Model Predictive Control with Time Varying Parameters for Plasma Shape and Current in a Tokamak

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Abstract: Synthesis and modeling of the advanced time-varying magnetic control system for plasma shape and current on the base of the Model Predictive Control principle in a tokamak is considered. The study was done for the Globus-M2 tokamak (Ioffe Institute, St. Petersburg, RF). The system presented has a hierarchical structure with a time-varying Model Predictive Controller at the top level and robust time-invariant controllers at the low level. The approach of adaptation of the plasma magnetic axis position to the shape parameters is used to resolve the contradiction between plasma position and shape. Numerical modeling of the control system synthesized has shown the efficiency of the proposed approach and a possibility to apply it in physical experiments.

Keywords: tokamak, plasma, predictive control, time-varying systems, adaptive control.

1. INTRODUCTION
Tokamaks are the closest to the solution of the controlled thermonuclear fusion problem (Wesson, 2011) in comparison with other devices in this field: stellarators, open magnetic traps, lasers. In tokamaks, two types of plasma control systems are applied: plasma magnetic and kinetic systems. Plasma magnetic control systems confine plasma in magnetic field and kinetic control systems provide necessary profiles of plasma parameters: plasma current, safety factor q, density, temperature, and have to control some full plasma parameters specifically plasma density by gas-pumping control, plasma thermonuclear power in thermonuclear tokamak-reactors (ITER, DEMO) etc.

Different approaches for plasma magnetic control are used at operating tokamaks and in simulations for constructed tokamaks (Mitrishkin et al., 2018 a,b). The basic directions of such applications are robust and adaptive control in hierarchical control structures. In robust control systems, feedback controllers have time invariant parameters in the presence of a time varying plant with uncertainties (Konkov, et al., 2020, Mitrishkin et al., 2020) and provide the required performance and robust stability margins for any representative from uncertainties. In the adaptive control systems, the controllers in hierarchical structures adapt to the time-varying plants to provide required performance and stability during operation (Kartsev et al., 2017). The most competitive among adaptation approaches is Model Predictive Control (MPC) (Gossner et al., 1999, Maciejowski, 2002, Rossiter, 2003, Wang, 2009) which gives a chance to use time-varying plant model in the predictions at each step of current control. This caused the direction of the given work for application of the MPC for plasma current and shape control in the spherical tokamak Globus-M2 with the usage of Linear Time-Varying (LTV) plasma models obtained on the base of experimental data (Mitrishkin et al., 2019). Since the plant model used is time-varying, the target predictive controller (as model-based controller) also has a time-varying form.

In Section 2, the used linear time-varying plasma models are briefly discussed. The plasma vertical and horizontal position robust time-invariant controller synthesis by the H∞ approach named Normalized Coprime Factorization (NCF) (McFarlane and Glover, 1989) is given in Section 3. Section 4 is devoted to the Poloidal Field (PF) and Central Solenoid (CS) coil currents tracking with decoupling. Section 5 presents the synthesis of the model predictive controller with time-varying plant model to stabilize plasma current and gap deviations about zero. In Section 6, the results of numerical simulations are shown and discussed.

2. MODELS OF THE PLASMA IN GLOBUS-M2
The plasma dynamics is described by Faraday’s equations for plasma, magnetic coils and vacuum vessel, the motion equation, and expression for the gaps between tokamak’s vessel and plasma surface. The equations are linearized around the set of MHD equilibria reconstructed from experimental data of Globus-M2 tokamak (Mitrishkin et al., 2019), producing linear equations with time-varying parameters,

\[ M \delta I + R \delta I + \frac{\partial V}{\partial R} (I) \delta R + \frac{\partial V}{\partial Z} (I) \delta Z = \delta U, \]

\[ m \delta \delta Z = \frac{\partial F_m}{\partial I} (I) \delta I + \frac{\partial F_m}{\partial R} (I) \delta R + \frac{\partial F_m}{\partial Z} (I) \delta Z, \]
The plasma equilibrium is stable in regards to the radial plasma displacements \( \partial Z / \partial R < 0 \), which allows neglecting of the small plasma mass \( m \) in the radial motion equation.

Defining the state vector \( x = [\delta f^T \, \delta Z^T]^T \), the output vector \( y = [\delta Z \, \delta R \, \delta g^T \, \delta g^T]^T \) and the input vector \( u = u(t) \), the model equations can be written in the standard state-space form of linear time-varying system,

\[
\dot{x} = A(t)x + B(t)u, \quad y = C(t)x. \tag{1}
\]

The Globus-M2 poloidal system consists of eight coils, the currents in vacuum vessel are modelled as the linear combination of 15 orthogonal eigenmodes (Lazarus, 1990), and the six gaps between the plasma and vacuum vessel are considered. Thus, the obtained model set consists of the linear time-invariant models, each of that has 26 states, 8 inputs and 17 outputs. The models are calculated for time points from 0.175 s to 0.2 s of the plasma discharge with time step of 1 ms.

3. PLASMA POSITION CONTROL

The direct control of the plasma vertical position together with the plasma horizon position, instead of the speed stabilization around zero, in a tokamak improves the plasma magnetic control system stability and reliability (Mitrichshkin and Kartsev, 2011, Mitrichshkin et al., 2014, Kartsev et al., 2017, and Mitrichshkin et al., 2019). Such a system uses the adaptive approach of automatic tuning of the plasma magnetic axis position to parameters of the plasma shape. Thus, the used control structure of the plasma position loops is the reference tracking. The plasma position control at the Globus-M2 tokamak is realised by means of thyristor Current Invertors (CI) as fast-acting actuators which apply voltages to the coils controlling the vertical (VFC) and horizontal (HFC) magnetic fields. To control the CI thyristor bridges the signal of the HFC current and VFC current are used as nonlinear feedback (Fig. 2). The detailed description of the CI and its identified linear model is represented in Kuznetso et al., 2019.

3.1 Vertical Plasma Position Control

Linear plasma models in the Globus-M2 tokamak has the state space form shown in Section 2. The one linear time-invariant model, named nominal model, at the time of 0.185 s of plasma discharge is used to perform H∞ synthesis using the approach based on NCF (McFarlane and Glover, 1989) and open loop shaping (McFarlane and Glover, 1992) methods. Unstable nominal model from the CI output voltage \( U_{HFC} \) to the plasma vertical position displacement \( \delta Z \) is scaled by input and output factors of 1.1 kV and 0.01 m respectively. Weighting function used for open loop shaping procedure is 0.2 (0.0002 \( s^{-1} \)) / s. The final linear controller \( K_{PL}(s) \) is reduced up to order 3. Step response settling time is less than 0.01 s to eliminate the beat effect from the nonlinear self-oscillating mode of operation of the CI (Kuznetso et al., 2019).

3.2 Horizontal Plasma Position Control

The plasma horizontal position controller is synthesized using the same approach, except that the plant nominal model from the CI output voltage \( U_{VFC} \) to the plasma horizontal position displacement \( \delta R \) includes the plasma vertical position stabilization controller \( K_{PL}(s) \) in the closed feedback loop and thus is stable. Weighting function for open loop shaping procedure is 1.1 (0.0002 \( s^{-1} \)) / s. Final controller \( K_{G}(s) \) is also reduced up to the order 3.

Fig. 2 shows the closed-loop control system of the plasma vertical and horizontal position tracking.
Poloidal Field (PF) and Central Solenoid (CS) coil current deviations, gaps, and plasma current deviation respectively.

4. PF&CS COIL CURRENTS DECOUPLING

The PF&CS coil current control system (Fig. 3) tracks the corresponding current references \( \delta I_{PF}^{REF} \). Multivariable controller \( K_{PF}(s) \) consists of the six scalar PD-controllers tuned to the target bandwidth of 500 s\(^{-1} \) iteratively to achieve dynamical decoupling between the current signals \( \delta I_{PF} \). The detailed description of the used decoupling approach is represented in Kartsev et al., 2017.

\[
\begin{align*}
\delta I_{PF}^{REF} & \quad K_{PF}(s) & \quad \text{Rectifiers} & \quad G_{PF}(s) & \quad \delta I_{pl} \\
\end{align*}
\]

Fig. 3. Closed-loop system of the PF&CS coil currents tracking with decoupling controller and the plant that includes the plasma position stabilization loops \( G_{PL} \).

5. MPC OF PLASMA SHAPE AND CURRENT

The time-varying nature of the plasma model (1) arises the need for usage of a time-varying controller to achieve better performance especially for the gaps as the safety parameters of tokamak operation (Kartsev et al., 2017). Model predictive control (Wang, 2009) is one of the most known and efficient approaches to control of plants with time-varying parameters.

5.1 Time-Varying MPC Plant Model

Since the MPC works in discrete time, that allows handling of signal saturation cases (Wang, 2009), the model (1) is transformed to the form of linear time-varying difference equation system,

\[
x[n+1] = A_x x[n] + B_u u[n], \quad y[n] = C_x x[n]. \tag{2}
\]

For computational purposes, the system (2) is represented as the time-series of discrete time-invariant linear models, one model for one discrete time point. The model time-series is calculated with time step of 1 ms while the discretization time of the MPC is of 0.5 ms.

The controllers of the closed-loop system of the PF&CS coil currents tracking \( G_{PF}(s) \) (with the \( G_{PL}(s) \) inside) synthesized for the one nominal time-invariant plasma model are applied to the time-varying model (2) that leads to the linear time-varying system \( G_{PF} \) (Fig. 3). To improve the MPC performance, the output signal \( [\delta I_{PF} \delta g]^{T} \) of the \( G_{PF} \) closed-loop system is weighted with the transfer function \( W_{out}(s) = (s + 0.3) / s \text{ } s^{-7} \), where \( s^{-7} \) is the identity matrix of size 7. The input signals of the plasma position control systems \( \delta Z_{REF} \) and \( \delta R_{REF} \) respectively with the transfer function \( W_{in}(s) = 1 / (0.003 s + 1) \text{ } s^{5} \) to protect the inputs of the nonlinear self-oscillating CI (Kuznetsov et al., 2019) from a wide-band dynamics. That leads to the weighted system \( G_{PFw} \) (Fig. 5). Input and output signals of the \( G_{PFw} \) named with subscript "w" correspond to the signals of the \( G_{PF}(s) \) system. All elements of the \( G_{PFw} \) are discretised respectively. The resulting discrete time-varying model \( G_{PFd} \) is the plant under control for the MPC synthesis.

5.2 Model Predictive Control Basics with Time Varying parameters

The time-varying MPC approach is based on the use of a time-varying plant model in discrete time. To explain the idea of the time-varying MPC, let us consider the control signal \( u = \begin{bmatrix} \delta Z_{REF} \delta R_{REF} \delta I_{PFw}^{REF} \end{bmatrix}^{T} \) and the feedback signal \( y = \begin{bmatrix} \delta I_{PF} \delta g \end{bmatrix}^{T} \) of the plant \( G_{PFd} \) shown in Fig. 4 (the weighted plasma current signal \( \delta I_{plw} \) is not shown). It is possible to predict a feedback signal of a time-varying plant by a finite number of steps ahead (named Prediction Horizon, see Fig. 4) having its time-varying discrete model. The number of steps for which control signals are considered is a Control Horizon.

Fig. 4. Discrete input and output signals of the model predictive controller.

Let us denote the forecast of the feedback signal at the current moment \( n \) as \( \hat{y}[n + 1 \mid n] \) ... \( \hat{y}[n + p \mid n] \), where \( p \) is the prediction horizon. The \( y \) signal forecast depends on the future control sequence \( u[n], u[n + 1], ... u[n + m] \), where \( m \) is the control horizon, and \( p > m \), otherwise the last values in the control sequence will not be taken into account. The time-varying MPC task is to find the optimal control sequence \( u \) in the sense of the quality criterion \( J \). Feedback in the system is closed by repeating the search for optimal control sequence at each step, taking into account the current real value of the system output. At the same time, only the first value of the sequence of optimal controls is applied to the plant, this approach is named Receding Horizon Principle that allows to achieve good performance of the algorithm in presence of disturbances and inconsistencies between the real plant and its predictive model.

Let us introduce the current moment \( n \) control value variation as \( \Delta u[n] = u[n] - u[n - 1] \). Considering the plant model,
\[ x[n + 1] = A_n x[n] + B_n u[n - 1] + B_n \Delta u[n], \quad y[n] = C_n x[n], \]

one can obtain the forecast of the model output \( \hat{y} \) for a finite number of steps ahead,

\[
\hat{x}[n + p | n] = \left( \prod_{i=0}^{p-1} A_{n+i} \right) \hat{x}[n | n] + \left( \prod_{i=0}^{p-1} A_{n+i} \right) B_n u[n - 1] + B_n \Delta u[n | n],
\]

\[
\hat{y}[n + p | n] = C_{n+p} \hat{x}[n + p | n].
\]

This leads to the following matrix equation,

\[
\hat{Y}_{ap} = S_{ap,n+1} \hat{x}[n | n] + S_{ap,n+1} u[n - 1] + S_{ap,n+1} \Delta U_n,
\]

where \( S_{ap,n+1} \), \( S_{ap,n+1} \), and \( S_{ap,n+1} \) are the matrices that depend on the time-varying matrices of the plant model in the current \( n \) and subsequent \( p \) steps. The control variation matrix is \( \Delta U_n = \{ \Delta u[n], \Delta u[n - 1 + m], \Delta u[k - 1 + m] \ldots \} \) for the moment \( n \). The output signal forecast matrix is \( \hat{Y}_{ap} = \{ \hat{y}[n + 1], \ldots, \hat{y}[n + p] \} \). The plant model state vector \( x[n] \) is estimated by the state observer, and the control value at the previous step \( u[n - 1] \) is contained in the MPC memory. The quality criterion \( J \) used contains the square of the mismatch between the predicted feedback \( \hat{y} \) and the reference signal \( y_{REF} = [\delta g_{REF} \delta \Delta R_{REF}]^T \). In addition, the quality criterion includes terms that take into account the smoothness and costs of control,

\[
J_n = (\hat{Y}_{ap} - Y_{REF})^T W_r (\hat{Y}_{ap} - Y_{REF}) + \Delta U_n^T \Delta U_n + U_n^T U_n
\]

where \( W_r \) is the positive definite weighting matrix, all signals \( \hat{y}, y_{REF}, \) and \( u \) are scaled with the corresponding scale factors. The reference signal and control signal matrices are,

\[
Y_{REF} = \{ y_{REF}[n + 1], \ldots, y_{REF}[n + p]\},
\]

\[
U = \{ u[n], u[n - 1 + m], \ldots, u[n - 1 + m] \}.
\]

Substituting the value (3) into (4), one can obtain the expression for \( J(\Delta U_n) \). Then the MPC optimization problem will consist in solving the system of the following algebraic equations,

\[
dJ(\Delta U_n) / d(\Delta U_n) = 0.
\]

After finding the optimal control variations \( \Delta U_{n}^* \), at the current step \( n \), only the value \( u[n] = \Delta u[n] + u[n - 1] \) is given to the plant under control. The actuators have saturation leading to the restrictions on the control matrix \( U_{\text{min}} \leq U_n \leq U_{\text{max}} \) that can be represented as \( M_{n,n} \Delta U_n \leq M_{\text{fin}} \), where matrix inequalities are elementwise. As the result, the optimal control variations \( \Delta U_{n}^* \) can be found by solving the problem (5) under constraints specified above.

### 5.3 Model Predictive Controller Synthesis

The approach of adaptation of the plasma magnetic axis position to the shape parameters is used to resolve the contradiction between plasma position and shape (Kartsev et al., 2017). The model predictive controller \( K_{MPC} \) additionally computes the \( \delta \Delta Z_{REF} \) and \( \delta \Delta R_{REF} \) adaptation signals (Fig. 5) for the plasma vertical and horizontal position tracking system.

The plasma shape and current control system tracks the corresponding references of the desired gaps \( \delta g_{REF} \) and the plasma current deviation \( \delta I_{PREF} \) (Fig. 5). Then, the weighted discrete model \( G_{PF} \) is used to synthesize the model predictive controller \( K_{MPC} \) (Wang, 2009) with the dimensionless output variables weighting vector of \([0.5 \ 1 \ 1 \ 4 \ 1 \ 1 \ 4]\) tuned by the results of the several numerical simulations. The prediction horizon is of 50 time steps ahead and the control horizon is of 10 time steps. The signal scaling is of 2 kA for the plasma current deviation and 0.05 m for the gap deviations. The signal constraints are of \( \pm 200 \) kA for the \( \delta I_{P} \), \( \pm 0.05 \) m for the \( \delta g \), \( \pm 200 \) kA for the \( \delta I_{PREF} \), and \( \pm 0.1 \) m for \( \delta \Delta Z_{REF} \) and \( \delta \Delta R_{REF} \).

During the trial and error tuning process, it was revealed that the PF&CS coil current control loop determines the major time constant for the gaps and plasma current response.

**6. SIMULATIONS AND THE RESULTS**

The linear time-varying model (1) of the plasma in Globus-M2 tokamak in the form of the time-series of the linear models (Mitrichkin et al., 2019) is used for numerical simulations. The models in the time-series are switched by the time of simulation with the shape preserving piecewise cubic spline interpolation (Fritsch and Carlson, 1980) of the model matrices. On each time interval, the models matrices are interpolated by the cubic Hermit interpolating polynomials with continuous value and first derivative. The polynomials preserve shape in terms of the monotonicity of the data and themselves. This approach used allows avoiding the fatal computational errors in linear interpolation of matrices inherent in the linear interpolation method. The full nonlinear models of the thyristor CIs as the actuators in the plasma vertical and horizontal position tracking loops are used for numerical simulations. The CIs work in the self-oscillating mode, thus, they are the source of the all signal oscillations at frequencies at about 3 kHz (Kuznetsov et al., 2019). In the simulations presented, the plasma current deviation is \( \delta I_{PREF} = 0 \) A, and the desired gap deviations are \( \delta g_{REF} = 0.02 \times [0.775 \ 0.775 \ -1.0 \ -7.25 \ 0.0 \ 0.1] \) m.

Fig. 6 and Fig. 7 show the plasma vertical and horizontal position tracking for the adaptation signals \( \delta \Delta Z_{REF} \) and \( \delta \Delta R_{REF} \) computed by the \( K_{MPC} \). Fig. 8 presents the PF&CS coil current deviations response. Fig. 9 and Fig. 10 display the gaps and their tracking errors respectively. The response time is about of 10 ms, the worse tracking error is about of 0.025 m for the \( g_2 \) signal. Fig. 11 illustrates the plasma current deviation response. The performance is not quite good because the tuning is focused on the plasma shape represented by the gaps as the safety parameters of tokamak operation.
Fig. 6. Plasma vertical position tracking.

Fig. 7. Plasma horizontal position tracking.

Fig. 8. PF&CS coil current deviations.

Fig. 9. Gap tracking with reference vector $0.02 \times [0.775 0.775 -1.0 -7.25 0.0 1.0]$ m.

Fig. 10. Gap tracking error signals.

Fig. 11. Stabilization of the plasma current.
7. CONCLUSIONS

In this paper, the MPC method is used for the plasma current and shape control in the tokamak Globus-M2 taking into account the time-varying models of the controlled plant which have been created for the specific plasma scenario. The simulations of the control system have been done with the full models of the nonlinear current inverters as plasma position control actuators. The MPC robustness property is not the focus of the paper; this is the proof-of-concept of using the time-varying MPC approach to the plasma magnetic control in a tokamak. The results of the simulation confirmed the efficiency of the proposed time-varying MPC approach. The resulting control system can be compared with magnetic plasma control systems synthesized in the following articles: Kartsev et al., 2017, Mitrishkin et al., 2019, Konkov, et al., 2020, Mitrishkin et al., 2020, where the different robust time-invariant controllers are used. In the future, the adaptation is supposed to be applied on the base of the MPC when plant linear models will be obtained on line at each discrete moment around plasma equilibrium reconstructions.

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