Voltage Synchronization of Discrete-time Microgrids over Analog Fading Channels

Jingang Lai[*] Xiaoqing Lu[**]

[*] *E.ON Energy Research Center, RWTH Aachen University, Aachen 52074, Germany (e-mail: jinganglai@126.com).*
[**] *School of Electrical Engineering and Automation, Wuhan University, Wuhan 430072, China (e-mail:luxq@whu.edu.cn).*

Abstract: In this paper, a robust distributed leader-follower tracking synchronization control scheme is developed for discrete-time DC microgrids over analog fading channels. The specific objective is to synchronize the output voltage of microgrids to the reference value by employing a fading communication network with multiplicate noises. For efficient coordination of distributed energy resources (DER), a distributed leader-follower tracking synchronization control protocol with shared communication framework and considering multiplicative noise disturbances is considered. Specifically, in order to restore the output voltage sag of DERs caused by the local loads changes, the proposed distributed cooperative controllers are designed to synchronize the output voltage of DERs to the desired value. Then the closed-loop stability analysis of the whole system is carried out, accordingly we derive the sufficient conditions to achieve all the DERs' output voltage synchronization in mean square. Moreover, the control gains have been designed to guarantee the convergence. With the proposed scheme, control derivation of voltage produced during the primary control stage can be well remedied even if the multiplicative communication noises exist. Simulation results on a DC microgrid control system are provided to show the effectiveness of the proposed control protocols against multiplicative communication noises effect.

Keywords: Cooperative voltage control, distributed communication, analog fading channel, discrete-time microgrid.

1. INTRODUCTION

Microgrids usually consisting of different types of power sources (e.g., distributed energy resources (DER) and energy storage systems) interfaced to it using power electronics converters, can realize superior power management within its distribution area no matter in grid connected mode or island mode Yu (2016), Lai (2020). More recently, DC microgrids have gained much attention due to their increased efficiency in delivering power and flexibility for the integration of power sources with DC nature (e.g., photovoltaic and battery energy storage systems, electric vehicles) Yu (2016), Lai (2020). In general, DC microgrids deliver power through a dc line, so there are no reactive power and frequency synchronization issues. Therefore, dc microgrids have relatively good power quality, high power conversion efficiency, and convenience of control compared to ac microgrid Yang et al (2011), Lu et al (2018), Logenthiran et al (2015).

Many studies have investigated different aspects of advanced control strategies for stable and efficient operation of DC microgrids. Conventional control strategies can be classified into centralized control, distributed control, and decentralized control Kwasinski et al (2011), Lu et al (2018). A centralized control system acquires information on the DERs through a communication system, and uses an energy management system to deliver the appropriate control commands Liang et al (2013), Simpson-Porco et al (2015). However, centralized schemes may not be able to operate under the significantly increased number of DER units, since this demands a high-speed communication network based on the single master-multiple slave structure, and the single point of failure in the communication network may destroy the reliability of the controlled system.

Due to inherently distributed and heterogeneous nature of the microgrid, it becomes an ideal platform for applications of distributed cooperative control algorithms. The distributed cooperative control scheme employs advanced information and communication technologies for data transmission and distributed decision-making of DERs, which improves the performance of frequency/voltage regulation, and load sharing of microgrids Coelho et al (2015), Meng et al (2018), Chen et al (2019), Lu et al (2017). The output information of the DERs is exchanged by a sparse communication network, and this is used as a control variable of the secondary controller to improve the control performance. Recently, several distributed control approaches have been introduced for the voltage/frequency regulation, and load power sharing of AC and DC microgrids. For restoring the voltage and fre-

quency and maintaining the power sharing of a microgrid, Dou et al (2016) presented a distributed control scheme for DC microgrids regulation under time delay communication conditions. The distributed control for average load sharing and voltage regulation in DC microgrids was proposed in Anand et al (2013), however, for convenience of analysis, they neglected the consideration for noise disturbances. A control scheme based on the average value of the DC output current was useful for restoring the DC microgrid voltage in Nasirian et al (2015), while the effect of the enhancement of current sharing accuracy was not obvious enough. Through a ideal spare communication network, a distributed voltage restoration control method was proposed to sustain DC bus voltage by employing law of fuzziness in Kakigano et al (2013).

These above mentioned methods all lies in that a distributed sparse communication network has been taken as a basis for designing control strategies. From a practical point of view, uncertainties always exist in the information transmission process, and their influence on the system cannot be ignored. An additive noise channel model was considered in Lai et al (2019), where a decreasing consensus gain is proposed to attenuate the effect of additive noises and strong consensus, and the frequency synchronization and optimal power sharing of DC microgrids were achieved in mean square. Furthermore, sufficient condition for mean square voltage synchronization under additive noises was proposed in Lai et al (2019). Besides the additive noise case, multiplicative noise channel model is another kind of model in the information transmission process Huang et al (2014), Lanzisera et al (2011), especially for fading communication channels. However, only a few papers studied the voltage synchronization with multiplicative noises.

Accordingly, we develop a robust distributed leader-follower tracking synchronization control scheme for DC microgrids over analog fading channels that will synchronize the terminal voltages of DERs to the desired value in a sparse communication network over analog fading channels. In this paper, the fading factor is essentially stochastic, which makes the problem more difficult. Furthermore, by properly choosing a stochastic Lyapunov function, a single recursion is constructed to describe the expectation of the stochastic Lyapunov function. Moreover, a sufficient condition is given to guarantee the mean square synchronization of the controlled DC microgrid systems are presented by employing martingale convergence theorem, furthermore a consensus gain is designed.

The rest of paper is organized as follows, Section 2 introduces preliminaries of problem and its formulation and droop-based distributed cooperative control of a DC microgrid. The proposed distributed voltage protocol and the closed-loop stability analysis of the whole system are presented and in Sections 3. Verification of the performance of the proposed control strategy based on offline digital time-domain simulations are provided in Section 4. Finally, conclusions and discussions are stated in Section 5.

The following notation will be used throughout this paper. Let $|\cdot|$ be the Euclidean norm. If A is a vector or a matrix, then A^T denotes its transposes. If A is a matrix, denote by $\|A\|$ the operator norm of A, i.e.,

$\|A\| = \sup\{|Ax| : |x| = 1\}$. Moreover, let $\lambda_i(A)$ be the ith smallest eigenvalues of symmetric matrix A. Denote $\bar{A}$ as $\bar{A} = \frac{A+A^T}{2}$. Ω represents the set of events. The σ-algebra $\sigma\{\{\varsigma_l \in C\}, C \in \mathcal{C}, l = 1, 2, \ldots\}$ is denoted by F. P is used to denote the probability measure. $P(\omega)$ is the probability of event ω. For a given random variable ς, its expectation is denoted by $E(\varsigma)$. For a family of random variables $\{\varsigma_l, l = 1, 2, \ldots\}$, the σ-algebra is denoted by $\sigma\{\varsigma_l, l = 1, 2, \ldots\}$, where $\mathcal{C}$ is a Borel set. We say that a random variable is adapted to a σ-algebra F if ς is F-measurable.

2. PROBLEM FORMULATION

System architecture of the proposed distributed voltage control scheme is illustrated by Fig. 1, which consists of droop-based local control and communication-based distributed cooperative control. Specifically, a typical converter-based DC microgrid control system is consists of three major parts: 1) the physical layer containing DC energy source, DC/DC converter, resistance-inductor-capacitor (RLC) filter, etc., 2) the control layer, which itself consists of primary and secondary controllers responsible for the voltage regulation in different operation modes, and 3) the communication layer, which is modeled as a directed graph among DER units.
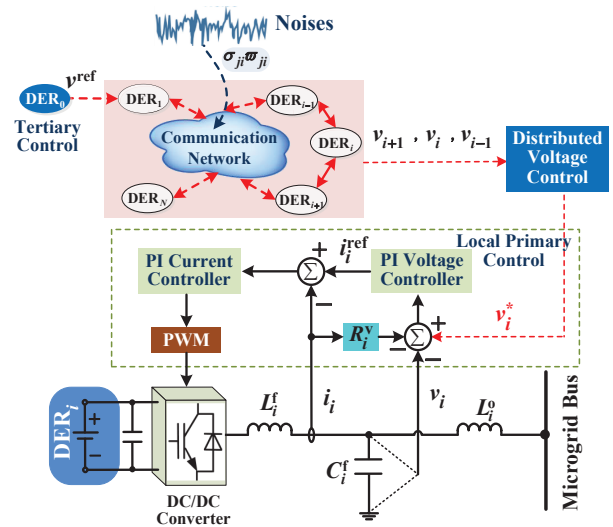

Fig. 1. Distributed leader-follower tracking synchronization control scheme for voltage synchronization in DC microgrids over analog fading channels.

The droop technique employed in primary controller for DERs can be given by

$$v_i = v_i^* - R_i^{\mathrm{v}} i_i \qquad (1)$$

where v_i^* is chosen from the nominal voltage set point of the DER_i, i_i are the measured currents at the DER terminal, R_i^{v} is the virtual resistance which is calculated by the maximum allowed voltage deviation Δv_i and maximum output current $i_{i,\max}$. Detailed expressions, and DER's parameters for the whole DC microgrid system, can be found in Huang et al (2014), Lai et al (2018).

Compared with the existing centralized secondary control mode, a novel distributed cooperative control scheme is

proposed for voltage synchronization of DC microgrids, which is based on the information exchanges among DERs and their neighbors to update v_i^* in each primary control process. Accordingly, the terminal voltage outputs v_i can be synchronized to their reference value v^{ref} which is supplied by a virtual leader DER$_0$.

According to algebra graph theory, the required sparse communication network for DC microgrids can be modeled by a digraph $\mathcal{G}(\mathcal{V}, \varepsilon, A)$, where $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ is set of nodes and $\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$ stands for set of edges. The adjacency matrix is defined as $A = (a_{ij})_{N \times N}$ where $a_{ij} > 0$ if DER$_i$ receives information from DER$_j$ and otherwise $a_{ii} = 0$. Also, the DER$_j$ is called neighbor of DER$_i$ if $(v_i, v_j) \in \varepsilon$ and the set of neighbors for DER$_i$ is defined as $N_i = v_j \in \mathcal{V}$. The Laplacian matrix is defined as $L = D - A$, where $D = \mathrm{diag}\{d_1, \cdots, d_N\}$ is the input degree matrix with $d_i = \sum_{j \in N_i} a_{ij}$. The Laplacian matrix has all row sums equal to zero, that is $L1_N = 0$, with $1_N = (1, \cdots, 1)^T \in R^N$. A directed graph has a spanning tree if every agent except one (which called the root), has at least input degree equal to one. the digraph $\bar{\mathcal{G}}$ is used to describe the interconnection topology of a microgrid consisting of one virtual leader-DER, denoted by 0, and follower DERs, denoted by $1, \cdots, N$. Now, let define $B = \mathrm{diag}\{a_{10}, \cdots, a_{N0}\}$, where $a_{i0} > 0$ if DER$_i$ which receives information from the leader and otherwise, $a_{i0} = 0$. In the following, we give some lemmas for the proof of our main results.

Lemma 1(Lai et al (2014), Lai et al (2018)) If the Laplacian matrix $L = (l_{ij}) \in R^{N \times N}$ corresponding to the undirected graph $\mathcal{G}$ is irreducible, then all the eigenvalues of $L + B$ have positive real parts and are in the open right half plane, where $B = \mathrm{diag}\{a_{10}, \ldots, a_{N0}\}$, $\sum_{i=1}^{N} a_{i0} > 0$ and $a_{i0} \geq 0$ for $i = 1, 2, \ldots, N$. Bear it in mind that the graph $\mathcal{G}$ is undirected, then $L + B$ is symmetrical and all the eigenvalues of the matrix $L + B$ are positive. Here, we denote these eigenvalues $0 < \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$.

Lemma 2(Williams (1911)) Let $(\varsigma_l)_{l \in N}$ be a martingale adapted to the filtration $(F_l)_{l \in N}$ on the probability space (Ω, F, P), and $\sup_{l \in N} E(\|\varsigma_l\|^2) < \infty$. Then, this martingale converges to a random variable in mean square sense.

3. DISTRIBUTED LEADER-FOLLOWER TRACKING SYNCHRONIZATION FOR VOLTAGE RESTORATION OF DC MICROGRIDS OVER ANALOG FADING CHANNELS

To ensure the DC bus voltage synchronization, a discrete-time distributed leader-follower tracking synchronization control scheme will be developed, which is fully distributed and robust to fading communication channels. In the following, we will discuss the development of the leader-follower tracking synchronization controller for voltage regulation. Specifically, the proposed robust distributed voltage control algorithmsynchronize the terminal voltage v_i of each DER to its reference value v^{ref} by choosing the control input v_i^* in (1).

The distributed voltage controller $u_i^v(t)$ is designed based on the ith DER communication with its neighbors, the discrete-time system states are updated as

$$v_i(t+1) = v_i(t) + u_i^v(t). \tag{2}$$

where u_i^v the leader-follower tracking synchronization controller for voltage regulation can be presented as

$$u_i^v(t) = k \sum_{j \in N_i} a_{ij} [\varpi_{ji}(t) - v_i(t)] + a_{i0} [\varpi_{i0} - v_i(t)] \tag{3}$$

such that $\lim_{t \to +\infty} E|v_i(t) - v^{\mathrm{ref}}|^2 = 0$ for $i = 1, \cdots, N$, where

$$\varpi_{ji}(t) = v_j(t) + |v_j(t) - v_i(t)|\xi_{ji}(t)$$

and

$$\varpi_{i0}(t) = v_0^{\mathrm{ref}} + |v_0^{\mathrm{ref}} - v_i(t)|\xi_{i0}(t)$$

respectively represent the state information received from DER$_i$'s neighbors and accessed the voltage reference v^{ref} corrupted by communication noises, the leader adjacency matrice $B = \mathrm{diag}\{a_{10}, \cdots, a_{N0}\} \in R^{N \times N}$. v_i is the measured voltage of DER$_i$ at time t, N_i is the set of its neighbors of DER$_i$, $A = (a_{ij})_{N \times N}$ is the adjacency matrix of DER$_i$, corresponding to the communication network $\mathcal{G}$. DER$_i$ can access the voltage reference, v^{ref}, if and only if $a_{i0} > 0$. That is to say, DER$_i$ belongs to the set of pinned DERs in DC microgrid.

Furthermore, we give the following Assumption 1 that $\{\xi_{ji}(t), t \in N, j \in \{0\} \cup \mathcal{V}, i \in \mathcal{V}\}$ need be satisfied. $\{\xi_{ji}(t), t \in N, j \in \{0\} \cup V, i \in V\} \subset \psi$ where $\psi = \{\varsigma | (\varsigma(l) \in R, F^\varsigma(l)) \mathrm{isamartingaledifference}, \sigma_\varsigma \triangleq \sup_{l \geq 0} E\|\varsigma(l)\|^2 < \infty\}$ with $F^\varsigma(l) = \sigma\{\varsigma(0), \ldots, \varsigma(l)\}$. $\xi_{ji}(t), i \in \{0\} \cup V, j \in V, l \in N$ are mutually independent and also independent of the initial conditions $v_i(0)$ and v^{ref}.

Remark 1: From distributed leader-follower tracking synchronization control protocol (3), we can find that the intensities of the multiplicate noises are dependent on the relative states of the DERs. This assumption is reasonable in many practical applications. For instance, in location technology based on time of arrival and time difference of arrival, the measurement error becomes larger when the distance from a location measurement unit is larger Huang et al (2014), Lanzisera et al (2011).

Denote $v(t) = (v_1(t), v_2(t), \ldots, v_N(t))^T \in R^N$, $v^{\mathrm{ref}} = 1_N \otimes v^{\mathrm{ref}} \in R^N$, $\varpi(t) = \mathrm{diag}\{\varpi_1(t), \ldots, \varpi_N(t)\}$, $\varpi_i(t) = \mathrm{diag}\{|v_1(t) - v_i(t)|, \ldots, |v_N(t) - v_i(t)|\}$ for $i = 1, \ldots, N$, $\xi(t) = (\xi_1^T(t), \ldots, \xi_N^T(t))^T$, $\xi_i^T(t) = (\xi_{1i}(t), \ldots, \xi_{Ni}(t))^T$, $\varpi_0(t) = \mathrm{diag}\{|v_0(t) - v_1(t)|, \ldots, |v_0(t) - v_1(t)|\}$ and $\xi_0(t) = (\xi_{01}(t), \ldots, \xi_{0N}(t))^T$. With these denotations, substituting protocol (3) into (2), the closed loop system can be rewritten in a compact form:

$$\begin{aligned} v(t+1) = &(I_N - kH)v(t) + kD\varpi(t)\xi(t) \\ &+ kB\varpi_0(t)\xi_0(t). \end{aligned} \tag{4}$$

where $H = L + B$, $D = \mathrm{diag}\{a(1, \cdot), \ldots, a(N, \cdot)\}$ is a $N \times N^2$ block diagonal matrix with $a(i, \cdot)$ being the ith row of the adjacency matrix $A = (a_{ij})_{N \times N}$ of the digraph $\mathcal{G}$. Then one gives the following result.

Theorem 1: Suppose that the digraph $\mathcal{G}$ is balanced and contains a spanning directed tree and there exists at least one $i \in \{1, \cdots, N\}$ such that $a_{i0} > 0$. If the control gain k satisfying $0 < k < \frac{2\lambda_1(\bar{H})}{\|H\|^2 + 4N\bar{\sigma}^2 + \bar{\sigma}_0^2}$, then the distributed

leader-follower tracking synchronization controllers (3) can synchronize the DER output voltage v_i to the reference state v^{ref} in mean square, even if the communication over analog fading channels exist.

Proof. By defining $e(t) = v(t) - v^{\text{ref}}$, we have

$$e(t+1) = (I - kL - kB)e(t) + kD\varpi(t)\xi(t) \\ + kB\varpi_0(t)\xi_0(t). \tag{5}$$

Then take the Lyapunov candidate as $V(v(t)) = e(t)^T e(t)$. By Ito fomula, one obtains

$$\begin{aligned} V(v(t+1)) = {} & V(v(t)) - 2ke^T(t)\bar{H}e(t) \\ & + k^2 e^T(t)H^T He(t) \\ & + k^2 e^T(t)\xi^T(t)\varpi^T(t)D^T D\varpi(t)\xi(t)e(t) \\ & + k^2 B\varpi_0(t)\xi_0(t) \\ & + 2k\xi^T(t)\varpi^T(t)D^T(I_N - kH)e(t) \end{aligned} \tag{6}$$

Due to the digraph $\mathcal{G}$ is balanced, we know that $\bar{H}$ is the Laplacian matrix of the symmetrized graph $\bar{\mathcal{G}}$ of $\mathcal{G}$. Due to the digraph $\mathcal{G}$ contains a spanning directed tree, we know that $\bar{\mathcal{G}}$ is connected and $\lambda_1(\bar{H})$. Therefore, from Lemma (1) and (6), we have

$$\begin{aligned} V(v(t+1)) \leq {} & (1 - 2k\lambda_1(\bar{H}) + k^2||H||^2)V(v(t)) \\ & + k^2 e^T(t)\xi^T(t)\varpi^T(t)D^T D\varpi(t)\xi(t)e(t) \\ & + k^2 B\varpi_0(t)\xi_0(t) \\ & + 2k\xi^T(t)\varpi^T(t)D^T(I_N - kH)e(t). \end{aligned} \tag{7}$$

Let $E\left[\xi_{0i}(t)^2\right] = \sigma_{0i}^2(t)$ and $\bar{\sigma}^2 = \sup_{t\geq 0}\{a_{0i}^2\sigma_{0i}^2(t)|i \in v\}$. Taking expectation on both sides of the above inequality and considering Assumption 1, we have

$$\begin{aligned} E[V(v(t+1))] \leq {} & (1 - 2k\lambda_1(\bar{H}) + k^2||H||^2)E[V(v(t))] \\ & + k^2\bar{\sigma}^2 E[\sum_{i=1}^{N}\sum_{j=1}^{N}(v_i(t) - v_j(t))] \\ & + k^2\bar{\sigma}_0^2 E[V(v(t))]. \end{aligned} \tag{8}$$

Note that $(v_i(t) - v_j(t))^2 \leq 2(v_i(t) - v^{\text{ref}})^2 + 2(v_j(t) - v^{\text{ref}})^2$. Therefore, we have

$$E[V(v(t+1))] \leq \eta E[V(v(t))], \tag{9}$$

where $\eta = 1 - 2k\lambda_1(\bar{H}) + k^2(||H||^2 + 4N\bar{\sigma}^2 + \bar{\sigma}_0^2)$.

Let $g(k) = 1 - 2k\lambda_1(\bar{H}) + k^2(||H||^2 + 4N\bar{\sigma}^2 + \bar{\sigma}_0^2)$. It is easy to get that the minimal value of $g(k)$ is $1 - 2k\lambda_1(\bar{H}) + k^2(||H||^2 + 4N\bar{\sigma}^2 + \bar{\sigma}_0^2)$ when $k = \frac{2\lambda_1(\bar{H})}{||H||^2 + 4N\bar{\sigma}^2 + \bar{\sigma}_0^2}$. By Lemma 2, we know that $\frac{\lambda_0^2(\bar{H})}{||H||^2} \leq 1$. Therefore $-1 < \frac{-\lambda_0^2(\bar{H})}{||H||^2} < 0$. Thus, it is easy to check that if k satisfies $0 < k < \frac{2\lambda_1(\bar{H})}{||H||^2 + 4N\bar{\sigma}^2 + \bar{\sigma}_0^2}$, then $0 < \eta < 1$. It follows from $E[V(v(t))] \to 0$ as $t \to \infty$ that $\lim_{t\to\infty} E\left|v(t) - v^{\text{ref}}\right|^2 = 0$ for $i = 1, \ldots, N$. The proof is thus completed.

Remark 2: Several existing related works merely consider the influences from additive noises Coelho et al (2015), Lai et al (2019), Lanzisera et al (2011), whose intensities are independent from the state variables. Thus, the consensus gain should be square summable and be decreasing to attenuate the influence from additive noises. However, the consensus gain should not decrease so fast

that the states of nodes converge prematurely to different values. As a result, a relatively slow convergence rate will result in by this decreasing gain. Compared with the additive noise case, the multiplicative noises are vanishing when the voltage synchronization is realized. Thus, voltage synchronization can be guaranteed by carefully choosing a constant gain, and all DERs will stay at the reference point once the voltage synchronization is realized. Finally, a geometric convergence can be realized due to the constant gain.

4. SIMULATION RESULTS

The performance of the proposed robust distributed voltage protocol is verified through simulation of an autonomous 48 V DC microgrid shown in Fig. 2, with parameters presented in Table I, in MATLAB/Simulink environment. The communication topology $\mathcal{G}$ is shown in Fig.2. The associated adjacency matrix A and Laplacian matrix L of $\mathcal{G}$ are respectively

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$L = \begin{pmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix},$$

and the corresponding leader adjacency matrix can be chosen as

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

ξ_{ij} are Gaussian white noises with distribution $N(0, 3)$. The distributed protocol (3) is applied with gain $k = 0.5$.
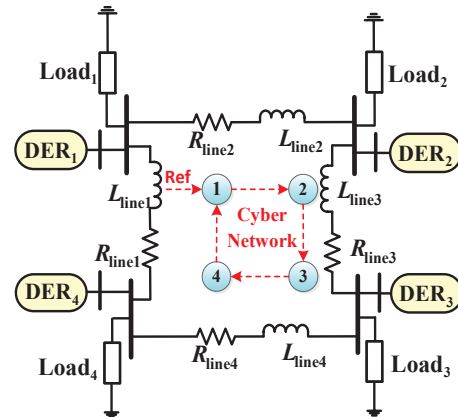

Fig. 2. Schematic diagram of the test DC microgrid, where the red dotted lines express the communication network $\mathcal{G}$.

Fig. 2 shows the directed communication topology $\mathcal{G}$ for four DERs. The comparison between terminal voltage restoration processes under multiplicate communication noises and ideal communication environments are shown in Fig. 3. Note that both local Load_1 and Load_3 are suddenly decreased from full load to 0 at $t = 0.4s$.

Table 1. Parameters of the DC Test System

DER$_1$ & DER$_3$		DER$_2$ & DER$_4$	
V_{DC}	100V	V_{DC}	80V
L^{f}	2 mH	L^{f}	1.95 mH
L^{o}	2 mH	L^{o}	1.95 mH
C^{f}	1.6 μF	C^{f}	1.5 μF
K^v_{PI}	1.2/560	K^v_{PI}	4/800
K^i_{PI}	5/560	K^i_{PI}	1/97
R^{v}	0.5 Ω	R^{v}	0.2 Ω
Load$_1$	Load$_2$	Load$_3$	Load$_4$
18Ω	21Ω	23Ω	19Ω
Line$_1$	Line$_2$	Line$_3$	Line$_4$
0.64Ω	0.51Ω	0.51Ω	0.58Ω
1.32 mH	1.05 mH	1.05 mH	1.21 mH

It can be found that, with the load varying, the output voltage v of each DER run to different values, which is less than reference voltage 48V in Fig. 3. Please note that due to the existence of impedance of transmission lines in the low voltage test microgrid system, the voltage of each DER is a local variable, which results in the output voltage v of each DER running to different values. Subsequently, at $t = $ 1s, we enable the proposed secondary controllers (3), which restores the output voltages to their desired reference values after 0.2s. Comparing Fig. 3 with Fig.4, it can be found that the existence of communication noises results in the convergence process of terminal voltages becoming jittery, but the proposed control algorithm can still achieve good control effect among DERs, when they receive state information corrupted by multiplicative noises from its neighbors.
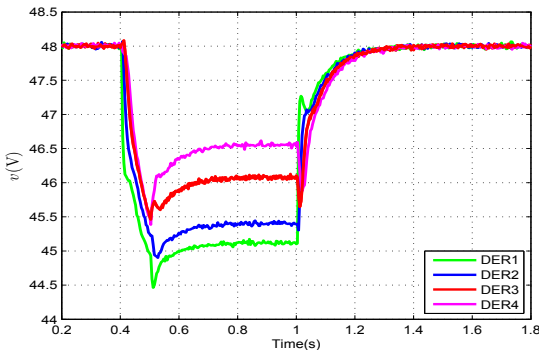

Fig. 3. The output voltage restoration processes over analog fading channels.

5. CONCLUSION

We have discussed the voltage restoration problem of DC microgrids over analog fading channels in this paper. The proposed robust distributed cooperative control protocols can only receive state information corrupted by multiplicative noises from its neighbors. Accordingly, all DERs' output voltages can be synchronized to the reference value in mean square convergence. By employing Lyapunov functional method and martingale convergence theorem, we analyzed stability of the resulted closed loop stochastic systems and derived sufficient conditions that all the DERs in the network achieve voltage restoration in mean square. To this end, we have obtained the control gains to guarantee the convergence. Eventually, we have
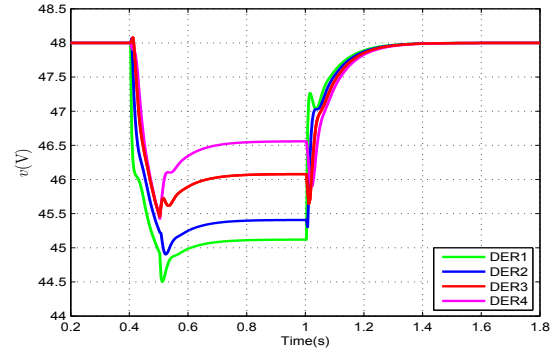

Fig. 4. The output voltage restoration under ideal communication. (i.e., without multiplicate noises).

validated the performance of the proposed controller by MATLAB/SimPowerSystems.

REFERENCES

X. Yu, and Y. Xue, Smart grids: a cyber-physical systems perspective. *P. IEEE*, 104(5):-1070, 2016.

J. Lai, X. Lu, X. Yu, and A. Monti, "Stochastic distributed secondary control for ac microgrids via event-triggered communication," *IEEE Trans. Smart Grid*, 2020, to be published, doi:10.1109/TSG.2020.2966691.

Q. Yang, J. A. Barria, T. C. Green, "Communication infrastructures for distributed control of power distribution networks," *IEEE Trans. Ind. Infor.*, vol.7, no.2, pp. 316-327, 2011.

X. Lu, J. Lai, X. Yu, Y. Wang, and J.M. Guerrero, "Distributed coordination of islanded microgrid clusters using a two-layer intermittent communication network," *IEEE Trans. Ind. Infor.*, vol. 14, no. 9, pp.3956-3969, Sep. 2018.

T. Logenthiran, R. T. Naayagi, W. L. Woo, V. T. Phan, and K. Abidi, "Intelligent control system for microgrids using multiagent system," *IEEE J. Em. Sel. Top. P.*, vol.3, no.4, pp.1036-1045, 2015.

A. Kwasinski, and C. N. Onwuchekwa, "Dynamic behavior and stabilization of dc microgrids with instantaneous constant-power loads," *IEEE Trans. Power Electron.*, vol. 26, no. 3, pp. 822-834, Mar. 2011.

J. Lai, and X. Lu, "Nonlinear mean-square power sharing control for ac microgrids under distributed event detection," *IEEE Trans. Ind. Infor.*, 2020, to be published, doi: 10.1109/TII.2020.2969458.

G. Chen, and Z. Guo, "Distributed secondary and optimal active power sharing control for islanded microgrids with communication delays," *IEEE Trans. Smart Grid*, vol. 10, no. 2, pp. 2002-2014, Mar. 2019.

W. Meng, X. Wang, and S. Liu, "Distributed load sharing of an inverterbased microgrid with reduced communication," *IEEE Trans. Smart Grid*, vol.9, no.2, pp. 1354-1364, Mar. 2018.

J.W. Simpson-Porco, Q. Shafiee, F. Dörfler, J.M. Guerrero, and F. Bullo, "Secondary frequency and voltage control of islanded microgrids via distributed averaging," *IEEE Trans. Ind. Electron*, vol. 62, no. 11, pp.7025-7038, Nov., 2015.

E. A. Coelho, D. Wu, J. M. Guerrero, J. C. Vasquez, T. Dragicevic, C. Stefanovic, and P. Popovski, "Small-

signal analysis of the microgrid secondary control considering a communication time delay," *IEEE Trans. Ind. Electron*, vol. 63, no. 10, pp.6257 - 6269, 2016.

H. Liang, B.J. Choi, et al, "Stability enhancement of decentralized inverter control through wireless communications in microgrids," *IEEE Trans. Smart Grid*, vol.4, no.1, pp. 321-331, Mar., 2013.

X. Lu, X. Yu, J. Lai, J.M. Guerrero, and H. Zhou, "Distributed secondary voltage and frequency control for islanded microgrids with uncertain communication links," *IEEE Trans. Ind. Infor.*, vol. 13, no. 2, pp.448-460, Apr. 2017.

H. Liang, B.J. Choi, W. Zhuang, and X. Shen, "Stability enhancement of decentralized inverter control through wireless communications in microgrids," *IEEE Trans. Smart Grid*, vol.4, no.1, pp. 321-331, Mar. 2013.

C. Dou, D. Yue, et al, "Multiagent system-based distributed coordinated control for radial dc microgrid considering transmission time delays," *IEEE Trans. Smart Grid*, vol. 8, no. 5, pp. 2370-2381, 2016.

S. Anand, B.G. Fernandes, and J. M. Guerrero,"Distributed control to ensure proportional load sharing and improve voltage regulation in low-voltage DC microgrids," *IEEE Trans. Power Electrion.*, vol.28, no.4, pp.1900-1913, Apr. 2013.

V. Nasirian, S. Moayedi, A. Davoudi, and F. Lewis, "Distributed cooperative control of dc microgrids," *IEEE Trans. Power Electron.*, vol. 30, no. 4, pp. 2288-2303, Apr. 2015.

H. Kakigano, Y. Miura, and T. Ise, "Distribution voltage control for dc microgrids using fuzzy control and gain-scheduling technique," *IEEE Trans. Power Electron.*, vol.28, no.5, pp.2246-2258, May, 2013.

J. Lai, X. Lu, A. Monti, and G.P. Liu, "Stochastic distributed pinning control for co-multi-inverter networks with a virtual leader," *IEEE Trans. Circuits and Systems-II*, 2019, to be published, doi: 10.1109/TC-SII.2019.2950764.

J. Lai, X. Lu, and A. Monti, "Distributed secondary voltage control for autonomous microgrids under additive measurement noises and time delays," *IET Gener. Transm. Dis.*, vol.13, no.14, pp.2976-2985, July, 2019.

J. Lai, X. Lu, and S.Chen, "Tracking consensus of nonlinear mass with asymmetric communication delays in noisy environments," *Commun Nonlinear Sci. Numer. Simulat.*, vol.19, no.7, pp.2334-2344, 2014.

D. Williams *Probability with Martingales*. Cambridge University Press: Cambridge, 1991.

B. Huang, L. Xie, and Z. Yang, "Analysis of TOA localization with heteroscedastic noises," *Chinese Control Conference*, pp. 327-332, 2014.

S. Lanzisera, D. Zats, K.S.J. Pister, "Radio frequency time-of-flight distance measurement for low-cost wireless sensor localization," *IEEE Sensors J.* vol. 11, no.3, pp. 837-845, 2011.

J. Lai, X. Lu, et al, "Synchronization of hybrid microgrids with communication latency," *Math. Probl. Eng.* vol. 586260, pp. 1-10, 2015.

J. Lai, X. Lu, et al, "Distributed multiagent-oriented average control for voltage restoration and reactive power sharing of autonomous microgrids," *IEEE Access* vol. 6, pp.25551-25561, 2018.

J. Lai, S. Chen, and X. Lu, "Formation tracking for nonlinear multi-agent systems with delays and noise disturbance," *Asian J. Control* vol. 17, no.3, pp. 879-891, 2015.