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On the Selection of Lambda in Lambda Tuning for PI(D) Controllers

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Abstract: Lambda tuning is frequently employed for PI(D) controllers in the process industry because of its simplicity and intuitiveness for the user. In this paper we analyze the fundamental choice of the tuning parameter λ, that is, of the desired closed-loop time constant, by considering different trade-offs that can be posed by the user in the controller design. In particular, we consider the trade-off between bandwidth and gain or phase margin and that between performance and maximum sensitivity. The achievement of a specified maximum sensitivity is also addressed. Simple analytical formulas are determined so that they can be used in order to provide an optimized tuning in different contexts.

Keywords: PID controllers, Tuning, Optimization.

1. INTRODUCTION

Proportional-Integral-Derivative (PID) controllers are widely used in the process industry because of their ability to provide a satisfactory performance in spite of their relative simplicity (Åström and Hägglund, 2006). Indeed, the cost/benefit ratio they ensure is difficult to be improved by other more sophisticated control algorithms. Actually, the great majority of the controllers employed in practice is of PI type, as the derivative action is often switched off in order to avoid problems related to the amplification of noise and, in general, to the increased complexity in the overall controller design (Visioli, 2006).

Many tuning rules have been devised in the last century in order to simplify the task required by the user to determine the most suitable PI(D) parameters for a given application (O'Dwyer, 2006). Between the many different available techniques, one of the most employed in industry is surely the so-called λ-tuning (see, for example, (McMillan, 1999; Rice, 2010)). In fact, despite the method is based on pole-zero cancellation and therefore it is not suitable for the load disturbance rejection task in lag-dominant processes (Shinskey, 1994), its ease of use and its capability to set the speed of response of the control loop in a very intuitive way makes it a preferred choice for the user in many situations (Olsen and Bialkowski, 2002).

Basically, λ-tuning, as in the Internal Model Control (IMC) approach (Morari and Zafiriou, 1989; Rivera et al., 1986), consists in cancelling the dominant pole(s) of the process dynamics so that the closed-loop time constant λ can be arbitrarily specified. In other words, starting from a simple first-order-plus-dead-time (FOPDT) process model, the PI parameters are automatically determined by using very simple formulas (see Section 2) once the desired closed-loop time constant λ is selected by the user. It clearly appears that the overall design effort is therefore restricted to the suitable choice of λ, by taking into account that its value handles the trade-off between aggressiveness and robustness (and control effort) of the control system. This can be indeed a critical issue and methods currently used in industry links the appropriate value of λ to the process parameters. For example, in (Coughran, 2013) λ has to be selected as three times the dead time of the process, while in (Smuts, 2011), it is suggested that its value is less than three times the value of the dominant time constant of the process. The well-known SIMC tuning rules proposed in (Skogestad, 2003) are based on selecting a value of λ equal to the dead time of the process. Software tools are also available to help the user to effectively accomplish this task (see, for example, Figure 1 where a practical probe is available to help the user to select the most suitable value of λ based on an estimated process model).

In this paper we provide different rules for the choice of λ by considering different requirements made by the user. In fact, it has to be taken into account that a (possibly skilled) user can be more familiar with some control concept (for example: bandwidth, phase margin, maximum sensitivity, etc.) than other ones and he/she may want to exploit these concepts in selecting the optimized value of λ for a given application. In particular, we consider different trade-offs between performance and robustness measures that typically arise in industrial applications. The devised rules are mostly based on the exploitation of an analytical approach than can be applied thanks to the simplicity of the mathematical expressions obtained by means of the pole-zero cancellation.

The paper is organized as follows. The problem is formulated in Section 2, where the λ-tuning approach is also briefly reviewed mainly for the purpose of introducing the notation. The different rules for the selection of λ are presented in Section 3. Worked examples are given in Section 4 and conclusions are drawn in Section 5.

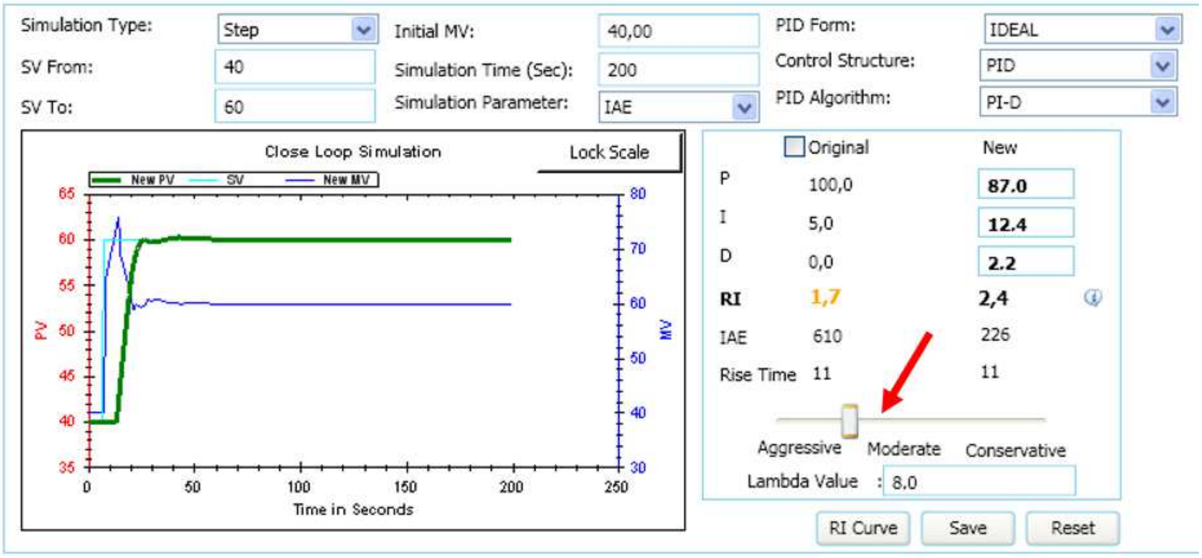

Fig. 1. Panel of the TuneVP software package related to λ-tuning for PID control (courtesy of Yokogawa Italy).

2. PROBLEM FORMULATION

We consider the standard unity-feedback control scheme of Figure 2. As it is typical for industrial plants, the process dynamics is described by a FOPDT model as

$$P(s) = \frac{\mu}{\tau s + 1} e^{-\theta s} \tag{1}$$

where μ is the gain, τ is the (dominant) time constant and θ is the (apparent) dead time of the process. The controller is chosen as a PI controller whose transfer function is

$$C(s) = K_p\left(1 + \frac{1}{T_i s}\right) \tag{2}$$

where K_p is the proportional gain and T_i is the integral time constant. The λ tuning rules consists in selecting the PI parameters as

$$\begin{aligned} K_p &= \tfrac{\tau}{\mu(\theta+\lambda)} \\ T_i &= \tau \end{aligned} \tag{3}$$

In this way, the loop transfer function $L(s) := C(s)P(s)$ results in

$$L(s) = \frac{e^{-\theta s}}{(\theta+\lambda)s} \tag{4}$$

This implies that the closed-loop transfer function is

$$F(s) = \frac{L(s)}{1+L(s)} = \frac{e^{-\theta s}}{(\theta+\lambda)s + e^{-\theta s}} \tag{5}$$

By considering the approximation (to be used in the denominator of (5))

$$e^{-\theta s} \cong 1 - \theta s \tag{6}$$

we obtain

$$F(s) \cong \frac{e^{-\theta s}}{\lambda s + 1} \tag{7}$$

It turns out that the only design parameter λ represents the desired closed-loop time constant, which has to be suitably selected depending on the given control requirements and on the required robustness. In fact, the process has unavoidable uncertainties and, in any case, the achieved performance is never exactly as expected because of the approximation (6). In the next section we give some methodologies for the sound selection of λ, where the previous approximation is not used.

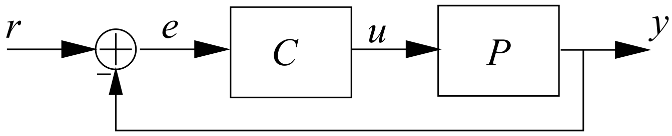

Fig. 2. The considered standard unity-feedback control system.

Remark 1. In case the process is modelled as a second-order-plus-dead-time (SOPDT) transfer function, a PID controller can be used so that the two zeros of the controller cancel the two poles of the process and the complementary sensitivity transfer function $F(s)$ is the same as (5).

Remark 2. Transfer functions (4) and (5) depends only on the dead time θ and on the design parameter λ, so that it is sensible to find tuning rules that relate λ to the only process parameter θ.

Remark 3. As already mentioned in the introduction, being based on pole-zero cancellation, the approach could not be suitable for lag-dominant processes (*i.e.*, when $\tau > 8\theta$) as in this case the load disturbance response is very sluggish. In general, the λ tuning approach is particularly suitable when the set-point following task is of major concern (for example, in batch processes, or when a feedforward controller is effectively used to handle the load disturbance task).

3. TUNING RULES

3.1 *Tuning to avoid overshoot and for minimum IAE*

Although, in principle, the set-point step response of system (7) has no overshoot, it has to be taken into account that the approximation (6) has been employed so that in practical cases there is an overshoot for small values of λ. In (Guzman et al., 2015) it has been determined that, by approximating $e^{-s\theta}$ with the first-order Padé approximation $(1 - s\frac{\theta}{2})/(1 + s\frac{\theta}{2})$, the denominator of the closed-loop transfer function does not exhibit complex poles if $\lambda \geq \left(\frac{1}{2} + \sqrt{2}\right)\theta$. The absence of overshoot implies also that the integrated absolute error (IAE) is equal to the integrated error (IE), that is

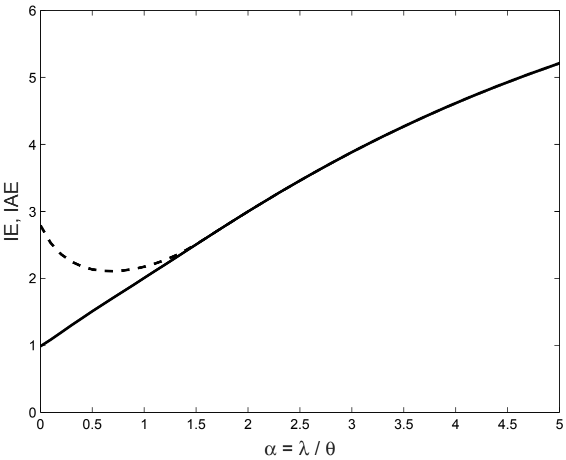

Fig. 3. Resulting values of IE (solid line) and IAE (dashed line) for different values of $\alpha = \lambda/\theta$.

$$\int_0^{\infty} |e(t)|dt = \int_0^{\infty} e(t)dt. \tag{8}$$

The values of IAE and IE can be evaluated numerically for different values of the ratio $\alpha := \lambda/\theta$, thus removing any kind of approximation. Results are shown in Figure 3 (the specific values of IE and IAE are related to the case of $\theta = 1$ but the results are analogous for the other cases) and clearly indicates that there is no overshoot for $\lambda \geq \frac{3}{2}\theta$ and the minimum value of the integrated absolute error is achieved for $\lambda = 0.7\theta$.

3.2 Trade-off between bandwidth and gain (phase) margin

The speed of response of a system is clearly related to its bandwidth, that is, to the gain crossover frequency ω_c of $L(s)$. By imposing $|L(j\omega_c)| = 1$, we obtain

$$\omega_c = \frac{k_p\mu}{T_i} = \frac{1}{\theta+\lambda} \tag{9}$$

Thus, the phase margin can be computed as

$$\varphi_m = \frac{\pi}{2} - \omega_c\theta = \frac{\pi}{2} - \frac{\theta}{\theta+\lambda} \tag{10}$$

Then, by imposing $\arg L(j\omega_\pi)| = -\pi$, we obtain

$$\omega_\pi = \frac{\pi}{2\theta} \tag{11}$$

so that the gain margin results

$$\kappa_m = \frac{\pi T_i}{2k_p\mu\theta} = \frac{\pi(\theta+\lambda)}{2\theta} = \frac{\pi}{2\theta\omega_c} \tag{12}$$

It is clear there is a trade-off between ω_c and κ_m and a Pareto front can be determined. Thus, a typical bargaining problem can be posed. Assuming that the user selects the minimum acceptable values for the gain crossover frequency and for the gain margin, and denoting them as ω_c^d and κ_m^d, respectively, the so-called Nash point can be determined as the optimal trade-off. In fact, we can determine the value of λ that corresponds to the point on the Pareto front for which the area of the rectangle having vertexes on it and in the disagreement point (ω_c^d, κ_m^d) is the largest (Sanchez et al., 2017). The situation is depicted in Figure 4.

The area of the rectangle is

$$A = \left(\omega_c - \omega_c^d\right)\left(\kappa_m - \kappa_m^d\right) = \left(\omega_c - \omega_c^d\right)\left(\frac{\pi}{2\theta\omega_c} - \kappa_m^d\right) \tag{13}$$

and, by solving

$$\frac{dA}{d\omega_c} = \frac{\pi}{2\theta\omega_c} - \kappa_m^d - \frac{\pi\left(\omega_c - \omega_c^d\right)}{2\theta\omega_c^2} = 0 \tag{14}$$

we have that A is maximum for (note that the second derivative of A is always less than zero):

$$\begin{aligned} \omega_c &= \omega_c^o := \frac{1}{2}\sqrt{\frac{2\pi\omega_c^d}{\theta\kappa_m^d}} \\ \kappa_m &= \kappa_m^o := \frac{1}{2}\sqrt{\frac{2\pi\kappa_m^d}{\theta\omega_c^d}} \\ \lambda &= \lambda_o := \sqrt{\frac{2\theta\kappa_m^d}{\pi\omega_c^d}} - \theta \end{aligned} \tag{15}$$

If, on the contrary, the user prefers to specify the desired gain crossover frequency and gain margin (we denote them as ω_c^u and κ_m^u, respectively, as they represent the utopia point), then we have

$$\begin{aligned} \kappa_m^d &= \frac{\pi}{2\theta\omega_c^u} \\ \omega_c^d &= \frac{\pi}{2\theta\kappa_m^u} \end{aligned} \tag{16}$$

Thus, by replacing (16) in (15), we have

$$\begin{aligned} \omega_c &= \omega_c^o := \frac{1}{2}\sqrt{\frac{2\pi\omega_c^u}{\theta\kappa_m^u}} \\ \kappa_m &= \kappa_m^o := \frac{1}{2}\sqrt{\frac{2\pi\kappa_m^u}{\theta\omega_c^u}} \\ \lambda &= \lambda_o := \sqrt{\frac{2\theta\kappa_m^u}{\pi\omega_c^u}} - \theta \end{aligned} \tag{17}$$

A similar reasoning can be applied by considering the trade-off between the gain crossover frequency and the phase margin. In this case the relationship is linear and expressed by (10). By denoting as φ_m^d the minimum acceptable value of the phase margin, the Nash point is determined as:

$$\begin{aligned} \omega_c &= \omega_c^o := \frac{\pi - 2\varphi_m^d + 2\theta\omega_c^d}{4\theta} \\ \varphi_m &= \varphi_m^o := \frac{\pi + 2\varphi_m^d - 2\theta\omega_c^d}{4} \\ \lambda &= \lambda_o := \frac{4\theta}{\pi + 2\varphi_m^d - 2\theta\omega_c^d} - \theta \end{aligned} \tag{18}$$

If the user might specifies his desired phase margin φ_m^u together with the desired value of ω_c, then we have

$$\begin{aligned} \omega_c^d &= \frac{\left(\frac{\pi}{2} - \varphi_m^u\right)}{\theta} \\ \varphi_m^d &= \frac{\pi}{2} - \theta\omega_c^u \end{aligned} \tag{19}$$

and, by replacing (19) in (18), we have

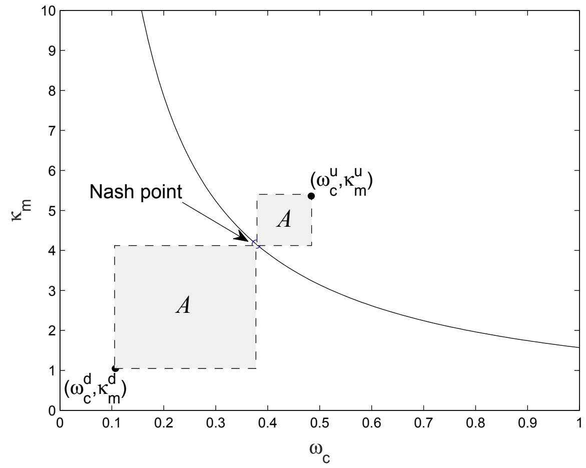

Fig. 4. Illustrative picture regarding the selection of the Nash point given the disagreement or the utopia point considering the trade-off between the gain crossover frequency and the phase margin.

$$\begin{aligned} \omega_c = \omega_c^o &:= \frac{\pi - 2\varphi_m^u + 2\theta\omega_c^u}{4\theta} \\ \varphi_m = \varphi_m^o &:= \frac{\pi + 2\varphi_m^u - 2\theta\omega_c^u}{4} \\ \lambda = \lambda_o &:= \frac{4\theta}{\pi + 2\varphi_m^u - 2\theta\omega_c^u} - \theta \end{aligned} \tag{20}$$

3.3 Tuning for specified maximum sensitivity

A typical approach for the tuning of a PID controller is to consider the resulting maximum sensitivity as a measure of the system robustness (see, for example, (Padula and Visioli, 2011; Åström et al., 2015; Alfaro and Vilanova, 2016)). In λ-tuning this can be relevant, for example, in order to compensate for an imperfect pole-zero cancellation or a imperfect dead time estimation (as uncertainties in the process parameters cannot be avoided in practice). In this context, defining the maximum sensitivity as

$$M_s := \max_w \left| \frac{1}{1+L(j\omega)} \right| \tag{21}$$

its value can be determined numerically for different values of α (see Figure 5). Then, a fitting function can be determined in order to obtain a relationship between α and M_s. It results:

$$M_s = \frac{\alpha + 1.2256}{\alpha + 0.3943} \tag{22}$$

Therefore, once a desired value of M_s has been selected by the user, then the corresponding value of λ can be obtained as:

$$\alpha = \frac{1.2256 - 0.3943 M_s}{M_s - 1} \tag{23}$$

3.4 Trade-off between performance and maximum sensitivity

In Veronesi and Visioli (2009, 2010), the set-point following performance of a PID controller is assessed by comparing the obtained integrated absolute error with a benchmark one calculated as $2A_s\theta$, that is, the one given by the response of (7), where $\lambda = \theta$ as suggested in (Skogestad, 2003), to a set-point signal of amplitude A_s.

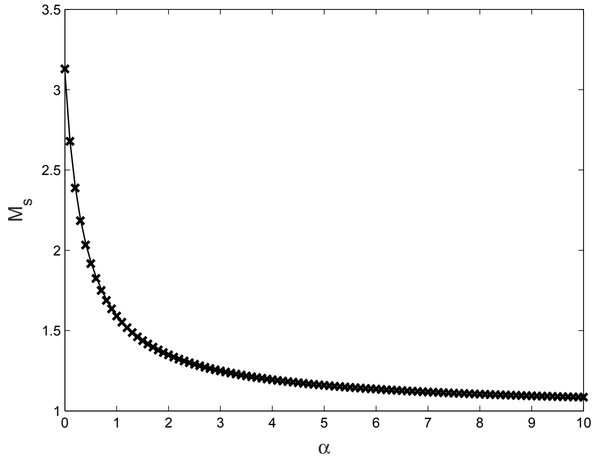

Fig. 5. Values of M_s for different values of α (black stars) and the determined fitting function (solid line).

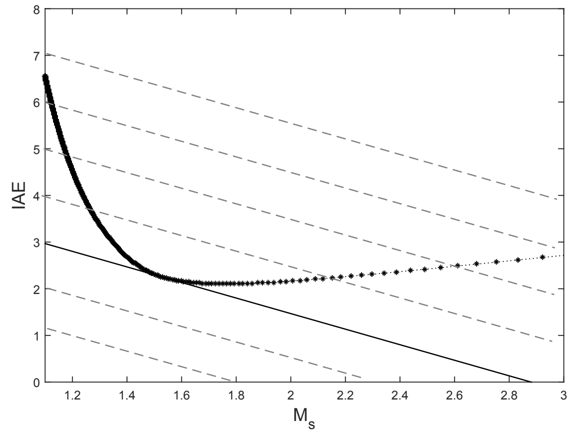

Fig. 6. Minimization of the cost function J. The dashed straight lines represent the points corresponding to constant values of J (the solid line corresponds to the optimal value). Values of (IAE,M_s) for different values of α are shown with the symbol ‘*’.

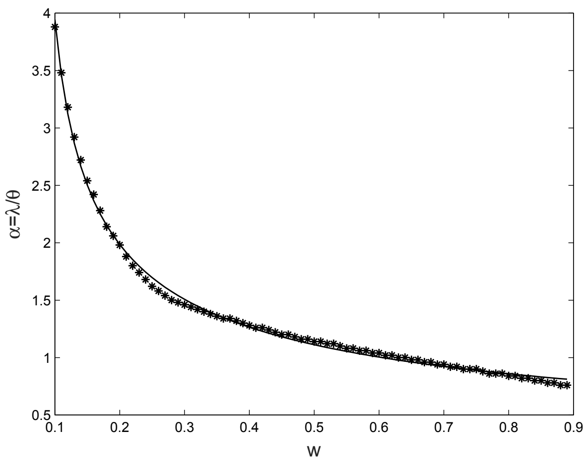

Fig. 7. Values of α (black stars) for different values of w and the determined fitting function (solid line).

It is therefore sensible to consider the trade-off between the performance assessed in this way and the achieved robustness, which can be considered as the ratio between the value of M_s obtained with a given value of λ and a target one equal to 1.2. Thus, the following new performance index can be minimized:

$$J = w\frac{IAE}{2A_s\theta} + (1-w)\frac{M_s}{1.2} \tag{24}$$

where the user can select a value of w between 0 and 1 in order to suitably weight the two terms. It is worth noting that, in the plane M_s-IAE, the considered cost function is represented by straight lines corresponding to constant values of J and, of course, the bigger is the value of J the worse is the overall performance. On the other hand, for a given value of w, the trade-off between IAE and M_s can be obtained by simulation for different values of α and drawn as a curve in the same plane. Since the overall cost function has to be minimized, the best achievable point achievable by λ-tuning is the one where that line is tangent to the constraint represented by the trade-off: the situation is shown in Figure 6, where the slope of the straight lines depends on the value of w.

By fitting the optimal values of α obtained for different values of w (see Figure 7), we obtain:

$$\alpha = \frac{0.7375}{(w-0.0673)^{0.4903}} \tag{25}$$

4. ILLUSTRATIVE EXAMPLES

Some results are shown hereafter only with the aim to illustrate the use of the devised formulas, rather than showing the effectiveness of the λ-tuning and the role of λ in the performance, as these are already well known. For this reason, we focus only of the set-point step response, even if load disturbance rejection is also relevant in general and has not to be neglected. We consider the process with $\mu = 1$, $\tau = 4$, and $\theta = 1$, that is,

$$P(s) = \frac{1}{4s+1}e^{-s} \tag{26}$$

Note that, in the process model, only the the dead time is relevant in the set-point step response. However, the PI controller, and therefore also the control variable, depends also on the process time constant and on the process gain.

4.1 Example 1

As a first case we consider the trade-off between the bandwidth (that is, the gain crossover frequency), and the gain margin. By setting the minimum acceptable values (that is, the disagreement point) as $\omega_c^d = 0.1/\theta$ and $\kappa_m^d = 1.1$, by applying (15) we obtain $\omega_c^o = 0.3779$, $\kappa_m^o = 4.1568$, $\lambda_o = 1.6463$. This gives $K_p = 1.5115$ and $T_i = 4$. On the other hand, if we set the desirable values of the gain crossover frequency and of the gain margin (utopia point) as $\omega_c^u = 1/\theta$ and $\kappa_m^u = 4$, it results (see (17)) $\omega_c^o = 0.6267$, $\kappa_m^o = 2.5066$, and $\lambda_o = 0.5958$, which gives $K_p = 2.5066$ and $T_i = 4$. The set-point unit step responses with these two tunings are shown for the sake of completeness in Figure 8 where also the minimum IAE case ($\lambda = 0.7$, which gives $K_p = 2.3529$ and $T_i = 4$) is plotted. Of course, as it is well known, results confirm that a smaller value of λ implies a smaller rise time but also a bigger control effort and a larger overshoot.

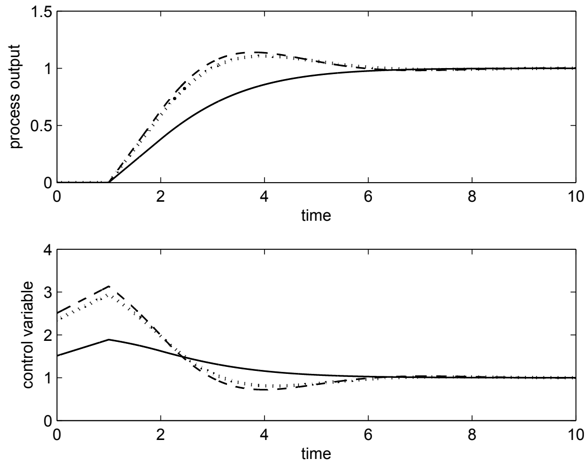

Fig. 8. Results related to Example 1. Solid line: $\lambda = 1.6463$. Dashed line: $\lambda = 0.5958$. Dotted line: $\lambda = 0.7$.

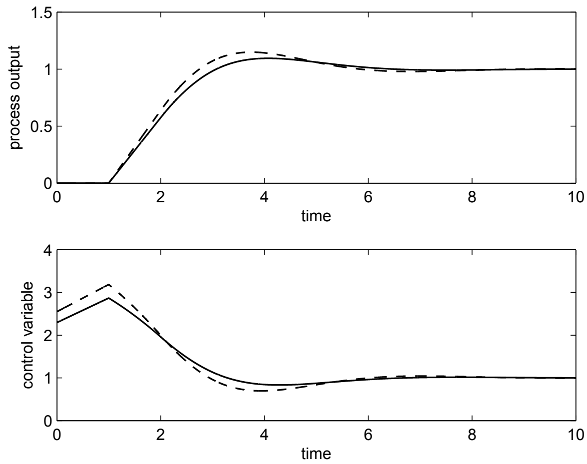

Fig. 9. Results related to Example 2. Solid line: $\lambda = 0.7434$. Dashed line: $\lambda = 0.570$.

4.2 Example 2

In the second example we consider the trade-off between the gain crossover frequency), and the phase margin. By setting the minimum acceptable values as $\omega_c^d = 0.1/\theta$ and $\varphi_m^d = \pi/6$, by applying (18) we obtain $\omega_c^o = 0.5736$, $\varphi_m^o = 0.9972$, and $\lambda_o = 0.7434$. This gives $K_p = 2.2943$ and $T_i = 4$. Alternatively, if we set the desirable values of the gain crossover frequency and of the phase margin as $\omega_c^u = 0.75/\theta$ and $\varphi_m^u = \pi/3$, it results (see (20)) $\omega_c^o = 0.6368$, $\varphi_m^o = 0.934$, and $\lambda_o = 0.570$, which gives a value of the proportional gain $K_p = 2.547$. The set-point unit step responses in the two cases are shown in Figure 9.

4.3 Example 3

In the third example we consider the tuning based on a specification of a desired maximum sensitivity. By setting the maximum sensitivity as $M_s = 1.2$, $M_s = 1.6$, and $M_s = 2$ (which are typical values), we obtain through (23) the values $\lambda = 3.7622$, $\lambda = 0.9912$, and $\lambda = 0.437$ respectively. The corresponding values of the proportional gain of the PI controller are, respec-

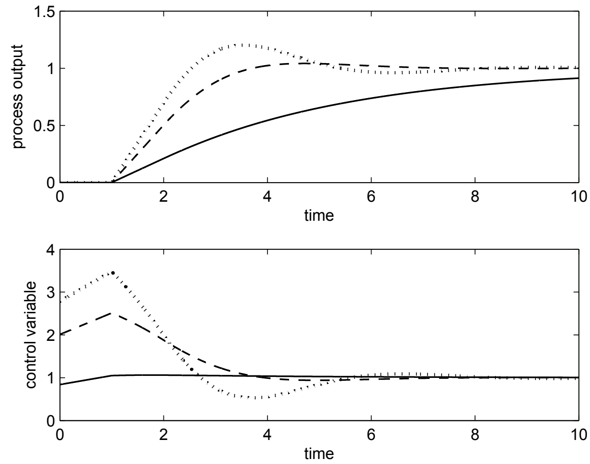

Fig. 10. Results related to Example 3. Solid line: $\lambda = 3.7622$. Dashed line: $\lambda = 0.9912$. Dotted line: $\lambda = 0.437$.

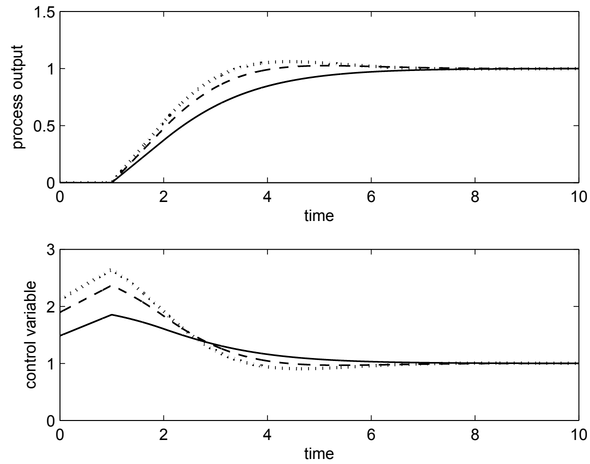

Fig. 11. Results related to Example 4. Solid line: $\lambda = 1.6972$. Dashed line: $\lambda = 1.1121$. Dotted line: $\lambda = 0.8893$.

tively ($T_i = 4$): $K_p = 0.8399$, $K_p = 2.0088$, and $K_p = 2.7835$. The set-point unit step responses in the three cases are shown in Figure 10.

4.4 Example 4

In the last example we consider the tuning based on the trade-off between performance and maximum sensitivity. By setting the value of the weight w as $w = 0.25$, $w = 0.5$, and $w = 0.75$, we obtain through (25) the values $\lambda = 1.6972$ (which corresponds to $K_p = 1.4830$), $\lambda = 1.1121$ (which corresponds to $K_p = 1.8938$), and $\lambda = 0.8893$ (which corresponds to $K_p = 2.1172$), respectively. The set-point unit step responses are plotted in Figure 11.

5. CONCLUSIONS

In this paper we have presented simple rules to determine the value of the desired closed-loop time constant in the λ-tuning of PI(D) controllers, starting from specifications given by the user in different forms. Although the rationale of λ-tuning is well known (and this is one of the reasons for its success in industry), the devised formulas are very useful because they can be used by users with different skills and different knowledge of the given industrial process. It is therefore believed that they can be exploited to optimize the performance and to obtain a fast commissioning of the control loop, which is one of the main features to improve the cost/benefit ratio of PID controllers.

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