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Robust Output Regulation of Permanent Magnet Synchronous Motors by Enhanced Extended Observer ⋆

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Abstract: This paper presents an application of the enhanced extended observer design with regard to the problem of robust output regulation of permanent magnet synchronous motors. The control framework is further advanced by means of the internal model so as to cope with disturbances represented by the time-varying load torque. The paper shows simulation of regulating a motor for different cases of torque loads and references and provides an explicit procedure of the recursive calculation of the enhanced extended observer coefficients.

Keywords: Dayawansa's lemma, Robust observers, Nonlinear systems, High-gain observers, Permanent magnet synchronous motors

1. INTRODUCTION

In recent years there is an increasing demand of high-efficiency motors — in particular, permanent magnet synchronous motors (PMSM) — as actuation components of industrial robots, unmanned aerial vehicles, electric cars, home appliances, and many others. This is due to their advantages over other types of actuation (such as hydraulic or pneumatic actuators, internal combustion engines) in terms of preciseness, serviceability, cost, and environmental friendliness. In addition, all the above mentioned robotic applications are being actively developed, their operations are being complicated, that constantly poses new challenges of upgrading all their components. This retains interest among scientists and engineers world-wide in developing new motor control techniques and enhancing or generalizing already existing ones so as to increase actuator performance, relax requirements to hardware components (e.g. such as reduction of physical sensors e.g. suggested in Bobtsov et al. (2015)), and, as a consequence, further advance the functionality of the overall system.

This paper is focused on design of a controller achieving output regulation of a PMSM. The controller consists, as usual, of an internal model (designed by means of standard methods) and of a robust stabilizer, that in the present case is designed by means of a technique suggested in Freidovich and Khalil (2008). We stress out that in this paper we do not propose any new method in robust output regulation but rather show a new application of the existing theories to the problem of PMSM regulation. We are appealing to the theory of output regulation, as available in various standard textbooks, such as Huang (2004) and Isidori et al. (2003), as well the theory of robust stabilization via extended observers, namely the result of Freidovich and Khalil (2008) that is further enhanced in Isidori et al. (2019). The latter enhancement turns out to be appropriate when certain "gains" that appear in the chain of integrators between input and output are time-varying, as for example in the case of quadrotors recently shown in Borisov et al. (2019). The main contribution of this paper is a new change of coordinates that puts the PMSM model into the normal form and makes possible to control it by means of the geometry-based tools.

The parametrization of the PMSM model derived in the paper explicitly shows a specificity of the dq-frame, namely it reveals that the full dynamics can be decoupled on the zero dynamics and the output dynamics, which both can be controlled independently by means the d- and q-component, respectively. This makes possible to theoretically confirm empirically designed approaches used to control PMSMs. In this sense, the control framework of this paper can be seen as a generalization of existing tools, such as PID controllers (see an application of the PI controller to a PMSM in Ortega et al. (2018)), for nonlinear systems. In fact, it generalizes them and gives a rigorous proof of the effectiveness of the proposed approach in a general setup with uncertain and time-varying parameters. From this point of view, standard PID controllers can be seen as a particular case of the control framework of this paper, which, moreover, guarantees achievement of stability properties in the case of a time-varying moment

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of inertia of the PMSM rotor, that has not been considered so far.

2. PROBLEM FORMULATION

Consider the two-phase $\alpha\beta$ model of the unsaturated non-salient PMSM suggested in Nam (2018)

$$\begin{aligned}\dot{\lambda} &= v - Ri,\\ j\dot{\omega} &= -B\omega + \tau_e - \tau_L,\\ \dot{\theta} &= \omega,\end{aligned}$$

where $\lambda = (\lambda_\alpha\ \lambda_\beta)^{\mathrm{T}} \in \mathbb{R}^2$ is the total flux, $i = (i_\alpha\ i_\beta)^{\mathrm{T}} \in \mathbb{R}^2$ are the currents, $v = (v_\alpha\ v_\beta)^{\mathrm{T}} \in \mathbb{R}^2$ are the voltages, $R > 0$ is the stator windings resistance, $j > 0$ is the rotor inertia, $\theta \in \mathbb{S} := [0, 2\pi)$ is the rotor phase, $\omega \in \mathbb{R}$ is the mechanical angular velocity, $B \geq 0$ is the viscous friction coefficient, τ_e is the torque of electrical origin, given by

$$\tau_e = n_p i^{\mathrm{T}} J \lambda,$$

where $n_p \in \mathbb{N}$ is the number of pole pairs and $J \in \mathbb{R}^{2\times 2}$ is the rotation matrix

$$J = \begin{pmatrix} 0 & -1\\ 1 & 0\end{pmatrix},$$

$\tau_L \in \mathbb{R}$ is the time-varying load torque that can be modeled by the exosystem

$$\begin{aligned}\dot{w} &= Sw,\\ \tau_L &= \Psi w,\end{aligned} \tag{1}$$

with known $S \in \mathbb{R}^{n_w\times n_w}$ and $\Psi \in \mathbb{R}^{1\times n_w}$ and unknown initial conditions $w(0)$ of the state vector $w \in \mathbb{R}^{n_w}$.

For surface-mounted PMSM's the total flux verifies

$$\lambda = Li + \lambda_m \mathcal{C}(\theta),$$

where $L > 0$ is the stator inductance, $\lambda_m > 0$ is the magnetic flux constant, and

$$\mathcal{C}(\theta) = \begin{pmatrix}\cos(n_p\theta)\\ \sin(n_p\theta)\end{pmatrix}.$$

Hence, a state-space model of the PMSM is given as

$$\begin{aligned}L\frac{di}{dt} &= -Ri - \lambda_m n_p \omega J\mathcal{C}(\theta) + v,\\ j\dot{\omega} &= -B\omega + \lambda_m n_p i^{\mathrm{T}} J\mathcal{C}(\theta) - \tau_L,\\ \dot{\theta} &= \omega.\end{aligned} \tag{2}$$

The goal is to design a controller fed only by the measurements of $\theta(t)$ such that for a given reference θ^* the relation

$$\lim_{t\to\infty} |\tilde{\theta}(t)| = 0, \tag{3}$$

with $\tilde{\theta}(t) = \theta(t) - \theta^*$ holds.

3. NORMAL FORM

Define new variables $\xi = (\xi_1\ \xi_2\ \xi_3)^{\mathrm{T}}$ and z as

$$\begin{aligned}\xi_1 &= n_p\theta,\\ \xi_2 &= \omega,\\ \xi_3 &= -B\omega + \lambda_m n_p i^{\mathrm{T}} J\mathcal{C}(\theta) - \tau_L,\\ z &= \lambda_m n_p i^{\mathrm{T}}\mathcal{C}(\theta),\end{aligned}$$

which is a globally defined change of variables.

Taking the derivatives of $\xi = (\xi_1\ \xi_2\ \xi_3)^{\mathrm{T}}$ and z one obtains the following

$$\begin{aligned}\dot{\xi}_1 &= n_p\dot{\theta} = n_p\omega = n_p\xi_2,\\
\dot{\xi}_2 &= \dot{\omega} = \frac{1}{j}\left[-B\omega + \lambda_m n_p i^{\mathrm{T}} J\mathcal{C}(\theta) - \tau_L\right] = \frac{1}{j}\xi_3,\\
\dot{\xi}_3 &= -B\dot{\omega} + \lambda_m n_p \frac{di^{\mathrm{T}}}{dt} J\mathcal{C}(\theta) + \lambda_m n_p^2 i^{\mathrm{T}} JJ\mathcal{C}(\theta)\dot{\theta} - \dot{\tau}_L\\
&= -B\dot{\omega} + \frac{\lambda_m n_p}{L}\left[-Ri - \lambda_m n_p\omega J\mathcal{C}(\theta) + v\right]^{\mathrm{T}} J\mathcal{C}(\theta)\\
&\quad -n_p \underbrace{\lambda_m n_p i^{\mathrm{T}}\mathcal{C}(\theta)}_{z}\omega - \dot{\tau}_L\\
&= -\frac{B}{j}\xi_3 + \frac{\lambda_m n_p}{L}\left[-Ri^{\mathrm{T}} J\mathcal{C}(\theta) - \lambda_m n_p\xi_2 + v^{\mathrm{T}} J\mathcal{C}(\theta)\right]\\
&\quad -n_p z\xi_2 - \dot{\tau}_L\\
&= -\frac{B}{j}\xi_3 - \underbrace{\frac{\lambda_m n_p}{L} Ri^{\mathrm{T}} J\mathcal{C}(\theta)}_{\frac{R}{L}[\xi_3 + B\xi_2 + \tau_L]} - \frac{\lambda_m^2 n_p^2}{L}\xi_2 + \frac{\lambda_m n_p}{L} v^{\mathrm{T}} J\mathcal{C}(\theta)\\
&\quad -n_p z\xi_2 - \dot{\tau}_L\\
&= -\frac{RB + \lambda_m^2 n_p^2}{L}\xi_2 - \frac{LB + Rj}{Lj}\xi_3 - n_p z\xi_2\\
&\quad -\frac{R}{L}\tau_L - \dot{\tau}_L + \frac{\lambda_m n_p}{L}\mathcal{C}^{\mathrm{T}}(\theta) J^{\mathrm{T}} v,\\
\dot{z} &= \lambda_m n_p\left[\frac{di^{\mathrm{T}}}{dt}\mathcal{C}(\theta) + n_p i^{\mathrm{T}} J\mathcal{C}(\theta)\dot{\theta}\right]\\
&= \frac{\lambda_m n_p}{L}\left[-Ri - \lambda_m n_p\omega J\mathcal{C}(\theta) + v\right]^{\mathrm{T}}\mathcal{C}(\theta)\\
&\quad +\lambda_m n_p^2 i^{\mathrm{T}}\omega J\mathcal{C}(\theta)\\
&= -\frac{R}{L}\underbrace{\lambda_m n_p i^{\mathrm{T}}\mathcal{C}(\theta)}_{z} + \frac{\lambda_m n_p}{L} v^{\mathrm{T}}\mathcal{C}(\theta) + \underbrace{\lambda_m n_p^2 i^{\mathrm{T}}\omega J\mathcal{C}(\theta)}_{n_p\xi_2[\xi_3 + B\xi_2 + \tau_L]}\\
&= -\frac{R}{L}z + n_p\xi_2\xi_3 + n_p B\xi_2^2 + n_p\tau_L\xi_2 + \frac{\lambda_m n_p}{L}\mathcal{C}^{\mathrm{T}}(\theta) v.\end{aligned}$$

Define the notations

$$g_2 = n_p, \quad g_3 = \frac{1}{j}, \quad b = \frac{\lambda_m n_p}{L},$$

and

$$\begin{aligned}q(z, \xi, \tau_L, \dot{\tau}_L) &= -\frac{RB + \lambda_m^2 n_p^2}{L}\xi_2 - \frac{LB + Rj}{Lj}\xi_3\\
&\quad -n_p z\xi_2 - \frac{R}{L}\tau_L - \dot{\tau}_L,\\
f_0(z, \xi, \tau_L) &= -\frac{R}{L}z + n_p\xi_2\xi_3 + n_p B\xi_2^2 + n_p\tau_L\xi_2,\end{aligned}$$

and write the system (2) in the new coordinates as

$$\begin{aligned}\dot{\xi}_1 &= g_2\xi_2,\\
\dot{\xi}_2 &= g_3\xi_3,\\
\begin{pmatrix}\dot{\xi}_3\\ \dot{z}\end{pmatrix} &= \begin{pmatrix} q(z, \xi, \tau_L, \dot{\tau}_L)\\ f_0(z, \xi, \tau_L)\end{pmatrix} + b\begin{pmatrix} -\sin\xi_1 & \cos\xi_1\\ \cos\xi_1 & \sin\xi_1\end{pmatrix} v.\end{aligned} \tag{4}$$

Since the signal ξ_1 is measurable, apply the Park's transformation as

$$v = \begin{pmatrix} v_\alpha\\ v_\beta\end{pmatrix} = \begin{pmatrix}\cos\xi_1 & -\sin\xi_1\\ \sin\xi_1 & \cos\xi_1\end{pmatrix}\begin{pmatrix} v_d\\ v_q\end{pmatrix},$$

in which $(v_d\ v_q)^{\mathrm{T}}$ is the new control signal in the rotating dq-frame. This transformation puts the system (4) into the form

$$\begin{aligned}\dot{z} &= f_0(z, \xi, \tau_L) + bv_d,\\
\dot{\xi}_1 &= g_2\xi_2,\\
\dot{\xi}_2 &= g_3\xi_3,\\
\dot{\xi}_3 &= q(z, \xi, \tau_L, \dot{\tau}_L) + bv_q.\end{aligned} \tag{5}$$

Choose $v_d = 0$, in which case (5) becomes

$$\begin{aligned} \dot{z} &= f_0(z, \xi, \tau_L), \\ \dot{\xi}_1 &= g_2\xi_2, \\ \dot{\xi}_2 &= g_3\xi_3, \\ \dot{\xi}_3 &= q(z, \xi, \tau_L, \dot{\tau}_L) + bv_q, \end{aligned} \tag{6}$$

Remark 1. Note that the upper subsystem, seen as a system with state z and inputs ξ_1, ξ_2, ξ_3 is input-to-state stable, because the exogenous signal $\tau_L(t) = \Psi w(t)$ is a bounded periodic input. We will exploit this property later. From physical point of view, the dynamics of z is fast due to the coefficient $-\frac{R}{L}$. If the current i is measurable, so as the variable z is available, such dynamics can be further accelerated by the appropriate control law chosen for v_d.

4. INTERNAL MODEL DESIGN

As it is well-known, the first step in solving a problem of output regulation consists in determining the solution pair $\pi(w), \psi(w)$ of the nonlinear regulator equations

$$\frac{\partial \pi}{\partial w} s(w) = f(w, \pi(w), \psi(w)), \qquad h_e(w, \pi(w)) = 0.$$

Inspection of (6) reveals that

$$\begin{aligned} \pi(w) &= 0, \\ \psi(w) &= -\frac{1}{b} q(0, 0, \tau, \dot{\tau}) = \frac{1}{b}\left(\frac{R}{L}\tau_L + \dot{\tau}_L\right) \\ &= \frac{1}{b}\left(\frac{R}{L}\Psi + \Psi S\right) w := \bar{\Psi} w. \end{aligned}$$

For the rejection of the exogenous input w we add the internal model of the form

$$\begin{aligned} \dot{\eta} &= F\eta + G[\Gamma\eta + \bar{u}], \\ v_q &= \Gamma\eta + \bar{u}, \end{aligned} \tag{7}$$

in which $\bar{u}$ is a control law to be defined later, $F \in \mathbb{R}^{n_w \times n_w}$ is a Hurwitz matrix in the companion form, the vectors $G = (0\ 0\ \cdots\ 0\ 1)^{\mathrm{T}} \in \mathbb{R}^{n_w}$ and $\Gamma \in \mathbb{R}^{1 \times n_w}$ are such that the pair (F, G) is controllable and the matrix $\Phi = F + G\Gamma$ has the same eigenvalues as the matrix S in the exosystem (1). It is known that there is a matrix Σ such that

$$\begin{aligned} \Sigma S &= (F + G\Gamma)\Sigma, \\ \bar{\Psi} &= \Gamma\Sigma. \end{aligned}$$

Adding such internal model we obtain the augmented system

$$\begin{aligned} \dot{z} &= f_0(z, \xi, \tau_L), \\ \dot{\eta} &= F\eta + G[\Gamma\eta + \bar{u}], \\ \dot{\xi}_1 &= g_2\xi_2, \\ \dot{\xi}_2 &= g_3\xi_3, \\ \dot{\xi}_3 &= q(z, \xi, \tau_L, \dot{\tau}_L) + b[\Gamma\eta + \bar{u}]. \end{aligned} \tag{8}$$

As a preliminary step in the design of the control $\bar{u}$, it is convenient to change variables as

$$\tilde{\eta} = \eta - \Sigma w,$$

taking derivative of which we obtain

$$\begin{aligned} \dot{\tilde{\eta}} &= F\eta + G[\Gamma\eta + \bar{u}] - \Sigma S w \\ &= (F + G\Gamma)\eta + G\bar{u} - (F + G\Gamma)\Sigma w \\ &= F\tilde{\eta} + G(\Gamma\tilde{\eta} + \bar{u}) \end{aligned}$$

and

$$\begin{aligned} \dot{\xi}_3 &= q(z, \xi, \tau_L, \dot{\tau}_L) + b[\Gamma\tilde{\eta} + \bar{\Psi} w + \bar{u}] \\ &= \tilde{q}(z, \xi, w) + b[\Gamma\tilde{\eta} + \bar{u}], \end{aligned}$$

in which $\tilde{q}(z, \xi, w) = q(z, \xi, \tau_L, \dot{\tau}_L) + b\bar{\Psi} w$ vanishes at $(z, \xi) = (0, 0)$ by definition of $\bar{\Psi}$.

In summary, system (8) can be put in the form

$$\begin{aligned} \dot{z} &= f_0(z, \xi, \Psi w), \\ \dot{\tilde{\eta}} &= F\tilde{\eta} + G[\Gamma\tilde{\eta} + \bar{u}], \\ \dot{\xi}_1 &= g_2\xi_2, \\ \dot{\xi}_2 &= g_3\xi_3, \\ \dot{\xi}_3 &= \tilde{q}(z, \xi, w) + b[\Gamma\tilde{\eta} + \bar{u}]. \end{aligned} \tag{9}$$

If we choose the control law for $\bar{u}$ as

$$\bar{u} = -\Gamma\tilde{\eta} + \frac{1}{b}[-\tilde{q}(z, \xi, w) + K\xi], \tag{10}$$

then the system (9) reduces to

$$\begin{aligned} \dot{z} &= f_0(z, \xi, \Psi w), \\ \dot{\tilde{\eta}} &= F\tilde{\eta} + G\frac{1}{b}[-\tilde{q}(z, \xi, w) + K\xi], \\ \dot{\xi}_1 &= g_2\xi_2, \\ \dot{\xi}_2 &= g_3\xi_3, \\ \dot{\xi}_3 &= K\xi. \end{aligned}$$

The subsystem consisting of the lower three equations is a linear system that, if K is appropriately chosen, can be made asymptotically stable. Moreover, it is immediate to see that the upper system, namely

$$\begin{aligned} \dot{z} &= f_0(z, \xi, \Psi w), \\ \dot{\tilde{\eta}} &= F\tilde{\eta} + G\frac{1}{b}[-\tilde{q}(z, \xi, w) + K\xi], \end{aligned} \tag{11}$$

seen as a system with state $(z, \tilde{\eta})$ and input ξ is input-to-state stable. In fact, the upper subsystem of (11), seen a system with state z and inputs ξ_1, ξ_2, ξ_3 is input-to-state stable, as observed before. Also the lower subsystem of (11), seen as a system with state $\tilde{\eta}$ and inputs $\tilde{q}(z, \xi, w)$ and ξ is input-to-state stable. Moreover, $\tilde{q}(z, \xi, w)$ vanishes at $(z, \xi) = (0, 0)$. Thus, by known properties, (11) is input-to-state stable.

In summary, if K is appropriately chosen, the closed-loop system (9) appears as a globally stable system that drives the input-to-state stable system (11). By known facts, the entire system is globally asymptotically stable.

The control $\bar{u}$ thus defined is not implementable because it is based on availability of $z, w, \tilde{\eta}$. Instead of that, in the next section we will replace it with its "robust replica" based on the enhanced extended observer.

5. ENHANCED EXTENDED OBSERVER DESIGN

Following the design paradigm proposed in Freidovich and Khalil (2008) and enhanced in Isidori et al. (2019), choose the control law for $\bar{u}$ as

$$\bar{u} = \mathrm{sat}_N\, \hat{u} = \mathrm{sat}_N[\bar{b}^{-1}(-\sigma + K\hat{\xi})], \tag{12}$$

where $K = (k_1\ k_2\ k_3)$ and $N > 0$ are the design parameters, $\bar{b}$ is a nonzero variable, for which the following relation holds

$$|[b - \bar{b}]\bar{b}^{-1}| \leq \delta < 1,$$

and $\hat{\xi} = \left(\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3\right)^{\mathrm{T}}$ and σ are the states of the extended observer defined by the equations

$$\begin{aligned}\dot{\hat{\xi}}_1 &= g_2\hat{\xi}_2 + \kappa a_3(g_2\tilde{\theta} - \hat{\xi}_1),\\ \dot{\hat{\xi}}_2 &= g_3\hat{\xi}_3 + \kappa^2 a_2(g_2\tilde{\theta} - \hat{\xi}_1),\\ \dot{\hat{\xi}}_3 &= \sigma + \bar{b}\bar{u} + \kappa^3 a_1(g_2\tilde{\theta} - \hat{\xi}_1),\\ \dot{\sigma} &= \kappa^4 a_0(g_2\tilde{\theta} - \hat{\xi}_1),\end{aligned} \tag{13}$$

in which $a = (a_0\ a_1\ a_2\ a_3)$ and κ are the design parameters.

Remark 2. The parameter $g_3 = 1/j$, that has been considered so far as a constant, in general can be time-varying, but bounded (see Liu and Zhu (2017)).

We stress that global asymptotic stability, that was previously achieved by means of the non-implementable control law (10) based on actual (non-available) terms, now is no longer ensured, since in (12) we have replaced these terms with their estimates obtained by the high-gain observer (13) and, as originally suggested in Freidovich and Khalil (2008), used the saturation function $\mathrm{sat}_N(\cdot)$ to prevent finite escape times caused by peaking of the state due to high values of κ. However, as it is shown in Freidovich and Khalil (2008), for any fixed arbitrarily large compact set of initial conditions we still can guarantee boundedness of all the trajectories of the system and their convergence to any arbitrarily small set, which establishes the property of semiglobal practical stability. If in addition the control law (10) renders the system locally exponentially stable, as it is in our case, we can achieve semiglobal asymptotic stability. This being the case, the following conclusion can be drawn.

Proposition 3. Given an arbitrary compact set of initial conditions there is a choice of the design parameters such that, for $t \geq 0$ all trajectories of the closed-loop system (6), (7), (12), (13) remain bounded and $\tilde{\theta}(t)$ tends to zero as time tends to infinity, which accomplishes the goal (3).

6. SIMULATION

This section addresses simulation of the proposed control framework in the problem of output regulation of a PMSM. Consider the motor, which parameters are listed in Table 1.

Table 1. Parameters of the motor used in the simulation

Parameter [units]	Value
Inductance L [mH]	20
Resistance R [Ω]	7.2
Drive inertia j [$\mathrm{kg}\cdot\mathrm{cm}^2$]	0.55367
Viscous friction coefficient B [$\mathrm{N}\cdot\mathrm{m}\cdot\mathrm{s/rad}$]	0.006
Pairs of poles n_p	8
Magnetic flux λ_m [Wb]	0.06

Recalling Remark 2 in order to provide complete picture of the enhanced extended observer design framework let us consider a general case when the parameter g_3 is time-varying. Let us consider it, as shown in Fig. 1, in the form of a biased sinusoidal function

$$g_3 = \mathcal{B}_g + \mathcal{A}_g \sin \Omega_g t,$$

with $\mathcal{B}_g = 1/j$, $\mathcal{A}_g = 10^4$, and $\Omega_g = 1$. Note that g_3 is bounded as

$$0 < g_3^{\min} \leq g_3(t) \leq g_3^{\max},$$

with $g_3^{\min} = \mathcal{B}_g - \mathcal{A}_g$ and $g_3^{\max} = \mathcal{B}_g + \mathcal{A}_g$.

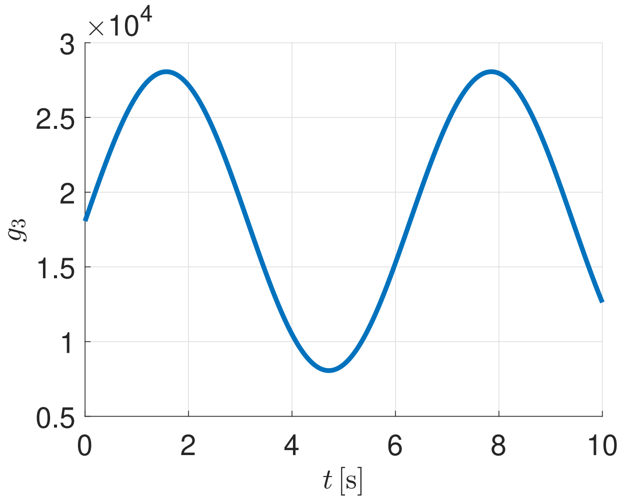

Fig. 1. The time-varying function g_3

Use the control law (12), (13) with the parameters

$$\bar{b} = 20,\ K = (-1\ -3\ -3),\ N = 10^7,\ \kappa = 10^3,$$

and the coefficients

$$a = (a_0\ a_1\ a_2\ a_3)$$

recursively calculated following the design framework of Isidori et al. (2019) (see details in Appendix A) as

$$\begin{aligned}a_3 &= b_3,\\ a_2 &= b_3 b_2,\\ a_1 &= b_3 b_2 b_1,\\ a_0 &= b_3 b_2 b_1 b_0,\end{aligned}$$

in which b_3, b_2, b_1, b_0 are chosen as

$$\begin{aligned}b_0 &= 1,\\ b_1 &= \frac{L_1 b_0 g_4^{\max} + b_0 g_4^{\max}}{g_3^{\min}},\\ b_2 &= \frac{L_2(g_4^{\max}\sqrt{2} + 2b_1 b_0 g_3^{\max}) + b_1 b_0 g_3^{\max}}{g_2^{\min}},\\ b_3 &= \frac{L_3(g_3^{\max}\sqrt{3} + 3b_2 b_1 b_0 g_2^{\max}) + b_2 b_1 b_0 g_2^{\max}}{g_1^{\min}},\end{aligned}$$

with $L_1 = L_2 = L_3 = 1$, $g_1^{\min} = g_4^{\max} = 1$, $g_2^{\max} = g_2$.

The controller then cascaded with the internal model (7) with the parameters

$$F = \begin{pmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ -1 & -3 & -3 \end{pmatrix}, \quad G = \begin{pmatrix} 0\\ 0\\ 1 \end{pmatrix}, \quad \Gamma = (1\ 2\ 3)\,.$$

Simulation results are shown in Fig 2–5 for different types of τ_L, namely

$$\begin{aligned}\tau_L &= 0,\\ \tau_L &= \mathcal{B},\\ \tau_L &= \mathcal{B} + \mathcal{A}\sin(\Omega t + \varphi),\\ \tau_L &= \mathcal{B} + \mathcal{A}\sin(\Omega t + \varphi) + \Delta,\end{aligned}$$

where $\mathcal{B}$ is the bias, $\mathcal{A}$ is the amplitude, Ω is the frequency, φ is the phase, Δ is the band-limited white noise. The height of the power spectral density of Δ is set as 0.001.

As the figures show, the controller steers the output $\theta(t)$ to the given reference θ^* as well as the internal model rejects effects caused by the nonzero load torque τ_L. Despite of the presence of the time-varying parameter g_3, the system remains stable due to the use of the enhanced structure

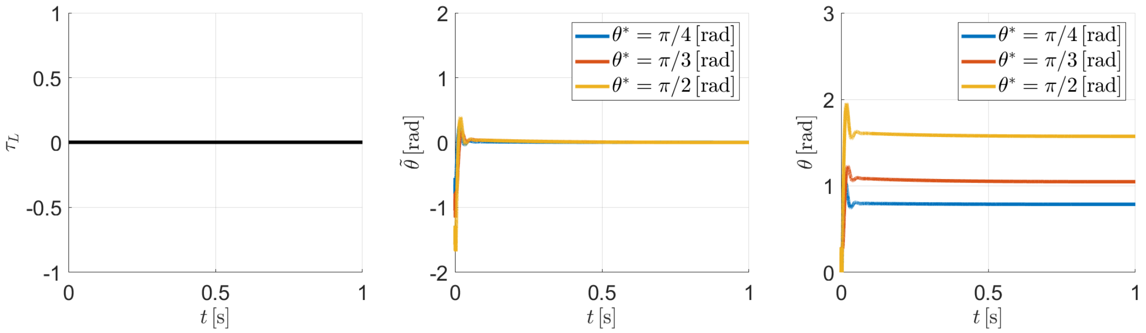

Fig. 2. Simulation results for the load-free case

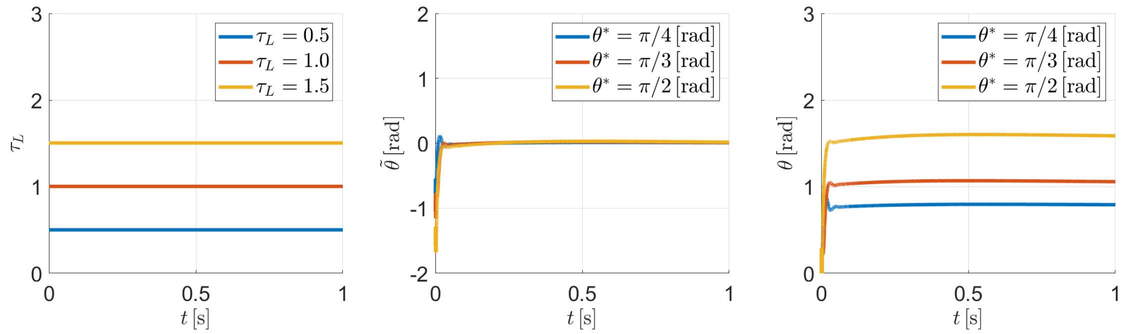

Fig. 3. Simulation results for the static load

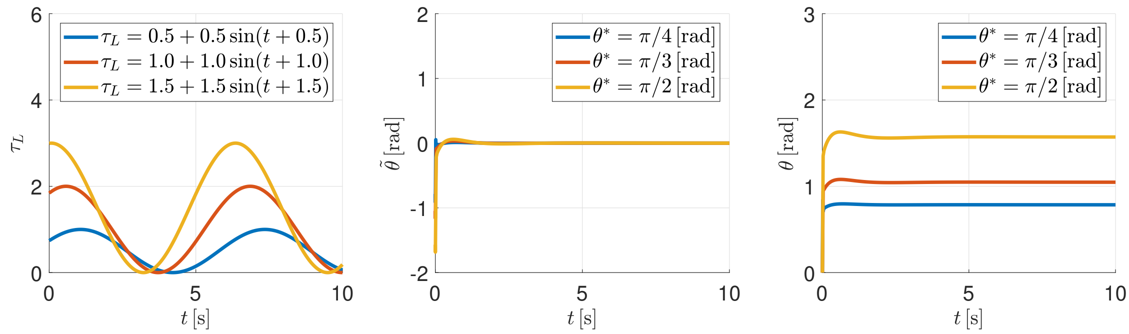

Fig. 4. Simulation results for the biased sinusoidal load

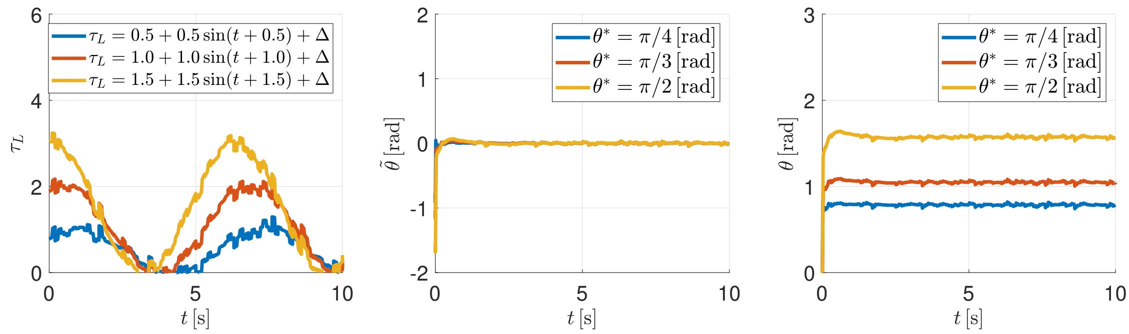

Fig. 5. Simulation results for the biased sinusoidal load with the band-limitted white noise

of the extended observer (13). The performance of the system is reasonably affected by the white noise in the fourth simulation, however the oscillations of the output remain bounded.

7. CONCLUSION

This applied research study addresses the output regulation problem of the PMSM and in addition to Borisov et al. (2019) illustrates another application of the enhanced extended observer design proposed in Isidori et al. (2019). This paper, moreover, shows benefits of cascading the controller with the internal model, that cancels any deterministic external disturbance, represented in the considered application by the time-varying load torque. As a direction of the future work, the authors keep working on advanced sensorless control techniques, for which measurements of regulated signals such as the rotor position or its velocity are no longer required.

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Appendix A. RECURSIVE CALCULATION OF THE ENHANCED EXTENDED OBSERVER COEFFICIENTS

As shown in the proof of the extended Dayawansa's Lemma given in Isidori et al. (2019), the coefficients $a_0, a_1, \dots a_{n-2} a_{n-1}$ can be taken as

$$\begin{aligned} a_{n-1} &= b_{n-1}, \\ a_{n-2} &= b_{n-1} b_{n-2}, \\ &\cdots \\ a_1 &= b_{n-1} b_{n-2} \cdots b_1, \\ a_0 &= b_{n-1} b_{n-2} \cdots b_1 b_0, \end{aligned}$$

in which b_0 can be taken equal to 1 and all other b_k's are coefficients, recursively calculated from $k = 1$ to $k = n-1$, required to respect inequalities of the form

$$\frac{\delta \|K_k\|}{d b_k g_{n-k}(t)} \ll 1, \tag{A.1}$$

in which $d = b_{k-1} \cdots b_1 b_0$, the vector K_k is of the form

$$K_k = \begin{pmatrix} b_{k-1} \\ b_{k-1} b_{k-2} \\ \vdots \\ b_{k-1} b_{k-2} \cdots b_1 \\ b_{k-1} b_{k-2} \cdots b_1 b_0 \end{pmatrix},$$

and

$$\delta = L_k ||A_k(t) - K_k C_k(t)|| + d||C_k(t)||,$$

with $L_k > 0$ and

$$A_k(t) = \begin{pmatrix} 0 & g_{n-k+2}(t) & 0 & \cdots & 0 & 0 \\ 0 & 0 & g_{n-k+3}(t) & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & 0 & g_n(t) \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix},$$

$$C_k(t) = \begin{pmatrix} g_{n-k+1} & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}.$$

Using the fact that

$$0 < g_{n-k}^{\min} \leq g_{n-k}(t) \leq g_{n-k}^{\max},$$

we replace (A.1) by

$$\frac{\delta \|K_k\|}{d b_k g_{n-k}^{\min}} \ll 1. \tag{A.2}$$

Consider the bound for δ as

$$\delta \leq L_k(\max\{g_r^{\max}\} + \|K_k\| g_{n-k+1}^{\max}) + d g_{n-k+1}^{\max},$$

with L_k being a fixed number and $r = \{n-k+2, n-k+3, \dots, n\}$. Assuming, w.l.o.g., that $b_k \geq 1$ for all k's, we have

$$\|K_k\| = d\sqrt{k}.$$

Hence, we have

$$\delta \leq L_k(\max\{g_r^{\max}\} + d\sqrt{k} g_{n-k+1}^{\max}) + d g_{n-k+1}^{\max}$$

and then (A.2) can be replaced by

$$\frac{L_k(\max\{g_r^{\max}\}\sqrt{k} + dk g_{n-k+1}^{\max}) + d g_{n-k+1}^{\max}}{b_k g_{n-k}^{\min}} \ll 1$$

or

$$b_k \gg \frac{L_k(\max\{g_r^{\max}\}\sqrt{k} + dk g_{n-k+1}^{\max}) + d g_{n-k+1}^{\max}}{g_{n-k}^{\min}},$$

which represents a final condition to be respected in order to appropriately set the coefficients $b_0, b_1, \dots, b_{n-1}$ and hence $a_0, a_1, \dots, a_{n-1}$ by increasing the parameter L_k.