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Robust Fault Detection for switched Takagi-Sugeno systems with unmeasurable premise variables: Interval-Observer-based approach

Yosr Garbouj * **Thach Ngoc Dinh** **,† **Zhenhua Wang** ***
Talel Zouari ****,* **Moufida Ksouri** * **Tarek Raïssi** **

* *University of Tunis El Manar, National Engineering School of Tunis (ENIT), Analysis, Conception and Control of Systems Laboratory (LR-11-ES20), BP 37, Le Belvedere 1002, Tunis, Tunisia.*
** *Conservatoire National des Arts et Métiers (CNAM), Cedric-Laetitia, 292 rue St-Martin, 75141 Paris Cedex 03, France.*
*** *School of Astronautics, Harbin Institute of Technology, Harbin 150001, PR China.*
**** *Department of Electromechanical Engineering, ESPRIT School of Engineering, Tunis, Tunisia.*
† *Corresponding author (e-mail: ngoc-thach.dinh@lecnam.net).*

Abstract: This paper deals with the problem of robust fault detection for continuous-time switched Takagi-Sugeno (T-S) fuzzy models. A procedure based on interval observers is proposed. First, an interval observer is designed under the assumption that the disturbances as well as the uncertainties are unknown but bounded. Stability and nonnegativity properties are given in terms of Linear Matrix Inequalities (LMIs) taking into account disturbances attenuation. Next, residual intervals generated by the interval observer are used for fault detection decision. Finally, a numerical example is provided to show the usefulness of this approach.

Keywords: Nonlinear switched system, interval observer design, Takagi-Sugeno models, unmeasurable premise variables, stability, robustness.

1. INTRODUCTION

Over the past decades, we have witnessed growing interests in *Takagi-Sugeno (T-S) fuzzy models* (Takagi and Sugeno (1993)). They have been considered as a powerful tool to cope with nonlinearities. T-S fuzzy systems use the center-of-gravity method for defuzzification (Nguyen et al. (2019)) decomposing a nonlinear system into different zones. The validity of each one is quantified by a nonlinear weighting function which depends on the so-called premise variables. As many nonlinear systems with switching features can be modeled as switching fuzzy systems, it is obvious that the class of switched fuzzy systems can describe more precisely both continuous and discrete dynamics as well as their interactions in complex real-world systems, see e.g. (Zouari et al. (2014); Garbouj et al. (2019); Ojleska and Stojanovski (2008)). In the literature, few works are devoted to this family of systems (Benzaouia (2012)) and the present paper is so motivated. Actually, our objective is to consider the following compact form

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i^{\sigma}(x(t))(A_i^{\sigma}x(t) + B_i^{\sigma}u(t)) + d(t) \\ y(t) = C^{\sigma}x(t) \end{cases}, \quad (1)$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input, $y(t) \in \Re^p$ is the output, $d(t) \in \Re^n$ is the bounded additive disturbances, r is the number of local models and σ is the switching law such that $\sigma \in \{1,...,N\}$ is the index of the active mode with N is a positive integer. The weighting function $\mu_i^{\sigma}(x(t))$ depending on the premise variables which are composed of the system state $x(t)$. We assume that they are *unmeasurable*. The following convex sum property is satisfied:

$$\begin{cases} 0 \le \mu_i^{\sigma}(x(t)) \le 1, \forall \sigma \in \{1,...,N\}, \forall i \in \{1,\dots r\} \\ \sum_{i=1}^{r} \mu_i^{\sigma}(x(t)) = 1 \end{cases}. \quad (2)$$

Furthermore, *model-based fault detection* represents an important research field and many results have been obtained in this context. Basically, the key of model-based diagnosis approaches is to use a mathematical model representation of the system to generate fault indicators called *residuals*. These signals are obtained by the comparison of the system and its fault-free model. Set-theoretic approaches for *fault detection* have been recently developed (Puig (2010); Stoican and Olaru (2013)). Two main techniques are proposed: Set-invariance method (Hanafi et al. (2015)) and set-membership approach (Fernández-Cantí et al. (2016)). Among several set-membership methods, *interval observers* are often used thanks to their aibility to generate adaptive thresholds for the system's outputs under the common assumption that the disturbances and the uncertainties are unkonw but bounded (Gouzé et al. (2000)). Consistency checks between the measurements outputs and the interval observer outputs provide robust residuals which are used for robust fault detection (Raïssi et al. (2010)). According to the above mentioned studies, this paper deals with the problem of designing T-S interval observer based fault detection for a class of switched fuzzy systems. It is worth noting that most of existing works in the literature related to interval estimator design for fuzzy systems, even in the non-switching case, handled only the *measurable* premise variables case (Li et al. (2019); Menasria et al. (2017); Martínez García et al. (2017)). To the best of the author's knowledge, *unmeasurable* premise variables have not been yet fully investigated. The main feature of our design is in fact, to transform the considered system into an uncertain system subject to unknown but bounded disturbances through some changes of variables. Thanks to the convex properties (2), we can assume that uncertain terms including unmeasurable premise variables are bounded by known bounds. Then, interval observer can be constructed.

For the rest, preliminaries and the problem formulation are presented in Section 2. Section 3 provides the main results for designing robust fault detection based on T-S interval observer. A numerical example is given in section 4 followed by a conclusion in section 5.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Preliminaries

The set of real numbers is denoted by $\Re$ and the set of nonnegative real numbers is denoted by $\Re_{\geq 0}$, i.e., $\Re_{\geq 0} := [0, +\infty)$. Inequalities are understood *component-wise*, i.e., for $x_a = [x_{a,1}, ..., x_{a,n}]^\top \in \Re^n$ and $x_b = [x_{b,1}, ..., x_{b,n}]^\top \in \Re^n$, $x_a \leq x_b$ if and only if, for all $i \in \{1, ..., n\}$, $x_{a,i} \leq x_{b,i}$. The symbol $P \succ 0$ (resp. $P \prec 0$) means that the symmetric matrix P is positive (resp. negative) definite. E_p is a $(p \times 1)$ vector whose elements are equal to 1. I_n is the identity matrix with dimension $n \times n$. The symbol * denotes the transposed element in the symmetric positions of a matrix. The left and right endpoints of an interval $[x(t)]$ are denoted by $\underline{x}(t)$ and $\overline{x}(t)$ such as $[x(t)] = [\underline{x}(t), \overline{x}(t)]$. A matrix $A \in \Re^{n\times n}$ is called Metzler if all the off-diagonal elements are nonnegative. A matrix $A \in \Re^{n\times n}$ is said to be nonnegative if each entry of A is nonnegative. Given a matrix $A \in \Re^{m\times n}$, we define $A^+ = \max\{0, A\}$, $A^- = A^+ - A$ and denote the absolute value of a matrix by $|A| = A^+ + A^-$ (similarly for vectors). For square matrices T_i, we define $diag([T_1 \dots T_N]) = \begin{bmatrix} T_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & T_N \end{bmatrix}$.

Lemma 1. (Farina and Rinaldi (2011)). The system described by:

$$\dot{x}(t) = Ax(t) + u(t) \tag{3}$$

is said to be nonnegative if A is a Metzler matrix and $u(t) \geq 0$. For any initial condition $x(0) \geq 0$, the solution of (3) satisfies $x(t) \geq 0, \forall t \geq 0$.

Lemma 2. (Chebotarev et al. (2015)). Let $x \in \Re^n$ be a vector such that $\underline{x} \leq x \leq \overline{x}$.

(1) if $A \in \Re^{m\times n}$ is a constant matrix, then

$$A^+\underline{x} - A^-\overline{x} \leq Ax \leq A^+\overline{x} - A^-\underline{x}. \tag{4}$$

(2) if $A \in \Re^{m\times n}$ is a matrix satisfying $\underline{A} \leq A \leq \overline{A}$, for some $\underline{A}, \overline{A} \in \Re^{m\times n}$, then

$$\begin{aligned} &\underline{A}^+\underline{x}^+ - \overline{A}^+\underline{x}^- - \underline{A}^-\overline{x}^+ + \overline{A}^-\overline{x}^- \leq Ax \\ &\leq \overline{A}^+\overline{x}^+ - \underline{A}^+\overline{x}^- - \overline{A}^-\underline{x}^+ + \underline{A}^-\underline{x}^-. \end{aligned} \tag{5}$$

Lemma 3. (Rami et al. (2008)). A matrix $A \in \Re^{n\times n}$ is Metzler if and only if there exists $\eta \in \Re_{\geq 0}$ such that $A + \eta I \in \Re^{n\times n}_{\geq 0}$.

Consequently, if there exist a positive diagonal matrix $P \in \Re^{n\times n}$ and a constant $\eta > 0$ such that

$$PA + \eta P \geq 0, \tag{6}$$

then, A is Metzler.

Lemma 4. (Boyd et al. (1994); Jiang et al. (2002)). Consider x and y with appropriate dimensions and Ω a positive definite matrix. The following property is verified:

$$x^T y + y^T x \leq x^T \Omega x + y^T \Omega^{-1} y. \tag{7}$$

Consequently, let $\lambda > 0$ be a scalar and $P \in \Re^{n\times n}$ be a symmetric positive definite matrix, then:

$$2x^T y \leq \frac{1}{\lambda} x^T P x + \lambda y^T P^{-1} y, \quad \forall x, y \in \Re^n. \tag{8}$$

2.2 Problem formulation

For robust fault detection purpose, an interval observer is first designed for the continuous-time switched T-S fuzzy system (1). Let introduce some assumptions which are used throughout the paper.

Assumption 1.

$$\underline{d} \leq d(t) \leq \overline{d}, \; \forall t \geq 0 \tag{9}$$

where $\underline{d} = -\overline{d} \in \Re^n$.

Assumption 2. The state of the system $x(t)$ and the known input vector $u(t)$ are supposed to be bounded in norm.

Now, given the lower and upper bounds $\underline{x}(t), \overline{x}(t), \in \Re^n$ of the state $x(t)$, the system (1) can be rewritten equivalently to the two following forms $\forall \sigma \in \{1, 2, \dots, N\}$ and $\forall i \in \{1, \dots, r\}$:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i^\sigma(\overline{x}(t))(A_i^\sigma x(t) + B_i^\sigma u(t)) \\ \quad - \sum_{i=1}^{r} \overline{\delta}_i^\sigma(t)(A_i^\sigma x(t) + B_i^\sigma u(t)) + d(t) \\ \text{or} \\ \dot{x}(t) = \sum_{i=1}^{r} \mu_i^\sigma(\underline{x}(t))(A_i^\sigma x(t) + B_i^\sigma u(t)) \\ \quad + \sum_{i=1}^{r} \underline{\delta}_i^\sigma(t)(A_i^\sigma x(t) + B_i^\sigma u(t)) + d(t) \\ y(t) = C^\sigma x(t) \end{cases} \tag{10}$$

with $\overline{\delta}_i^\sigma(t) = \mu_i^\sigma(\overline{x}(t)) - \mu_i^\sigma(x(t))$ and $\underline{\delta}_i^\sigma(t) = \mu_i^\sigma(x(t)) - \mu_i^\sigma(\underline{x}(t))$.

Let us define:

$$\begin{aligned} \overline{\Delta A}^\sigma(t) &= \sum_{i=1}^{r} \overline{\delta}_i^\sigma(t) A_i^\sigma, & \underline{\Delta A}^\sigma(t) &= \sum_{i=1}^{r} \underline{\delta}_i^\sigma(t) A_i^\sigma \\ &= \mathscr{A}^\sigma \overline{\Sigma}_A^\sigma(t) \mathrm{E}_A & &= \mathscr{A}^\sigma \underline{\Sigma}_A^\sigma(t) \mathrm{E}_A \end{aligned} \tag{11}$$

$$\mathscr{A}^\sigma = \begin{bmatrix} A_1^\sigma & \cdots & A_r^\sigma \end{bmatrix}, \; \mathrm{E}_A = \begin{bmatrix} I_n & \cdots & I_n \end{bmatrix}^T \tag{12}$$

$$\overline{\Sigma}_A^\sigma(t) = diag([\overline{\delta}_1^\sigma(t) I_n \dots \overline{\delta}_r^\sigma(t) I_n]) \tag{13}$$

$$\underline{\Sigma}_A^\sigma(t) = diag([\underline{\delta}_1^\sigma(t) I_n \dots \underline{\delta}_r^\sigma(t) I_n]) \tag{14}$$

$$\begin{aligned} \overline{\Delta B}^\sigma(t) &= \sum_{i=1}^{r} \overline{\delta}_i^\sigma(t) B_i^\sigma, & \underline{\Delta B}^\sigma(t) &= \sum_{i=1}^{r} \underline{\delta}_i^\sigma(t) B_i^\sigma \\ &= \mathscr{B}^\sigma \overline{\Sigma}_B^\sigma(t) \mathrm{E}_B & &= \mathscr{B}^\sigma \underline{\Sigma}_B^\sigma(t) \mathrm{E}_B \end{aligned} \tag{15}$$

$$\mathscr{B}^\sigma = \begin{bmatrix} B_1^\sigma & \cdots & B_r^\sigma \end{bmatrix}, \; \mathrm{E}_B = \begin{bmatrix} I_m & \cdots & I_m \end{bmatrix}^T \tag{16}$$

$$\overline{\Sigma}_B^\sigma(t) = diag([\overline{\delta}_1^\sigma(t) I_m \dots \overline{\delta}_r^\sigma(t) I_m]) \tag{17}$$

$$\underline{\Sigma}_B^\sigma(t) = diag([\underline{\delta}_1^\sigma(t) I_m \dots \underline{\delta}_r^\sigma(t) I_m]) \tag{18}$$

Thus by using the convex sum properties given in (2) for all $\underline{x}(t) \in \Re^n$ and $\overline{x}(t) \in \Re^n$, the system (10) becomes :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i^\sigma(\overline{x}(t))[(A_i^\sigma - \overline{\Delta A}^\sigma(t))x(t) + \\ \quad (B_i^\sigma - \overline{\Delta B}^\sigma(t))u(t)] + d(t) \\ \text{or} \\ \dot{x}(t) = \sum_{i=1}^{r} \mu_i^\sigma(\underline{x}(t))[(A_i^\sigma + \underline{\Delta A}^\sigma(t)x(t) + \\ \quad (B_i^\sigma + \underline{\Delta B}^\sigma(t)u(t)] + d(t) \\ y(t) = C^\sigma x(t) \end{cases} \quad (19)$$

Remark 1. Due to the convex property of the weighting functions (2), we have $-1 \leq \overline{\delta}_i^\sigma(t) \leq 1$ and $-1 \leq \underline{\delta}_i^\sigma(t) \leq 1$. The terms $\overline{\Sigma}_A^\sigma(t)$, $\underline{\Sigma}_A^\sigma(t)$, $\overline{\Sigma}_B^\sigma(t)$ and $\underline{\Sigma}_B^\sigma(t)$ satisfy $\overline{\Sigma}_A^{\sigma T}(t)\overline{\Sigma}_A^\sigma(t) \leq I_{nr}$, $\underline{\Sigma}_A^{\sigma T}(t)\underline{\Sigma}_A^\sigma(t) \leq I_{nr}$, $\overline{\Sigma}_B^{\sigma T}(t)\overline{\Sigma}_B^\sigma(t) \leq I_{mr}$ and, $\underline{\Sigma}_B^{\sigma T}(t)\underline{\Sigma}_B^\sigma(t) \leq I_{mr}$.

Assumption 3. There exist known constant matrices $\overline{\Delta\chi}_{\min}^\sigma$, $\overline{\Delta\chi}_{max}^\sigma$, $\underline{\Delta\chi}_{\min}^\sigma$ and $\underline{\Delta\chi}_{\max}^\sigma$ where χ denotes the letter A or B, such that, for all $t \geq 0$, for all $\sigma \in \{1, 2, \ldots, N\}$,

$$\overline{\Delta\chi}_{\min}^\sigma \leq \overline{\Delta\chi}^\sigma(t) \leq \overline{\Delta\chi}_{\max}^\sigma,$$
$$\underline{\Delta\chi}_{\min}^\sigma \leq \underline{\Delta\chi}^\sigma(t) \leq \underline{\Delta\chi}_{\max}^\sigma.$$

3. MAIN RESULTS

3.1 Interval observer synthesis

Consider the following upper and lower dynamics with $\sigma \in \{1, 2, \ldots, N\}$ and $i \in \{1, \ldots, r\}$:

$$\begin{cases} \dot{\overline{x}}(t) = \sum_{i=1}^{r} \mu_i^\sigma(\overline{x}(t))[(A_i^\sigma - L_i^\sigma C^\sigma)\overline{x}(t) + B_i^\sigma u(t) + \\ \quad L_i^\sigma y(t) + \overline{d} - \overline{\varphi}_{A,\min}^\sigma(t) - \overline{\varphi}_{B,\min}^\sigma(t) \\ \dot{\underline{x}}(t) = \sum_{i=1}^{r} \mu_i^\sigma(\underline{x}(t))[(A_i^\sigma - L_i^\sigma C^\sigma)\underline{x}(t) + B_i^\sigma u(t) + \\ \quad L_i^\sigma y(t) + \underline{d} + \underline{\varphi}_{A,\min}^\sigma(t) + \underline{\varphi}_{B,\min}^\sigma(t) \end{cases} \quad (20)$$

where

$$\overline{\varphi}_{A,\min}^\sigma(t) = \overline{\Delta A}_{\min}^{\sigma+}\underline{x}^+(t) - \overline{\Delta A}_{\max}^{\sigma+}\underline{x}^-(t) - \overline{\Delta A}_{\min}^{\sigma-}\overline{x}^+(t) + \overline{\Delta A}_{\max}^{\sigma-}\overline{x}^-(t) \quad (21)$$

$$\overline{\varphi}_{B,\min}^\sigma(t) = \overline{\Delta B}_{\min}^\sigma u^+(t) - \overline{\Delta B}_{\max}^\sigma u^-(t) \quad (22)$$

$$\underline{\varphi}_{A,\min}^\sigma(t) = \underline{\Delta A}_{\min}^{\sigma+}\underline{x}^+(t) - \underline{\Delta A}_{\max}^{\sigma+}\underline{x}^-(t) - \underline{\Delta A}_{\min}^{\sigma-}\overline{x}^+(t) + \underline{\Delta A}_{\max}^{\sigma-}\overline{x}^-(t) \quad (23)$$

$$\underline{\varphi}_{B,\min}^\sigma(t) = \underline{\Delta B}_{\min}^\sigma u^+(t) - \underline{\Delta B}_{\max}^\sigma u^-(t). \quad (24)$$

Note that the superscripts + in the equations from (21) to (24) have been defined in the preliminaries. Let introduce the upper and lower observation errors $\overline{e}(t) = \overline{x}(t) - x(t)$ and $\underline{e}(t) = x(t) - \underline{x}(t)$. To obtain $\overline{e}(t)$ in (25), we use the first expression of $x(t)$ given in (19) while to obtain $\underline{e}(t)$ in (26), the second form of (19) is used. Hence,

$$\dot{\overline{e}}(t) = \sum_{i=1}^{r} \mu_i^\sigma(\overline{x}(t))((A_i^\sigma - L_i^\sigma C^\sigma)\overline{e}(t) + \overline{d} - d(t) + \overline{\Delta A}^\sigma(t)x(t) + \overline{\Delta B}^\sigma(t)u(t) + \overline{\psi}(t) \quad (25)$$

$$\dot{\underline{e}}(t) = \sum_{i=1}^{r} \mu_i^\sigma(\underline{x}(t))((A_i^\sigma - L_i^\sigma C^\sigma)\underline{e}(t) + d(t) - \underline{d} + \underline{\Delta A}^\sigma(t)x(t) + \underline{\Delta B}^\sigma(t)u(t) + \underline{\psi}(t) \quad (26)$$

where $\overline{\psi}(t) = -\overline{\varphi}_{A,\min}^\sigma(t) - \overline{\varphi}_{B,\min}^\sigma(t)$ and $\underline{\psi}(t) = -\underline{\varphi}_{A,\min}^\sigma(t) - \underline{\varphi}_{B,\min}^\sigma(t)$ with $\overline{\varphi}_{A,\min}^\sigma(t)$, $\overline{\varphi}_{B,\min}^\sigma(t)$, $\underline{\varphi}_{A,\min}^\sigma(t)$ and $\underline{\varphi}_{B,\min}^\sigma(t)$ defined in (21)-(24).

It is clear that the dynamics of the error estimation given in (25) and (26) depend on the state $x(t)$ and the disturbance $d(t)$, then the problem of designing the interval observer (20) is reduced to finding appropriate gains $L_i^\sigma \in \Re^{n\times p}$ for each mode $\sigma \in \{1, 2, \ldots, N\}$ in order to ensure the global asymptotic stability and the nonnegativity property of the errors dynamics and to minimize the influence of $d(t)$ on the upper and lower errors dynamics $\overline{e}(t)$ and $\underline{e}(t)$.

Let define the augmented upper and lower vectors as $\overline{e}_a(t) = \begin{bmatrix} \overline{e}^T(t) & x^T(t) \end{bmatrix}^T$ and $\underline{e}_a(t) = \begin{bmatrix} \underline{e}^T(t) & x^T(t) \end{bmatrix}^T$ from which the following dynamics are obtained with $\sigma \in \{1, 2, \ldots, N\}$ and $i, j \in \{1, \ldots, r\}$:

$$\dot{\overline{e}}_a(t) = \sum_{i=1}^{r} \mu_i^\sigma(\overline{x}(t)) \sum_{j=1}^{r} \mu_j^\sigma(x(t))(\overline{A}_{ij}^\sigma(t)\overline{e}_a(t) + \overline{B}_{ij}^\sigma(t)u(t)) + \overline{E}\overline{d} + \overline{F}d(t) + G\overline{\psi}(t) \quad (27)$$

$$\dot{\underline{e}}_a(t) = \sum_{i=1}^{r} \mu_i^\sigma(\underline{x}(t)) \sum_{j=1}^{r} \mu_j^\sigma(x(t))(\underline{A}_{ij}^\sigma(t)\underline{e}_a(t) + \underline{B}_{ij}^\sigma(t)u(t)) + \underline{F}d(t) + \underline{E}\underline{d} + G\underline{\psi}(t) \quad (28)$$

where

$$\begin{cases} \overline{A}_{ij}^\sigma(t) = \begin{bmatrix} A_i^\sigma - L_i^\sigma C^\sigma & \overline{\Delta A}^\sigma(t) \\ 0 & A_j^\sigma \end{bmatrix}, \overline{B}_{ij}^\sigma = \begin{bmatrix} \overline{\Delta B}^\sigma(t) \\ B_j^\sigma \end{bmatrix} \\ \underline{A}_{ij}^\sigma(t) = \begin{bmatrix} A_i^\sigma - L_i^\sigma C^\sigma & \underline{\Delta A}^\sigma(t) \\ 0 & A_j^\sigma \end{bmatrix}, \underline{B}_{ij}^\sigma = \begin{bmatrix} \underline{\Delta B}^\sigma(t) \\ B_j^\sigma \end{bmatrix} \\ \overline{E} = [\,I\ 0\,]^T, \underline{E} = [\,-I\ 0\,]^T \\ \overline{F} = [\,-I\ I\,]^T, \underline{F} = [\,I\ I\,]^T, G = [\,I\ 0\,]^T \end{cases}$$

Theorem 1. Let the system (19) satisfy Assumptions 1-3 and assume that $\underline{x}(0)$, $\overline{x}(0)$ are known and the initial state $x(0)$ verifies $\underline{x}(0) \leq x(0) \leq \overline{x}(0)$. If there exist a diagonal positive matrix $P_1 \in \Re^{n\times n}$, a positive definite matrix $P_2 \in \Re^{n\times n}$ and strictly positive scalars η^σ, ρ_1^σ, $\overline{\gamma}$ and λ^σ for all $\sigma \in \{1, \ldots, N\}$ such that for all $i, j \in \{1, \ldots, r\}$, the following constrained minimization problem

$$\underset{P_1, P_2, K_i^\sigma, \rho_1^\sigma}{\text{minimize}} \quad \overline{\gamma}$$
$$\text{subject to} \quad \begin{bmatrix} \phi_i^\sigma & 0 & P_1 & P_1\mathscr{A}^\sigma \\ * & \Upsilon_j^\sigma & 0 & 0 \\ * & * & -\overline{\gamma}I & 0 \\ * & * & * & -\frac{1}{\rho_1^\sigma}I \end{bmatrix} \prec 0 \quad (29)$$
$$P_1A_i^\sigma - K_i^\sigma C^\sigma + \eta^\sigma P_1 \geq 0.$$

where

$$\phi_i^\sigma = A_i^{\sigma T}P_1 + P_1A_i^\sigma - C^{\sigma T}K_i^{\sigma T} - K_i^\sigma C^\sigma + \frac{3}{\lambda^\sigma}P_1 + I_n$$
$$K_i^\sigma = P_1L_i^\sigma$$
$$\Upsilon_j^\sigma = A_j^{\sigma T}P_2 + P_2A_j^\sigma + \frac{3}{\lambda^\sigma}P_2 + \rho_1^\sigma \mathrm{E}_A^T\mathrm{E}_A,$$

with $\mathscr{A}^\sigma$, E_A defined in (12), is solvable, then (20) is an optimal interval observer for the system (1) that guarantees the attenuation of additive disturbances effect with the cost function computed by $\gamma = \sqrt{\overline{\gamma}}$.

Remark 2. Notice that the terms η^σ are fixed before solving the LMIs (29) by Matlab. Thus, it is not a nonlinear optimization problem.

Proof.

(1) Stability property

Consider the following common Lyapunov function for the augmented upper dynamic (27):

$$V(\overline{e}_a(t)) = \overline{e}_a{}^T(t)P\overline{e}_a(t),\ P = diag([\,P_1\ \ P_2\,]) \succ 0 \qquad (30)$$

Taking the derivative of the Lyapunov function (30) along all trajectories of (27), then $\forall \sigma \in \{1,2,\ldots,N\}$ and $\forall i,j \in \{1,\ldots,r\}$:

$$\begin{aligned}\dot{V}(\overline{e}_a(t)) &= \dot{\overline{e}}_a^T(t)P\overline{e}_a(t) + \overline{e}_a{}^T(t)P\dot{\overline{e}}_a(t)\\ &= \sum_{i=1}^{r}\mu_i^\sigma(\overline{x}(t))\sum_{j=1}^{r}\mu_j^\sigma(x(t))(\overline{e}_a^T(t)\overline{A}_{ij}^{\sigma T}(t)P\overline{e}_a(t)\\ &\quad+\overline{e}_a^T(t)P\overline{A}_{ij}^{\sigma}(t)\overline{e}_a(t) + 2\overline{e}_a^T(t)P\overline{B}_{ij}^{\sigma}(t)u(t))\\ &\quad+\overline{d}^T\overline{E}^TP\overline{e}_a(t) + \overline{e}_a^TP\overline{E}\overline{d}\\ &\quad+2\overline{e}_a^T(t)P\overline{F}d(t) + 2\overline{e}_a^T(t)PG\overline{\psi}(t))\end{aligned} \qquad (31)$$

Based on Lemma 4, the following inequalities are deduced where $\lambda^\sigma > 0$ for all $\sigma \in \{1,\ldots,N\}$ can be selected arbitrarily

$$\begin{aligned}2\overline{e}_a^T(t)P\overline{B}_{ij}^\sigma(t)u(t) &\le \frac{1}{\lambda^\sigma}\overline{e}_a^T(t)P\overline{e}_a(t)\\ &\quad+u^T(t)\overline{B}_{ij}^{\sigma T}(t)\,[\lambda^\sigma P]\,\overline{B}_{ij}^\sigma(t)u(t)\\ 2\overline{e}_a^T(t)P\overline{F}d(t) &\le \frac{1}{\lambda^\sigma}\overline{e}_a^T(t)P\overline{e}_a(t)\\ &\quad+d^T(t)\overline{F}^T\,[\lambda^\sigma P]\,\overline{F}d(t)\\ 2\overline{e}_a^T(t)PG\overline{\psi}(t) &\le \frac{1}{\lambda^\sigma}\overline{e}_a^T(t)P\overline{e}_a(t)\\ &\quad+\overline{\psi}^T(t)G^T\,[\lambda^\sigma P]\,G\overline{\psi}(t)\end{aligned} \qquad (32)$$

Using the property of the weighting function given in (2), the combination of (31) and (32) leads to:

$$\dot{V}(\overline{e}_a(t)) \le \overline{e}_a^T(t)\Gamma^\sigma\overline{e}_a(t) + \overline{d}^T\overline{E}^TP\overline{e}_a(t) + \overline{e}_a^TP\overline{E}\overline{d} + \upsilon^\sigma \qquad (33)$$

where for $\sigma \in \{1,\ldots,N\}$, $i,j \in \{1,\ldots,r\}$

$$\begin{aligned}\Gamma^\sigma &= \overline{A}_{ij}^{\sigma T}P + P\overline{A}_{ij}^\sigma + \frac{3}{\lambda^\sigma}P,\\ \upsilon^\sigma &=\\ &\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i^\sigma(\overline{x}(t))\mu_j^\sigma(x(t))\left(u^T(t)\overline{B}_{ij}^{\sigma T}(t)\,[\lambda^\sigma P]\,\overline{B}_{ij}^\sigma(t)u(t)\right)\\ &\quad+d^T(t)\overline{F}^T\,[\lambda^\sigma P]\,\overline{F}d(t) + \overline{\psi}^T(t)G^T\,[\lambda^\sigma P]\,G\overline{\psi}(t)\end{aligned} \qquad (34)$$

Based on Assumption 2, $\overline{\psi}(t)$ given in (25) is bounded and based on Assumption 1, it follows that υ^σ in (33) is bounded for all $\sigma \in \{1,\ldots,N\}$. Besides, the upper estimation error given in (25) can be seen as

$$\overline{e}(t) = H\overline{e}_a(t),\ H = [\,I\ \ 0\,] \qquad (35)$$

In the system (25), the effect of the known bound of the additive disturbances $d(t)$ on the upper observation error $\overline{e}$ is bounded by the positive real number $\overline{\gamma} = \gamma^2$ if the following condition holds (Boyd et al. (1994)):

$$\dot{V}(\overline{e}_a(t)) + \overline{e}^T(t)\overline{e}(t) - \overline{\gamma}\overline{d}^T\overline{d} \le 0 \qquad (36)$$

Since ϑ^σ given in (33) is bounded, thus, by substituting (33) and (35) in (36), (27) is *Input to State Stable (ISS)* (Vu et al. (2007)) if the following inequality holds

$$\begin{aligned}&\overline{e}_a^T(t)\Gamma^\sigma\overline{e}_a(t) + \overline{d}^T\overline{E}^TP\overline{e}_a(t) + \overline{e}_a^TP\overline{E}\overline{d}\\ &+\overline{e}_a^T(t)H^TH\overline{e}_a - \overline{\gamma}\overline{d}^T\overline{d} \le 0\end{aligned} \qquad (37)$$

or equivalently

$$\begin{bmatrix}\overline{e}_a\\ \overline{d}\end{bmatrix}^T\begin{bmatrix}\Gamma^\sigma + H^TH & P\overline{E}\\ * & -\overline{\gamma}I\end{bmatrix}\begin{bmatrix}\overline{e}_a\\ \overline{d}\end{bmatrix} \le 0 \qquad (38)$$

Replacing the term Γ^σ by its expression given in (34), the inequality (38) holds if the subsequent one is satisfied

$$\begin{bmatrix}\overline{A}_{ij}^{\sigma T}P + P\overline{A}_{ij}^\sigma + \frac{3}{\lambda^\sigma}P + I_n & P\overline{E}\\ * & -\overline{\gamma}I\end{bmatrix} \prec 0 \qquad (39)$$

Recall that $P = diag([\,P_1\ \ P_2\,])$, and replacing $\overline{A}_{ij}^{\sigma T}$ by its expression given in (27), we have:

$$\begin{bmatrix}\Theta_i^{\sigma T}P_1 + P_1\Theta_i^\sigma + \frac{3}{\lambda^\sigma}P_1 + I_n & P_1\overline{\Delta A}^\sigma(t) & P\bar{E}\\ * & A_j^{\sigma T}P_2 + P_2A_j^\sigma + \frac{3}{\lambda^\sigma}P_2 & 0\\ * & * & -\bar{\gamma}I\end{bmatrix} \prec 0 \qquad (40)$$

where $\Theta_i^\sigma = (A_i^\sigma - L_i^\sigma C^\sigma)$. Let rewrite (40) by separating the time-depending term $P_1\overline{\Delta A}^\sigma(t)$, we obtain

$$\begin{bmatrix}\Theta_i^{\sigma T}P_1 + P_1\Theta_i^\sigma + \frac{3}{\lambda^\sigma}P_1 + I_n & 0 & P\bar{E}\\ * & A_j^{\sigma T}P_2 + P_2A_j^\sigma + \frac{3}{\lambda^\sigma}P_2 & 0\\ * & * & -\bar{\gamma}I\end{bmatrix} + \overbrace{\begin{bmatrix}0 & P_1\overline{\Delta A}^\sigma(t) & 0\\ * & 0 & 0\\ * & * & 0\end{bmatrix}}^{\mathscr{W}} \prec 0 \qquad (41)$$

The matrix $\mathscr{W}$ can be decomposed such that $\mathscr{W} = \mathscr{Q} + \mathscr{Q}^T$ where

$$\mathscr{Q} = \begin{bmatrix}0 & P_1\overline{\Delta A}^\sigma(t) & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}. \qquad (42)$$

Using the definition of the uncertainty $\overline{\Delta A}^\sigma(t)$ given in (11), it yields the following partition of $\mathscr{Q}$

$$\mathscr{Q} = \overbrace{\begin{bmatrix}P_1\mathscr{A}^\sigma & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}}^{X}\overbrace{\begin{bmatrix}0 & \overline{\Sigma}_A^\sigma(t)\mathrm{E}_A & 0\\ 0 & 0 & 0\end{bmatrix}}^{Y}. \qquad (43)$$

Choosing $\Omega = diag([\,\rho_1^\sigma I_n\ \ \rho_2^\sigma I_n\,]) \succ 0$ with ρ_1^σ, ρ_2^σ are any strictly positive scalars for all $\sigma \in \{1,\ldots,N\}$. Applying Lemma 4 to (43) yields

$$\mathscr{W} \le X\Omega^{-1}X^T + Y^T\Omega Y. \qquad (44)$$

Bearing in mind that $\overline{\Sigma}_A^{\sigma T}(t)\overline{\Sigma}_A^\sigma(t) \le I_{nr}$ (see Remark 1), the following inequality holds

$$\mathscr{W} \le diag([\,\frac{1}{\rho_1^\sigma}P_1\mathscr{A}^\sigma\mathscr{A}^{\sigma T}P_1\ \ \rho_1^\sigma\mathrm{E}_A^T\mathrm{E}_A\ \ 0\,]) \qquad (45)$$

Substituting (45) in (41) leads to:

$$\begin{bmatrix}\Xi_i^\sigma & 0 & P\overline{E}\\ * & \Upsilon_j^\sigma & 0\\ * & * & -\overline{\gamma}I\end{bmatrix} \prec 0 \qquad (46)$$

where

$$\begin{aligned}\Xi_i^\sigma &= \Theta_i^{\sigma T}P_1 + P_1\phi_i^\sigma + \frac{3}{\lambda^\sigma}P_1 + I_n + \frac{1}{\rho_1^\sigma}P_1\mathscr{A}^\sigma\mathscr{A}^{\sigma T}P_1\\ \Upsilon_j^\sigma &= A_j^{\sigma T}P_2 + P_2A_j^\sigma + \frac{3}{\lambda^\sigma}P_2 + \rho_1^\sigma\mathrm{E}_A^T\mathrm{E}_A\end{aligned}$$

From LMI (29), based on the Schur complement (Boyd et al. (1994)) with $K_i^\sigma = P_1L_i^\sigma$ we can conclude that from (46), the augmented upper dynamic (27) is *ISS*. Similarly one can prove that the augmented lower dynamic (28) is *ISS*.

2 Nonnegativity property

First, from (21)-(24) and (5) of Lemma 2, the following inequalities hold

$$\begin{array}{l} \overline{\Delta A}^{\sigma}(t)x(t) \geq \overline{\varphi}^{\sigma}_{A,\min}(t),\ \underline{\Delta A}^{\sigma}(t)x(t) \geq \underline{\varphi}^{\sigma}_{A,\min}(t) \\ \overline{\Delta B}^{\sigma}(t)u(t) \geq \overline{\varphi}^{\sigma}_{B,\min}(t),\ \underline{\Delta B}^{\sigma}(t)u(t) \geq \underline{\varphi}^{\sigma}_{B,\min}(t) \end{array} \tag{47}$$

From Assumption 1, we have for all $\sigma \in \{1,\dots,N\}$, $i \in \{1,\dots,r\}$, $\overline{d} - d \geq 0$ and $d - \underline{d} \geq 0$. Thus from (25)-(26), it holds that

$$\begin{array}{l} \overline{\psi}(t) + \overline{d} - d(t) + \overline{\Delta A}^{\sigma}(t)x(t) + \overline{\Delta B}^{\sigma}(t)u(t) \geq 0 \\ d(t) - \underline{d} + \underline{\Delta A}^{\sigma}(t)x(t) + \underline{\Delta B}^{\sigma}(t)u(t) + \underline{\psi}(t) \geq 0 \end{array} \tag{48}$$

Subsequently, thanks to (29) and Lemma 3, one can ensure that $(A_i^{\sigma} - L_i^{\sigma}C^{\sigma})$ is Metzler for all $\sigma \in \{1,\dots,N\}$, for all $i \in \{1,\dots,r\}$ since $P_1(A_i^{\sigma} - L_i^{\sigma}C^{\sigma}) + \eta^{\sigma}P_1 \geq 0, \forall \sigma \in \{1,\dots,N\}, \forall i \in \{1,\dots,r\}$ and P_1 is diagonal positive matrix. Lastly, according to Lemma 1, if $\overline{x}(0)$ and $\underline{x}(0)$ are supposed to be known such that

$$\begin{cases} \overline{e}(0) = \overline{x}(0) - x(0) \geq 0 \\ \underline{e}(0) = x(0) - \underline{x}(0) \geq 0 \end{cases},$$

then the dynamics of the estimation errors given in (25)-(26) stay positive and consequently, $\underline{x}(t) \leq x(t) \leq \overline{x}(t)$ which completes the proof.

Remark 3. For ensuring the stability property, we employ a common Lyapunov function (30) which is restrictive but standard in designing interval observer for switched systems, and it has previously been used by many works in the same context, see e.g., (Ethabet et al. (2018); Dinh et al. (2019)).

3.2 Robust Fault Detection based on interval observer

In this section, the previous results are used to generate residuals for fault detection. Under the presence of a sensor fault, system (1) can be represented by:

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{r} \mu_i^{\sigma}(x(t))(A_i^{\sigma}x(t) + B_i^{\sigma}u(t)) + d(t) \\ y(t) = C^{\sigma}x(t) + f(t) \\ \forall \sigma \in \{1,2,\dots,N\}, \forall i \in \{1,\dots,r\}, \end{cases} \tag{49}$$

where $f(t) \in \Re^p$ denotes p^{th} sensor fault. The principle of model-based fault approaches is to compare the measurements $y(t)$ with their estimates $\widehat{y}(t)$ provided by a faultless model. The comparison leads to the generation of a residual $r(t) \in \Re^p$ given by:

$$r(t) = \widehat{y}(t) - y(t). \tag{50}$$

In fault-free operation, the residual are arround zero. Nevertheless, when considering a system's model affected by perturbations and uncertainties given in (19), the residuals deviate from zero even in the fault-free scenario. To cope with this problem, a passive approach is used based on the interval observer (20) designed in the previous section.

Based on (4) of *Lemma* 2, the lower and upper outputs of the system (1) are given by:

$$\begin{cases} \overline{y}(t) = C^{\sigma+}\overline{x}(t) - C^{\sigma-}\underline{x}(t) \\ \underline{y}(t) = C^{\sigma+}\underline{x}(t) - C^{\sigma-}\overline{x}(t) \end{cases} \tag{51}$$

Let $\left[\underline{y}(t), \overline{y}(t)\right]$ be the domain of the output $y(t)$, the fault detection test can be formulated as $y(t) \notin \left[\underline{y}(t), \overline{y}(t)\right]$ which is equivalent to:

$$0 \notin [\underline{r}(t), \overline{r}(t)]. \tag{52}$$

where

$$\begin{cases} \underline{r}(t) = \underline{y}(t) - y(t) = -C^{\sigma+}(x(t) - \underline{x}(t)) - C^{\sigma-}(\overline{x}(t) - x(t)) \\ \overline{r}(t) = \overline{y}(t) - y(t) = C^{\sigma+}(\overline{x}(t) - x(t)) - C^{\sigma-}(x(t) - \underline{x}(t)) \end{cases} \tag{53}$$

Thus, the residual is described by and adaptative threshold.

Remark 4. The considered system in this paper is affected by unknown but bounded disturbances. If these bounds are large, so does the width of the interval observer and it may lead to misdetection of the small faults. The proposed T-S interval observer design method given in (20) allows to compute optimal gains which attenuate the effect of the system's disturbances and ensure a tighter interval width which make it possible to detect low amplitude faults.

4. NUMERICAL EXAMPLE

To show the effectiveness of the proposed fault detection method, a switched system described by (49) is considered as follows

$$A_1^1 = \begin{bmatrix} -0.9 & 0 & -0.45 \\ 0 & -2.1 & 0 \\ 0 & 0 & -0.1 \end{bmatrix}, A_2^1 = \begin{bmatrix} -3.86 & 0 & 1.22 \\ 0 & -0.15 & 0 \\ 0 & 0 & -0.1 \end{bmatrix}$$
$$A_1^2 = \begin{bmatrix} -5.5 & 0 & 1.5 \\ 0 & -1.1 & 0 \\ 0 & 0 & -0.1 \end{bmatrix}, A_2^2 = \begin{bmatrix} -2.6 & 0 & 0.3 \\ 0 & -0.15 & 0 \\ 0 & 0 & -0.1 \end{bmatrix}$$
$$B_1^1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, B_2^1 = B_1^2 = B_2^2 = B_1^1,$$
$$C^1 = [\,0\ 0\ 1.2\,],\ C^2 = [\,0\ 0\ 1.7\,]. \tag{54}$$

The weighting functions are hyperbolic tangent functions and depend on the unmeasured state x_1:

$$\begin{cases} \xi(t) = x_1(t) \\ \mu_1^{\sigma}(x(t)) = \frac{1}{2}(1 - \tanh(x_1(t)),\ \forall \sigma \in \{1,2\} \\ \mu_2^{\sigma}(x(t)) = 1 - \mu_1^{\sigma}(x_1(t)),\ \forall \sigma \in \{1,2\} \end{cases} \tag{55}$$

For the simulation, the disturbances are chosen such as: $d(t) = 0.1[\,\cos(3.5t)\ \cos(3.5t)\ \cos(3.5t)\,]^T$, $\overline{d} = [\,0.1\ 0.1\ 0.1\,]^T$ and $\underline{d} = -\overline{d}$. Thus, Assumption 1 is satisfied. The switching signal between the two modes of the considered system is plotted in Figure 1. The fault signal is set up as:

$$f(t) = \begin{cases} 0.03,\ 2s \leq t \leq 4s \\ 0.02,\ 8s \leq t \leq 9s \\ 0 \text{ otherwise} \end{cases} \tag{56}$$

The initial conditions are $x(0) = [\,0\ 0\ 0\,]^T$ and $\overline{x}(0) = -\underline{x}(0) = [\,0.1\ 0.1\ 0.1\,]^T$. By fixing $\lambda^1 = 85.96$, $\lambda^2 = 85.76$, $\eta^1 = 10$ and $\eta^2 = 26$, the solution of LMIs (29) of Theorem 1 are obtained using the package CVX (Grant et al. (2008)). The values of the optimal gains are given by:

$$L_1^1 = \begin{bmatrix} -0.3750 \\ 0.0000 \\ 16.4867 \end{bmatrix}, L_2^1 = \begin{bmatrix} 1.0167 \\ 0.0000 \\ 16.4867 \end{bmatrix},$$
$$L_1^2 = \begin{bmatrix} 0.0824 \\ 0.0000 \\ 11.6377 \end{bmatrix}, L_2^2 = \begin{bmatrix} 0.1765 \\ 0.0000 \\ 11.6377 \end{bmatrix}$$

The attenuation level is $\overline{\gamma} = 7.5443$. We verify that the matrices $A_i^{\sigma} - L_i^{\sigma}C^{\sigma}$ are Metzler for all $\sigma \in \{1,2\}$ and for all $i \in \{1,2\}$. In Figure 2, it is clear that the relations $\underline{y}(t) \leq y(t) \leq \overline{y}(t)$ and $0 \in [\underline{r}(t), \overline{r}(t)]$ hold in fault-free case while these relations are broken when the fault occurs. It should be noted that despite the low values of the considered fault (56), the detection is successful. At the instant $t = 4.01s$, the fault is still detected, which is explained by the fault extension because of the fact that the switching instant $t = 3s$ happens during the faulty period $2s \leq t \leq 4s$.

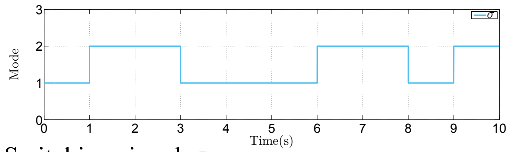

Fig. 1. Switching signal σ

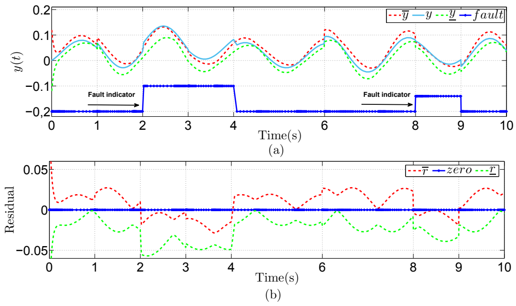

Fig. 2. Interval estimation of (a) the output and (b) the residual

5. CONCLUSION

In this paper, an interval observer has been designed to cope with the problem of robust fault detection for T-S switched fuzzy systems where the premise variables are unmeasurable. Through changes of variables, we can transform the considered system into uncertain one before the interval observer can be designed. The attenuation of the disturbances effects to optimize the interval length is also taken into account. Based on this methodology, the interval of the residual is generated to be able to use for fault detection. Many extensions of this approach are possible. We plan to investigate the Fault Tolerant Control (FTC). Moreover, employing multiple Lyapunov functions to ensure stability property of the proposed interval observer may be expected.

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