Servo Velocity Control using a P+ADOB controller *

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Abstract: This paper describes preliminary results on a Proportional plus Adaptive Disturbance Observer (P+ADOB) controller applied to velocity regulation tasks in a servo system. Adaptation law is obtained to estimate the servo system input gain, which is subsequently employed in the design of a Disturbance Observer. Compared with previous approaches, this feature relaxes the assumption on exact knowledge on the input gain, and only upper and lower bounds on this term are assumed known. A stability proof assuming constant disturbances allows concluding that the estimate of the input gain is bounded, and the velocity tracking error converges to zero. Real-time experiments illustrate the performance of the proposed controller.

Keywords: ADRC, DOB, adaptive control, servo systems, real-time control.

1. INTRODUCTION

The principle of Active Disturbance Rejection Control (ADRC) scheme was first proposed in 1829 by the french engineer and mathematician Jean-Victor Poncelet through the so-called Invariance Principle [Preminger and Rootenberg (1964)]. The idea behind this approach is to counteract the effects of both internal and external disturbances, as well as the effects of parametric uncertainties affecting the response of a plant. One of the most important works on ADRC was carried out by C.D. Johnson [Johnson (1971)], who develops what he calls Disturbance Accommodation Control (DAC), which is based on the Poncelet's Invariance Principle. The goal of the DAC is to estimate the external disturbances that affect a plant, and the corresponding estimate is used to exactly or approximately cancel out through the plant control inputs the effects of the real disturbances.

It is worth mentioning that among the first regulators aimed to cancel out the effect of constant disturbances is the ubiquitous Proportional-Integral-Derivative (PID) controller, where the Integral part is responsible for compensating constant perturbations [Åström et al. (2006)] and it can be considered as the most basic form of a Disturbance Observer [Johnson (2008)]. Certain PID topologies such as the setpoint weighted PID Control [Hägglund and Åström (1985)], [Visioli (2012)] compensate for constant and time-varying perturbations depending on how the setpoint weights are adjusted. These forms of PID control are applied to first and second-order systems, as in the case of servo systems for speed and position control [Luna and Garrido (2018)],[Garrido and Luna (2018)].

The work of Han and Gao [Han (2009)], [Gao et al. (2001)] analyzes how the ADRC shares some properties of the

* The support given by CONACyT-MEXICO to the first and second author through a Ph.D. scholarship is also recognized. PID controller, and also describes how ADRC incorporates from Modern Control theory the use of an extended state observer for estimating disturbances and for compensating the effects of real disturbances on the response of a plant, whose model may also contain parametric uncertainties. The use of ADRC and nonlinear PID controllers are also proposed in the reference, as mentioned above, to improve the performance of the PID controller applied to perturbed plants.

Another ADRC method that addresses the problem of disturbance rejection is based on the Generalized Proportional Integral (GPI) observer. The GPI observer estimates endogenous and exogenous disturbances, it may resort to high gain, and it can estimate simultaneously the phase variables related to the plant output and a disturbance term produced by lumping all the disturbances. The GPI observer also provides estimates of the time derivatives of the disturbance term and use a time polynomial as a model of the disturbance term [Sira-Ramírez et al. (2010)].

Using a Disturbance Observer (DOB) by Ohishi and Ohnishi [Ohishi et al. (1987)], [Ohishi et al. (1988)] is another philosophy pursuing ADRC, and it has been applied mainly in motion control. In this scheme, input and output measurements and a nominal model of an unperturbed plant are used to reconstruct the disturbances. The disturbance estimate is then injected into the plant input to counteract the effects of the real disturbance [Ohishi et al. (2000)].

It is worth remarking that in the ADRC schemes described previously, the input gain of the plant under control is assumed exactly known. On the other hand, in some references [Yao et al. (2013)] and [Duan et al. (2019)], an adaptive mechanism is added to the ADRC framework. However, none of them explicitly adapts the parameters used in the ADRC design.

In this work, a Proportional plus Adaptive Disturbance Observer (P+ADOB) controller is proposed for controlling the speed of a servo system. An adaptive algorithm estimates the input gain of the nominal model of the servo system used in the design of the DOB. Exact knowledge of the input gain is dispensed, and it is assumed that only an upper and a lower bound of this term are known. Furthermore, it is shown that the proposed adaptive ADRC scheme guarantees that all the signals of the closed-loop system remain bounded, and the velocity error converges to zero in the case of constant disturbances.

The outline of this exposition is as follows. After introducing the mathematical model of a servo system, a DOB is designed when the servo system input gain is known. Subsequently, an adaptation law is obtained for estimating this term. A stability proof of the closed-loop system is also provided. Real-time experiments on a laboratory prototype allow assessing the performance of the proposed P+ADOB controller.

2. MATHEMATICAL MODEL OF A SERVO SYSTEM

Consider a servo system composed of a DC motor driving a brass disk inertia, a power amplifier working in current mode and whose task is to keep the armature current proportional to the control voltage u, and a velocity sensor. A model of this system is

$$J\dot{\omega} + F\omega = ku + \phi \tag{1}$$

where the variables ω and $\dot{\omega}$ are respectively the velocity and acceleration of the servo system, u is the control input voltage, J is the sum of the servomotor inertia, the brass disk, and the sensor inertia, k is a parameter related to the amplifier gain and the motor torque constant, and the term ϕ is a constant external disturbance.

The model (1) has the next alternative writing if a := F/J, b := k/J and $\bar{\phi} := \phi/J$

$$\dot{\omega} = -a\omega + bu + \bar{\phi} \tag{2}$$

Furthermore, friction torques can be lumped if they are unknown with the term $\overline{\phi}$ as a single disturbance d

$$\dot{\omega} = bu + d \tag{3}$$

$$d := \bar{\phi} - a\omega \tag{4}$$

3. DOB-BASED CONTROL: KNOWN INPUT GAIN CASE

Consider the plant given in (3). Assume that the input gain b is known and the Laplace transform for d exists. Therefore, these assumptions allow obtaining the following expression

$$s\Omega(s) = D(s) + bU(s)$$

where $D(s) = \mathcal{L} \{d\}$, $U(s) = \mathcal{L} \{u\}$ and $\Omega(s) = \mathcal{L} \{\omega\}$. The notation $\mathcal{L} \{\cdot\}$ stands for the Laplace operator.

In a standard DOB [Ohishi et al. (1988)] the disturbance estimation is performed as follows

$$\hat{D}(s) = [s\Omega(s) - bU(s)]F(s)$$
(5)

The DOB filter F(s) used for disturbance estimation is defined as

$$F(s) = \frac{\beta}{s+\beta} \tag{6}$$

with cutoff frequency $\beta > 0$.

Then, substituting (6) into (5) leads to

$$\hat{D}(s) = \Omega(s)\frac{\beta s}{s+\beta} - U(s)\frac{\beta b}{s+\beta}$$
(7)

Therefore, a Proportional controller with a Disturbance Observer (P+DOB) is employed to the servo system for velocity control as is shown in Fig. 1, and the control law is given by

$$U(s) = \frac{1}{b} \left\{ K_p E(s) - \hat{D}(s) \right\}$$
(8)

where $E(s) = R(s) - \Omega(s)$ is the velocity error and $K_p > 0$. It is worth noting that the disturbance estimate $\hat{D}(s)$ is employed to compensate for the real disturbance effects. Moreover, The reference R(s) is assumed constant.

4. DOB-BASED CONTROL: UNKNOWN INPUT GAIN CASE

Consider again (7) and (8) written in the time domain

$$\hat{l} = -\beta \hat{d} + \beta \left[\dot{\omega} - bu \right] \tag{9}$$

$$u = \frac{1}{b} \left\{ K_p e - \hat{d} \right\} \tag{10}$$

with $e = r - \omega$. Substituting (10) into (9) yields

$$\hat{l} - \beta \dot{\omega} = -\beta K_p e \tag{11}$$

Defining

$$x := \hat{d} - \beta \omega \tag{12}$$

produces the next equality by substituting the time derivative of x into (11)

$$\dot{x} = -\beta K_p e \tag{13}$$

Therefore, the disturbance \hat{d} is computed as

$$\hat{d} = \beta \omega + x \tag{14}$$

$$\dot{x} = -\beta K_p e \tag{15}$$



Fig. 1. Proportional plus Disturbance Observer (P+DOB) controller with known input gain applied to a servo system for velocity control.

Substituting (14) into (10) leads to

$$u = \frac{1}{b} \left[K_p e - \beta \omega - x \right] \tag{16}$$

On the other hand, if b is unknown and an estimate \hat{b} is available, then the latter allows writing (16) as

$$u = \frac{1}{\hat{b}} \left[K_p e - \beta \omega - x \right] \tag{17}$$

Adding and subtracting $\hat{b}u$ in the serve system model (3), defining $\tilde{b} = \hat{b} - b$ and substituting (17) into (3) yield

$$\dot{\omega} = K_p e - \beta \omega - x + d - \tilde{b}u \tag{18}$$

Two modifications of equation (18) corresponding to the closed-loop system are needed to perform a stability analysis. First, the term βr is added and subtracted in (18) using the fact that $e = r - \omega$ thus yielding

$$\dot{\omega} = (K_p + \beta) e - x + d - \beta r - \tilde{b}u \tag{19}$$

Second, regarding the disturbance d defined in (4), note that it is time-varying because it depends on the servo system speed ω . To overcome this problem note that d has the next alternative writing by nothing that $\omega = r - e$

$$d = \bar{\phi} - ar + ae \tag{20}$$

The term $\bar{\phi} - ar$ in the above equality is the constant part of the disturbance.

Substituting (20) into (19) produces

$$\dot{\omega} = (K_p + \beta + a) e - x + \bar{\phi} - ar - \beta r - \tilde{b}u \qquad (21)$$

Defining $\alpha := K_p + \beta + a$, $p := \overline{\phi} - ar - \beta r$ and z := x - p permits writing (21) as follows

$$\dot{\omega} = \alpha e - z - \tilde{b}u \tag{22}$$

5. STABILITY ANALYSIS

Consider the following Lyapunov function candidate

$$V = \frac{1}{2}e^2 + \frac{1}{2\beta K_p}z^2 + \frac{1}{2\gamma}\tilde{b}^2$$
(23)

The time derivative of (23) is

$$\dot{V} = e(-\dot{\omega}) + \frac{1}{\beta K_p} z\dot{z} + \frac{1}{\gamma} \tilde{b}\dot{\tilde{b}}$$
(24)

where $\dot{e} = -\dot{\omega}$. Substituting (22) into (24) and nothing that $\dot{z} = \dot{x} = -\beta K_p e$ produces

$$\dot{V} = -\alpha e^2 + \tilde{b} \left[ue + \frac{1}{\gamma} \dot{\tilde{b}} \right]$$
(25)

If the following update law

$$\dot{\tilde{b}} = \dot{\tilde{b}} = -\gamma u e \tag{26}$$

is used for estimating b, then (25) becomes

$$\dot{V} = -\alpha e^2 \tag{27}$$

Therefore, $V(0) \geq V$, and variables e, z and \hat{b} are bounded so do x and ω . Consequently, if $\hat{b} \neq 0$, then the control signal u and \dot{e} also remain bounded. Applying Barbalat's lemma allows showing that e converges to zero [Sastry and Bodson (2011)]. To this end, integrating (27) with respect to time yields

$$V - V(0) = -\alpha \int_0^t e^2(\rho) d\rho$$

Since that $V(0) \ge V$, it follows that:

$$\alpha \int_0^t e^2(\rho) d\rho = V(0) - V \le 2V(0)$$

from which the following inequality holds

$$\int_0^t e^2(\rho) d\rho \le \frac{2V(0)}{\alpha} < \infty$$

From the above and the boundedness of e and \dot{e} , it follows that e converges to zero.

The existence of a singularity in control law (17) is a problem, it happens when $\hat{b} = 0$. This issue is solved by means of a parameter projection procedure applied to the gradient algorithm (26). To this end, let the set $\hat{\Omega}$ containing Ω and assume that $b \in \Omega \subset \mathbb{R}$ where Ω is the convex hypercube

$$\Omega = \{ b \mid 0 < b_{min} \le b \le b_{max} \}$$

$$(28)$$

Define the set

$$\Omega_{\delta} = \{ b \mid b_{min} - \delta \le b \le b_{max} + \delta \}$$
(29)

where $\delta > 0$ is chosen such that $\Omega_{\delta} \subset \hat{\Omega}$. Therefore, the projection operator $\Pr(\gamma \xi)$ is defined as [Khalil (1996)]:

$$\Pr\left(\gamma\xi\right) = \begin{cases} \gamma\xi, \text{ if } b_{min} \leq b \leq b_{max} \text{ or} \\ \text{ if } \hat{b} > b_{max} \text{ and } \xi \leq 0 \text{ or} \\ \text{ if } \hat{b} < b_{min} \text{ and } \xi \geq 0 \\ \gamma\bar{\xi}, \text{ if } \hat{b} > b_{max} \text{ and } \xi > 0 \\ \gamma\check{\xi}, \text{ if } \hat{b} < b_{min} \text{ and } \xi < 0 \end{cases}$$
(30)

where

$$\bar{\xi} = \left[1 + \frac{b_{max} - \hat{b}}{\delta}\right]\xi, \quad \check{\xi} = \left[1 + \frac{\hat{b} - b_{min}}{\delta}\right]\xi$$

The choice of δ such that $\Omega_{\delta} \subset \hat{\Omega}$ make sure that $\hat{b} \neq 0 \forall b \in \Omega_{\delta}$.

Consequently, the update law for estimating the input gain b is taken as

$$\hat{b} = \Pr\left(\gamma\xi\right); \quad \xi = -ue \tag{31}$$

It is worth noting that using the update law (31) for estimating b does not affect the stability result previously obtained without parameter projection. To prove this statement the term $\gamma\xi$ is decomposed into terms as follows

$$\gamma \xi = \Pr\left(\gamma \xi\right) + (\gamma \xi)_{\perp} \tag{32}$$

and the term
$$(\gamma \xi)_{\perp}$$
 is defined as

$$(\gamma\xi)_{\perp} = \begin{cases} 0 & , \text{ if } b_{min} \leq \hat{b} \leq b_{max} \text{ or} \\ & \text{ if } \hat{b} > b_{max} \text{ and } \xi \leq 0 \text{ or} \\ & \text{ if } \hat{b} < b_{min} \text{ and } \xi \geq 0 \end{cases}$$
$$\left[\frac{\hat{b} - b_{max}}{\delta} \right] \gamma\xi, \text{ if } \hat{b} > b_{max} \text{ and } \xi > 0 \\ \left[\frac{\hat{b}_{min} - \hat{b}}{\delta} \right] \gamma\xi, \text{ if } \hat{b} < b_{min} \text{ and } \xi < 0 \end{cases}$$
(33)



Fig. 2. Proportional plus Adaptive Disturbance Observer (P+ADOB) controller with known input gain applied to a servo system for velocity control.

From the above definition it is straighfoward to conclude that

$$(b-b)(\gamma\xi)_{\perp} = b(\gamma\xi)_{\perp} \ge 0 \tag{34}$$

On the other hand, replacing (31) into the Lyapunov time derivative (25) and remembering that $\dot{\tilde{b}} = \dot{\tilde{b}}$ boils down to

$$\dot{V} = -\alpha e^2 + \frac{1}{\gamma} \tilde{b} \left[-\gamma \xi + \Pr\left(\gamma \xi\right) \right]$$
(35)

Substituting $\gamma \xi$ given in (32) into (35) leads to

$$\dot{V} = -\alpha e^2 - \frac{1}{\gamma} \tilde{b}(\gamma \xi)_{\perp} \tag{36}$$

Finally, inequality (34) allows concluding that

$$\dot{V} = -\alpha e^2 \le 0 \tag{37}$$

From the above, the control law applied to the servo system (3) is

$$u = \frac{1}{\hat{b}} \left\{ K_p e - \hat{d} \right\}$$
(38)

$$\hat{d} = -\beta \hat{d} + \beta \left[\dot{\omega} - \hat{b}u \right]$$
(39)

$$\hat{b} = \Pr(\gamma \xi); \quad \xi = -ue$$
 (40)

The next proposition resumes the stability results described in this section.

Proposition 1. Consider the servo system model (3) in closed-loop with the control law (38), (39), (40), and assume that only upper and lower bounds of the input gain b are known. Then, all the closed-loop signals remain bounded and the velocity error e converges asymptotically to zero

6. EXPERIMENTS

6.1 Experimental setup

The Fig. 3 depicts the experimental setup which consists of a brushed DC motor JDTH-2250-DQ-1C from Clifton Precision with a brass disk acting as load, a SA-7388F-1 Servotek tachogenerator model is used to obtain velocity measurements, a Copley Controls power amplifier model 413 working in current mode is employed to feed the DC motor, and a box that galvanically isolates a data acquisition card from the power amplifier. For performing



Fig. 3. Experimental setup.

data acquisition a Servo To Go data card is mounted inside a personal computer with an Intel Core 2 Quad processor. All the programming is done using the Mathworks Matlab/Simulink software, together with the WINCON real-time software from Quanser Consulting. The Simulink diagrams use a sampling period of one millisecond and the integration method corresponds to Euler-ode1 .

The servo motor angular velocity ω is filtered to attenuate the noise in the measurements of the tachogenerator using the next second-order filter.

$$\frac{\mathcal{L}\left\{\omega_{f}\right\}}{\mathcal{L}\left\{\omega\right\}} = \left[\frac{90}{s+90}\right]^{2}$$

6.2 Experimental results

In this section, the P+ADOB controller studied in Sections 4 and 5 is experimentally assessed. The input gain b of the linear model (3) fulfills $b_{min} = 5$ and $b_{max} = 120$. The value of $\delta = 0.01$. Several initial conditions for $\hat{b}(0)$ are tested to show the performance of the P+ADOB control scheme.

The performance of the closed-loop system is evaluated through the Integral Squared Error (ISE), the Integral of the Absolute value of the Error (IAE), the Integral of the Absolute value of the Control (IAC) and the Integral of the Absolute value of the Control Variation (IACV)indexes that are defined as

$$ISE = \int_{\tau_1}^{\tau_2} 100 [e]^2 dt \qquad IAE = \int_{\tau_1}^{\tau_2} 100 |e| dt$$
$$IACV = \int_{\tau_1}^{\tau_2} \left| \frac{du}{dt} \right| dt \qquad IAC = \int_{\tau_1}^{\tau_2} |u| dt$$

which are evaluated at $\tau_1 = 15$ and $\tau_2 = 20$ seconds.

Table 1. Experimental results using $K_p = 3$, $\gamma = 10$ and $\beta = 10$ for the P+ADOB controller.



Fig. 4. Velocity responses for 930 and 990 r/min.



Fig. 5. Control signals for 900 and 1140 r/min.



Fig. 6. Velocity errors for 900 and 1140 r/min.

The reference r is a filtered pulse train switching between 930 r/min and 990 r/min.

6.3 Performance of the P+ADOB controller

Table 1 resumes the outcomes of the experiments. The closed-loop responses, the control signals, and velocity error graphs are depicted in Fig. 4, Fig. 5 and Fig. 6 respectively. The estimated gains \hat{b} are shown in Fig. 7, as well as the estimated disturbances in Fig. 8.

From Table 1, it is clear that good performance is obtained using large values in the initial condition, that is, when



Fig. 7. Evolution of the estimated gains \hat{b} for different initial conditions $\hat{b}(0)$.



Fig. 8. Estimated disturbance \hat{d} for different initial conditions $\hat{b}(0)$.

using 80, 60 and 40. However, for lower values of $\hat{b}(0)$ produce lower steady-state values of \hat{b} and performance degrades. In this regard, note that a lower value of \hat{b} implies a large value of $1/\hat{b}$ in the control law (38). Therefore, it would produce large control signals, as is observed in Fig. 5 for $\hat{b}(0) = 20$. The closed-loop responses, as well as the velocity errors in Fig. 4 and Fig. 6 also reflect the effects of this small initial condition as oscillating behavior.

Note also that the ISE and IAE indexes have their lowest values when $\hat{b}(0) = 60$. Hence, it suggests that this could be the best input gain estimated because, for all other conditions, those indexes have higher values.

7. CONCLUSION

This paper presents preliminary results on a Proportional plus Adaptive Disturbance Observer (P+ADOB) controller for velocity regulation tasks in a servo system. An update law estimates its input gain, which is subsequently employed in the design of a Disturbance Observer. It is shown that all the closed-loop system signals are bounded, and the velocity error converges to zero. Real-time experiments carried out in a laboratory prototype illustrate the performance of the proposed controller.

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