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Sensor Fault Identification in Nonlinear Dynamic Systems ⋆

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Abstract: The problem of sensor fault identification in engineering systems described by nonlinear dynamic models is considered. To solve the problem, sliding mode observers are used. The feature of the suggested solution is using the reduced order model of the initial system to design sliding mode observer. This allows to obtain sliding mode observer of reduced dimension and extend a class of systems for which sliding mode observer can be constructed.

Keywords: Nonlinear dynamic systems, faults, identification, sliding mode observers, reduced order models.

1. INTRODUCTION

This paper is devoted to the problem of fault diagnosis in engineering systems. This problem was extensively investigated for the past 30 years, see, e.g., Blanke et al. (2006); Ding (2014); Samy et al. (2011) where different tools for fault diagnosis have been developed, in particular, identification. There are many methods of identification, one is based on sliding mode observers and uses peculiarities of sliding motion Utkin (1992).

Sliding mode observers are used for fault identification (reconstruction) in linear systems Edwards and Spurgeon (1994); Edwards et al. (2000); Tan and Edwards (2009); Chandra et al. (2015); Zhirabok et al. (2019), nonlinear systems Yan and Edwards (2007); He and Zhang (2012); Brahim et al. (2017), and descriptor systems Chan et al. (2017), for fault tolerant control Corradini et al. (2005); Alwi and Edwards (2008), in practical applications Chandra et al. (2015); Meziane et al. (2015); Mohamed et al. (2016); Yang and Yin (2020); Zhang et al. (2016).

In this paper, the method based on sliding mode observers is used to solve the problem of sensor fault identification in nonlinear systems under disturbances. This problem was studied in Edwards et al. (2000); Tan and Edwards (2002, 2003); Filasova and Krokavec (2010); Kalsi et al. (2011).

To discuss features of some of these papers, consider system described by nonlinear dynamic model

$$\begin{aligned} \dot{x}(t) &= Fx(t) + Gu(t) + C\Psi(x(t), u(t)) + L\rho(t), \\ y(t) &= Hx(t) + \sum_{i=1}^{l} D_i d_i(t). \end{aligned} \quad (1)$$

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Here $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^l$ are vectors of state, control, and output, F, G, H, C, and L are constant matrices; D_i and $d_i(t) \in R$ are matrix and function describing faults: if faults are absent, $d_i(t) = 0$, if a fault occurs in the ith sensor, $d_i(t)$ becomes an unknown bounded function of time, $i = 1, ..., l$; $D_1 = (1\,0\,...\,0)^T$, ... $D_l = (0\,0\,...\,1)^T$; $\rho(t) \in R^p$ is the disturbance, we assume that $\rho(t)$ is an unknown bounded function of time; $\Psi(x, u)$ is nonlinear term:

$$\Psi(x, u) = \begin{pmatrix} \varphi_1(A_1 x, u) \\ ... \\ \varphi_q(A_q x, u) \end{pmatrix},$$

A_1, ..., A_q are constant row matrices, φ_1, ..., φ_q are nonlinear functions.

For simplicity, consider the case when the fault can occur in single sensor only with the number j, appropriate matrix $D := D_j$, and function $d(t) := d_j(t)$, that is the jth component of the vector y is subjected to the fault.

To solve the problem of sensor fault identification, the methods suggested in Tan and Edwards (2003) and similar papers assume that a new state vector being a filtered version of $y(t)$ is introduced and special system of the dimension $n + l$ is constructed. It is known that under some conditions the extended system is transformed into decomposition of two subsystems having the following peculiarities: the functions $d(t)$ and $\rho(t)$ enter in one of the subsystem only, the output vector $y(t)$ depends on the state vector of this subsystem, and the second subsystem is stable. Based on this decomposition, sliding mode observer is designed. The methods suggested in Edwards et al. (2000); Kalsi et al. (2011) provide only approximate solution of the sensor fault identification problem since the final expressions contain the derivative $\dot{d}(t)$.

The contribution of the present paper is that in contrast to these methods, we construct sliding mode observer based on the reduced order model of the initial system of the dimension $k < n$ invariant with respect to the disturbance which does not contain the derivative $\dot{d}(t)$. Besides, the problem of sensor fault identification is solved without conditions imposed in Tan and Edwards (2003).

As usual, it is assumed that the function $C\Psi(x,u)$ satisfies Lipschitz condition about x uniformly for t and u:

$$\|C(\Psi(x,u) - \Psi(x',u))\| \leq N\|x - x'\|, \qquad (2)$$

where N is some positive constant. This condition is not satisfied for so phenomenons as Coulomb friction, backlash, and square root. To take into account such phenomenons, we introduce the generalized Lipschitz condition:

$$\|C(\Psi(x,u) - \Psi(x',u))\| \leq N\|x - x'\| + M, \qquad (3)$$

N, $M \geq 0$ are some constants.

The rest of the paper is organized as follows. In Section 2, the reduced order model of the initial system is constructed. Section 3 presents the FD problem different solutions. Practical example is considered in Section 4. Section 5 concludes the paper.

2. REDUCED ORDER MODEL DESIGN

Depending on how the signal of faulty sensor enters in sliding mode observer, we consider three variants.
1. This signal enters in the linear terms of the observer only.
2. This signal enters in the nonlinear terms of the observer only.
3. This signal enters both in the linear and nonlinear terms of the observer.

Consider initially the first variant. In this case solution of the problem is based on the reduced model of the initial system (1):

$$\begin{aligned} \dot{x}_*(t) &= F_*x_*(t) + G_*u(t) + J_*Hx(t) \\ &\quad + C_*\Psi(x_*(t), H^{(j)}x(t), u(t)) + L_*\rho(t), \\ y_*(t) &= H_*x_*(t) + D_*d(t), \end{aligned} \qquad (4)$$

where $x_*(t) \in R^k$ is the state vector, F_*, G_*, J_*, H_*, D_*, and L_* are some matrices to be determined,

$$C_*\Psi(x_*, H^{(j)}x, u) = \begin{pmatrix} \varphi_{i_1}(A_{*1i_1}x_* + A_{*2i_1}H^{(j)}x, u) \\ ... \\ \varphi_{i_k}(A_{*1i_k}x_* + A_{*2i_k}H^{(j)}x, u) \end{pmatrix}, \qquad (5)$$

A_{*1i_1}, A_{*2i_1}, ..., A_{*1i_k}, A_{*2i_k} are matrices to be determined, $y^{(j)} = H^{(j)}x$ is the vector y without jth element corresponding to the faulty sensor, $H^{(j)}$ is the corresponding matrix. This means that the nonlinear term in (4) does not depend on the jth component of the vector y subjected to the fault.

We assume that $x_*(t) = \Phi x(t)$ and $y_*(t) = R_*y(t)$ for matrices Φ and R_* under $d(t) = 0$ and $\rho(t) = 0$. These matrices satisfy the conditions Zhirabok et al. (2017a,b)

$$\begin{aligned} &\Phi F = F_*\Phi + J_*H, \quad R_*H = H_*\Phi, \quad \Phi G = G_*, \\ &A_i = (A_{*1i} \;\; A_{*2i})\begin{pmatrix} \Phi \\ H^{(j)} \end{pmatrix}, \quad i = i_1, ..., i_k, \\ &\Phi C = C_*, \quad R_*D = D_*, \quad \Phi L = L_*. \end{aligned} \qquad (6)$$

Consider the method to construct system (4) insensitive to the disturbance which will be used for sliding mode observer design. The matrices F_* and H_* are sought in the canonical form

$$F_* = \begin{pmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & \\ 0 & 0 & 0 & ... & 0 \end{pmatrix}, \quad H_* = (\,1 \; 0 \; 0 \; ... \; 0\,). \qquad (7)$$

Based on the matrices F_* and H_* in (7), equations for rows of the matrices Φ and J_* are obtained from (6):

$$\begin{aligned} &\Phi_1 = R_*H, \quad \Phi_iF = \Phi_{i+1} + J_{*i}H, \quad i = 1, ..., k-1, \\ &\Phi_kF = J_{*k}H, \end{aligned} \qquad (8)$$

where Φ_i and J_{*i} are ith rows of the matrices Φ and J_*, $i = 1, ..., k$, k is the model (4) dimension.

To construct sliding mode observer, the condition $R_*D = 0$ should be satisfied. To take it into account, introduce the matrix D^0 of maximal rank such that $D^0D = 0$, then $R_* = SD^0$ is true for some matrix S.

It is shown in Zhirabok et al. (2017a) that (8) can be rewritten in the form of single equation

$$SD^0F^k = J_{*1}HF^{k-1} + J_{*2}HF^{k-2} + ... + J_{*k}H.$$

Rewrite it in as

$$(\,S \;\; -J_{*1} \;\; ... \;\; -J_{*k}\,)V^{(k)} = 0, \qquad (9)$$

where

$$V^{(k)} = \begin{pmatrix} D^0HF^k \\ HF^{k-1} \\ ... \\ H \end{pmatrix}.$$

As is known Zhirabok et al. (2017a,b), the condition of insensitivity to the disturbance $\rho(t)$ is of the form $L_* = \Phi L = 0$; it can be shown Zhirabok et al. (2017a,b) that this condition can be presented in the form

$$(\,S \;\; -J_{*1} \;\; ... \;\; -J_{*k}\,)L^{(k)} = 0, \qquad (10)$$

where

$$L^{(k)} = \begin{pmatrix} D^0HL & D^0HFL & D^0HF^2L & ... & D^0HF^{k-1}L \\ 0 & HL & HFL & ... & HF^{k-2}L \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 0 \end{pmatrix}.$$

Since the row $(\,S \;\; -J_{*1} \;\; ... \;\; -J_{*k}\,)$ satisfies the condition (9), it follows from (9) and (10)

$$(\,S \;\; -J_{*1} \;\; ... \;\; -J_{*k}\,)(V^{(k)} \;\; L^{(k)}) = 0. \qquad (11)$$

Equation (11) has a nontrivial solution if

$$rank(V^{(k)} \;\; L^{(k)}) < l(k+1) - 1.$$

To construct the model (4) of minimal dimension, find minimal k for which equation (11) has a solution and find a solution of (11), then obtain from (8) the matrix Φ, set $C_* := \Phi C$, calculate (5), and check the condition

$$rank\begin{pmatrix} \Phi \\ H^{(j)} \end{pmatrix} = rank\begin{pmatrix} \Phi \\ H^{(j)} \\ A_i \end{pmatrix}, \quad i = i_1, ..., i_k. \qquad (12)$$

If it is true, set $G_* := \Phi G$ and $D_* := \Phi D$; the matrices A_{*1i} and A_{*2i}, $i = i_1, ..., i_k$, entering in the nonlinear function $C_*\Psi(x_*, H^{(j)}x, u)$, are found from (6). If (12) is not true, one finds another solution of (11) with former or incremented dimension k.

As a result, the model (4) takes the form

$$\begin{aligned} \dot{x}_*(t) &= F_*x_*(t) + G_*u(t) + J_*Hx(t) \\ &\quad + C_*\Psi(x_*(t), H^{(j)}x(t), u(t)), \\ y_*(t) &= H_*x_*(t) = R_*y(t), \end{aligned} \tag{13}$$

3. SLIDING MODE OBSERVER DESIGN

3.1 Problem Solution: the Main Relations

Analogously to He and Zhang (2012), sliding mode observer should be written in the form

$$\begin{aligned} \dot{\hat{x}}_*(t) &= F_*\hat{x}_*(t) + G_*u(t) + J_*y(t) - J_*Dv(t) \\ &\quad + C_*\Psi(\hat{x}_*(t), y^{(j)}(t), u(t)) - Ke_y(t), \\ \hat{y}_*(t) &= H_*\hat{x}_*(t). \end{aligned} \tag{14}$$

Here the matrix K is chosen so that $F_{**} = F_* - KH_*$ to be stable matrix, the discontinuous vector $v(t)$ is defined by

$$v(t) = \begin{cases} g\dfrac{Qe_y(t)}{\|Qe_y(t)\|}, & \text{if } e_y(t) \neq 0, \\ 0, & \text{otherwise,} \end{cases} \tag{15}$$

$e_y(t) = \hat{y}_*(t) - y_*(t) = \hat{y}_*(t) - R_*y(t)$ is the output estimation error; the rules to choose the matrix Q and positive scalar g are discussed below. Note that since the matrices F_* and H_* are in the canonical form (7), the matrix K always exists.

Define $e(t) = \hat{x}_*(t) - x_*(t)$; since $R_*H = H_*\Phi$ and $R_*D = 0$, then

$$\begin{aligned} e_y &= \hat{y}_* - R_*y = H_*\hat{x}_* - R_*(Hx + Dd) \\ &= H_*\hat{x}_* - R_*Hx + R_*Dd = H_*e. \end{aligned}$$

Based on (13) and (14), one obtains the equation for $e(t)$:

$$\begin{aligned} \dot{e}(t) &= F_*e(t) + J_*(y(t) - Hx(t)) \\ &\quad - J_*Dv(t) + \Delta\Psi(t) - Ke_y(t) \\ &= F_{**}e(t) + J_*D(d(t) - v(t)) + \Delta\Psi(t), \end{aligned} \tag{16}$$

where

$$\begin{aligned} \Delta\Psi(t) &= C_*(\Psi(\hat{x}_*(t), y^{(j)}(t), u(t)) \\ &\quad - \Psi(x_*(t), H^{(j)}x(t), u(t))). \end{aligned}$$

Since the function $\Psi(x, u)$ satisfies the generalized Lipschitz condition (3) about x, then the function $C_*\Psi(x_*(t), H^{(j)}x(t), u(t))$ satisfies this condition about x_* and

$$\begin{aligned} \|\Delta\Psi(t)\| &= \|C_*(\Psi(\hat{x}_*(t), y^{(j)}(t), u(t)) \\ &\quad - \Psi(x_*(t), H^{(j)}x(t), u(t)))\| \\ &\leq N_*\|e(t)\| + M_* \end{aligned} \tag{17}$$

for some scalars $N_*, M_* \geq 0$. Note that the nonzero value of M_* is used to provide existence of sliding motion in the observer (14) (see the condition (19)). On the other hand, it is known Edwards et al. (2000) that a sliding motion takes place forcing $e(t) = 0$, that is $\hat{x}_*(t) = x_*(t)$, and one may set here $M_* := 0$.

Since the matrix F_{**} is stable, then for arbitrary symmetric positive defined matrix W there exists the symmetric positive defined matrix P such that $F_{**}^T P + PF_{**} = -W$. By analogy with He and Zhang (2012), we assume that there exists the matrix Q so that

$$PJ_*D = H_*^T Q^T. \tag{18}$$

Theorem. If $\lambda_{min}(W) > 2N_*\|P\|$ and scalar g satisfies the condition

$$g > \|d(t)\| + M_*\frac{\|P\|}{\|QH_*\|}, \tag{19}$$

then the sliding motion of system (16) is asymptotically stable.

Proof. Consider the following Lyapunov function

$$V(t) = e^T(t)Pe(t)$$

and using (16) find its derivative with respect to time:

$$\begin{aligned} \dot{V}(t) &= (F_{**}e(t) + J_*D(d(t) - v(t)) \\ &\quad + \Delta\Psi(t))^T Pe(t) + e^T(t)P(F_{**}e(t) \\ &\quad + J_*D(d(t) - v(t)) + \Delta\Psi(t)) \\ &= e^T(t)(F_{**}^T P + PF_{**})e(t) \\ &\quad + (J_*D(d(t) - v(t)))^T Pe(t) \\ &\quad + e^T(t)PJ_*D(d(t) - v(t)) + 2(Pe(t))^T\Delta\Psi(t). \end{aligned} \tag{20}$$

Clearly, there exists positive defined matrix W such that the first addend in (20) takes the form $-e^T(t)We(t)$.

Using (15) and (18), one transforms the expression $(J_*D(d(t) - v(t)))^T Pe(t) + e^T(t)PJ_*D(d(t) - v(t))$:

$$\begin{aligned} &(J_*D(d(t) - v(t)))^T Pe(t) \\ &\quad + e^T(t)PJ_*D(d(t) - v(t)) \\ &= 2e^T(t)PJ_*D(d(t) - v(t)) \\ &= -2e^T(t)H_*^T Q^T v(t) + 2e^T(t)H_*^T Q^T d(t) \\ &= -2ge^T(t)H_*^T Q^T\frac{Qe_y(t)}{\|Qe_y(t)\|} + 2(QH_*e(t))^T d(t) \\ &= -2g(QH_*e(t))^T\frac{QH_*e(t)}{\|QH_*e(t)\|} + 2(QH_*e(t))^T d(t) \\ &= -2g\|QH_*e(t)\| + 2(QH_*e(t))^T d(t). \end{aligned} \tag{21}$$

Combine the obtained expression with $-e^T(t)We(t)$ and the last addend in (20) $2(Pe(t))^T\Delta\Psi(t)$ and transform the result:

$$\begin{aligned} \dot{V}(t) &= -e^T(t)We(t) - 2g\|QH_*e(t)\| \\ &\quad + 2(QH_*e(t))^T d(t) + 2(Pe(t))^T\Delta\Psi(t) \\ &\leq -\lambda_{min}(W)\|e(t)\|^2 - 2g\|QH_*e(t)\| \\ &\quad + 2\|QH_*e(t)\|\|d(t)\| + 2\|P\|\|\Delta\Psi(t)\|\|e(t)\| \\ &\leq -\lambda_{min}(W)\|e(t)\|^2 - 2\|QH_*\|\|e(t)\| \\ &\quad \times(g - \|d(t)\|) + 2\|P\|\|e(t)\|(N_*\|e(t)\| + M_*) \\ &= -(\lambda_{min}(W) - 2\|P\|N_*)\|e(t)\|^2 \\ &\quad - 2\|QH_*\|\|e(t)\| \\ &\quad \times\left(g - \|d(t)\| - M_*\frac{\|P\|}{\|QH_*\|}\right) < 0; \end{aligned} \tag{22}$$

one accounts the conditions $\lambda_{min}(W) > 2N_*\|P\|$ and (19) in the last inequality. Hence $\dot{V}(t) < 0$ that completes the proof.

It is known Edwards et al. (2000) that a sliding motion takes place forcing $\dot{e}(t) = e(t) = 0$, then (16) implies

$$0 = D_*(v(t) - d(t)) + \Delta\Psi(t).$$

Since in sliding motion $\|\Delta\Psi(t)\| \leq N_*\|e(t)\| = 0$ (see remark after (17)), the function $d(t)$ can be estimated in the form

$$\hat{d}(t) = g\frac{Qe_y(t)}{\|Qe_y(t)\| + \delta},$$

where $\delta > 0$ is small scalar. Note that right-hand side of the last equation depends only on the output estimation error $e_y(t) = \hat{y}_*(t) - R_*y(t)$.

3.2 Problem Solution: the Second Variant

Consider the case when the jth component of the vector y, subjected to the fault, enters in the nonlinear term only. In this case it is suggested to find the approximate solution as follows. Assume that the function $d(t)$ is sufficiently small, then

$$\begin{aligned} C_*\Psi(\hat{x}_*, y, u) &\approx C_*\Psi(\hat{x}_*, Hx, u) + \frac{\partial C_*\Psi(\hat{x}_*, y, u)}{\partial y}d \\ &= C_*\Psi(\hat{x}_*, Hx, u) + d_*, \end{aligned} \quad (23)$$

where

$$d_* = C_{**}d, \quad C_{**} = \frac{\partial C_*\Psi(\hat{x}_*, y, u)}{\partial y}.$$

In this case (14), (16), and (18) should be modified as

$$\begin{aligned} \dot{\hat{x}}_*(t) &= F_*\hat{x}_*(t) + G_*u(t) + J_*y(t) \\ &\quad + C_*\Psi(\hat{x}_*(t), Hx(t), u(t)) - v(t) - Ke_y(t), \\ \hat{y}_*(t) &= H_*\hat{x}_*(t), \\ \dot{e}(t) &= F_{**}e(t) + d_*(t) - v(t) + \Delta\Phi_*(t), \\ P &= H_*^TQ^T, \end{aligned} \quad (24)$$

respectively, where

$$\begin{aligned} \Delta\Phi_*(t) &= C_*(\Psi(\hat{x}_*(t), Hx(t), u(t)) \\ &\quad - \Psi(x_*(t), Hx(t), u(t))). \end{aligned} \quad (25)$$

Then (21) results in

$$-2g_2\|QH_*\|e(t) + 2(QH_*e(t))^Td_*(t)$$

and (22) is modified as follows:

$$\begin{aligned} \dot{V}(t) &\leq -(\lambda_{min}(W) - 2\|P\|N_*)\|e(t)\|^2 \\ &\quad - 2\|QH_*\|\|e(t)\| \\ &\quad \times \left(g_2 - \|d_*(t)\| - M_*\frac{\|P\|}{\|QH_*\|}\right). \end{aligned}$$

Clearly, $\dot{V}(t) < 0$ if $\lambda_{min}(W) > 2N_*\|P\|$ and

$$g_2 > \|d_*(t)\| + M_*\frac{\|P\|}{\|QH_*\|}.$$

Then the function $d(t)$ can be estimated as

$$\hat{d}(t) = g_2(C_{**}^TC_{**})^{-1}C_{**}^T\frac{Qe_y(t)}{\|Qe_y(t)\| + \delta}.$$

Note that for some types of nonlinearities one may obtain exact solution (see Example).

3.3 Problem Solution: the Third Variant

As before, we find the approximate solution. In this case combination of (14), (16), and (18) results in

$$\begin{aligned} \dot{\hat{x}}_*(t) &= F_*\hat{x}_*(t) + G_*u(t) + J_*y(t) - v(t) \\ &\quad + C_*\Psi(\hat{x}_*(t), y(t), u(t)) - Ke_y(t), \\ \hat{y}_*(t) &= H_*\hat{x}_*(t), \\ \dot{e}(t) &= F_{**}e(t) + C_0d(t) - v(t) + \Delta\Psi_*(t), \\ P &= H_*^TQ^T, \end{aligned}$$

where $\Delta\Psi_*(t)$ is given by (25), $C_0 = J_*D + C_{**}$. Then (21) results in

$$-2g_3\|QH_*\|e(t) + 2e(t)^TPC_0d(t)$$

and (22) is modified as follows:

$$\begin{aligned} \dot{V}(t) &\leq -(\lambda_{min}(W) - 2\|P\|N_*)\|e(t)\|^2 \\ &\quad - 2\|QH_*\|\|e(t)\| \\ &\quad \times \left(g_3 - \|d(t)\|\frac{\|PC_0\|}{\|QH_*\|} - M_*\frac{\|P\|}{\|QH_*\|}\right). \end{aligned}$$

Clearly, $\dot{V}(t) < 0$ if $\lambda_{min}(W) > 2N_*\|P\|$ and

$$g_3 > \|d(t)\|\frac{\|PC_0\|}{\|QH_*\|} + M_*\frac{\|P\|}{\|QH_*\|}. \quad (26)$$

Then the function $d(t)$ can be estimated as

$$\hat{d}(t) = g_3(C_0^TC_0)^{-1}C_0^T\frac{Qe_y(t)}{\|Qe_y(t)\| + \delta}.$$

3.4 Problem Solution: Special Case

Consider a special case when the nonlinear term in (4) $C_*\Psi(x_*(t), H^{(j)}x(t), u(t))$ does not depend on the variable $x_*(t)$. Clearly, the equation (16) becomes linear due to $\Delta\Psi = 0$, and a solution is simplified. Really, construct sliding mode observer in the form

$$\begin{aligned} \dot{\hat{x}}_*(t) &= F_*\hat{x}_*(t) + G_*u(t) + J_*y^{(j)}(t) \\ &\quad + \Psi_*(y^{(j)}(t), u(t)) + D_*v(t) - Ke_y(t), \\ \hat{y}_*(t) &= H_*\hat{x}_*(t), \end{aligned} \quad (27)$$

where the matrix K is chosen such that $F_{**} = F_* - KH_*$ to be stable matrix, the discontinuous vector $v(t)$ is defined by (15), the function $\Psi_*(H^{(j)}x, u)$ by assumption depends only on the variables $y^{(j)} = H^{(j)}x$ and u, the matrix Q and scalar g are chosen as above. Note that if $C_* = 0$, the observer (27) does not contain the nonlinear term.

Then the equation for $e(t)$ takes the form

$$\begin{aligned} \dot{e}(t) &= F_*e(t) + D_*(v(t) - d(t)) - Ke_y(t) \\ &= F_{**}e(t) + D_*(v(t) - d(t)). \end{aligned}$$

It can be shown that the sliding motion of system (16) is asymptotically stable if $g > \|d(t)\|$ since $N_* = M_* = 0$ in (19).

There are two sufficient criteria of independence of nonlinear term $C_*\Psi(x_*, y, u)$ of the variable x_* based on (12):
1) each row A_i linearly depends on rows of the matrix $H^{(j)}$ which is equivalent to

$$rank(H^{(j)}) = rank\begin{pmatrix} H^{(j)} \\ A_i \end{pmatrix}, \quad i = i_1, ..., i_k;$$

2) if some row A_i does not depend on rows of the matrix $H^{(j)}$, then ith row of the matrix C_* is equal to zero.

When the jth component of the vector y, subjected to the fault, is in the nonlinear term only, the condition above is replaced by

$$rank(H) = rank\begin{pmatrix} H \\ A_i \end{pmatrix}, \quad i = i_1, ..., i_k. \quad (28)$$

4. EXAMPLE

Consider the control system

$$\begin{aligned} \dot{x}_1 &= u/\vartheta_1 - a_4\sqrt{x_1 - x_2}, \\ \dot{x}_2 &= a_4\sqrt{x_1 - x_2} - a_5\sqrt{x_2 - x_3}, \\ \dot{x}_3 &= a_5\sqrt{x_2 - x_3} - a_6\sqrt{x_3 - \vartheta_7}, \\ y_1 &= x_1, \quad y_2 = x_2, \end{aligned} \tag{29}$$

where $a_4 = \vartheta_4\sqrt{2\vartheta_8}/\vartheta_1$, $a_5 = \vartheta_5\sqrt{2\vartheta_8}/\vartheta_2$, and $a_6 = \vartheta_6\sqrt{2\vartheta_8}/\vartheta_3$.

The equations (29) constitute a model of the example of three tank system (Patton and Chen (1991)). The system consists of three consecutively united tanks with areas of the cross-section ϑ_1, ϑ_2, and ϑ_3. The tanks are linked by pipes with areas of the cross-section ϑ_4 and ϑ_5. The liquid flows into the first tank and follows from the third one through the pipe with area of the cross-section ϑ_6 which is located at height ϑ_7; ϑ_8 is the gravitational constant. The levels of liquid in the tanks are x_1, x_2, and x_3, respectively. Assume for simplicity that $\vartheta_1 = 1$ and $\vartheta_7 = 0$.

Clear, $F = 0$ in the model (1). To overcome this difficulty, transform the initial equations by entering formal addends $-(x_1 - x_2) + (x_1 - x_2)$, $(x_1 - 2x_2 + x_3) - (x_1 - 2x_2 + x_3)$, and $(x_2 - 2x_3) - (x_2 - 2x_3)$ in the first, second, and third equations, respectively. As a result, the system is described by matrices and nonlinearities as follows:

$$F = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix}, \quad G = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\Psi(x,u) = \begin{pmatrix} -a_4\sqrt{A_1 x} + A_1 x \\ a_4\sqrt{A_1 x} - a_5\sqrt{A_2 x} - (A_1 x - A_2 x) \\ a_5\sqrt{A_2 x} - a_6\sqrt{A_3 x} - (A_2 x - A_3 x) \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad D_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad D_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad L = 0,$$

$$A_1 = (1\ -1\ 0), \quad A_2 = (0\ 1\ -1), \quad A_3 = (0\ 0\ 1),$$

Consider the fault in the second sensor where $D^0 = (1\ \ 0)$. Equation (11) with $k = 1$ takes the form

$$(S\ \ -J_*) = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

and has a solution $S = 1$, $J_* = (-1\ 1)$. As a result, $R_* = (1\ 0)$, $\Phi = (1\ 0\ 0)$, $G_* = (1\ 0)$, and $C_* = (1\ 0\ 0)$. It can be shown that the condition (12) is satisfied for A_1 contained in $C_*\Psi(x,u)$ and $A_{*1} = (0\ 0\ 1\ -1)$. As a result, the model (13) is of the form

$$\begin{aligned} \dot{x}_* &= -(H_1 x - H_2 x) + u - a_4\sqrt{H_1 x - H_2 x} \\ &\quad + H_1 x - H_2 x = u - a_4\sqrt{H_1 x - H_2 x}, \\ y_* &= x_* = y_1. \end{aligned}$$

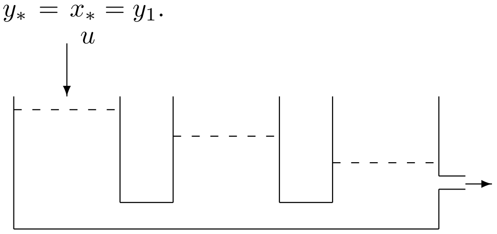

Fig. 1. Three-tank system

Since A_1 satisfies the condition (28) and $H_* = 1$, one may set $P = Q = 1$ based on (24). Sliding mode observer is described by

$$\begin{aligned} \dot{\hat{x}}_* &= u - a_4\sqrt{y_1 - y_2} - v - b e_y, \\ \hat{y}_* &= \hat{x}_* = y_1, \end{aligned} \tag{30}$$

where $b > 0$, $e_y(t) = \hat{y}_*(t) - y_1(t) = e(t) = \hat{x}_*(t) - x_*(t)$,

$$v(t) = \begin{cases} g\dfrac{e_y(t)}{\|e_y(t)\|}, & \text{if } e_y(t) \neq 0, \\ 0, & \text{otherwise,} \end{cases}$$

$g > \|d_2(t)\|$; clearly, $N_* = M_* = 0$.

Note that nonlinearity "square root" allows to obtain exact solution. Write down the express for the error e:

$$\begin{aligned} \dot{e} &= \dot{\hat{x}}_* - \dot{x}_* \\ &= u - a_4\sqrt{y_1 - y_2} - v - b e_y - (u - a_4\sqrt{H_1 x - H_2 x}). \end{aligned}$$

Since $y_1 = H_1 x$ and $y_2 = H_2 x + d_2$, then

$$\dot{e} = -a_4\sqrt{y_1 - y_2} - v - b e_y + a_4\sqrt{y_1 - y_2 + d_2}.$$

A sliding motion takes place forcing $\dot{e}(t) = e(t) = 0$, then

$$0 = a_4\sqrt{y_1 - y_2} - v - a_4\sqrt{y_1 - y_2 + d_2}$$

and

$$y_1 - y_2 + d_2 = \left(\sqrt{y_1 - y_2} + v/a_4\right)^2.$$

As a result, the function $d_2(t)$ can be estimated as

$$\hat{d}_2(t) = 2\sqrt{y_1(t) - y_2(t)}\ \frac{v(t)}{a_4} + \left(\frac{v(t)}{a_4}\right)^2.$$

It can be shown that $D^0 = (0\ \ 1)$ for the fault in the first sensor and

$$\begin{aligned} \dot{x}_{*1} &= a_4\sqrt{H_1 x - H_2 x} - a_5\sqrt{H_2 x - x_{*2}}, \\ \dot{x}_{*2} &= a_5\sqrt{H_2 x - x_{*2}} - a_6\sqrt{x_{*2} - \vartheta_7}. \end{aligned}$$

Since $H_* = (1\ \ 0)^T$, there is no exist the matrix Q to obtain symmetric positive define matrix P, and the function $d_1(t)$ cannot be estimated by the suggested method.

For simulation, consider system (29) and the observer (30) with the control $u(t) = sin(t)$. The fault is presented by signal $d_2(t) = sin(\pi t - 2\pi)$ on time intervals $t = 4 \div 8\,c$. Simulation results are shown in Figs. 2 and 3. Figs. 2 and 3 show behavior of the estimation $\hat{d}_2(t)$ and the estimation error $\hat{d}_2(t) - d_2(t)$.

5. CONCLUSION

In this paper, the problem of sensor fault identification in technical systems described by nonlinear models under the disturbance is solved using methods using sliding mode observers. In contrast to the known methods, sliding mode observer is constructed based on the reduced order model of the initial system that allows to extend a class of systems for which sliding mode observer can be constructed and the problem without conditions imposed in known papers. Future research direction is constructing adaptive sliding mode observers for sensor fault identification.

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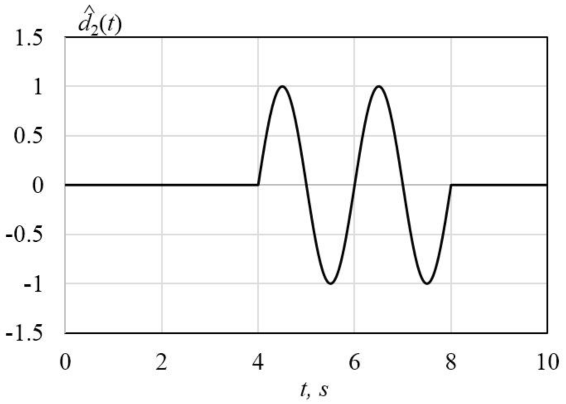

Fig. 2. Estimation of the function $d_2(t)$

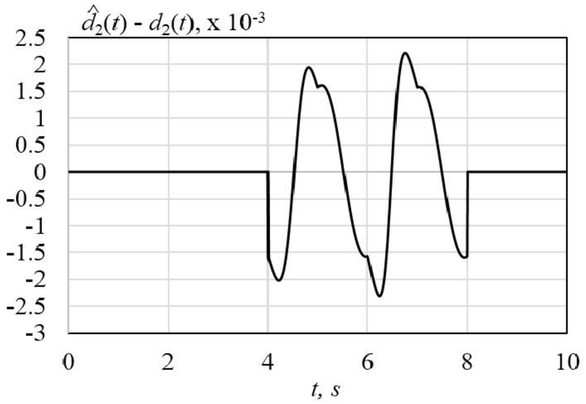

Fig. 3. Error of estimation of the function $d_2(t)$

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