{M}_i} \left(A_{ij}x_j(k) + B_{ij}u_j(k) \right), \quad (1)$$

where $x_i \in \mathbb{R}^{q_i}$ and $u_i \in \mathbb{R}^{r_i}$ are, respectively, the states and inputs of each subsystem $i \in \mathcal{N}$, which are assumed to be constrained in convex sets containing the origin $\mathcal{X}_i$ and $\mathcal{U}_i$. The variable w_i indicates disturbances caused by the coupling through states x_j and inputs u_j in the set of neighbors $\mathcal{M}_i$, where $\mathcal{M}_i \triangleq \{j \in \mathcal{N} \setminus \{i\} : A_{ij} \neq 0 \vee B_{ij} \neq 0\}$. Note that w_i is bounded due to the system constraints $w_i \in \mathcal{W}_i \triangleq \bigoplus_{j \in \mathcal{M}_i} \left(A_{ij}\mathcal{X}_j \oplus B_{ij}\mathcal{U}_j \right)$. Furthermore, note that it is straightforward to extend this set to account for external disturbances.

Aggregating local states and inputs into $x_{\mathcal{N}} = (x_i)_{i \in \mathcal{N}}$ and $u_{\mathcal{N}} = (u_i)_{i \in \mathcal{N}}$, the system evolution is given by

$$x_{\mathcal{N}}(k + 1) = A_{\mathcal{N}}x_{\mathcal{N}}(k) + B_{\mathcal{N}}u_{\mathcal{N}}(k), \quad (2)$$

where $A_{\mathcal{N}} = [A_{ij}]_{i,j \in \mathcal{N}}$ and $B_{\mathcal{N}} = [B_{ij}]_{i,j \in \mathcal{N}}$. Global states and inputs are constrained in the sets $\mathcal{X}_{\mathcal{N}} = \times_{i \in \mathcal{N}} \mathcal{X}_i$ and $\mathcal{U}_{\mathcal{N}} = \times_{i \in \mathcal{N}} \mathcal{U}_i$, respectively. Note that the disturbance term $w_{\mathcal{N}}$ is not in (2) because the coupling is implicitly included in the dynamics. Moreover, external disturbances will not be considered for simplicity.

Each subsystem $i \in \mathcal{N}$ is assumed to be governed by a local controller with exclusive access to its local state x_i. To control the overall system, agents can communicate within a network described by the graph $(\mathcal{N}, \mathcal{L})$, where the set of subsystems is $\mathcal{N}$ and the set of links is $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$. Each link $l_{ij} = \{i, j\} = \{j, i\} = l_{ji} \in \mathcal{L}$ is assumed to provide a bidirectional information flow, which can be enabled or disabled according to the control scheme requirements.

At time instant k, the set of active links defines the control network topology $\Lambda \subseteq \mathcal{L}$. In total, there are $2^{|\mathcal{L}|}$ different topologies: $\mathcal{T} = \{\Lambda_0, \Lambda_1, \ldots, \Lambda_{2^{|\mathcal{L}|}-1}\}$, where $\Lambda_0 = \Lambda_{dec}$ corresponds to the decentralized topology (all communication links disabled), and $\Lambda_{2^{|\mathcal{L}|}-1} = \Lambda_{cen}$ denotes the centralized topology (full network cooperation). The disjoint set of cooperation clusters of agents, referred to as coalitions, resulting from a given topology $\Lambda \in \mathcal{T}$ is denoted by $\mathcal{N}/\Lambda = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{|\mathcal{N}/\Lambda|}\}$. Note that these sets satisfy that $\bigcup_{\mathcal{C} \in \mathcal{N}/\Lambda} \mathcal{C} = \mathcal{N}$ (Rahwan et al., 2012). The dynamics of each coalition $\mathcal{C} \in \mathcal{N}/\Lambda$ are

$$x_{\mathcal{C}}(k + 1) = A_{\mathcal{C}}x_{\mathcal{C}}(k) + B_{\mathcal{C}}u_{\mathcal{C}}(k) + w_{\mathcal{C}}(k),$$
$$w_{\mathcal{C}}(k) = \sum_{j \in \bigcup_{i \in \mathcal{C}} \mathcal{M}_i \setminus \mathcal{C}} \left(A_{\mathcal{C}j}x_j(k) + B_{\mathcal{C}j}u_j(k) \right), \quad (3)$$

where $x_{\mathcal{C}} = (x_i)_{i \in \mathcal{C}}$ and $u_{\mathcal{C}} = (u_i)_{i \in \mathcal{C}}$ are, respectively, the aggregate state and input vectors of the subsystems in $\mathcal{C}$, which are constrained in the sets $\mathcal{X}_{\mathcal{C}} = \times_{i \in \mathcal{C}} \mathcal{X}_i$ and $\mathcal{U}_{\mathcal{C}} = \times_{i \in \mathcal{C}} \mathcal{U}_i$. The vector $w_{\mathcal{C}} = (w_i)_{i \in \mathcal{C}}$ indicates the disturbances due to the coupling of $\mathcal{C}$ with other agents.

From an overall viewpoint, the dynamics of all coalitions are aggregated as (2), with $x_{\mathcal{N}} = (x_{\mathcal{C}})_{\mathcal{C} \in \mathcal{N}/\Lambda}$ and $u_{\mathcal{N}} = (u_{\mathcal{C}})_{\mathcal{C} \in \mathcal{N}/\Lambda}$.

3. COALITIONAL CONTROL ALGORITHM

A hierarchical coalitional MPC scheme with a top-down approach is presented in this section. Every T_{up} time instants, an upper control layer receives all the current subsystems states and computes the best communication topology according to a trade-off between control performance and cooperation costs. To this end, it is assumed that each communication link connected $l_{ij} \in \mathcal{L}$ has a cost $c_{link} \geqslant 0$. Note that if $c_{link} = 0$, there will be no incentive to adopt a topology different from the centralized one because it provides the best results. Once the new topology is selected, it is sent to the lower layer, where agents must adopt it. Unlike previous works (Núñez et al., 2013, 2015; Maestre et al., 2014a), the new communication network is considered N_t samples ahead to smooth the evolution of the system variables and improve performance.

For each coalition $\mathcal{C} \in \mathcal{N}/\Lambda$, let us define the control and topology sequences as $U_{\mathcal{C}}(k) \triangleq [u_{\mathcal{C}}(k), \ldots, u_{\mathcal{C}}(k + N_p - 1)]$ and $\Lambda_{\mathcal{C}}(k) \triangleq [\Lambda_{\mathcal{C}}(k), \ldots, \Lambda_{\mathcal{C}}(k + N_p)]$, respectively. The goal of coalition $\mathcal{C}$ is to minimize the sum of a stage cost function $l_{\mathcal{C}}(\cdot)$, a cooperation cost function $g_{\mathcal{C}}(\cdot)$ and a terminal cost function $f_{\mathcal{C}}(\cdot)$ over the N_p-horizon, i.e.,

$$J_{\mathcal{C}}\left(x_{\mathcal{C}}(k), U_{\mathcal{C}}(k), \Lambda_{\mathcal{C}}(k)\right) = \sum_{t=0}^{N_p-1} l_{\mathcal{C}}\left(x_{\mathcal{C}}(k + t), u_{\mathcal{C}}(k + t)\right)$$
$$+ \sum_{t=0}^{N_p} g_{\mathcal{C}}\left(|\Lambda_{\mathcal{C}}(k + t)|\right) + f_{\mathcal{C}}\left(x_{\mathcal{C}}(k + N_p)\right), \quad (4)$$

with

$$l_{\mathcal{C}}\left(x_{\mathcal{C}}(k + t), u_{\mathcal{C}}(k + t)\right) =$$
$$x_{\mathcal{C}}(k + t)^{\top}Q_{\mathcal{C}}x_{\mathcal{C}}(k + t) + u_{\mathcal{C}}(k + t)^{\top}R_{\mathcal{C}}u_{\mathcal{C}}(k + t),$$
$$g_{\mathcal{C}}\left(|\Lambda_{\mathcal{C}}(k + t)|\right) = c_{link}|\Lambda_{\mathcal{C}}(k + t)|,$$
$$f_{\mathcal{C}}\left(x_{\mathcal{C}}(k + N_p)\right) = x_{\mathcal{C}}(k + N_p)^{\top}P_{\mathcal{C}}\left(\Lambda_{\mathcal{C}}(k + N_p)\right)x_{\mathcal{C}}(k + N_p),$$

where $Q_{\mathcal{C}}$ and $R_{\mathcal{C}}$ are positive-definite weighting matrices; $|\Lambda_{\mathcal{C}}(k + t)|$ represents the number of links in coalition $\mathcal{C}$ at time step $k + t$; and $P_{\mathcal{C}}$ is a terminal cost positive-definite matrix, which depends on the topology at the end of the prediction horizon N_p.

The proposed hierarchical coalitional MPC algorithm is summarised in *Algorithm 1*.

Algorithm 1

Step 1: Every T_{up} time samples, the upper control layer solves the following mixed-integer optimization problem to find optimal values for $U_{\mathcal{N}}(k)$, N_{t}, and Λ:

$$\{U_{\mathcal{N}}^*(k), \Lambda_{\mathrm{new}}^*, N_{\mathrm{t}}^*\} = \arg \min_{U_{\mathcal{N}}(k), \Lambda_{\mathrm{new}}, N_{\mathrm{t}}} J_{\mathcal{N}}\big(x_{\mathcal{N}}(k), U_{\mathcal{N}}(k), \boldsymbol{\Lambda}(k)\big), \tag{5}$$

subject to

$$x_{\mathcal{C}}(k+t+1) = A_{\mathcal{C}} x_{\mathcal{C}}(k+t) + B_{\mathcal{C}} u_{\mathcal{C}}(k+t),$$
$$x_{\mathcal{C}}(k) = \tilde{x}_{\mathcal{C}}(k), \ \ \forall \mathcal{C} \in \mathcal{N}/\Lambda_{\mathcal{C}},$$
$$x_{\mathcal{C}}(k+t) \in \mathcal{X}_{\mathcal{C}}\big(\Lambda_{\mathcal{C}}(k+t)\big) \ominus \mathcal{W}_{\mathcal{C}}\big(\Lambda_{\mathcal{C}}(k+t)\big), \ t=1,\dots,N_{\mathrm{p}}-1,$$
$$x_{\mathcal{C}}(k+N_{\mathrm{p}}) \in \mathcal{X}_{\mathcal{C}f}\big(\Lambda_{\mathcal{C}}(k+N_{\mathrm{p}})\big),$$
$$u_{\mathcal{C}}(k+t) \in \mathcal{U}_{\mathcal{C}}\big(\Lambda_{\mathcal{C}}(k+t)\big), \ t=0,\dots,N_{\mathrm{p}}-1,$$

for all $\mathcal{C} \in \mathcal{N}/\Lambda$, and

$$\Lambda(k+t) = \begin{cases} \Lambda_{\mathrm{cur}} & \text{if } t < N_{\mathrm{t}} \\ \Lambda_{\mathrm{new}} & \text{if } t \geqslant N_{\mathrm{t}} \end{cases}, \quad t = 0,\dots,N_{\mathrm{p}},$$
$$\Lambda_{\mathrm{new}} \subseteq \mathcal{L},$$

where the global cost function is

$$J_{\mathcal{N}}\big(x_{\mathcal{N}}(k), U_{\mathcal{N}}(k), \boldsymbol{\Lambda}(k)\big) = \sum_{\mathcal{C} \in \mathcal{N}/\Lambda} J_{\mathcal{C}}\big(x_{\mathcal{C}}(k), U_{\mathcal{C}}(k), \boldsymbol{\Lambda}_{\mathcal{C}}(k)\big), \tag{6}$$

$\tilde{x}_{\mathcal{C}}$ represents the measured state; $\mathcal{W}_{\mathcal{C}}$ refers to the disturbances caused by the coupling of $\mathcal{C}$ with other neighboring subsystems; the terminal set of the coalition is $\mathcal{X}_{\mathcal{C}f}$, which depends on the topology at instant $k + N_{\mathrm{p}}$; and the current and the new topology are denoted respectively as Λ_{cur} and Λ_{new}. Furthermore, we require the global cost to decrease to ensure stability and feasibility in the case of a switch of topology. Otherwise, the same topology can be maintained, i.e., $\Lambda_{\mathrm{new}} = \Lambda_{\mathrm{cur}}$, because it guarantees stability and recursive feasibility. See section 4 for details.

Step 2: Each coalition $\mathcal{C}$ computes its control sequence at each time instant and implements the topology chosen by the upper layer. In particular, each coalition $\mathcal{C}$ solves

$$U_{\mathcal{C}}^*(k) = \arg \min_{U_{\mathcal{C}}(k)} J_{\mathcal{C}}\big(x_{\mathcal{C}}(k), U_{\mathcal{C}}(k), \boldsymbol{\Lambda}_{\mathcal{C}}(k)\big), \tag{7}$$

subject to

$$x_{\mathcal{C}}(k+t+1) = A_{\mathcal{C}} x_{\mathcal{C}}(k+t) + B_{\mathcal{C}} u_{\mathcal{C}}(k+t),$$
$$x_{\mathcal{C}}(k) = \tilde{x}_{\mathcal{C}}(k), \ \ \forall \mathcal{C} \in \mathcal{N}/\Lambda_{\mathcal{C}},$$
$$x_{\mathcal{C}}(k+t) \in \mathcal{X}_{\mathcal{C}}\big(\Lambda_{\mathcal{C}}(k+t)\big) \ominus \mathcal{W}_{\mathcal{C}}\big(\Lambda_{\mathcal{C}}(k+t)\big), \ t=1,\dots,N_{\mathrm{p}}-1,$$
$$x_{\mathcal{C}}(k+N_{\mathrm{p}}) \in \mathcal{X}_{\mathcal{C}f}\big(\Lambda_{\mathcal{C}}(k+N_{\mathrm{p}})\big),$$
$$u_{\mathcal{C}}(k+t) \in \mathcal{U}_{\mathcal{C}}\big(\Lambda_{\mathcal{C}}(k+t)\big), \ t=0,\dots,N_{\mathrm{p}}-1,$$

and

$$\Lambda_{\mathcal{C}}(k+t) = \begin{cases} \Lambda_{\mathcal{C},\mathrm{cur}} & \text{if } t < N_{\mathrm{t}} \\ \Lambda_{\mathcal{C},\mathrm{new}} & \text{if } t \geqslant N_{\mathrm{t}} \end{cases}, \quad t = 0,\dots,N_{\mathrm{p}},$$

where $\Lambda_{\mathcal{C},\mathrm{cur}}$ and $\Lambda_{\mathcal{C},\mathrm{new}}$ denote coalition $\mathcal{C}$ in the current and the new topology, respectively.

Remark 1. The transition from Λ_{cur} to Λ_{new} is not instantaneous. Due to the receding horizon strategy, if a change of topology is requested at time step $k + N_{\mathrm{t}}$, at sample $k+1$ the controller will consider Λ_{new} from sample $k + N_{\mathrm{t}} - 1$; at time instant $k + 2$, Λ_{new} will start from sample $k + N_{\mathrm{t}} - 2$, and so on. After N_{t} samples, the change of topology will be completed.

4. STABILITY AND CONTROLLER DESIGN

The stability of the coalitions is assured through a terminal invariant set approach (Amrit et al., 2011; Blanchini, 1999). For each topology Λ, the invariant set $\Omega_{\mathcal{C}}^{\Lambda}$ for each coalition $\mathcal{C}$ is calculated considering its disturbances, which stem from the coupling with its neighbors.

Assumption 1. For each coalition $\mathcal{C} \in \mathcal{N}/\Lambda$ described by (3), there is a feedback $K_{\mathcal{C}}^{\Lambda}$ that ensures that all eigenvalues of $(A_{\mathcal{C}} + B_{\mathcal{C}} K_{\mathcal{C}}^{\Lambda})$ are within the unit circle for all $\Lambda \in \mathcal{T}$. Likewise, the same holds for the corresponding global feedback $K_{\Lambda} = (K_{\mathcal{C}}^{\Lambda})_{\mathcal{C} \in \mathcal{N}/\Lambda}$.

Assumption 2. There is a robust positively invariant (RPI) set $\Omega_{\mathcal{C}}^{\Lambda}$ that satisfies

$$(A_{\mathcal{C}} + B_{\mathcal{C}} K_{\mathcal{C}}^{\Lambda})\Omega_{\mathcal{C}}^{\Lambda} \oplus \mathcal{W}_{\mathcal{C}}^{\Lambda} \subseteq \Omega_{\mathcal{C}}^{\Lambda}, \ \Omega_{\mathcal{C}}^{\Lambda} \subseteq \mathcal{X}_{\mathcal{C}}, \ K_{\mathcal{C}}^{\Lambda}\Omega_{\mathcal{C}}^{\Lambda} \subseteq \mathcal{U}_{\mathcal{C}}, \tag{8}$$

under the linear control law $u_{\mathcal{C}} = K_{\mathcal{C}}^{\Lambda} x_{\mathcal{C}}$, with $\mathcal{W}_{\mathcal{C}}^{\Lambda}$ being a convex set that contains the origin in its interior.

Remark 2. There are numerous procedures to find an RPI set given a bounded disturbance, see, for example, the work of Blanchini (1999). Here, the Multi-Parametric Toolbox of MATLAB® (Herceg et al., 2013) is employed to compute the maximal RPI set for each coalition $\mathcal{C}$ of the topology $\Lambda \in \mathcal{T}$ taking into account its constraints and the disturbances arising from its neighbors.

Assumption 3. The terminal cost $f\big(x_{\mathcal{N}}(k)\big)$ is a Lyapunov function: $x_{\mathcal{N}}(k)^{\top} P_{\Lambda} x_{\mathcal{N}}(k)$, with $P_{\Lambda} = (P_{\mathcal{C}}^{\Lambda})_{\mathcal{C} \in \mathcal{N}/\Lambda}$, for the overall system controlled by $K_{\Lambda} = (K_{\mathcal{C}}^{\Lambda})_{\mathcal{C} \in \mathcal{N}/\Lambda}$, which also satisfies

$$f\big(x_{\mathcal{N}}(k)\big) \geqslant \sum_{t=0}^{\infty} l_{\mathcal{N}}\big(x_{\mathcal{N}}(k+t), u_{\mathcal{N}}(k+t)\big). \tag{9}$$

Observe that, in accordance with (9) and (4), it can be deduced that $J_{\mathcal{N}}(\cdot)$ in (6) satisfies

$$J_{\mathcal{N}}(\cdot) \geqslant \sum_{t=0}^{\infty} l_{\mathcal{N}}\big(x_{\mathcal{N}}(k+t), u_{\mathcal{N}}(k+t)\big) + \sum_{t=0}^{N_{\mathrm{p}}} g_{\mathcal{N}}(|\Lambda(k+t)|)$$
$$\geqslant \sum_{t=0}^{\infty} l_{\mathcal{N}}\big(x_{\mathcal{N}}(k+t), u_{\mathcal{N}}(k+t)\big). \tag{10}$$

In other words, $J_{\mathcal{N}}(\cdot)$ is bound on the control cost-to-go of the overall system.

Remark 3. The size of a coalition $\mathcal{C}$ can range from a singleton $\mathcal{C} = \{i\}$ to even the global system $\mathcal{C} = \mathcal{N}$. Hence, these assumptions require defined feedbacks for all cases from decentralized to centralized topologies. Nevertheless, it is possible to implement this method with just a subset of topologies to avoid combinatorial explosion issues.

The design of the overall matrices K_{Λ} and P_{Λ} to guarantee stability for each topology $\Lambda \in \mathcal{T}$ is based on the following linear matrix inequality (LMI) (Maestre et al., 2014a).

Theorem 1. Let the system be divided into N subsystems connected by a set of active links $\Lambda \subseteq \mathcal{L}$. The discrete system matrices are $A_{\mathcal{N}} = [A_{ij}]_{i,j \in \mathcal{N}}$ and $B_{\mathcal{N}} = [B_{ij}]_{i,j \in \mathcal{N}}$, and the stage cost matrices $Q_{\mathcal{N}} = \mathrm{diag}(Q_i)_{i \in \mathcal{N}}$ and $R_{\mathcal{N}} = \mathrm{diag}(R_i)_{i \in \mathcal{N}}$. If there are matrices $H_{\Lambda} = H_{\Lambda}^{\top} = [H_{ij}]_{i,j \in \mathcal{N}}$, where $H_{ij} \in \mathbb{R}^{q_i \times q_j}$, and $Y_{\Lambda} = [Y_{ij}]_{i,j \in \mathcal{N}}$, where $Y_{ij} \in \mathbb{R}^{r_i \times q_j}$ with $H_{ij} = 0$ and $Y_{ij} = 0$ if the link $l_{ij} \in \Lambda$ is disabled, i.e., if $i \in \mathcal{C}$ and $j \notin \mathcal{C}$, in such that the following constraints are satisfied

$$\begin{bmatrix} H_\Lambda & H_\Lambda A_{\mathcal{N}}^\top + Y_\Lambda^\top B_{\mathcal{N}}^\top & H_\Lambda Q_{\mathcal{N}}^{1/2} & Y_\Lambda^\top R_{\mathcal{N}}^{1/2} \\ A_{\mathcal{N}} H_\Lambda + B_{\mathcal{N}} Y_\Lambda & H_\Lambda & 0 & 0 \\ Q_{\mathcal{N}}^{1/2} H_\Lambda & 0 & I & 0 \\ R_{\mathcal{N}}^{1/2} Y_\Lambda & 0 & 0 & I \end{bmatrix} \geqslant 0, \tag{11}$$

then there is a $P_\Lambda = H_\Lambda^{-1}$ that satisfies (9), and a feedback control law $K_\Lambda = Y_\Lambda H_\Lambda^{-1}$ that stabilizes the closed-loop system for the topology Λ.

Note that the controllers are designed maximizing the trace of H_Λ in (11), i.e.,

$$\max \operatorname{tr}(H_\Lambda), \tag{12}$$

as a way of minimizing the trace of $P_\Lambda = H_\Lambda^{-1}$ and, therefore, the cost-to-go of the system.

Assumption 4. *Algorithm 1* has an overall feasible control sequence at time instant $k = 0$ for the initial topology.

Theorem 2. Let the N-agent system be connected by the set of active links of Λ and let the matrices K_Λ and P_Λ be obtained according to (11), the coalitional MPC controllers can guarantee recursive feasibility and stability of the closed-loop system.

5. APPLICATION

An eight-coupled tanks plant, which is an extension of the quadruple-tanks process proposed by Johansson (2000), has been designed to test the proposed algorithm. As shown in Fig. 1, four upper tanks (3, 4, 7, and 8) discharge caudal flows into lower tanks (1, 2, 5, and 6), and these, in turn, into sinking tanks. They are controlled by four pumps that transport water from sinks to tanks, and the coupling is set by six three-way valves that divide into two ways the pumped flows on arrival.
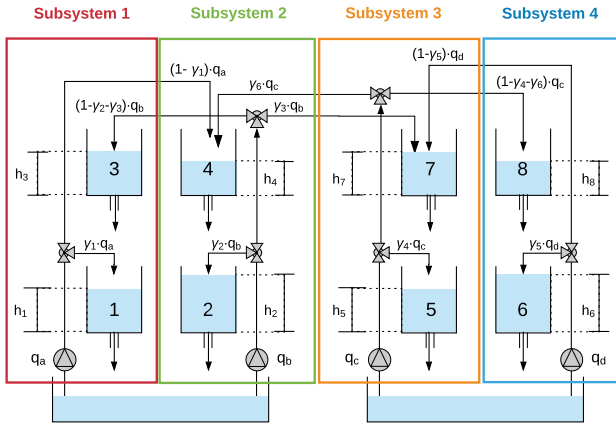

Fig. 1. Schematic diagram of the eight-coupled tanks plant

The overall system is divided into $N = 4$ subsystems: the first one is composed of tanks 1 and 3; the second subsystem is formed by tanks 2 and 4; the third one by tanks 5 and 7; and the fourth subsystem by tanks 6 and 8. The control goal is to make lower tanks reach the desired target levels taking into account control and cooperation costs, and operational constraints. Hence, it is a multi-variable problem with four inputs (q_a, q_b, q_c, q_d) and four outputs (h_1, h_2, h_5, h_6).

5.1 Plant model

The non-linear model used in simulations can be obtained on applying mass balance and Bernoulli's law. Hence, the plant is described by the following equations:

$$\begin{aligned} S_1 \frac{dh_1}{dt} &= a_3\sqrt{2gh_3} - a_1\sqrt{2gh_1} + \gamma_1 \frac{q_\text{a}}{3600}, \\ S_2 \frac{dh_2}{dt} &= a_4\sqrt{2gh_4} - a_2\sqrt{2gh_2} + \gamma_2 \frac{q_\text{b}}{3600}, \\ S_3 \frac{dh_3}{dt} &= -a_3\sqrt{2gh_3} + (1 - \gamma_2 - \gamma_3)\frac{q_\text{b}}{3600}, \\ S_4 \frac{dh_4}{dt} &= -a_4\sqrt{2gh_4} + (1 - \gamma_1)\frac{q_\text{a}}{3600} + \gamma_6 \frac{q_\text{c}}{3600}, \\ S_5 \frac{dh_5}{dt} &= a_7\sqrt{2gh_7} - a_5\sqrt{2gh_5} + \gamma_4 \frac{q_\text{c}}{3600}, \\ S_6 \frac{dh_6}{dt} &= a_8\sqrt{2gh_8} - a_6\sqrt{2gh_6} + \gamma_5 \frac{q_\text{d}}{3600}, \\ S_7 \frac{dh_7}{dt} &= -a_7\sqrt{2gh_7} + (1 - \gamma_5)\frac{q_\text{d}}{3600} + \gamma_3 \frac{q_\text{b}}{3600}, \\ S_8 \frac{dh_8}{dt} &= -a_8\sqrt{2gh_8} + (1 - \gamma_4 - \gamma_6)\frac{q_\text{c}}{3600}, \end{aligned} \tag{13}$$

where h_n is the water level of tank $n \in \{1, 2, \ldots, 8\}$; $S_n = 13.89 \cdot 10^{-3}[\text{m}^2]$ is its corresponding cross-section, which is assumed to be equal for all tanks; and $a_n = 50.265 \cdot 10^{-6}[\text{m}^2]$ stands for the cross-section of all the outlet pipes. The parameters $\gamma_i \in [0,1]$, $\forall i \in \{1, 2, \ldots, 6\}$ refer to the opening of the six three-way valves ($\gamma_1, \gamma_4 = 0.3$; $\gamma_2, \gamma_5 = 0.4$ and $\gamma_3, \gamma_6 = 0.1$); gravity is $g = 9.81[\text{m/s}^2]$; and q_m is the caudal flow pumped by pump $m \in \{\text{a, b, c, d}\}$.

Let us define the operating point of each tank, measured in meters, by water levels $h_1^0 = 2.77$, $h_2^0 = 2.659$, $h_3^0 = 2.524$, $h_4^0 = 0.129$, $h_5^0 = 2.77$, $h_6^0 = 2.659$, $h_7^0 = 1.242$, $h_8^0 = 1.209$; and the operating point of each pump, measured in cubic meters per hour, by flow rates $q_1^0 = 0.1920$, $q_2^0 = 2.4194$, $q_3^0 = 1.3953$, $q_4^0 = 1.0113$, to obtain the control model. The corresponding overall discrete linear state-space model is

$$\bar{x}_{\mathcal{N}}(k+1) = A_{\mathcal{N}} \bar{x}_{\mathcal{N}}(k) + B_{\mathcal{N}} \bar{u}_{\mathcal{N}}(k), \tag{14}$$

where $\bar{x}_{\mathcal{N}}(k) = [h_1(k) - h_1^0, \ldots, h_8(k) - h_8^0]^\top$ is the state vector, and $\bar{u}_{\mathcal{N}}(k) = [q_\text{a}(k) - q_\text{a}^0, \ldots, q_\text{d}(k) - q_\text{d}^0]^\top$ is the input vector. Likewise, the state evolution of each subsystem $i \in \mathcal{N} \triangleq \{1, 2, 3, 4\}$ is given by

$$\begin{aligned} \bar{x}_i(k+1) &= A_{ii}\bar{x}_i(k) + B_{ii}\bar{u}_i(k) + \bar{w}_i(k), \\ \bar{w}_i &= \sum_{j \in \mathcal{M}_i} A_{ij}\bar{x}_j(k) + B_{ij}\bar{u}_j(k), \end{aligned} \tag{15}$$

where the subsystem matrices are given by

$$A_{11} = \begin{bmatrix} 0.9762 & 0.0246 \\ 0 & 0.9751 \end{bmatrix}, B_{11} = \begin{bmatrix} 0.0296 \\ 0 \end{bmatrix}, B_{12} = \begin{bmatrix} 0.0006 \\ 0.0494 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} 0.9757 & 0.1041 \\ 0 & 0.8946 \end{bmatrix}, B_{21} = \begin{bmatrix} 0.0037 \\ 0.0662 \end{bmatrix}, B_{22} = \begin{bmatrix} 0.0395 \\ 0 \end{bmatrix}, B_{23} = \begin{bmatrix} 0.0005 \\ 0.0095 \end{bmatrix},$$

$$A_{33} = \begin{bmatrix} 0.9762 & 0.0349 \\ 0 & 0.9647 \end{bmatrix}, B_{32} = \begin{bmatrix} 0.0002 \\ 0.0098 \end{bmatrix}, B_{33} = \begin{bmatrix} 0.0296 \\ 0 \end{bmatrix}, B_{34} = \begin{bmatrix} 0.0011 \\ 0.0589 \end{bmatrix},$$

$$A_{44} = \begin{bmatrix} 0.9757 & 0.0354 \\ 0 & 0.9642 \end{bmatrix}, B_{43} = \begin{bmatrix} 0.0011 \\ 0.0589 \end{bmatrix}, B_{44} = \begin{bmatrix} 0.0395 \\ 0 \end{bmatrix},$$

$$A_{ij} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, i \neq j \ \forall i \in \mathcal{N}, j \in \mathcal{M}_i,$$

and the states and inputs are respectively subject to the constraints $0.02 < h_n \leqslant 5$ and $0 < q_m \leqslant 5$. Each local controller can communicate in a bidirectional way through the network formed by the edges $\mathcal{L} = \{\{1, 2\}, \{2, 3\}, \{3, 4\}\}$. As the number of links is $|\mathcal{L}| = 3$, there are 8 different cooperation topologies, $\mathcal{T} = \{\Lambda_1, \Lambda_2, \ldots, \Lambda_8\}$. In this

case, Λ_1 and Λ_8 refer to the decentralized and centralized topology, respectively.

From the global viewpoint, the objective of the proposed coalitional MPC is to optimize the sum of the performance cost J_{perf} and the cooperation cost J_{coop} subject to a set of constraints, i.e.,

$$\{U^*_{\mathcal{N}}(k), \Lambda^*_{\text{new}}, N^*_{\text{t}}\} = \arg \min_{U_{\mathcal{N}}, \Lambda_{\text{new}}, N_{\text{t}}} J_{\text{perf}} + J_{\text{coop}}, \quad (16)$$

with

$$J_{\text{perf}} = \sum_i^N \sum_{t=0}^{N_{\text{p}}-1} \left(\bar{x}_i(k+t+1)^\top Q_i \bar{x}_i(k+t+1) + \bar{u}_i(k+t)^\top R_i \bar{u}_i(k+t) \right) + \bar{x}_i(k+N_{\text{p}})^\top P_i \bar{x}_i(k+N_{\text{p}}),$$

$$J_{\text{coop}} = \sum_{t=1}^{N_{\text{t}}} c_{\text{link}} |\Lambda_{\text{cur}}| + \sum_{t=N_{\text{t}}+1}^{N_{\text{p}}} c_{\text{link}} |\Lambda_{\text{new}}|,$$

subject to the subsystem dynamics (15); the new topology $\Lambda_{\text{new}} \subseteq \mathcal{T}$ after $k + N_{\text{t}}$ time steps; the state terminal set $x_i(k+N_{\text{p}}) \in \Omega_i$; the state and input constraints; and the state and input weighting $Q_i = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $R_i = 0.1$, $\forall i \in \{1, 2, 3, 4\}$.

Additionally, local controllers $K_\Lambda = [K_{ij}]_{i,j \in \mathcal{N}}$ and the terminal cost matrices $P_\Lambda = [P_{ij}]_{i,j \in \mathcal{N}}$ for each topology are obtained solving (12). For example, for topology Λ_1, i.e., when no cooperation between agents is enabled, the following matrices are obtained

$$K_{\Lambda_1} = \begin{bmatrix} 0.020 & 0 & 0.024 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.051 & 0 & -0.049 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.035 & 0 & 0.039 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.036 & 0 & -0.033 \end{bmatrix},$$

$$P_{\Lambda_1} = \begin{bmatrix} 22.05 & 0 & 11.55 & 0 & 0 & 0 & 0 & 0 \\ 0 & 19.58 & 0 & 15.19 & 0 & 0 & 0 & 0 \\ 11.55 & 0 & 11.96 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15.19 & 0 & 14.97 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 22.49 & 0 & 19.93 & 0 \\ 0 & 0 & 0 & 0 & 0 & 19.95 & 0 & 11.01 \\ 0 & 0 & 0 & 0 & 13.93 & 0 & 14.47 & 0 \\ 0 & 0 & 0 & 0 & 0 & 11.01 & 0 & 10.67 \end{bmatrix}.$$

5.2 Simulation control methods

In order to assess our method, the following three control methods have been considered:

1. **CEN** consists of a centralized MPC scheme that has full system information.

2. **ABRUPT** presents a coalitional MPC algorithm where the topology Λ_{cur} is instantly switched to Λ_{new} such as in (Núñez et al., 2015) and (Fele et al., 2018), i.e., the transition horizon is $N_{\text{t}} = 0$. In this method, the topology Λ^*_{new} is chosen every $T_{\text{up}} = 20$ samples (from $k = 0$) after evaluating $|\mathcal{T}| = 8$ problems.

3. **PRED$_{N_{\text{t}}\text{up}}$** presents a coalitional MPC scheme, based on *Algorithm 1*, which solves $|\mathcal{T}| \cdot |N_{\text{t}}|$ convex problems every $T_{\text{up}} = 20$ samples (from $k = 10$), and chooses that with the lowest cost to find the pair $\{\Lambda^*_{\text{new}}, N^*_{\text{t}}\}$. The variable N_{t} can take values between 0 and 10, and the total number of topologies is $|\mathcal{T}| = 8$.

5.3 Simulation results

The sampling time used in the simulations is $T_{\text{m}} = 5$ s, with a simulation length of $N_{\text{sim}} = 240$ time instants. The

MPC methods consider a prediction horizon $N_{\text{p}} = 20$, and a cost per active cooperation link $c_{\text{link}} = 0.02$. The value of c_{link} has been chosen large enough to represent cooperation costs considering that a small value and a very large value will always lead to a centralized and a decentralized network, respectively. Table 1 displays a summary of the simulation results for each scheme.

Table 1. Simulation results for each method.

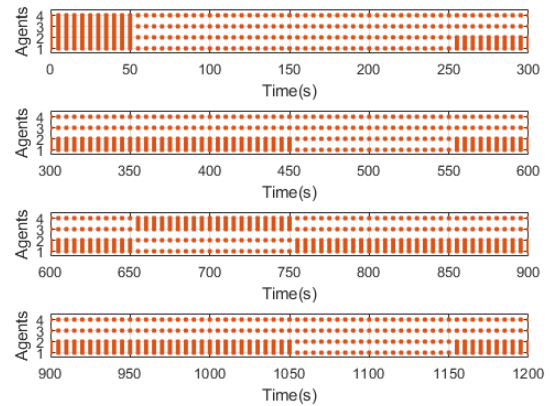|	CEN
Performance cost	1.1543
Cooperation cost	14.4000
Total cost	15.5543
	ABRUPT
Historical of topologies	[8, 1, 5, 1, 5, 5, 8, 7, 5, 2, 5, 1]
Performance cost	1.1582
Cooperation cost	6.2600
Total cost	7.4182
	PRED$_{N_{\text{t}}\text{up}}$
Historical of topologies	[8, 1, 1, 5, 5, 1, 5, 2, 5, 5, 5, 1, 5]
Performance cost	1.3469
Cooperation cost	4.3210
Total cost	5.6679

Fig. 2. Formation of coalitions in the scheme PRED$_{N_{\text{t}}\text{up}}$

Fig. 2 illustrates the formation of coalitions in the scheme PRED$_{N_{\text{t}}\text{up}}$, where the four agents have cooperated for the first $t = 50$ s (centralized topology Λ_8); then, the topology changes to Λ_1 and agents do not cooperate until $t = 250$ s; afterwards, agents 1 and 2 form a coalition, and agents 3 and 4 work independently, etc. As a example, figs. 3 and 4 depict the outputs of tanks 2 and 5 and their corresponding flow rates of pumps q_{b} and q_{c}. Regarding the total cost of CEN, ABRUPT has a reduction of 52.36 % and PRED$_{N_{\text{t}}\text{up}}$ a reduction of 63.56 %. As can be seen, the leeway when choosing the instant to introduce the new topology is essential in the reduction in the total cost.

6. CONCLUSIONS

In this work, a hierarchical coalitional MPC algorithm where local controllers can predict the topology transitions over the prediction horizon is presented. In particular, it is introduced a new variable, the transition horizon N_{t}, into the optimization problem to provide the best time instant to switch topology. Moreover, conditions for closed-loop stability and recursive feasibility are given.
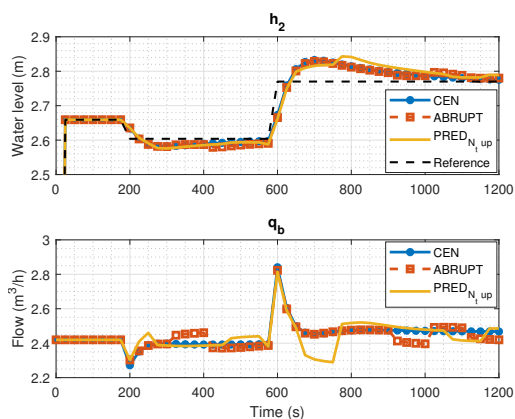
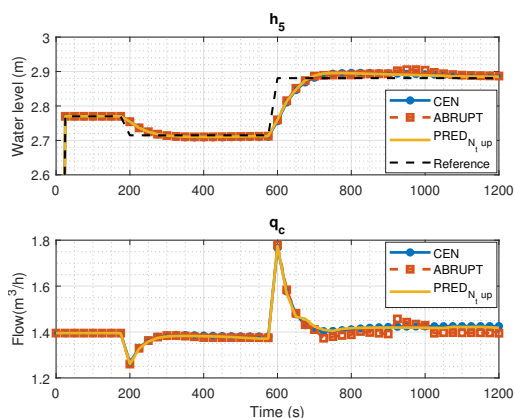Fig. 3. Output of tank 2, and flow rate of pump q_b

Fig. 4. Output of tank 5, and flow rate of pump q_c

Numerical results in a simulated eight-coupled tanks plant show improvements in the total cost after N_t is introduced in the optimization problem. In particular, our comparison shows that the method $\text{PRED}_{N_t \, up}$ provides the lowest accumulated total when cooperation costs are explicitly considered. Future work will include a fully distributed implementation of the proposed approach and plug-and-play features.

REFERENCES

Alejandro, J., Arce, A., and Bordons, C. (2014). Combined environmental and economic dispatch of smart grids using distributed model predictive control. *International Journal of Electrical Power & Energy Systems*, 54, 65–76.

Alvarado, I., Limón, D., Muñoz de la Peña, D., Maestre, J.M., Ridao, M.A., Scheu, H., Marquardt, W., Negenborn, R., De Schutter, B., Valencia, F., et al. (2011). A comparative analysis of distributed mpc techniques applied to the hd-mpc four-tank benchmark. *Journal of Process Control*, 21(5), 800–815.

Amrit, R., Rawlings, J.B., and Angeli, D. (2011). Economic optimization using model predictive control with a terminal cost. *Annual Reviews in Control*, 35(2), 178–186.

Arnold, M., Negenborn, R.R., Andersson, G., and De Schutter, B. (2010). Distributed predictive control for energy hub coordination in coupled electricity and gas networks. In *Intelligent Infrastructures*, 235–273. Springer.

Blanchini, F. (1999). Set invariance in control. *Automatica*, 35(11), 1747–1767.

Camacho, E.F. and Bordons, C. (2007). *Model predictive control*. Springer Verlag London Limited.

De Oliveira, L.B. and Camponogara, E. (2010). Multi-agent model predictive control of signaling split in urban traffic networks. *Transportation Research Part C: Emerging Technologies*, 18(1), 120–139.

Fele, F., Debada, E., Maestre, J.M., and Camacho, E.F. (2018). Coalitional control for self-organizing agents. *IEEE Transactions on Automatic Control*, 63(9), 2883–2897.

Fele, F., Maestre, J.M., Hashemy, S.M., Muñoz de la Peña, D., and Camacho, E.F. (2014). Coalitional model predictive control of an irrigation canal. *Journal of Process Control*, 24(4), 314–325.

Herceg, M., Kvasnica, M., Jones, C.N., and Morari, M. (2013). Multi-parametric toolbox 3.0. In *2013 European Control Conference (ECC)*, 502–510. IEEE.

Johansson, K.H. (2000). The quadruple-tank process: A multivariable laboratory process with an adjustable zero. *IEEE Transactions on control systems technology*, 8(3), 456–465.

Kulikowski, R. (1970). Optimization of large-scale systems. *Automatica*, 6(2), 315–331.

Maestre, J.M., Muñoz de la Peña, D., Jiménez Losada, A., Algaba, E., and Camacho, E.F. (2014a). A coalitional control scheme with applications to cooperative game theory. *Optimal Control Applications and Methods*, 35(5), 592–608.

Maestre, J.M., Muñoz de la Peña, D., Jiménez Losada, A., Algaba Durán, E., and Camacho, E.F. (2011). An application of cooperative game theory to distributed control. *IFAC Proceedings Volumes*, 44(1), 9121–9126.

Maestre, J.M., Negenborn, R.R., et al. (2014b). *Distributed model predictive control made easy*, volume 69. Springer.

Negenborn, R.R., van Overloop, P.J., Keviczky, T., De Schutter, B., et al. (2009). Distributed model predictive control of irrigation canals. *NHM*, 4(2), 359–380.

Núñez, A., Ocampo-Martinez, C., De Schutter, B., Valencia, F., López, J.D., and Espinosa, J. (2013). A multiobjective-based switching topology for hierarchical model predictive control applied to a hydro-power valley. *IFAC Proceedings Volumes*, 46(20), 534–539.

Núñez, A., Ocampo-Martinez, C., Maestre, J.M., and De Schutter, B. (2015). Time-varying scheme for non-centralized model predictive control of large-scale systems. *Mathematical Problems in Engineering*, 2015.

Rahwan, T., Michalak, T., Wooldridge, M., and Jennings, N.R. (2012). Anytime coalition structure generation in multi-agent systems with positive or negative externalities. *Artificial Intelligence*, 186, 95–122.

Scattolini, R. (2009). Architectures for distributed and hierarchical model predictive control–a review. *Journal of process control*, 19(5), 723–731.

Shneiderman, D. and Palmor, Z. (2010). Properties and control of the quadruple-tank process with multivariable dead-times. *Journal of Process Control*, 20(1), 18–28.

Trodden, P. and Richards, A. (2009). Adaptive cooperation in robust distributed model predictive control. In *2009 IEEE Control Applications,(CCA) & Intelligent Control,(ISIC)*, 896–901. IEEE.