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Application of the Dynamic Iterative Learning Control to the Heteroplanar Active Magnetic Bearing

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Abstract:
Heteroplanar active magnetic bearings have numerous applications, where one example is a high-temperature gas-cooled reactors. Rotor imbalance, however, may cause problems for critical parts of the system in the form of repetitive periodic vibrations. This is known problem and periodic component extraction is widely used in active magnetic bearing unbalance control laws. More recently, iterative learning control has been considered as an alternative and this paper gives new results on this approach. In particular, a new control law in the 2D systems setting is developed and the results of a simulation based study using the model of a test rig are given, where such a study is an essential step prior to experimental validation.

1. INTRODUCTION

An active magnetic bearing (AMB) provides has advantages for rotational machines such as reduced maintenance, high rotational speed, low wear and long life-time (Schweitzer and Maslen, 2009). There are different types of AMB models and one way to classify them is by division into current- and voltage-controlled models. Current-controlled models are often used in industrial applications, since they are easier to control. Voltage-controlled models are more accurate and preferred in some applications for reasons such as: higher overall robustness, very low stiffness values and a simpler power amplifier architecture. Although voltage-controlled models need more complex control laws, but improved performance should be obtained, even in the presence of large disturbances.

Features of this form of bearing that degrade performance include mass imbalance, sensor runout, an asymmetrical circuit and misalignment of the geometric centers between the rotor and the stator. The result can be asymmetrical magnetic forces produced by the active magnetic bearing, which could eventually lead to repetitive periodic rotor vibrations. Moreover, the application of feedback control action alone is also known to be problematic in at least some applications.

A considerable volume of literature exists on the problem of controlling the unbalance characteristic in active magnetic bearings. The performance of these bearings is also influenced by non-periodic disturbances and as a result most unbalance control designs must extract the periodic components from the rotor displacement signal by either direct or indirect means, which, in turn, is costly from a signal processing standpoint. In particular, the process of extracting the periodic components is difficult and hence applying rotor unbalance control is difficult.

The presence of repetitive disturbances in this application area strongly suggests that iterative learning control (ILC) may be applied to advantage. Previous research on the use of ILC in active bearings area includes (Zheng et al., 2020) with supporting experimental results. In this previous research the analysis is based on a discrete linear system sample varying model of the dynamics but for the experimental results the bearing dynamics are modeled by a continuous-time transfer-function with a pair of purely imaginary complex conjugate poles, i.e., simple harmonic motion.

The overall control scheme in (Zheng et al., 2020) applies a Proportional plus Integral plus Derivative (PID) control loop first and then applies ILC to the resulting controlled dynamics, i.e., a two step design. An alternative approach is to treat the design problem in the 2D systems setting, i.e., systems that propagate information in two independent directions and hence variables are described in terms of two indeterminates. This approach forms the basis for the results in this paper.

Throughout this paper, the null and identity matrices of compatible dimensions are denoted by 0 and I respectively. Also, a symmetric positive definite (respectively negative definite) matrix, say M, is denoted by $M \succ 0$ (respectively $M \prec 0$) and the spectral radius of a matrix is denoted by $\rho(\cdot)$.

This paper begins in the next section with the model of the bearing system considered and then the following section gives the necessary background on the ILC analysis used in this paper. Section 3 details the control law design and Section 4 the

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results of a simulation-based evaluation of performance when applied to a particular bearing. Finally, an objective overview of the progress reported in this paper is given in the concluding section together with discussion of planned future research.

2. BACKGROUND – THE SYSTEM CONSIDERED

The physical system considered in this paper is shown in the schematic diagram of Fig. 1. Various possibilities exist for the

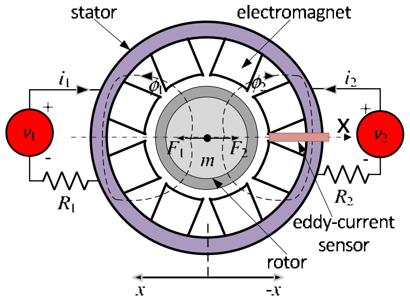

Fig. 1. Schematic of the AMB.

control of AMB dynamics, where the commonly considered approaches are based on either voltage or current actuated models. Moreover, the former is more precise than the latter. Voltage actuated control is considered in this paper, which is achieved by using a model of the dynamics where the coil current is taken as a state variable rather than the input. The linearized voltage-controlled AMB model is 3rd order dynamics as given next.

Suppose that the state variables are x_1, x_2 and x_3 representing, respectively, the rotor position [m], the rotor speed [m/s] and the coil current [A]. Also set $x = [\, x_1 \;\; x_2 \;\; x_3 \,]^T$. Then the state-space model of the dynamics to be controlled is

$$\begin{aligned} \dot{x}(t) &= \tilde{A}x(t) + \tilde{B}u(t), \\ y(t) &= \tilde{C}x(t), \end{aligned} \tag{1}$$

where $u(t)$ is the input voltage and

$$\tilde{A} = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{2k_s}{m} & 0 & \dfrac{2k_1}{m} \\ 0 & -\dfrac{k_i}{L_s+L_o} & -\dfrac{R}{L_s+L_o} \end{bmatrix},$$

$$\tilde{B} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{L_s+L_o} \end{bmatrix},$$

$$\tilde{C} = [\, 1 \;\; 0 \;\; 0 \,].$$

Table 1 defines the parameters in this model together with the values used in the simulation case study given later in the paper.

3. ILC DESIGN

3.1 Preliminaries

Many systems repeatedly perform the same task of finite duration, after which the system is returned to the initial location.

Table 1. Values and descriptions of parameters

Symbol	Value	Physical description
$s_0[m]$	0.0004	air gap
$m[kg]$	2.5	mass of the rotor in the bearing plane
$L_o[H]$	0.0025	coil inductance
$L_s[H]$	0.0005	coil inductance losses
$R[\Omega]$	0.5	coil resistance
N	108	number of turns of wire in the coil
$\mu_0[H/m]$	1.25×10^{-6}	permeability of free space
$A[m^2]$	0.0014	cross sectional area of air gap
$k_i[N/A]$	15.625	current stiffness
$k_s[N/m]$	97656.25	displacement stiffness

Each execution of the task is known as a trial or pass or iteration, where in this paper trial is exclusively used. The duration of each trial is termed the trial length. A typical example is a robot performing a 'pick and place' operation, i.e., i) collect a payload from a fixed location, ii) transfer it over a finite duration, iii) place it on a moving conveyor (under synchronization), iv) return to the original location to collect the next payload and then repeat i)-iv) for as many trials as needed or until a halt is required for maintenance or other reasons. It was applications such as this one that motivated the initial ILC research.

The earliest research on ILC is widely credited to (Arimoto et al., 1984) and focused on robotic applications. Since this early work, ILC has remained a very active research area both in terms of terms of theory/algorithm development and applications. The survey papers (Bristow et al., 2006) and (Ahn et al., 2007) are possible starting points for the earlier literature. More recent application areas include additive manufacturing (Sammons et al., 2019), center-articulated industrial vehicles (Dekker et al., 2019) and in healthcare, see, e.g. (Sakariya et al., 2020; Seel et al., 2016).

In ILC there is information propagation from trial-to-trial and along the trials. In this paper the nonnegative subscript k on a scalar or vector variable denotes the trial number and such a variable for discrete dynamics is written as $y_k(p)$, $0 \le p \le \alpha - 1$, where α denotes the number of samples along the trial and if the sampling period is T_s then $\alpha \times T_s$ gives the trial length. Hence the dynamics of a discrete linear time-invariant system in the ILC setting is described by the state-space model

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p) + Bu_{k+1}(p), \\ y_{k+1}(p) &= Cx_{k+1}(p) + Du_{k+1}(p), \end{aligned} \tag{2}$$

where $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the trial profile vector and $u_k(p) \in \mathbb{R}^r$ is the control input vector.

Let $y_{ref}(p)$ be a possibly vector-valued reference representing the desired output behavior and denote the system output on trial k as $y_k(p)$, $0 \le p \le \alpha - 1$, $k \ge 0$. Then the error on trial k is

$$e_k(p) = y_{ref}(p) - y_k(p). \tag{3}$$

The control design problem is to construct an input sequence $\{u_k\}$ that when applied forces the error sequence $\{e_k\}$ to converge in k, i.e.,

$$\lim_{k\to\infty} ||e_k|| = 0, \;\; \lim_{k\to\infty} ||u_k - u_\infty|| = 0, \tag{4}$$

where $||\cdot||$ is a signal norm in a suitably chosen function space with a norm-based topology and u_∞ is termed the learned control. As the trial length is finite, convergence in k can occur for unstable systems, i.e, $\rho(A) \ge 1$ in (2).

Once a trial is complete all information generated during its evolution is available for use in designing the control input for the next trial and thereby improve performance from trial-to-trial. Hence a commonly used form of an ILC law is

$$u_{k+1}(p) = u_k(p) + \Delta u_{k+1}(p), \tag{5}$$

where $\Delta u_{k+1}(p)$ is the correction term to be designed. Given that all previous trial is available the ILC law on trial $k + 1$ can access information from any sample instant on the previous trial, e.g., at sample instant p previous trial data at, say, sample instant $p+1$ can be used. The feature is sometimes termed 'non-causal' in the ILC literature and it can be shown that if such data is not used, i.e., at sample instant p on trial $k+1$ the control law only uses information at sample instant p,then the ILC law can be replaced by a standard feedback control loop.

If the dynamics along the trial are discrete then a commonly used setting for design is based on a form of lifting. Consider without loss of generality the single-input single-output case and focus on the trial output. Then the so-called supervector representation of this variable is formed by placing the sample values in turn as the entries in an $\alpha \times 1$ vector. Repeating this step for all other variables, the trial-trial error updating is represented by a standard difference equation, providing a setting for analysis and design based on standard discrete linear systems theory and design algorithms. This setting for discrete dynamics has been extensively used with many designs leading through to experimental validation and actual application, see, e.g., the survey papers by (Bristow et al., 2006; Ahn et al., 2007) are possible starting points for the literature.

As discussed above, an ILC system can converge from trial-to-trial even if the system is unstable. In lifting based design, the only option is to design a pre-stabilizing feedback control loop and then apply the ILC design to the resulting controlled dynamics. This is a two stage design procedure and also problems arise in robust control design as product terms arise formed by matrices from the nominal model state-space matrices and those describing the uncertainty description, .e.g., norm bounded or polytopic, used.

An alternative setting for ILC analysis and design is to use 2D systems theory, where the two directions of information propagation is from trial-to-trial (k) and along the trial (p). The first work on this approach is usually credited to (Kurek and Zaremba, 1993). This work used the Roesser model (Roesser, 1975), whose domain of operation is the complete upper-right quadrant of the 2D plane, i.e., $(k, p) \in [0, \infty] \times [0, \infty]$.

Repetitive processes are another class of 2D systems that make a series of sweeps, termed trials, also termed passes in some of the literature, through dynamics that are defined over a finite duration, which is known as the trial length. The output produced on each trial is termed the trial profile. On completion of a current trial, the process resets to the starting location and the next trial can begin, either immediately or after some further time has elapsed. The trial profile produced on the previous trial contributes to current trial dynamics and it is the finite duration of the trial length that is particularly relevant to ILC analysis.

Let $\{y_k\}$ denote the sequence of trial profiles generated by a repetitive process. Then this sequence can contain oscillations which increases in amplitude from trial-to-trial. Moreover, this unwanted behavior cannot be regulated by standard control action, e.g., static state or output feedback control activated, respectively, by the current trial state or trial profile vector. Instead, this must be augmented by a term activated by previous trial information, i.e., repetitive processes are a class of 2D systems and therefore require a 2D control law.

Discrete linear repetitive processes are described by the following state-space model over $0 \leq p \leq \alpha - 1,\ k \geq 0$,

$$\begin{aligned} x_{k+1}(p+1) &= \mathbb{A}x_{k+1}(p) + \mathbb{B}u_{k+1}(p) + \mathbb{B}_0 y_k(p), \\ y_{k+1}(p) &= \mathbb{C}x_{k+1}(p) + \mathbb{D}u_{k+1}(p) + \mathbb{D}_0 y_k(p), \end{aligned} \tag{6}$$

where on trial k, $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the trial profile vector and $u_k(p) \in \mathbb{R}^r$ is the control input vector.

The stability theory for linear repetitive processes (Rogers et al., 2007) is based on an abstract model of the dynamics in a Banach space setting that includes all examples as special cases. Given the unique control problem, this theory requires that a bounded initial trial profile is required to produce a bounded sequence of trial profiles $\{y_k\}$, where bounded is defined by the norm on the associated signal space. This property can be imposed over the finite and fixed trial length or uniformly, i.e., independent of the trial length, where this last property can be analyzed mathematically by considering $\alpha \to \infty$ and is termed stability along the trial.

In many applications the stronger stability property must be imposed and the following result characterizes stability along the trial of examples described by (6).

Lemma 1. (Rogers et al., 2007). A discrete linear repetitive processes described by (6) is stable along the trial if and only if (i) $\rho(\mathbb{D}_0) < 1$, (ii) $\rho(\mathbb{A}) < 1$, and (iii) all eigenvalues of $G(z) = \mathbb{C}(zI - \mathbb{A})^{-1}\mathbb{B}_0 + \mathbb{D}_0$ have modulus strictly less than unity for all $|z| = 1$.

Condition (ii) in this result requires frequency attenuation the complete spectrum of the previous trial profile and this may be restrictive in control design in many cases. This, in turn, has led to the use of sufficient conditions developed through the use of Lyapunov functions with computations that can be implemented using Linear Matrix Inequalities (LMIs) and this is the method used in the remainder of this paper.

Asymptotic stability for the processes considered holds if and only if the first condition of this last result holds. This will enforce convergence in k of the sequence $\{y_k\}$ where the resulting dynamics are described by a standard (i.e. one independent variable) discrete linear systems state-space model but this system can be unstable (due to the finite trial length). Hence, in general, stability along the trial must be enforced as per conditions (ii) and (iii) in this result.

In this last result, (ii), which governs the dynamics along a trial, is only a necessary condition for stability along the trial. Condition (iii) requires frequency attenuation the complete spectrum of the previous trial profile. Moreover, this condition can be 'difficult' to use in the control law design. This, in turn, has led to the use of sufficient conditions developed through the use of Lyapunov functions with computations that can be implemented using Linear Matrix Inequalities (LMIs).

3.2 Dynamic ILC

In some applications the previously published (see e.g. (Hladowski et al., 2010)) static ILC law suffers from possible high conservativeness and difficulties in obtaining required performance with allowable, by the actuators, control signals and sufficiently fast convergence etc. Some improvements in this area are possible by using optimization techniques to choose the "best" control law, but these are limited in their applicability. An alternative is to consider the use of dynamic ILC laws or schemes. Also, there are other ILC structures where dynamic controllers and filters have been used, e.g. (de Roover et al., 2000).

In this paper, a dynamic ILC law (or controller, see (Hladowski et al., 2016, 2017) for more background) is considered for a discrete linear time-invariant state-space model written in the ILC setting as in (2) with $D = 0$. The dynamic ILC controller considered has state dynamics governed by

$$\eta^c_{k+1}(p+1) = A_c\eta^c_{k+1}(p) + A_{0,c}\eta_{k+1}(p) + B_{0c}e_k(p), \tag{7}$$

where

$$\eta^c_{k+1}(p) = x^c_{k+1}(p-1) - x^c_k(p-1), \tag{8}$$

is the trial-to-trial increment of the controller internal state vector $x^c_k(p) \in \mathbb{R}^{n_c}$, where possibly $n \neq n_c$. The control input for the next trial is that used on the previous plus a correction term which is in this case the controller output, $\Delta u_{k+1}(p)$, i.e.,

$$\begin{aligned} \eta_{k+1}(p+1) &= x_{k+1}(p) - x_k(p), \\ u_{k+1}(p) &= u_k(p) + \Delta u_{k+1}(p). \end{aligned} \tag{9}$$

In the dynamic ILC case, the control input increment is given by

$$\begin{aligned} \Delta u_{k+1}(p) &= C_c\eta^c_{k+1}(p+1) + E_c\eta_{k+1}(p+1), \\ &+ D_c e_k(p+1). \end{aligned} \tag{10}$$

The resulting controller depends on the trial-to-trial increments of both the system to be controlled and the controller state vector and a phase advance term in the previous trial error. For implementation (using (9)) the current trial input is

$$\begin{aligned} u_{k+1}(p) &= C_c\eta^c_{k+1}(p+1) + E_c\Big(x_{k+1}(p) - x_k(p)\Big), \\ &+ D_c\left(y_{ref}(p+1) - y_k(p+1)\right) + u_k(p), \end{aligned} \tag{11}$$

where η^c is defined in (7). Introducing

$$\mathbb{X}_k(p) = \begin{bmatrix} \eta_k(p) \\ \eta^c_k(p) \end{bmatrix}, \tag{12}$$

the controlled ILC dynamics can be written as

$$\begin{aligned} \mathbb{X}_{k+1}(p+1) &= \mathbb{A}\mathbb{X}_{k+1}(p) + \mathbb{B}_0 e_k(p), \\ e_{k+1}(p) &= \mathbb{C}\mathbb{X}_{k+1}(p) + \mathbb{D}_0 e_k(p), \end{aligned}$$

where

$$\begin{aligned} \mathbb{A} &= \begin{bmatrix} A + BE_c & BC_c \\ A_{0,c} & A_c \end{bmatrix}, & \mathbb{B}_0 &= \begin{bmatrix} BD_c \\ B_{0c} \end{bmatrix}, \\ \mathbb{C} &= \begin{bmatrix} -C(A+BE_c) & -CBC_c \end{bmatrix}, & \mathbb{D}_0 &= I - CBD_c, \end{aligned} \tag{13}$$

which is a particular case of the discrete linear repetitive process state-space model (6) with no input terms and $e_k(p)$ is the trial profile vector. Hence stability along the trial in this case will guarantee monotonic (from trial-to-trial) error convergence, i.e., solve the ILC design problem.

One set of (sufficient) conditions for stability along the trial of a discrete linear repetitive process described by (13) is the following.

Lemma 2. (Rogers et al., 2007). A discrete linear repetitive process described (13) and (13) is stable along the trial if there exist

$$\mathbb{P}_1 = \begin{bmatrix} \hat{P}_{11} & 0 \\ 0 & \hat{P}_{22} \end{bmatrix} \succ 0,$$

where $\hat{P}_{11}$, $\hat{P}_{22}$ are, respectively, of the same dimensions as $\mathbb{A}, \mathbb{D}_0$ such that

$$\begin{bmatrix} \mathbb{P}_1 - W - W^T & W\Phi^T \\ \Phi W & -\mathbb{P}_1 \end{bmatrix} \prec 0, \tag{14}$$

where

$$\Phi = \begin{bmatrix} \mathbb{A} & \mathbb{B}_0 \\ \mathbb{C} & \mathbb{D}_0 \end{bmatrix} \tag{15}$$

and W is a compatibly dimensioned matrix.

This last result is used in the proof of the following theorem that enables controller design.

Theorem 1. (Hladowski et al., 2017). An ILC control configuration described by the discrete linear repetitive process state-space model (13) for the system of (2) is stable along the trial if the following LMI is feasible

$$\begin{bmatrix} \mathbb{P}_1 - W - W^T & (\hat{A}W + \hat{B}N)^T \\ \hat{A}W + \hat{B}N & -\mathbb{P}_1 \end{bmatrix} \prec 0, \tag{16}$$

where

$$\hat{A} = \begin{bmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ -CA & 0 & I \end{bmatrix}, \hat{B} = \begin{bmatrix} B & 0 \\ 0 & I \\ -CB & 0 \end{bmatrix} \tag{17}$$

$$\mathbb{P}_1 = \begin{bmatrix} \hat{P}_{11} & 0 \\ 0 & \hat{P}_{22} \end{bmatrix} \succ 0,$$

and $\hat{P}_{11}$ and $\hat{P}_{22}$ have the same dimensions as $\mathbb{A}$ and $\mathbb{D}_0$ respectively. If this LMI is feasible the dynamic controller matrices, which are the compatible blocks of the matrix

$$\tilde{K} = \begin{bmatrix} E_c & C_c & D_c \\ A_{0,c} & A_c & B_{0,c} \end{bmatrix}. \tag{18}$$

can be calculated as

$$\tilde{K} = NW^{-1}. \tag{19}$$

4. DYNAMIC ILC APPLIED TO THE BEARING SYSTEM

Discretizing (1) with discretization period of $T_s = 1 \cdot 10^{-4}$ yields the following state-space representation for the dynamic bearing system in the form of (2) with matrices

$$\begin{aligned} A &= \begin{bmatrix} 1 & 0.0001 & 3.108 \cdot 10^{-8} \\ 7.813 & 1 & 0.0006199 \\ -2.023 & -0.5166 & 0.9833 \end{bmatrix}, \\ B &= \begin{bmatrix} 3.458 \cdot 10^{-10} \\ 1.036 \cdot 10^{-5} \\ 0.03306 \end{bmatrix}, C = [1\,0\,0], D = [0]. \end{aligned} \tag{20}$$

In this section, the dynamic ILC controller of the previous section is applied to the bearing system. For comparison against a non-ILC design, a stabilizing state feedback control law is used, where applying one of the many pole placement design algorithms gives the stabilizing state feedback gain matrix as

$$K = \begin{bmatrix} 1.723 \cdot 10^4, 50.45, 0.7738 \end{bmatrix}. \tag{21}$$

This law places the system poles at $(-126.88 - 2.32j)$, $(-126.88 + 2.32j)$, -171.08 (for implementation as $u(p) = -Kx(p)$).

Fig. 2 gives a block diagram of the system used in the simulation, where the switch SW_1 is used to apply each of the two designs in turn. Moreover, z^{-1} denotes the backward shift operator in p and z_k^{-1} the same operator in k. The reference signal for the AMB is zero since it is required the AMB shaft (see Fig. 1) stays in the middle position, denoted by zero. In the simulations below, the starting position for the AMB was taken as $2.5 \cdot 10^{-4}$.

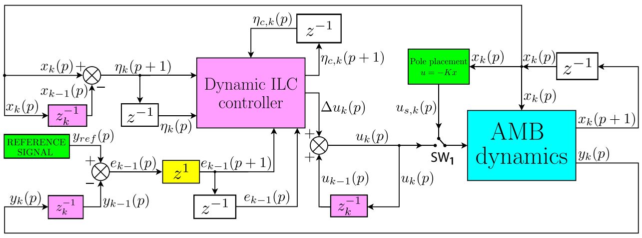

Fig. 2. Block diagram arrangement used to generate the simulation results.

Using the LMI and the associated optimization procedure given in the appendix, the following matrices defining the ILC dynamic controller (7) were selected as a starting point for investigation

$$\begin{aligned} A_c &= 0.9178, \\ A_{0,c} &= \left[\, 7.326 \cdot 10^4 \;\; -1.899 \cdot 10^6 \;\; -1.553 \cdot 10^4 \,\right], \\ B_{0,c} &= 2.784 \cdot 10^5, C_c = -0.0003081, D_c = 935, \\ E_c &= \left[\, -1024 \;\; -1.004 \cdot 10^4 \;\; -40.24 \,\right]. \end{aligned} \tag{22}$$

4.1 Simulation – Sample Results

An extensive set of simulations were performed, where in each case uniformly-distributed additive noise was added to both the input and output, both of amplitude 10^{-5}. This value for the noise was selected based on the signal measurements in input/output channels. The trial length was 0.2 [s], which corresponds to $\alpha = 2000$ samples.

A representative of the simulation results obtained with both designs is given Fig. 3 for the selected initial starting position. The pole placement design returns to zero after $0.18s$ and the ILC design after $0.03s$. Also the overshoot for the pole placement design is approximately 3.5 times large than that for the ILC design. Fig. 4 shows that both designs require a similar level of control input.

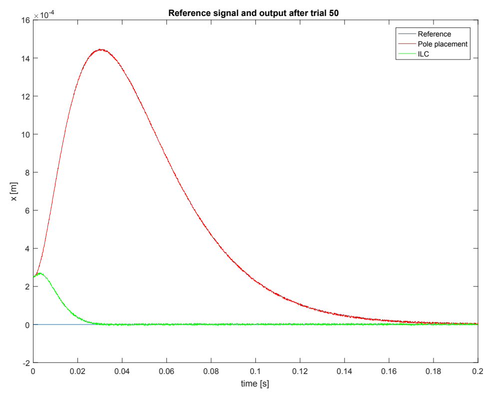

Fig. 3. Comparison of the outputs produced by the two designs.

5. CONCLUSIONS

This paper has continued the development of dynamic ILC design in the repetitive process setting. The particular focus here is an application to hetroplanar active magnetic bearings, where a core requirement is to keep a particular variable at a constant value, i.e., a regulator design to correct unwanted initial displacement. A comparative simulation study using a model of an actual bearing has demonstrated potentially strong benefits of the ILC design over a standard state feedback design. Planned future work includes experimental validation and also algorithm development for the ILC design to reduce the requirement to store the complete previous trial state vector.

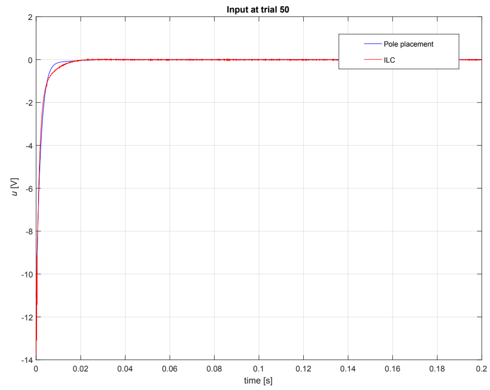

Fig. 4. Comparison of the inputs required by the two designs.

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APPENDIX – THE OPTIMIZATION ALGORITHM

(1) Assume a sufficiently large number of simulated trials β.
(2) Obtain a starting point of K_{opt}:
   (a) Solve the LMI of (16), if feasible, to calculate K using (18). Set $K_{opt} = K$. When (16) has no solutions, the optimization scheme can be rerun with random starting entries in K.
   (b) Simulate (2) over each trial using the control signal calculated from (11) to obtain the error defined by (3)) forming the vector $\mathbf{E} = \{e_1, e_2, \ldots, e_\beta\}$.
   (c) Calculate $f = f_{opt} = \sqrt{(\mathbf{E}\mathbf{E}^T)}$,
(3) repeat over the trials
   (a) Adjust $\tilde{K}$ according to Optimization Algorithm rule, taking into account the value of f
   (b) Simulate (2) using control matrices given by (18) and the control signal calculated using (11) to obtain the updated error signal vector $\mathbf{E}$.
   (c) Calculate the value of objective function $f = \sqrt{(\mathbf{E}\mathbf{E}^T)}$,
   (d) If $f < f_{opt}$, set $f_{opt} = f$ and $K_{opt} = K$
(4) Until (f_{opt} is small enough or a required number of iterations has been performed)
(5) (end of optimization phase) Use values from $K = K_{opt}$ to control the system
(6) (stability test) Calculate the matrices of (13) and apply Lemma 2. If the resulted system is stable, STOP and apply the calculated controller matrices. If stability cannot be guaranteed, restart the algorithm using a different starting point.