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Minimum-time feedforward control in ratio control systems

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Abstract: In this paper we propose a ratio control scheme for industrial processes that exploits a two-state feedforward action in order to achieve a minimum-time set-point transient response, by considering the actuator constraints. A Tracking Ratio Station is then used to achieve the desired ratio value when the two processes have a different dynamics. The control architecture can easily be implemented with standard industrial hardware. Simulation results show the effectiveness of the methodology.

Keywords: Ratio control, Feedforward, Constraints, PID control.

1. INTRODUCTION

Ratio control architectures are often applied in industry when the main control specification is to keep a predefined value of the ratio between two (or more) process variables (Seborg et al., 2004; Shinskey, 1994, 1996). Relevant applications where ratio control is important include blending (where the appropriate mixing of two components determines the quality of the final product) and combustion systems, whose efficiency is determined by the air-fuel ratio.

Different ratio control schemes are usually considered in industrial practice, each of them having its pros and cons, which can be analyzed in order to select the most suitable one for a given application (Visioli, 2006). Usually, each process variable is controlled by means of a standard feedback Proportional-Integral-Derivative (PID) controller and then additional blocks and connections implement the ratio control. In the parallel control scheme, the same set-point signal is applied to both the closed-loop control systems, where one of them is suitably scaled by the value of the required ratio, so that the ratio is perfectly kept during the transient responses if the dynamics of the two loops are the same. This implies that, in case the dynamics of the two processes are the same, the same tuning of the PID controllers should be applied. The main drawback of this technique is that the ratio is lost when a disturbance occurs in anyone of the two loops, and if anyone of the two control signals becomes saturated.

The problem of compensating for load disturbances and saturating control signals can be partially addressed by using the process variable of one (master) loop (suitably scaled by the value of the required ratio) as the set-point signal of the other (slave) loop (this is the so-called series metered control). In this way the ratio can be tracked also when a disturbance affects the master loop or when the control signal in the master loop becomes saturated. A drawback is that the performance in the set-point response deteriorates, as the dynamics of the slave control loop introduces some delay between the two process variables. Also in this case a suitable choice of the parameters of the two PID controllers (for example by detuning the master loop and by making the slave loop more aggressive) might help in satisfying the control requirements. Load disturbances and saturation of the control signal in the slave loop are still not treated in this approach.

An additional problem is also often present in combustion systems, where an excessive amount of fuel has to be avoided in any case for safety reasons (Gomes, 1985). To prevent this, min and max selectors can be used in order to easily implement a nonlinear control system (usually termed as cross-limiting control) suitable for this purpose.

With the aim of improving the previous classical methodologies, other ratio control schemes have recently been proposed in the literature. The so-called Blend Station has been proposed in (Hägglund, 2001), where the response of the system to set-point changes is improved by using a weighting factor to determine the relative influence of the set-point and of the output of the master loop in the determination of the set-point signal for the slave control loop. In other words, the set-point value of the slave loop is calculated from a linear combination of the set-point and of the process variable of the master loop. The basic Blend Station technique has been further improved by using a time-varying weighting factor (Visioli, 2005) or, alternatively, suitable dynamic systems in order to perfectly keep the desired ratio during transient responses when the set-point value changes. Other approaches based on the Blend Station concept to address load disturbances have been proposed in (Yesil et al., 2007) and (Jin and Peng, 2014). Actually, the design of these architectures are based on a model of the two processes, which poses some concerns about the robustness of the method.

A different approach is represented by the so-called Tracking Ratio Station, for which the master and the slave control loops can be swapped depending on the state of the overall system (Hägglund, 2017). In particular, at each time instant, the external set-point is applied to the loop with the largest control error and its output is used as the set-point (scaled by the ratio value) for the other loop. A hysteresis can then be suitably employed to avoid too frequent switchings. The main advantage of the Tracking Ratio Station is that it is capable to handle also load disturbances, control signals saturations, or the use of manual control for one of the loops.

In all the above described schemes, the main goal of the control architecture is to keep the ratio between the two process variables, while the performance of the (set-point) transient responses (for example, in terms of overshoot or settling time) is determined mainly by the PID controllers. Indeed, achieving a fast transient response can be important in many applications (for example, during the startup of the process operations in order to obtain the target production rate as soon as possible) but, as already mentioned, the use of aggressive controllers can be in conflict with the design for the ratio tracking task. For this reason, a ratio control scheme that exploits also feedforward actions has been proposed in (Visioli and Hägglund, 2019). In particular, the transient response is determined (in the nominal case) by the application of an inversion-based feedforward signal to each of the control loops. Then, the PID controllers and the Tracking Ratio Station are used to increase the robustness of the system.

In this paper we propose a modification of the scheme described in (Visioli and Hägglund, 2019) in order to minimize the transient time by taking into account the actuators constraints. In particular, the two-state feedforward action developed in (Visioli, 2004) is employed in the master loop in order fully exploit the actuator capabilities in achieving the fastest transient. Then, the ratio between the two process variables is controlled by using the Tracking Ratio Station, which also provides the required robustness.

The paper is organized as follows. In Section 2 the control architecture is described by reviewing the feedforward control design and the Tracking Ratio Station and by describing how they can be suitably combined in the overall architecture. Simulation results showing the effectiveness of the technique are presented and discussed in Section 3. Finally, concluding remarks are given in Section 4.

2. RATIO CONTROL ARCHITECTURE

2.1 Problem formulation

We consider a ratio control problem with two self-regulating processes P_1 and P_2 with different dynamics. As it is usual in process control, their dynamics are assumed to be described by means of first-order-plus-dead-time (FOPDT) transfer functions:

$$\bar{P}_1(s) = \frac{K_1}{T_1 s + 1} e^{-L_1 s} \tag{1}$$

$$\bar{P}_2(s) = \frac{K_2}{T_2 s + 1} e^{-L_2 s} \tag{2}$$

where K_1 and K_2 are the two process gains, T_1 and T_2 are the time constants, and L_1 and L_2 are the dead times.

We denote the output variables of the first and second process y_1 and y_2 , respectively, and the two control variables u_1 and u_2. We assume that the control variables have a saturation value equal to u_{max}, that is, we assume that

$$-u_{max} \leq u_i \leq u_{max}, \qquad i = 1, 2. \tag{3}$$

The aim of the control architecture is to obtain $y_2(t) = ay_1(t)$, where a is the value of the ratio. In particular, we want to track the ratio when the process variables are required to be driven from a steady-state value to another one (in other words, a set-point change is applied to the control system. For the sake of clarity, and without loss of generality, we consider null initial conditions. Further, we want to minimize the transient time of the set-point response by taking into account the actuator constraints.

Remark 1. We have considered the same symmetric saturation values for both the actuators for the sake of clarity of the presentation of the methodology. However, handling different saturation values implies only slight changes, which can be simply determined, in the following description of the methodology.

2.2 Control architecture

The devised new ratio control architecture is shown in Figure 1, where the master loop is selected in such a way that P_1 is the process with the slowest dynamics. In particular, we assume $L_1 > L_2$. The overall scheme appears to be similar to the one proposed in (Visioli and Hägglund, 2019) but there are important differences, even if both of them exploit a feedforward control approach to improve the transient performance. In this paper the feedforward control law is applied only to the master loop with the aim to minimize the set-point response transient time and the set-point signal is suitably modified in order to make the feedforward action effective and to improve the ratio tracking performance (for which, also the Tracking Ratio Station plays a key role). More details are given in the following subsections.

2.3 Feedforward control law

A two-state feedforward control law has been proposed in (Visioli, 2004) in order to obtain, for a single FOPDT process, a minimum-time process variable transition from a given steady-state value to another one. In particular, if we consider process P_1 and, without loss of generality, a process variable transition from 0 to $\bar{y}_1$ to be achieved in a time interval τ, we can determine a feedforward control law defined as

$$u_{ff}(t) = \begin{cases} \bar{u}_{ff} & \text{if } t < \tau \\ \bar{y}_1/K_1 & \text{if } t \geq \tau \end{cases} \tag{4}$$

where

$$\tau = -T_1 \log\left(1 - \frac{\bar{y}_1}{K_1 \bar{u}_{ff}}\right) \tag{5}$$

If $u_{ff}(t)$ is given as the only input to the process, in the nominal case we have that the corresponding output function is

$$y_1(t) = \begin{cases} 0 & \text{if } t < L_1 \\ K_1 \bar{u}_{ff}\left(1 - e^{-\frac{t-L_1}{T_1}}\right) & \text{if } L_1 \leq t \leq \tau + L_1 \\ \bar{y}_1 & \text{if } t \geq \tau + L_1 \end{cases} \tag{6}$$

It can easily be shown that, if we select $\bar{u}_{ff} = u_{max}$, then a minimum transition time τ is achieved by considering the actuator constraints. For this reason, this feedforward signal is applied to the master loop of the ratio control scheme.

Remark 2. From a practical point of view it is convenient to consider a value of $\bar{u}_{ff}$ slightly less than u_{max} so that the saturation value of the actuator is not exceeded by the overall control action when the feedback control variable is added to the feedforward one because of the presence of modelling uncertainties and because of the Tracking Ratio Station.

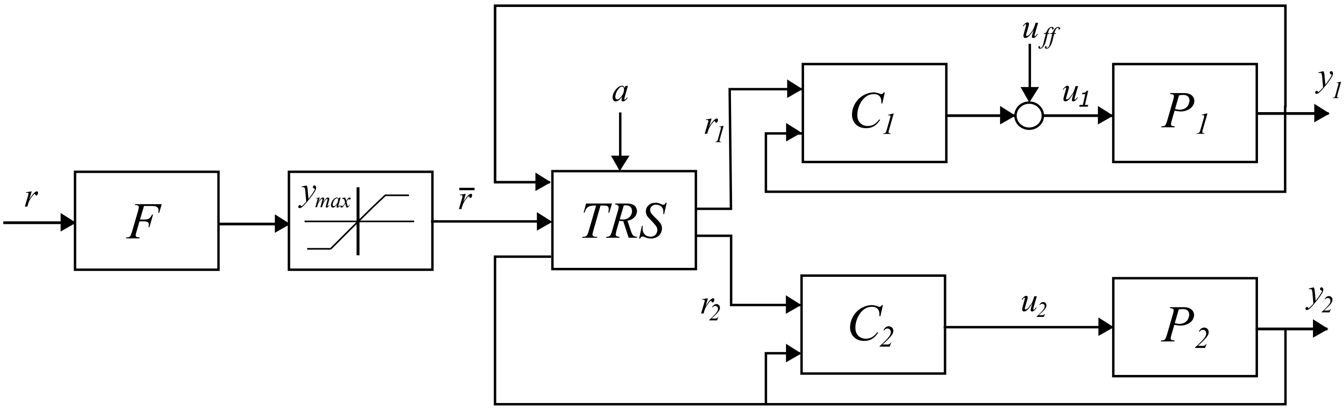

Fig. 1. Control scheme of the proposed ratio controller.

2.4 PI controllers tuning

As it is common practice in industry, the two feedback controllers are selected as PI controllers with transfer functions

$$C_1(s) = K_{p1}\left(1 + \frac{1}{T_{i1}s}\right) \tag{7}$$

$$C_2(s) = K_{p2}\left(1 + \frac{1}{T_{i2}s}\right) \tag{8}$$

where K_{p1} and K_{p2} the proportional gains and T_{i1} and T_{i2} are the integral time constants. They can be tuned by applying one of the many available tuning rules (O'Dwyer, 2006). Here we suggest to use the well-known AMIGO tuning rules (Åström and Hägglund, 2004, 2006), which are well known to be effective in providing a high robustness of the control loop. They are defined as ($j = 1,2$):

$$K_{pj} = \frac{0.15}{K_j} + \left(0.35 - \frac{L_jT_j}{(L_j+T_j)^2}\right)\frac{T_j}{K_jL_j}; \tag{9}$$

$$T_{ij} = 0.35L_j + \frac{13L_jT_j^2}{T_j^2 + 12L_jT_j + 7L_j^2} \tag{10}$$

2.5 Tracking Ratio Station

The Tracking Ratio Station is employed in order to force the process variable of the slave loop to track the process variable of the master loop. It consists of determining the loop with the largest control error and, for that loop, the signal $\bar{r}$ as the set-point and its process variable is chosen (after having been suitably scaled by the ratio value) as the set-point for the other loop. In this way there can be many switchings in a transient response and, in order to reduce their number, a hysteresis of width ε can be conveniently employed. Formally, we have:

$$r_1(t) = \begin{cases} \bar{r}(t) \text{ if } \begin{cases} |e_1(t)| - |e_2(t)| \geq \varepsilon/2 \\ |e_1(t)| - |e_2(t)| \geq -\varepsilon/2 \\ \text{and } r_1^{\text{old}}(t) = \bar{r}(t) \end{cases} \\ y_2(t)/a \text{ if } \begin{cases} |e_1(t)| - |e_2(t)| < \varepsilon/2 \\ \text{and } r_1^{\text{old}}(t) = y_2(t)/a \\ |e_1(t)| - |e_2(t)| < -\varepsilon/2 \end{cases} \end{cases} \tag{11}$$

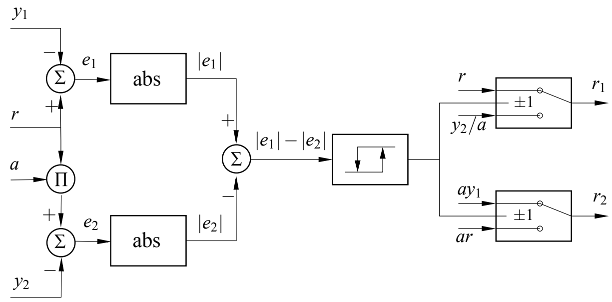

Fig. 2. Block diagram presentation of the Tracking Ratio Station.

$$r_2(t) = \begin{cases} \bar{r}(t) \text{ if } \begin{cases} |e_1(t)| - |e_2(t)| \geq \varepsilon/2 \\ |e_1(t)| - |e_2(t)| \geq -\varepsilon/2 \\ \text{and } r_2^{\text{old}} = \bar{r}(t) \end{cases} \\ y_2(t)/a \text{ if } \begin{cases} |e_1(t)| - |e_2(t)| < \varepsilon/2 \\ \text{and } r_2^{\text{old}}(t) = ay_1(t) \\ |e_1(t)| - |e_2(t)| < -\varepsilon/2 \end{cases} \end{cases} \tag{12}$$

where $e_1(t) = \bar{r}(t) - y_1(t)$ and $e_2(t) = \bar{r}(t) - y_2(t)$ are the control errors of the first and second loop, and $r_1^{\text{old}}(t)$ and $r_2^{\text{old}}(t)$ are the most recent values of the two set-point signals. A block scheme of the Tracking Ratio Station is shown in Figure 2.

2.6 Reference signal design

The presence of the slave loop and of the Tracking Ratio Station makes the output of the master loop, when the two-state feedforward action is applied, different from (6) even in the nominal case, that is, when there are no modelling uncertainties. This implies that the reference signal $\bar{r}$ of the ratio control system must not be selected as (6) as it should be done in the case of a single process control loop. On the contrary, it has to be designed in order to ensure the minimum time transition but, at the same time, to facilitate the tracking of the ratio value.

For this purpose, we calculate $\bar{r}$ so that the slave loop achieves the same transition time τ as the master loop. In particular, we calculate τ through (5) and then we calculate the value of the two-state feedforward control law that should be applied to the second process to obtain such a transition. By denoting it as $\bar{u}_{ff2}$, the following result is obtained:

$$\bar{u}_{ff2} = \frac{\bar{y}_2/K_2}{1 - e^{-\tau/T_2}} \tag{13}$$

where $\overline{y_2} = a\overline{y_1}$. Then, we set the filter F transfer function in such a way that the reference signal $\overline{r}$ becomes

$$\overline{r}(t) = \begin{cases} 0 & \text{if } t < L_1 \\ K_2\overline{u}_{ff2}\left(1 - e^{-\frac{t-L_1}{T_2}}\right) & \text{if } L_1 \le t \le \tau + L_1 \\ \overline{y_1} & \text{if } t \ge \tau + L_1 \end{cases} \tag{14}$$

Thus, we obtain:

$$F(s) = \frac{K_2\overline{u}_{ff2}/\overline{y_1}}{T_2 s + 1} e^{-L_1 s} \tag{15}$$

The value of the output of $F(s)$ is then saturated at the value $y_{max} = \overline{y_1}$.

Remark 3. Note that the value of the dead time of the master loop is used to calculate the reference signal in order to synchronize the two process variables.

Remark 4. As the dynamics of P_2 is faster than the dynamics of P_1, the rise time of the reference signal $\overline{r}$ will in general be lower than the rise time of nominal output signal of P_1 caused by the two-state feedforward action. This is indeed beneficial for the ratio control performance as it allows (thanks to the Tracking Ratio Station) the process variable of the slave loop to react quickly to the change of the master loop process variable. Eventually, this ensure that the transient time of both processes is minimized.

3. SIMULATION RESULTS

In this section we present some simulation examples in order to show the effectiveness of the methodology and to illustrate how the design issues can be considered by the user. For the sake of simplicity and clarity we consider a control task where $a = 1$ and the required transition for the process variables is from 0 to $\overline{y_1} = 1$. Further, we consider the same processes as in (Visioli and Hägglund, 2019) so that a comparison with the method where the feedforward action is based on system inversion can be done more easily.

In order to evaluate the performance of a considered control system, we consider, on the one hand, the 2% and 5% settling time defined as the time interval from the time instant of the application of set-point step signal to the time instant when both process variables reach and stay within a range of 2% and 5%, respectively, of their final value. The 2% settling time is denoted T_{s2}, while the 5% settling time is denoted T_{s5}.

On the other hand, the ratio control performance is given by the following integrated absolute error index, defined as

$$J = \int_0^\infty |ay_1(t) - y_2(t)|dt. \tag{16}$$

3.1 Example 1

As a first example, consider the following two FOPDT processes (so that there is no modelling uncertainty in the calculation of the two-state feedforward control law):

$$P_1(s) = \frac{1}{5s+1} e^{-2s} \tag{17}$$

$$P_2(s) = \frac{1}{s+1} e^{-0.2s} \tag{18}$$

Obviously, for the purpose of designing the control system, $\overline{P_1}(s) = P_1(s)$ and $\overline{P_2}(s) = P_2(s)$.

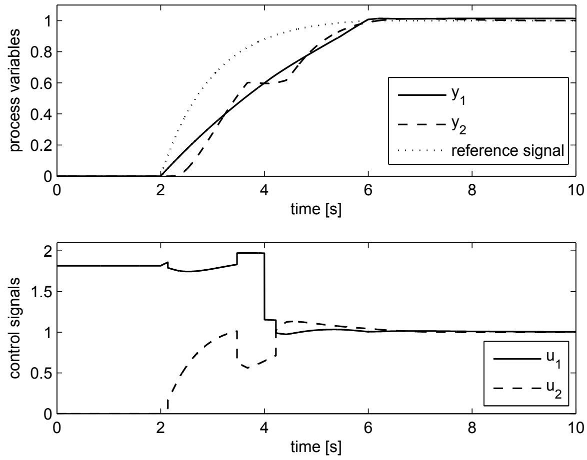

Fig. 3. Results related to Example 1 with the two-state feedforward action and $u_{max} = 2$.

The application of the AMIGO tuning rules result in $K_{p1} = 0.515$, $T_{i1} = 4.457$, $K_{p2} = 1.205$, and $T_{i2} = 0.777$.

As a first case we set the actuator maximum limit to $u_{max} = 2$. By taking into account the considerations done in Remark 2, we set $\overline{u}_{ff} = 1.816$ so that the transition time (see (5) is equal to $\tau = 4$. The hysteresis width is set to $\varepsilon = 0.1$. The ratio control result is shown in Figure 3, where both process variables and both control variables are plotted together with the reference signal $\overline{r}$. It can be noted that both the actuators (and, in particular, that of the first loop) does not exceed the saturation value. Indeed, it can be observed that, as expected, a fast transient is achieved by the first loop and then the Tracking Ratio Station is capable to recover the ratio tracking performance. The values of the settling times and of the performance index J are shown in Table 1.

For the sake of comparison, the ratio control scheme with the inversion-based feedforward signals applied to the two loops (Visioli and Hägglund, 2019) have also been simulated. In this case, the transition time has been selected equal to $\tau = 5.4$ in order to obtain the minimum transition time without exceeding the saturation value of $u_{max} = 2$ and therefore in order to provide a fair comparison. Results are shown in Figure 4, where it appears that, as expected since there is no model uncertainty, the ratio is perfectly tracked but the transient response is slower, as is confirmed by the achieved settling times shown in Table 1. Note that also the reference signal is overlapped with the two process variables, as required by the methodology.

In order to better illustrate how the increment of the actuator saturation limit influences the performance, we consider also the value $u_{max} = 3$. In this case we set a slightly conservative maximum value of the feedforward action equal to $\overline{u}_{ff} = 2.97$ so that we have a transition time equal to $\tau = 2.05$. The set-point response results are shown in Figure 5. It can be observed that while the rise time decreases, the robustness of the system and the ratio tracking performance decreases. This is confirmed by the analysis of the performance indices as the 5% settling time is $T_{s5} = 4.48$, which is lower than the previous case, but the 2% settling time increases to $T_{s5} = 12.88$ because of the overshoot and also the integrated absolute error increases to $J = 0.847$.

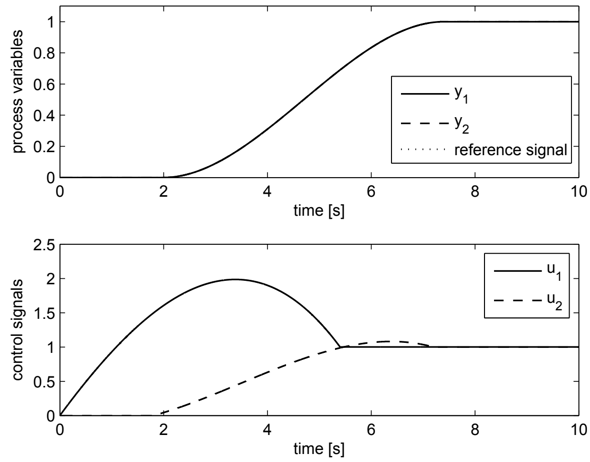

Fig. 4. Results related to Example 1 with the inversion-based feedforward action and $u_{max} = 2$.

Table 1. Values of performance indices for Example 1 with $u_{max} = 2$.

Performance index index	Two-state feedforward	Inversion-based feedforward
T_{s2}	5.85	6.95
T_{s5}	5.71	6.67
J	0.328	0

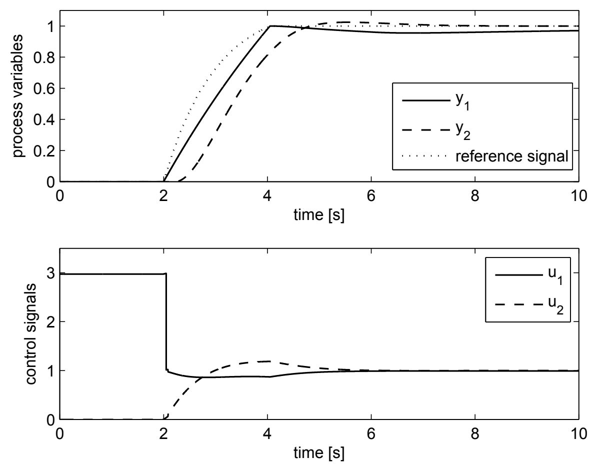

Fig. 5. Results related to Example 1 with the two-state feedforward action and $u_{max} = 3$.

3.2 Example 2

In order to evaluate the performance of the system in the presence of modelling uncertainties, as a second example, we consider the following processes:

$$P_1(s) = \frac{1}{(s+1)^8} \tag{19}$$

$$P_2(s) = \frac{1}{s+1}e^{-0.2s} \tag{20}$$

In order to calculate the feedforward action to be applied to the first loop, the first process has to be approximated as a FOPDT transfer function, resulting in

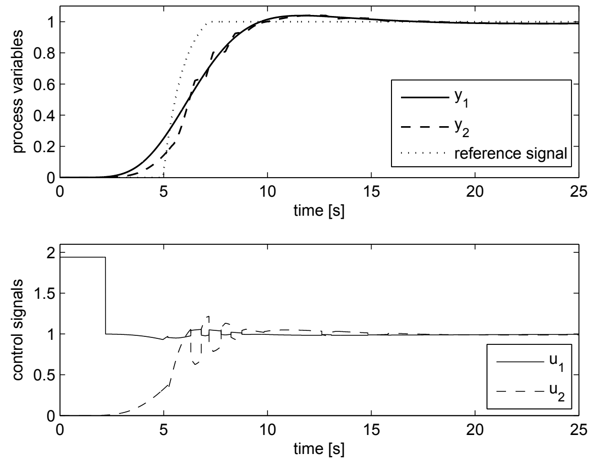

Fig. 6. Results related to Example 2 with the two-state feedforward action.

Table 2. Values of performance indices for Example 2 with $u_{max} = 2$.

Performance index	Two-state feedforward	Inversion-based feedforward
T_{s2}	14.37	16.91
T_{s5}	9.12	9.84
J	0.363	0.437

$$\bar{P}_1(s) = \frac{1}{3.04s+1}e^{-4.97s} \tag{21}$$

while $\bar{P}_2(s) = P_2(s)$. The tuning of the two PI controllers using the AMIGO tuning rules yields $K_{p1} = 0.220$, $T_{i1} = 3.382$, $K_{p2} = 1.205$, and $T_{i2} = 0.777$.

As in Example 1, we fix $u_{max} = 2$. By considering the transfer function (21) we set $\bar{u}_{ff} = 1.946$ so that we obtain a transition time $\tau = 2.2$. As in this second example the robustness issue is more relevant, the hysteresis amplitude has been reduced to $\varepsilon = 0.01$.

Results related to the ratio control scheme with the two-state feedforward controller are shown in Figure 6, while those related to the inversion-based control approach are shown in Figure 7 (in this latter case $\tau = 3.28$ has been selected in order to avoid to exceed the saturation limits of the actuators). It appears that also in this case the two-state control law allows a reduction of the settling time (see Table 2) In addition, because of the modelling uncertainties in the first process, the approach based on the two-state feedforward signal is capable to also improve the ratio tracking performance. This can be justified as the inversion-based approach is based on a nominal model of the process and the robustness of the system increases when the transition time increases. Thus, the reduction of the transition time to fully exploit the actuator capabilities (and therefore to achieve a minimum-time transition) might reduce the ratio tracking performance.

3.3 Discussion

From the presented results it appears that the use of the two-state feedforward action allows the minimization of the set-point step response and the Tracking Ratio Station avoids a significant decrement of the ratio control performance. Indeed,

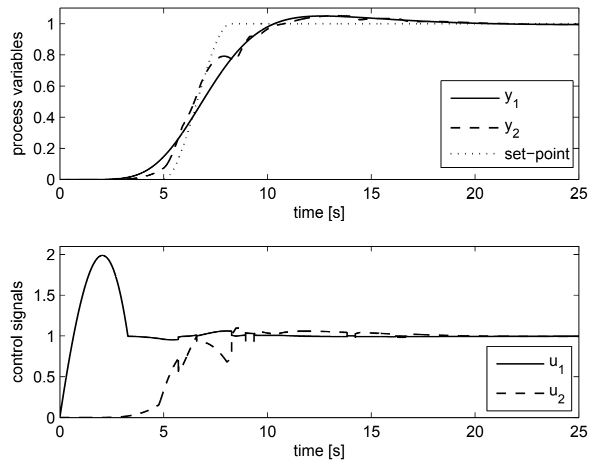

Fig. 7. Results related to Example 2 with the inversion-based feedforward action.

with respect to the inversion-based feedforward approach, the use of the full power of the actuator in the first part of the transient is obviously beneficial in achieving a fast transient (Pfeiffer, 2000). The Tracking Ratio Station and the suitable choice of the reference signal for the two loops allows the second process variable to track the first one despite the lag because of the dynamics of the second loop.

In any case, there is a the trade-off between the achievement of a fast transient and of a high performance in the ratio tracking task and this can be handled by the selection of the value of $\bar{u}_{ff}$, which has a clear physical meaning and whose value can be conveniently chosen by the user in order to meet the requirements of the given application.

4. CONCLUSIONS

In this paper we have presented a ratio control system where the use of a two-state feedforward action allows the minimization of the transient time in the set-point following task. The ratio control performance is recovered by using a Tracking Ratio Station. Simulation results have shown that this control architecture is an effective alternative option to the use of an inversion-based approach for the design of the feedforward control law. The main advantages of the method have been discussed and the physical meanings of the design parameters have been highlighted.

As results are very promising, future work will include the test of the methodology in real plants. In fact, the implementation of the methodology can be done with standard industrial hardware. Further, it will be worth investigating how a suitable combined design of the PI controller parameters and of the reference signal with the feedforward control law can improve the overall performance.

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