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Optimal Tyre Management of a Formula One car

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Abstract: Optimal control calculations are used to study the effect of tyre wear on race car performance. This is achieved by solving a minimum lap time optimal control problem over multiple laps using a dynamical model of a Formula One car. A previously developed thermodynamic model is enhanced by adding an additional state for the carcass temperature of the tyres. The tyre grip is modelled as a function of the tyres' wear and temperature. Grip reduces when tyres get worn, or the tyres are not operated within their optimal temperature window. Overheating the tyres can accelerate wear, which in turn, degrades performance. The optimal control problem solver needs to 'manage' the state of tyres throughout a race (not just a single lap) to ensure that optimal race performance is achieved. At some point during a race a pit stop may be required to change worn tyres so that tyre grip can be restored. It is essential to understand the wear characteristics of various tyre compounds in order to determine the point when the time needed for a pit stop is justified in terms of subsequent racing performance.

Keywords: Thermal tyre model, tyre wear, Formula One car modelling

1. INTRODUCTION

Racing teams perform optimal control simulations to tailor vehicle set-up parameters specific to each track, and to refine their racing strategy (Perantoni and Limebeer, 2014). An important component of race strategy is the understanding of optimal tyre management and how it can yield a competitive advantage. The forces produced at the contact patch of each tyre determine the motion (and therefore handling characteristics) of the car (Kelly, 2008). Vehicle performance is in turn dependent on the thermal and wear characteristics of the tyres. The frictional properties of racing tyres are especially sensitive to temperature, and maximum performance is only achievable in a relatively small temperature band (Kelly and Sharp, 2012). In the tyre model presented here, friction is modelled as a function of tread temperature. The carcass can be considered as a heat store for the tyres (Kelly, 2008), and plays an important role in keeping the tread in the optimal temperature window. Tyre wear also plays a big part in handling performance. As a tyre is worn, its ability to provide grip is compromised, thereby causing increases in lap time (Farroni et al., 2017).

Section 2 gives a thermodynamic tyre model, which keeps track of the tyre tread and carcass temperatures. These temperatures, in turn, influence both the tyres' grip and wear properties. Simulation results are given in Section 3, with conclusions drawn in Section 4.

2. TYRE THERMODYNAMIC MODEL

The tyre tread is a thin layer ($\approx 5\,\mathrm{mm}$) of rubber that makes contact with the track, while the carcass gives the tyre its mechanical strength. The carcass weighs approximately ten times more than the tread. A lumped parameter model is used to describe the thermodynamic behaviour of the tyres of which the states are the temperatures of the tyres' tread and carcass. Although the carcass comprises rubber and a metal core, it is modelled as a homogenous substance with physical properties representative of both materials. The two thermal states, together with the dominant heat flows, are shown in Fig. 1, with the temperatures of the tread and carcass given by T_{tread} and T_{carc} respectively.

2.1 Thermodynamic model

The six-parameter model described by Tremlett and Limebeer (2016) forms the basis of this work. The vehicle dynamics model used here is described in Appendix A of Tremlett and Limebeer (2016), and the interested reader is referred there. This model is now expanded by adding an additional carcass temperature state, with its associated heat transfer equations, to each wheel. All parameters and variables are defined in Appendix A. While the new model is apparently more complicated, the number of free parameters has been reduced to three [1].

[1] John von Neumann's warning bears repeating: '$\cdots$ with four parameters I can fit an elephant, and with five I can make him wiggle his trunk (Dyson, 2004).'

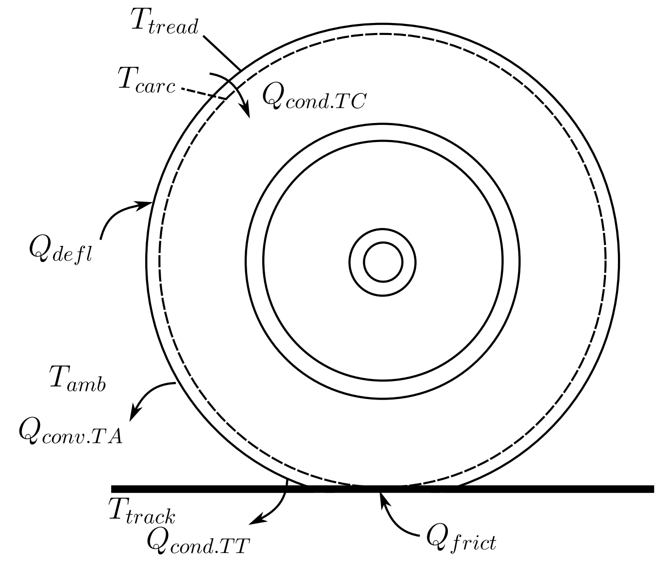

Fig. 1. Thermodynamic model with dominant heat flows

The heat generated by tyre friction is given by Equation (2) in Tremlett and Limebeer (2016)

$$Q_{frict} = p_1 u_n (|F_x \kappa| + |F_y \tan \alpha|), \tag{1}$$

where p_1 is the fraction of the frictional power lost to the track, and u_n, F_x, κ, F_y and α have their usual meaning; see Tremlett and Limebeer (2016). The heat transfer due to convection between the tyre tread and the surrounding ambient air is given by:

$$Q_{conv.TA} = h_{forc} A_{conv} (T_{tread} - T_{amb}), \tag{2}$$

where

$$A_{conv} = A_{tot} - c_w c_l \tag{3}$$

is the area available for convection. The first term on the right-hand side is the surface area of the tread

$$A_{tot} = \pi D W, \tag{4}$$

while the second term is the area of the tread in contact with the road. The tread contact patch width of the front and rear tyres are given by c_{wf} and c_{wr} respectively; see Table A.4. The length of the contact patch c_l is given by:

$$c_l = a_{cp} F_z^{0.7}. \tag{5}$$

where a_{cp} is a constant reflecting the difference between the front and rear contact patch lengths, and F_z is the normal force acting on the wheel. The heat transfer coefficient h_{forc} is determined using an empirical formula that correlates well with CFD simulation (Farroni et al. (2014) Equation 10):

$$h_{forc} = \frac{K_{air}}{L} \Big[0.0239 \Big(\frac{uL}{v_{air}} \Big)^{0.805} \Big], \tag{6}$$

where K_{air} is the conductivity, and v_{air} the kinematic viscosity, of air. Both are evaluated at the average temperature between the tread and ambient air using a material physical property database–see Table A–9 (Çengel and Cimbala, 2014); u is the forward velocity of the vehicle. The characteristic length of the heat exchange surface is given by:

$$L = \frac{1}{\frac{1}{D} + \frac{1}{W}}, \tag{7}$$

where D is the tyre diameter and W the tread width. The conduction between the track and tyre tread is given by:

$$Q_{cond.TT} = h_{tt} A_{cp} (T_{tread} - T_{track}) \tag{8}$$

Table 1. Thermodynamic constants

Parameter	Front left	Front Right	Rear left	Rear right
p_1	0.7291	0.5670	0.5071	0.6124
p_2	0.0060	0.0000	0.0125	0.0000
p_3	252.53	263.48	174.93	181.38

where h_{tt} is the heat transfer coefficient of conduction and A_{cp} is the non-sliding area of the contact patch, which is given by:

$$A_{cp} = c_w c_s(\alpha) c_l. \tag{9}$$

The function $c_s(\alpha)$ controls the relative proportion of sliding and non-sliding regions as the vehicle approaches its cornering limit and is given by

$$c_s(\alpha) = \frac{\alpha}{\alpha_c}(c_{s2} - c_{s1}) + c_{s1}; \tag{10}$$

with c_{s1} and c_{s2} being reference values, while α_c is the reference slip angle. Conduction between the tread and carcass layers is given by:

$$Q_{cond.TC} = p_3 A_{tot} (T_{carc} - T_{tread}), \tag{11}$$

where p_3 is the heat transfer coefficient of conduction, and A_{tot} is given in (4).

Heat is generated as the carcass deflects under load. This is assumed to be proportional to the square of longitudinal force acting on the tyre as per Clark and Dodge (1985):

$$Q_{defl} = p_2 \Big(\frac{u_n F_x^2}{|F_z|} \Big). \tag{12}$$

We can now describe the thermal dynamics of the tyre tread and carcass layers as follows

$$\begin{aligned} m_t c_t \frac{\mathrm{d}}{\mathrm{d}t} T_{tread}(t) = & \, Q_{frict} - Q_{cond.TT} \\ & - Q_{conv.TA} + Q_{cond.TC} \end{aligned} \tag{13}$$

$$m_c c_c \frac{\mathrm{d}}{\mathrm{d}t} T_{carc}(t) = Q_{defl} - Q_{cond.TC} \tag{14}$$

where m_t and m_c are the masses, and c_t and c_c the heat capacities of the tread and carcass respectively. Parameters p_1 to p_3 are fitted using a least-squares minimisation of the error between telemetry data taken around a lap of 'Circuit de Catalunya' in Barcelona and the tread temperature calculated by the thermodynamic model.

Fig. 2 shows a best-fit parameter set for the front-left tyre, which has a root mean square error of 6.8°C. No telemetry data was available for the carcass temperature, which shows only small variations. This is attributable to its greater thermal inertia as compared to the thin tread layer. In essence, the carcass acts as a heat store that prevents the tread from cooling down to the track temperature. Table 1 shows the parameters that resulted in the best fit for each tyre.

2.2 Tyre wear model

The model developed in Tremlett and Limebeer (2016) forms the basis of this aspect of the work. Wear rate ($\dot{w}$) is added as an extra state to be associated with each tyre, so that the total tyre wear can be calculated and minimised as part of the optimal control problem. The mechanical

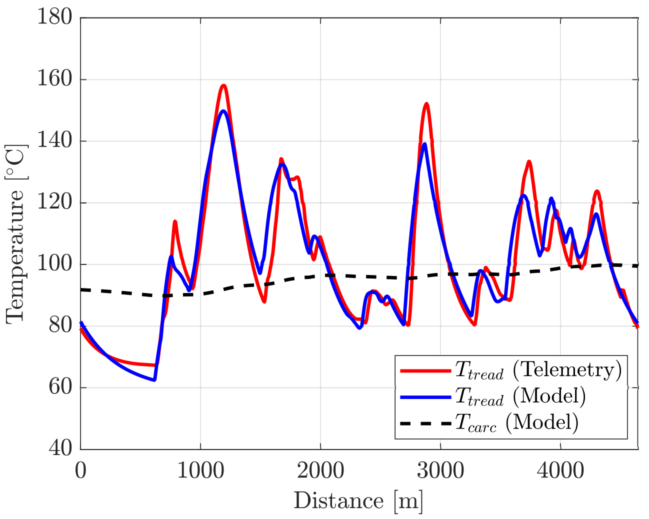

Fig. 2. Parameter fitting using telemetry data

abrasion between a tyre and track asperities is modelled using the power law relationship:

$$\dot{w}_p = w_{p1}\left(\frac{Q_{frict}}{Q_{ref}}\right)^{w_{p2}}, \tag{15}$$

where w_{p1}, w_{p2} and Q_{ref} are constants (Tremlett and Limebeer, 2016). If a tyre is overworked while cold, particles break away from the surface in a phenomenon called graining and is modelled by [2] :

$$\dot{w}_g = w_{g1}(\max(t_{tp} - T_{tread}), 0)^{w_{g2}}; \tag{16}$$

w_{g1} and w_{g2} are constants. If a tyre gets too hot, local hot spots will form and tyre wear is again accelerated. This phenomenon is called blistering and is modelled by [3] :

$$\dot{w}_b = w_{b1}(\max(T_{tread} - t_{tp}), 0)^{w_{b2}}; \tag{17}$$

w_{b1} and w_{b2} are constants. The transition temperature t_{tp} distinguishes between these two mechanisms: graining occurs below the transition temperature and blistering occurs above it. The cumulative wear rate is assumed to be a simple superposition of the above three wear mechanisms.

2.3 Performance alteration

Tyre degradation has an impact on race performance and is thus included in the optimal control problem. Both temperature and wear characteristics will be considered.

Temperature-grip characteristic The first mechanism that alters the car's performance is the tyres' temperature-grip characteristic. The temperature window in which the maximum achievable grip is obtained is shown in Fig. 3; this curve is representative of a super-soft tyre

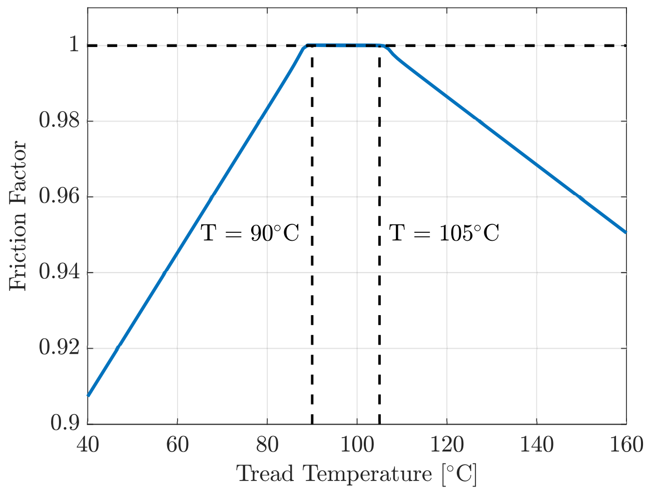

Fig. 3. Optimal temperature window for super-soft tyre compound

(the compound that will be used for the simulations in the following section). This window must be adjusted for each tyre compound. The optimal controller aims to get the tyre surface temperatures into this optimal operating window and hold it there as nearly as possible. An example of such a curve is given in Limebeer and Massaro (2018). In this case the optimal grip is achieved when the tread temperature lies between 90°C and 105°C.

Carcass temperature The tread-only model used in Tremlett and Limebeer (2016) is limited by the fact that it exaggerated the extent to which it is possible to cool the tread once it overheats. The optimal controller achieves maximum grip by raising the tread temperature into the optimal operating window. If one wants to reduce the temperature of an over-heated tread, one has to reduce the temperature of the tyre carcass—this process is slowed by the relatively high thermal inertia of the carcass as compared to that of the tyre tread. For this reason it is important to introduce a carcass temperature state that represents this thermal-inertia-related lag. The tread undergoes rapid temperature changes throughout a lap, while the carcass temperature varies more slowly. Once the carcass has heated up, and possibly over-heated, cooling it down again is also a slow process.

Tyre wear The tyre tread temperature alter performance, but in isolation, it is a fully reversible effect (as it is modelled here). This reversibility is not a tyre-wear property—once the tyre is worn out it stays worn out! It is thus necessary to add into the model a mechanism that recognises tyre degradation due to wear, and prevents the optimal controller from regaining maximum grip once degradation has occurred.

Examples of how different tyre types lose performance, as the tyre degrades, is given in Farroni et al. (2017). Passenger tyres are typically designed for longevity, whereas racing tyres are designed to offer enhanced grip, but for a shorter time. The wear curve of a super-soft tyre compound is shown in Fig. 4. The grip initially increases with wear before it starts to degrade, because racing tyres are not fully vulcanised during the manufacturing process (Haney, 2003), and go through a curing process during

[2] Most optimal control solvers deal only with 'smooth' functions. To that end (16) is approximated by

$$\dot{w}_g = w_{g1}\left((t_{tp} - T_{tread} + \sqrt{(t_{tp} - T_{tread})^2 + \epsilon})/2\right)^{w_{g2}}$$

for 'small' $\epsilon > 0$. See page 417 of Limebeer and Massaro (2018) for further details.

[3] A usable smooth approximation to (17) is

$$\dot{w}_b = w_{b1}\left((T_{tread} - t_{tp} + \sqrt{(T_{tread} - t_{tp})^2 + \epsilon})/2\right)^{w_{b2}}.$$

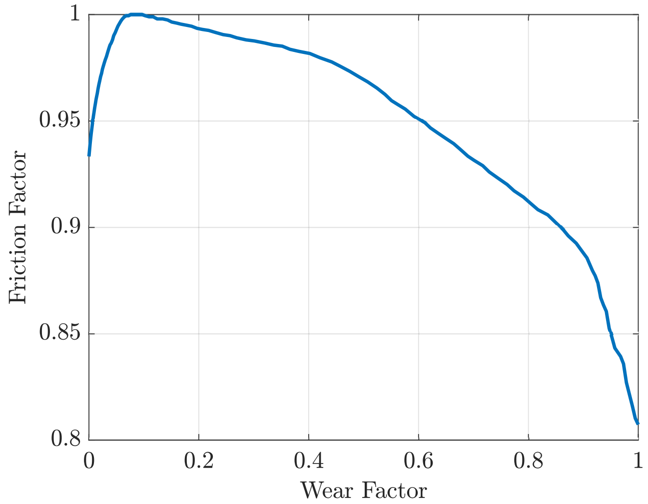

Fig. 4. Wear characteristics of super-soft tyre compound

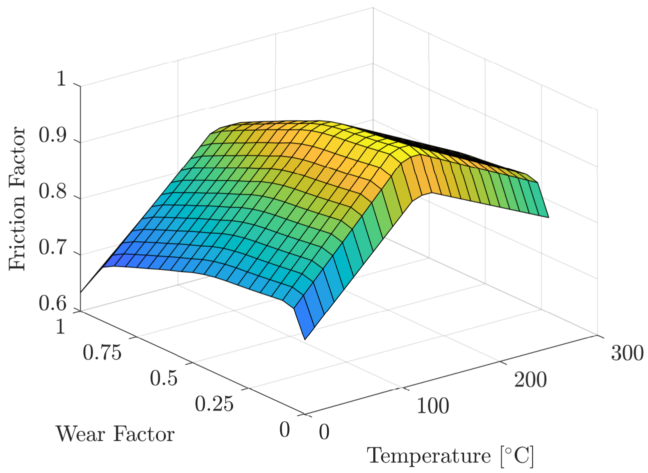

Fig. 5. Performance altering grip curve

the initial wear phase, which form cross-links between sections of the polymer chains. This increases the rigidity and durability of the rubber (Mark et al., 2013).

Tyre wear is introduced into the tyre model as a third dimension to the temperature-grip sensitivity curve. This allows the tyre grip to be modelled as a function of the tread temperature (a reversible process) and tyre wear (an irreversible process). This forces the optimal controller to manage the tyres usage throughout a race (assuming no pit stops); the three-dimensional grip-sensitivity curve used here is shown in Fig. 5. Slices along constant wear planes resemble Fig. 3, whereas slices along constant temperature planes resemble Fig. 4.

3. SIMULATION RESULTS

Two simulations are presented that illustrate the significance of the additions to the model as presented in Tremlett and Limebeer (2016). Firstly, a multi-lap simulation was performed (in the absence of tyre wear) that investigates the effect of the proposed tyre thermodynamic model on a multi-lap optimal control problem. After that, we investigate the influence of wear on the optimal control solution. The general-purpose optimal control problem

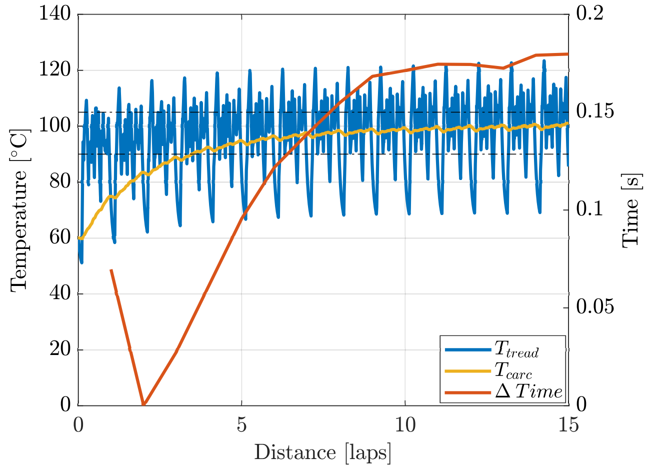

Fig. 6. Output of simulation without wear component

solver (GPOPS-II) by Patterson and Rao (2014) is used to perform the simulations.

3.1 Addition of tread and carcass temperatures

In this section the optimal control solver is used to minimise the time taken to complete fifteen laps around the Circuit de Catalunya in Barcelona. The initial tyre temperatures were set to 60°C, which is well below the optimal tyre operating temperature. For this simulation the super-soft compound was used that will achieve optimal grip when the tread temperature is between 90°C and 105°C. The temperature profiles for the rear-right tyre are shown in Fig. 6. The first lap is slow because the tyres are cold and thus unable to provide optimal grip. In the second lap the tread temperature is within the optimal operating window and the fastest lap of 81.08 s is recorded. The time difference relative to fastest lap (lap two) is shown as the orange graph. As the various laps elapse, the carcass temperature slowly increases, making it difficult for the optimal controller to keep the tread temperature in the optimal window. As a result thermal influences cause a loss of performance with each subsequent lap. The time delta curve settles out after approximately 10 laps whereafter the lap times remain roughly constant (≈0.18 s slower than the fastest lap).

3.2 Addition of tyre wear

Tyre wear influences are now introduced. To exaggerate the effect of tyre wear, the wear rate model was 'tweaked' so that a tyre is completely worn out within a single lap. The velocity profiles for various wear cases are shown in Fig. 7. There is no wear in the base case, which inevitably leads to the fastest lap. When the wear rates are set to their nominal values, only slight deviations relative to the base case are observed. This is expected seeing that racing tyres are designed to last for multiple laps. On new tyres the car's maximum speed builds up until the peak of the wear curve given in Fig. 4 is achieved. As the nominal wear rate is exaggerated, the vehicle speed is reduced so as to prevent the tyres from being fully worn before reaching the end of the lap.

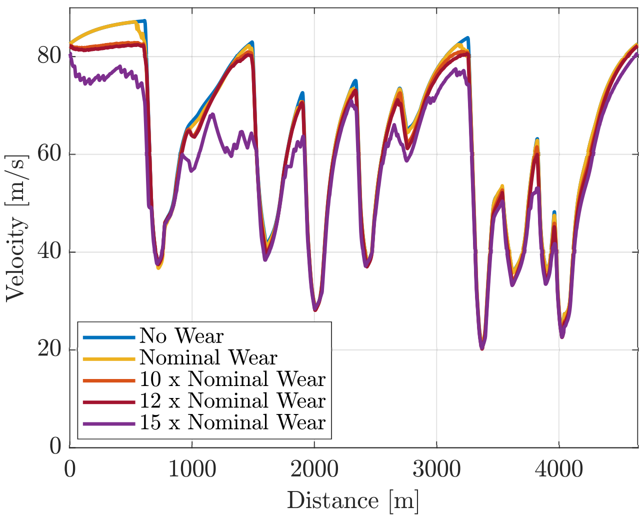

Fig. 7. Output of simulation with artificial tyre wear

3.3 Multi-lap simulation using wear model

The tyre wear model is now adjusted back to its nominal values to facilitate multiple laps. The results of a multi-lap simulation is shown in Fig. 8 and Fig. 9. Initially, the tyres are purposefully worn and heated so that they can quickly reach the peak of the friction factor curve. Over the first two laps the grip increases as the tyres wear and the tread temperatures are brought into the optimal window; as in Fig.6, the second lap is the fastest lap. Between laps three and eight, the lap times increase at a roughly constant rate. This is the result of a roughly constant decline in the friction factor characteristic over this tyre usage range. After lap eight, tyre wear affects performance more severely, and the lap times start to degrade more rapidly. This can be ascribed to the slope changes in the friction factor curve. Another consequence of having increasingly worn tyres, is that is becomes more difficult to keep the tread in the optimal temperature window. It is clear that there are increasing departures from the treads' optimal temperature operating window as the race progresses. In contrast to Fig.6, the lap time ΔT does not plateau, but continues to increases more rapidly as the tyres are worn. This speaks to the irreversible nature of tyre wear that has not been captured before. At some point the drop-off in performance degrades to the extent that the car is no longer competitive which will warrant a pit stop to change the tyres.

4. CONCLUSION

A thermodynamic tyre model has been presented in which the tread and carcass temperatures of the tyres are calculated. The tread temperature reacts quickly to driver inputs and can be easily manipulated so that it comes into the optimal temperature window, where maximum grip is achieved. The carcass on the other hand, has a higher thermal inertia, and its temperature increases only slowly over time, making its temperature more difficult to manage. A wear model is also introduced into the tyre model that accounts for the loss in performance due to tyres usage. Tyre wear is modelled as a function of temperature and frictional power, and describes qualitatively the way tyre

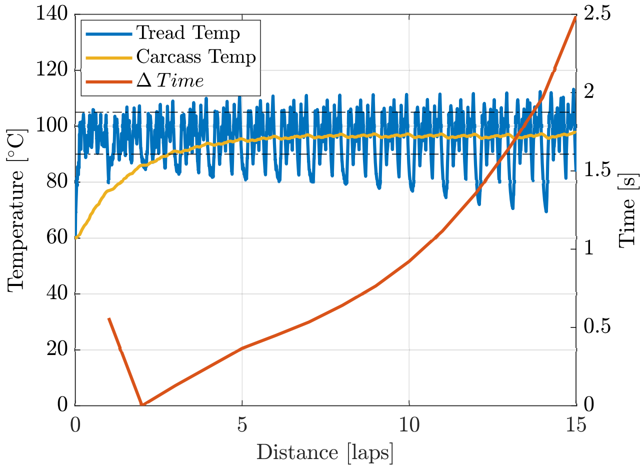

Fig. 8. Temperature profile for simulation with wear characteristics of the super-soft tyre compound

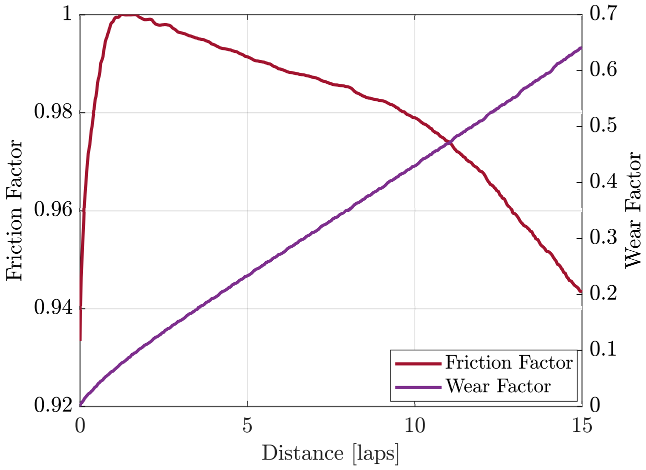

Fig. 9. Performance metrics for simulation with wear characteristics of the super-soft tyre compound

performance will degrade with use. Each tyre compound has its own characteristics, and will react differently to tyre use. The tyre model can be used to determine the point where tyre wear compromises performance to such an extent that the tyres are no longer competitive—this is a process not an instantaneous event. Our intention is that race teams can use the techniques introduced here to gain competitive advantage and improve their tyre management strategies.

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Appendix A. DEFINITION OF MODEL PARAMETERS

Table A.1. Vehicle dynamics variables

Parameter	Description	Units
u_n	Wheel longitudinal velocity	m/s
u	Car longitudinal velocity	m/s
F_x	Longitudinal tyre force	N
F_y	Lateral tyre force	N
F_z	Normal tyre force	N
α	Lateral slip angle	rad
κ	Longitudinal slip coefficient	-

Table A.2. Thermodynamic variables

Parameter	Description	Units
Q_{frict}	Heat generated by tyre friction	W
Q_{defl}	Heat generated by tyre deflection	W
$Q_{conv.TA}$	Convective heat transfer between tread and ambient air	W
$Q_{cond.TT}$	Conductive heat transfer between tread and track	W
$Q_{cond.TC}$	Conductive heat transfer between tread and carcass	W
A_{cp}	Area of the contact patch	m^2
A_{tot}	Total area of tyre tread	m^2

Table A.3. Tyre wear model parameters

Parameter	Description	Units	Value
w_{g1}	Graining gain factor	mm/°C s	0.4×10^{-5}
w_{g2}	Graining exponent	-	2
w_{b1}	Blistering gain factor	mm/°C s	0.8×10^{-5}
w_{b2}	Blistering exponent	-	2
w_{p1}	Frictional power gain factor	mm/s	0.09
w_{p2}	Frictional power exponent	-	1.6
Q_{ref}	Reference frictional power	kW	150
t_{tp}	Minimum wear temperature transition	°C	100
g_s	Temperature gain factor	-	0.5

Table A.4. Thermodynamic model parameters

Parameter	Description	Units	Value
T_{amb}	Track air temperature	°C	25
T_{track}	Track surface temperature	°C	35
m_t	Tyre tread mass	kg	0.5
m_c	Tyre carcass mass (front)	kg	9.5
	Tyre carcass mass (rear)	kg	11.5
c_t	Heat capacity of tread	kJ/kg K	2.4
c_c	Heat capacity of carcass	kJ/kg K	1.6
h_{tt}	Track-tyre heat transfer coefficient	kW/m^{2}K	12
c_{wf}	Contact patch width (front)	m	0.20
c_{wr}	Contact patch width (rear)	m	0.22
a_{cp}	Contact patch length constant	m/kN	0.056
α_c	Reference sliding/non-sliding slip angle	°	8
c_{s1}	Sliding/non-sliding reference 1	-	0.3
c_{s2}	Sliding/non-sliding reference 2	-	0.8
D	Tyre diameter	m	0.66
W	Tyre width (front)	m	0.355
	Tyre width (rear)	m	0.380