Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Design of Adaptive Cruise Controllers for Externally Positive Vehicles

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Abstract: This paper addresses the design of adaptive cruise controllers for guaranteed collision avoidance. The design problem is solved with distributed feedback controllers which work with locally measurable quantities and, hence, do not require a centralised coordinating unit or a communication system. The design objectives are achieved by placing the closed-loop eigenvalues in a proposed set so that the controlled vehicle is asymptotically stable and satisfies a sufficient condition on external positivity.

1. INTRODUCTION

Adaptive cruise control (ACC) is a driver assistance system that allows for automatically maintaining a safety distance to the predecessor vehicle. Figure 1 shows the basic structure of an ACC platoon with vehicles that are equipped with radar sensors which measure the distance $d_i(t)$ (illustrated by))))) to adapt the velocity $v_i(t)$ of each vehicle so that a desired distance is achieved and all vehicles travel with a common velocity asymptotically. To benefit from reduced fuel consumption due to exploitation of slipstream, the distance has to be small which leads to the fact that a driver in an automated vehicle is not able to react on a disturbance in time to prevent an imminent collision. Thus, the cruise controller has to guarantee collision avoidance for arbitrary length of the platoon.

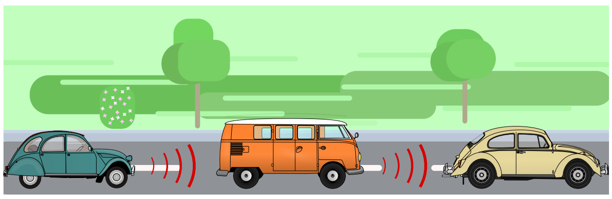

Fig. 1. Platoon with adaptive cruise control

From a control theoretic point of view, a set of vehicles organised in a platoon is represented by a multi-agent system as depicted in Fig. 2. This allows for using techniques for the design of multi-agent systems which aim at finding properties of the controlled vehicles so that the platoon shows a desired behaviour. It has been shown by Lunze (2018a) that collision avoidance can be guaranteed if the controlled vehicles have *externally positive* dynamics. This property characterises how the controlled vehicle reacts on the input of the physical coupling that can be interpreted as a disturbance since it is not known.

This paper addresses the question on how to achieve external positivity under the constraint that the ACC controller operates only with locally measurable quantities

$$\boldsymbol{y}_i(t) = \begin{pmatrix} v_i(t) \\ d_i(t) \end{pmatrix}$$

and under the circumstance that there is no global coordinator with overall knowledge. Furthermore, it will be discussed when an extension to cooperative adaptive cruise control (CACC) is reasonable, where the controllers have to communicate.

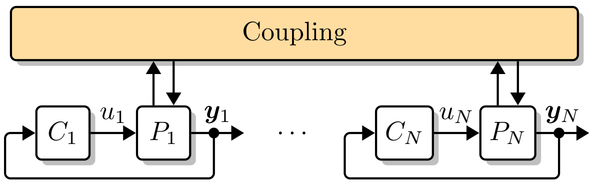

Fig. 2. Control structure of a multi-agent system

Literature. The notion of string stability was introduced to study the behaviour of a long concatenation of subsystems, e. g. a vehicle platoon. A popular sufficient condition on string stability published by Swaroop and Hedrick (1996) requires a bounded error transfer function and was experimentally evaluated Ploeg et al. (2011) with a few vehicles. However, it has been shown that string stability is only necessary for collision avoidance since it does not prevent overshooting. Lunze (2018a) proved that external positivity of the controlled vehicles guarantees collision avoidance which is a stricter condition than a bounded error transfer function.

A system is called externally positive if and only if its impulse response is non-negative (Farina and Rinaldi, 2011). Unfortunately, the design of externally positive control systems is not solved yet in general. There are publications that present sufficient conditions, for example, by Swaroop (2003) who gave a compensator structure that achieves a non-negative impulse response. However, the proposed approach does not achieve set-point following and is feedforward which is not robust. Rachid (1995) presented a more practically useful sufficient condition for externally positive systems with real poles and zeros based on an observation of El-Khoury et al. (1993) which provides an upper bound for the number of extrema of the step response based on the location of the zeros of a system.

The present paper uses the condition of Rachid (1995) to complete the previous work published by Schwab and Lunze (2019) by presenting a procedure to design externally positive vehicles as the main contribution.

Structure. This paper is organised as follows: Section 2 describes the design problem. In Section 3, the vehicle model and ACC structure is introduced and some properties of externally positive systems are presented in Section 4. The proposed design procedure that achieves externally positive dynamics is presented in Section 5 and the effectiveness is evaluated by a simulation study which is given in Section 6.

2. PROBLEM STATEMENT

This paper is concerned with the ACC design for platoons of N identical vehicles which are enumerated in ascending order beginning with $i = 1$ for the leader. The velocity of each vehicle is denoted by $v_i(t)$ and the longitudinal position $s_i(t)$ is determined by

$$s_i(t) = \int_0^t v_i(\tau)\,\mathrm{d}\tau + s_{i0}, \quad i = 1, 2, \ldots, N$$

with the initial position s_{i0}. The inter-vehicle distance of two consecutive vehicles is given by

$$d_i(t) = s_{i-1}(t) - s_i(t) - d_0, \quad i = 2, 3, \ldots, N$$

with d_0 denoting the minimum distance including the vehicle length and a least permitted separation as shown in Fig. 3.

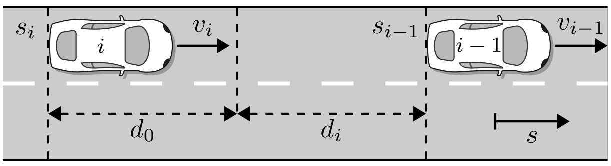

Fig. 3. Two consecutive vehicles

Desired platoon behaviour. In a steady state, all vehicles should move with the same (piecewise) constant reference velocity $v_0(t)$, i. e. the velocities

$$\lim_{t\to\infty} |v_i(t) - v_0(t)| = 0, \quad i = 1, 2, \ldots, N \tag{1}$$

are synchronised leading to a constant inter-vehicle distance. The desired spacing should increase with the velocity and satisfy the requirement

$$\lim_{t\to\infty} |d_i(t) - \beta v_i(t)| = 0, \quad i = 2, 3, \ldots, N \tag{2}$$

asymptotically with β denoting the time-headway coefficient. Additionally, there should be no situation in which a vehicle moves backwards, i. e.

$$v_i(t) \geq 0, \quad t \geq 0, \quad i = 1, 2, \ldots, N \tag{3}$$

and all vehicle distances should be non-negative

$$d_i(t) \geq 0, \quad t \geq 0, \quad i = 2, 3, \ldots, N \tag{4}$$

to ensure that all vehicles comply with a minimum distance $s_{i-1}(t) - s_i(t) \geq d_0$ and, thus, to guarantee collision avoidance.

Design objectives. Lunze (2018a) showed that the desired behaviour (1)–(4) is achieved if and only if the controlled vehicle

$$\bar{\Sigma}_i : \begin{cases} V_i(s) = \bar{G}(s)\, V_{i-1}(s) \\ D_i(s) = G_\mathrm{d}(s)\, V_{i-1}(s) \end{cases} \tag{5}$$

with $V_i(s)$ and $D_i(s)$ denoting the Laplace transforms of $v_i(t)$ and $d_i(t)$ has the following properties: $\bar{\Sigma}_i$ has to be be I/O-stable (D1) with the static reinforcements

$$\bar{G}(0) = 1 \tag{D2}$$

$$G_\mathrm{d}(0) = \beta \tag{D3}$$

and, furthermore, has to be externally positive, i. e.

$$\bar{G}(s) \bullet\!\!-\!\!\circ\, \bar{g}(t) \geq 0, \quad t \geq 0 \tag{D4}$$

with $\bullet\!\!-\!\!\circ$ denoting the Laplace transform correspondence of the transfer function $\bar{G}(s)$ and the impulse response $\bar{g}(t)$.

Problem 1. **(ACC design)** Find a feedback control law $u_i(t)$ so that $\bar{\Sigma}_i$ satisfies the design objectives (D1)–(D4) using local information.

The term *local information* refers to the signals that can be measured locally with sensors, namely the driven velocity $v_i(t)$ and the inter-vehicle distance $d_i(t)$. The present paper will propose a control structure and a design procedure to find a set of parameters that renders the controlled vehicle externally positive based on the vehicle model presented in the next section.

3. MODELS

3.1 Microscopic vehicle model

The dynamical behaviour of each vehicle ($i = 1, 2, \ldots, N$) is described by the first-order model

$$\Sigma_i : \dot{v}_i(t) = -\frac{c}{m} v_i(t) + \frac{1}{m} u_i(t), \quad v_i(0) = v_{i0}. \tag{6}$$

The inter-vehicle distance of all follower vehicles ($i = 2, 3, \ldots, N$) is governed by the velocity difference

$$\dot{d}_i(t) = v_{i-1}(t) - v_i(t). \tag{7}$$

A combination of (6) and (7) yields the extended model (cf. Fig. 4) for the vehicles ($i = 2, 3, \ldots, N$)

$$P_i : \begin{pmatrix} \dot{v}_i(t) \\ \dot{d}_i(t) \end{pmatrix} = \begin{pmatrix} -c/m & 0 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} v_i(t) \\ d_i(t) \end{pmatrix} + \begin{pmatrix} 1/m \\ 0 \end{pmatrix} u_i(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} v_{i-1}(t)$$

with the initial state

$$\begin{pmatrix} v_i(0) \\ d_i(0) \end{pmatrix} = \begin{pmatrix} v_{i0} \\ s_{i-1}(0) - s_i(0) - d_0 \end{pmatrix}.$$

The leader P_1 has the velocity dynamics (6), i. e. $P_1 = \Sigma_1$.

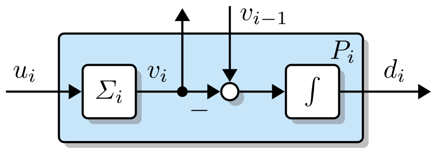

Fig. 4. Extended vehicle model ($i = 2, 3, \ldots, N$)

Discussion. The velocity dynamics (6) results from the physical equilibrium of forces

$$\underbrace{m\,\dot{v}(t)}_{F(t)} + \underbrace{c\,v(t)}_{F_\mathrm{r}(t)} = u(t)$$

of a straight moving rigid object with frictional force $F_\mathrm{r}(t)$ and, consequently, $u(t)$ describes a driving force from the power train. This model is a fundamental representation of the longitudinal dynamics of a vehicle and is commonly used in similar form, for example, by Ploeg et al. (2011) and Seiler et al. (2004). However, this model does not take the dynamics of the power train into account which is assumed to be significantly quicker than the acceleration of the vehicle mass. Lunze (2018b) presented a way to calculate the boundary of the deviation of a higher-order model from the ideal first-order closed-loop dynamics

$$\bar{\Sigma}_i : \begin{cases} \dot{v}_i(t) - \frac{1}{\beta} v_i(t) + \frac{1}{\beta} v_{i-1}(t) \\ d_i(t) = \beta\, v_i(t), \end{cases}$$

which satisfies the time-headway condition (2) permanently. The boundary can be used to show that the platoon of controlled vehicles is collision-free although the power train is neglected in model (6).

3.2 Control structure

This section summarises the ACC structure proposed by Schwab and Lunze (2019). An additional regulator state

$$\dot{z}_i(t) = \beta\, v_i(t) - d_i(t)$$

is introduced to satisfy the structural requirements of the time-headway policy (2) resulting in the open-loop model

$$P_{0i} : \begin{pmatrix} \dot{v}_i(t) \\ \dot{d}_i(t) \\ \dot{z}_i(t) \end{pmatrix} = \underbrace{\begin{pmatrix} -c/m & 0 & 0 \\ -1 & 0 & 0 \\ \beta & -1 & 0 \end{pmatrix}}_{\boldsymbol{A}_0} \begin{pmatrix} v_i(t) \\ d_i(t) \\ z_i(t) \end{pmatrix} + \underbrace{\begin{pmatrix} 1/m \\ 0 \\ 0 \end{pmatrix}}_{\boldsymbol{b}} u_i(t) + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} v_{i-1}(t). \quad (8)$$

The control loop is closed with the feedback

$$u_i(t) = -\boldsymbol{k}^{\mathrm{T}} \begin{pmatrix} v_i(t) \\ d_i(t) \\ z_i(t) \end{pmatrix} = -\left(k_{\mathrm{v}}\ k_{\mathrm{d}}\ k_{\mathrm{z}}\right) \begin{pmatrix} v_i(t) \\ d_i(t) \\ z_i(t) \end{pmatrix}, \quad (9)$$

which finally results in the closed-loop dynamics

$$\bar{\Sigma}_i : \begin{cases} \begin{pmatrix} \dot{v}_i(t) \\ \dot{d}_i(t) \\ \dot{z}_i(t) \end{pmatrix} = \bar{\boldsymbol{A}} \begin{pmatrix} v_i(t) \\ d_i(t) \\ z_i(t) \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}_{\boldsymbol{e}} v_{i-1}(t) \\ v_i(t) = \underbrace{\left(1\ 0\ 0\right)}_{\boldsymbol{c}^{\mathrm{T}}} \begin{pmatrix} v_i(t) \\ d_i(t) \\ z_i(t) \end{pmatrix} \end{cases} \quad (10)$$

with $\bar{\boldsymbol{A}} = \boldsymbol{A}_0 - \boldsymbol{b}\boldsymbol{k}^{\mathrm{T}} = \begin{pmatrix} -\frac{c+k_{\mathrm{v}}}{m} & -\frac{k_{\mathrm{d}}}{m} & -\frac{k_{\mathrm{z}}}{m} \\ -1 & 0 & 0 \\ \beta & -1 & 0 \end{pmatrix}$.

Lemma 2. **(Static reinforcement)** Given the control structure depicted in Fig. 5, the design objectives (D2) and (D3) are satisfied if the parameters of the feedback law (9) are chosen so that the controlled vehicle (10) is asymptotically stable.

Proof. The transfer functions (cf. eqn. (5)) of the controlled vehicle are explicitly given by

$$\bar{G}(s) = \boldsymbol{c}^{\mathrm{T}}(s\boldsymbol{I} - \bar{\boldsymbol{A}})^{-1}\boldsymbol{e} = \frac{-k_{\mathrm{d}}\, s + k_{\mathrm{z}}}{m\, s^3 + (c + k_{\mathrm{v}})\, s^2 + (\beta k_{\mathrm{z}} - k_{\mathrm{d}})\, s + k_{\mathrm{z}}} \quad (11)$$

and due to (7) by

$$G_{\mathrm{d}}(s) = \frac{1}{s}(1 - \bar{G}(s)) = \frac{m\, s^2 + (c + k_{\mathrm{v}})\, s + \beta k_{\mathrm{z}}}{m\, s^3 + (c + k_{\mathrm{v}})\, s^2 + (\beta k_{\mathrm{z}} - k_{\mathrm{d}})\, s + k_{\mathrm{z}}}.$$

Evaluation of the limit $s \to 0$ reveals that the transfer functions $\bar{G}(s)$ and $G_{\mathrm{d}}(s)$ satisfy (D2) and (D3) for

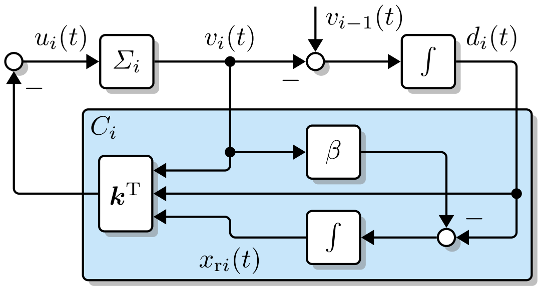

Fig. 5. Structure of the local ACC control loop

$k_{\mathrm{z}} \neq 0$. The necessary condition of the stability criterion by HURWITZ requires k_{z} to be positive. Thus, the design objectives (D2) and (D3) are satisfied if the controlled vehicle (10) is asymptotically stable. □

Lemma 2 shows that the requirement of stability (D1) is sufficient for (D2) and (D3) in the proposed control structure. This fact allows for reformulating Problem 1 as the following more specific design problem.

Problem 3. **(ACC design)** Given the parameters m, c and the time-headway coefficient β, find the feedback gain $\boldsymbol{k}$ that renders the controlled vehicle $\bar{\Sigma}_i$ asymptotically stable (D1) and the impulse response

$$\bar{g}(t) = \boldsymbol{c}^{\mathrm{T}} \mathrm{e}^{\bar{\boldsymbol{A}}t} \boldsymbol{e}$$

non-negative for $t \geq 0$ (D4).

In the next sections, conditions on external positivity are presented that will be used to develop a design procedure that solves Problem 3.

4. EXTERNALLY POSITIVE SYSTEMS

The monograph by Farina and Rinaldi (2011) gives a good overview on conditions, structures and realisability of externally positive systems which are defined as follows.

Definition 4. **(Externally positive system)** A system with state $\boldsymbol{x}(t)$, input $u(t)$ and output $y(t)$ is called *externally positive* if its output is non-negative for zero initial state and any non-negative input

$$\boldsymbol{x}(0) = \boldsymbol{0},\ u(t) \geq 0,\ t \geq 0 \implies y(t) \geq 0,\ t \geq 0.$$

A system is externally positive if and only if its impulse response $g(t)$ is non-negative

$$g(t) \geq 0, \quad t \geq 0. \quad (12)$$

The inequality (12) is the only necessary and sufficient condition known so far. Unfortunately, it is difficult to relate the controller parameters with the impulse response $g(t)$ in order to find parameters that satisfy (12). A practically useful sufficient condition for systems with *real* poles and zeros was presented by Rachid (1995) and considers a factorisation of a stable transfer function

$$g(t) \circ\!\!-\!\!\bullet\ G(s) = k \prod_{i=1}^{n} G_i(s) \quad (13)$$

with

$$G_i(s) = \begin{cases} \frac{s - s_{0i}}{s - s_i}, & i = 1, 2, \ldots, q \\ \frac{1}{s - s_i}, & i = q+1, q+2, \ldots, n \end{cases} \quad (14)$$

where q and n denote the number of zeros and the order of $G(s)$, respectively. The factorisation (13) represents a series connection of n first-order systems. Thus, if there exists a factorisation so that the elements (14) are externally positive, the impulse response $g(t)$ will be non-negative. The following lemma and theorem will give conditions under which the elements $(i = 1, 2, \ldots, q)$ of (14) and the factorisation (13) are externally positive.

Lemma 5. **(Externally positive first-order system)** The first-order system

$$\Sigma : \begin{cases} \dot{x}(t) = a\,x(t) + b\,u(t) \\ y(t) = c\,x(t) + d\,u(t) \end{cases}$$

with a feed-through $d \geq 0$ and the transfer function

$$G(s) = k\,\frac{s - s_{01}}{s - s_1}$$

is externally positive if and only if

$$s_{01} < s_1 \tag{15}$$

holds true, i. e. the zero is located to the left of the pole.

Proof. Explicit calculation of the transfer function yields

$$\begin{aligned} G(s) &= c(s-a)^{-1}b + d \\ &= d\,\frac{s - (a - cb/d)}{s - a}. \end{aligned}$$

Its corresponding impulse response

$$G(s) \bullet\!\!-\!\!\circ\, g(t) = c\,\mathrm{e}^{at}\,b + d\,\delta(t)$$

with $\delta(t)$ denoting the Dirac impulse is non-negative if and only if c and b have the same sign since d is assumed to be non-negative. Evaluation of the condition (15) yields

$$a - \frac{cb}{d} < a \iff \frac{cb}{d} > 0,$$

which is satisfied if $g(t)$ is non-negative since c and b have the same sign which proves the necessity. Sufficiency follows from the fact that all considerations can be applied in reverse order. □

Lemma 5 can be used to show under which condition a factorized transfer function (13) corresponds to a non-negative impulse response which was originally published by Rachid (1995) in similar form.

Theorem 6. **(Configuration of poles and zeros)** Consider a stable transfer function $G(s)$ with real poles and zeros. The impulse response $g(t) \circ\!\!-\!\!\bullet\, G(s)$ is non-negative if there exists a factorisation (13), so that the condition

$$s_{0i} < s_i, \quad i = 1, 2, \ldots, q \tag{16}$$

is satisfied.

Proof. The impulse response $g(t)$ can be written as

$$g(t) = k\,(g_1 * g_2 * \cdots * g_n)(t)$$

with $g_i(t) \circ\!\!-\!\!\bullet\, G_i(s)$. The first q elements of the convolution have a feed-through (cf. eqn. (14)) and are respectively non-negative under the condition (16) due to Lemma 5. The remaining $n - q$ impulse responses correspond to first-order lag systems which always have a non-negative impulse response (assuming positive static gain). Consequently, $G(s)$ represents an externally positive system since the convolution of non-negative functions is again non-negative. □

In the next section, a design procedure will be presented that renders the controlled vehicle externally positive using the sufficient condition of Theorem 6.

5. CONTROLLER DESIGN

5.1 State feedback gain

Since the control law (9) represents a full state feedback, Ackermann's formula

$$\boldsymbol{k}^{\mathrm{T}} = (\bar{a}_0\ \bar{a}_1\ \bar{a}_2\ 1) \underbrace{\begin{pmatrix} \boldsymbol{s}_{\mathrm{R}}^{\mathrm{T}} \\ \boldsymbol{s}_{\mathrm{R}}^{\mathrm{T}}\boldsymbol{A}_0 \\ \boldsymbol{s}_{\mathrm{R}}^{\mathrm{T}}\boldsymbol{A}_0^2 \\ \boldsymbol{s}_{\mathrm{R}}^{\mathrm{T}}\boldsymbol{A}_0^3 \end{pmatrix}}_{\boldsymbol{T}} \tag{17}$$

can be used to calculate the feedback gain with $\boldsymbol{A}_0$ from (8) and the last row of the inverse controllability matrix

$$\boldsymbol{s}_{\mathrm{R}}^{\mathrm{T}} := (0\ \cdots\ 0\ 1)\,\boldsymbol{S}^{-1},$$

which exists since the open-loop model (8) is completely controllable. The matrix $\boldsymbol{T}$ can be explicitly calculated as

$$\boldsymbol{T} = \begin{pmatrix} 0 & \beta m & m \\ 0 & -m & 0 \\ m & 0 & 0 \\ -c & 0 & 0 \end{pmatrix} \tag{18}$$

and the coefficients of the characteristic polynomial $p(\lambda) = \lambda^3 + \bar{a}_2\lambda^2 + \bar{a}_1\lambda + \bar{a}_0$ are given by

$$\bar{a}_0 = -\bar{\lambda}_1\bar{\lambda}_2\bar{\lambda}_3 \tag{19}$$

$$\bar{a}_1 = \bar{\lambda}_1\bar{\lambda}_2 + \bar{\lambda}_2\bar{\lambda}_3 + \bar{\lambda}_1\bar{\lambda}_3 \tag{20}$$

$$\bar{a}_2 = -(\bar{\lambda}_1 + \bar{\lambda}_2 + \bar{\lambda}_3), \tag{21}$$

where $\bar{\lambda}_i, (i = 1, 2, 3)$ denote the closed-loop eigenvalues. A combination of (17)–(21) yields

$$\boldsymbol{k} = \begin{pmatrix} -(\bar{\lambda}_1 + \bar{\lambda}_2 + \bar{\lambda}_3)\,m - c \\ -m\,(\beta\,\bar{\lambda}_1\bar{\lambda}_2\bar{\lambda}_3 + \bar{\lambda}_1\bar{\lambda}_2 + \bar{\lambda}_2\bar{\lambda}_3 + \bar{\lambda}_1\bar{\lambda}_3) \\ -\bar{\lambda}_1\bar{\lambda}_2\bar{\lambda}_3\,m \end{pmatrix}, \tag{22}$$

which allows to calculate the feedback gain explicitly for a given set of closed-loop eigenvalues. The arising question is how to find a set of eigenvalues that renders the closed-loop system externally positive. To answer this question, the zeros of the controlled vehicle will be examined since they play a significant role as shown by Theorem 6.

5.2 Zeros of the controlled vehicle

The Rosenbrock matrix of the closed-loop system

$$\boldsymbol{P}(\bar{\mu}) = \begin{pmatrix} \bar{\mu}\boldsymbol{I} - \bar{\boldsymbol{A}} & -\boldsymbol{e} \\ \boldsymbol{c}^{\mathrm{T}} & 0 \end{pmatrix}$$

allows for calculating all invariant zeros of the closed-loop system by solving

$$\det(\boldsymbol{P}(\bar{\mu})) = 0.$$

Straightforward calculation with the parameters of the model (10) yields

$$\bar{\mu} = \frac{k_{\mathrm{z}}}{k_{\mathrm{d}}}$$

and with (22)

$$\bar{\mu} = \frac{\bar{\lambda}_1\bar{\lambda}_2\bar{\lambda}_3}{\beta\,\bar{\lambda}_1\bar{\lambda}_2\bar{\lambda}_3 + \bar{\lambda}_1\bar{\lambda}_2 + \bar{\lambda}_2\bar{\lambda}_3 + \bar{\lambda}_1\bar{\lambda}_3}. \tag{23}$$

Thus, the system has a single real zero which depends on the set of desired eigenvalues of the controlled vehicle. Furthermore, $\bar{\mu}$ is the root of the nominator of the transfer function (11) and, hence, represents a transmission zero.

5.3 Design algorithm

The design approach is based on Theorem 6 which presents a sufficient condition for external positivity of systems with real poles and zeros. Without loss of generality, it is assumed that $\bar{\lambda}_1$ will be the dominant eigenvalue, i. e. the one with the largest real part. If the remaining eigenvalues $\bar{\lambda}_2$ and $\bar{\lambda}_3$ and the zero $\bar{\mu}$ are located to the left of $\bar{\lambda}_1$ in the stable half plane

$$\bar{\lambda}_2 < \bar{\lambda}_1 \tag{24}$$

$$\bar{\lambda}_3 < \bar{\lambda}_1 \tag{25}$$

$$\bar{\mu} < \bar{\lambda}_1, \tag{26}$$

the condition of Theorem 6 is satisfied and, thus, the controlled vehicle (10) externally positive.

In the following approach, the zero $\bar{\mu}$ will be set to be equal to $\bar{\lambda}_3$ in order to find a set of eigenvalues that satisfies the design objectives (24)–(26). The approach

$$\boxed{\bar{\lambda}_3 = \bar{\mu}} < 0 \tag{27}$$

and eqn. (23) yield the equivalent condition

$$0 = \bar{\lambda}_3 \left(\beta + \frac{1}{\bar{\lambda}_1} + \frac{1}{\bar{\lambda}_2} \right) \iff \beta + \frac{1}{\bar{\lambda}_1} + \frac{1}{\bar{\lambda}_2} = 0,$$

which can be rearranged to obtain

$$\boxed{\bar{\lambda}_2 = -\frac{\bar{\lambda}_1}{\beta \bar{\lambda}_1 + 1}}. \tag{28}$$

The relation (28) can be used to calculate $\bar{\lambda}_2$ for a given choice of $\bar{\lambda}_1$ and will ensure that the approach (27) holds true. To make sure that $\bar{\lambda}_2$ is stable, the condition

$$\beta \bar{\lambda}_1 + 1 < 0 \iff \boxed{\bar{\lambda}_1 < -\frac{1}{\beta}}$$

has to be satisfied. A second condition on the choice of $\bar{\lambda}_1$ arises from the design objective (24) combined with (28), which reads as

$$\bar{\lambda}_1 \left(1 + \frac{1}{\beta \bar{\lambda}_1 + 1} \right) > 0 \iff 1 + \frac{1}{\beta \bar{\lambda}_1 + 1} < 0$$

and is satisfied if

$$\boxed{\bar{\lambda}_1 > -\frac{2}{\beta}}$$

holds true. These results prove the following theorem.

Theorem 7. **(ACC controller design)**
The controlled vehicle (10) is externally positive with the parameters of the feedback gain (22) and a given time-headway coefficient β if the closed-loop eigenvalues are chosen according to

$$\bar{\lambda}_1 \in \left(-\frac{2}{\beta}, -\frac{1}{\beta} \right) \tag{29}$$

$$\bar{\lambda}_2 = -\frac{\bar{\lambda}_1}{\beta \bar{\lambda}_1 + 1} \tag{30}$$

$$\bar{\lambda}_3 = \bar{\mu} \tag{31}$$

for a given $\bar{\mu} < \bar{\lambda}_1$.

A set of eigenvalues determined according to Theorem 7 satisfies the conditions (24)–(26) and, hence, renders the closed-loop system asymptotically stable and externally positive which solves Problem 3, i. e. the design objectives (D1)–(D4) are satisfied. The controller design procedure is summarised in the following algorithm.

Algorithm 1. **(Design procedure)**

Given: Vehicle parameters m, c and time-headway β

1. Choose a dominant eigenvalue $\bar{\lambda}_1$ in the interval (29)
2. Choose the location of the zero so that $\bar{\mu} < \bar{\lambda}_1$ holds
3. Calculate $\bar{\lambda}_2$ and $\bar{\lambda}_3$ with (30) and (31)
4. Calculate the feedback gain $\boldsymbol{k}$ with (22)

Result: Externally positive controlled vehicle (10)

Discussion. The dominant eigenvalue of a linear system determines the largest time constant in the dynamical behaviour. In the presented procedure, the dominant time constant ranges in the interval from $\beta/2$ to β, i. e. the smaller the desired time-headway coefficient is chosen the quicker the controlled vehicle has to be. If it is not possible to implement a controller that satisfies condition (29) due to physical limitations (e. g. heavy vehicles, trucks), a cooperative adaptive cruise control (CACC) has to be used to solve the platooning problem using additional communication links. A method to design the communication structure in the specific case that β is too small for the intended application was presented by Lunze (2019b,a).

The choice of the location of the zero is arbitrary apart from the condition $\bar{\mu} < \bar{\lambda}_1$. This is due to the fact that the eigenvalue $\bar{\lambda}_3$ and the zero cancel each other and, hence, do not appear in the transient behaviour. However, the presented procedure guarantees that $\bar{\lambda}_3$ is stable which is important since a pole/zero cancellation is not robust in the sense of model uncertainties. A small deviation

$$|\bar{\mu} - \bar{\lambda}_3| < \epsilon$$

with $\epsilon > 0$ does not jeopardise the stability or external positivity of the controlled vehicle.

6. SIMULATION EXAMPLE

The proposed design method is demonstrated with a vehicle with the mass $m = 1000\,\mathrm{kg}$, friction constant $c = 200\,\mathrm{kg/s}$ and time-headway coefficient $\beta = 2\,\mathrm{s}$. The dominant eigenvalue is chosen to be

$$\bar{\lambda}_1 = -0.75\,\mathrm{s}^{-1},$$

which lies in the interval $(-1\,\mathrm{s}^{-1}, -0.5\,\mathrm{s}^{-1})$ given by (29). The closed-loop zero is chosen as

$$\bar{\mu} = 3\,\bar{\lambda}_1 = -2.25\,\mathrm{s}^{-1}$$

to satisfy the objective (26). The remaining eigenvalues

$$\bar{\lambda}_2 = -1.5\,\mathrm{s}^{-1} \quad \text{and} \quad \bar{\lambda}_3 = \bar{\mu} = -2.25\,\mathrm{s}^{-1}$$

are calculated with eqns. (30) and (31). The resulting feedback gain

$$\boldsymbol{k}^{\mathrm{T}} = \begin{pmatrix} 4300\,\mathrm{kg/s} & -1125\,\mathrm{kg/s^2} & 2531.25\,\mathrm{kg/s^3} \end{pmatrix} \tag{32}$$

obtained by eqn. (22) renders the controlled vehicle externally positive as shown by Fig. 6, where $\bar{g}(t) \circ\!\!-\!\!\bullet\ \bar{G}(s)$ and $g_{\mathrm{d}}(t) \circ\!\!-\!\!\bullet\ G_{\mathrm{d}}(s)$ denote the impulse responses of the controlled vehicle (cf. models (5) and (10)).

The transient behaviour of a set of $N = 20$ identical vehicles (10) with the controller (32) organised in a platoon is illustrated in Figure 7. All vehicles are starting from rest and follow the switching reference trajectory

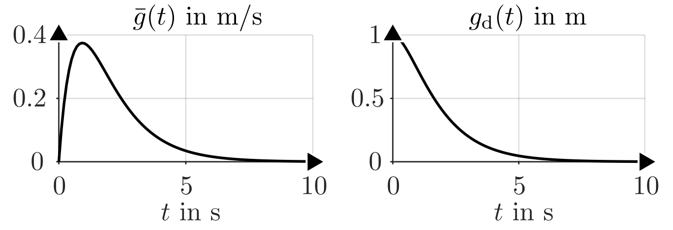

Fig. 6. Impulse responses of the controlled vehicle

$$v_0(t) = \begin{cases} 20\,\mathrm{m/s}, & 0\,\mathrm{s} \leq t < 30\,\mathrm{s} \\ 4\,\mathrm{m/s}, & 30\,\mathrm{s} \leq t < 60\,\mathrm{s} \\ 14\,\mathrm{m/s}, & 60\,\mathrm{s} \leq t \end{cases}$$

depicted in the middle graph of Fig. 7. The velocity and inter-vehicle distance shown in the bottom two graphs of Fig. 7 are always non-negative due to the external positivity of the individual vehicles. This behaviour guarantees collision avoidance which can be seen in the top graph with the positions of all vehicles which are separated

$$s_{i-1}(t) - s_i(t) \geq d_0, \quad i = 2, 3, \ldots, N$$

by at least $d_0 = 5\,\mathrm{m}$ at any time.

7. CONCLUSION

This paper has presented a design procedure for adaptive cruise controllers that guarantee collision avoidance. The proposed control structure is working with the driven velocity and inter-vehicle distance, which are locally measurable quantities. Based on a sufficient condition, calculation rules that yield a set of closed-loop eigenvalues are

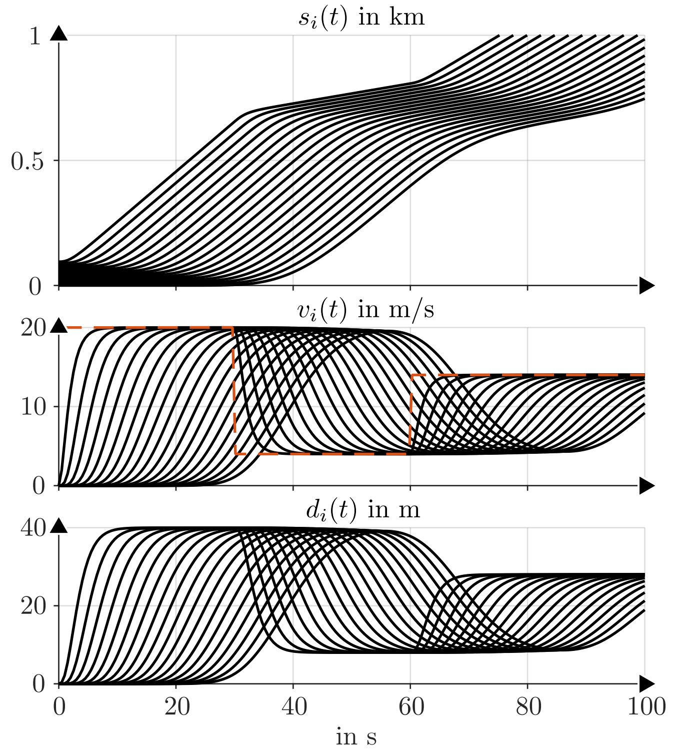

Fig. 7. Position, velocity and inter-vehicle distance of the platoon ($v_0(t)$ – –)

presented which can be placed via Ackermann's formula to achieve externally positive dynamics.

The proposed control scheme uses the multi-agent approach on the platooning problem, which is characterised by considering what properties the overall system should have to show a desired behaviour and then deriving what conditions have to be imposed on the subsystems to achieve said behaviour. This strategy with external positivity as the key property of the controlled vehicles allows for assembling arbitrarily long platoons with guaranteed collision avoidance.

In order to improve safety or to include slow vehicles in the platoon, the presented approach can be extended to CACC structures with communication systems. The design of the communication structure of CACC systems was discussed by Lunze (2019b,a).

Future work will include a generalisation of the design approach to higher-order vehicle models that take the dynamics of the power train with possibly complex conjugate poles into account.

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