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Hypertracking: a new approach to signals beyond the Nyquist frequency —a brief overview

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Abstract: Shannon's sampling theory has had a great impact not only on signal processing, but also on digital control theory. According to this theory it is universally believed that the so-called Nyquist frequency is the absolute upper bound for any control objectives. For example, tracking or rejection of signals that resides beyond the Nyquist frequency has been regarded impossible. On the other hand, such a demand can often be encountered in practice. Disturbance rejection of winds in hard disc drives where such a disturbance usually occurs at a frequency higher than the Nyquist frequency are such examples. The present paper summarizes the recent results obtained by the authors that show that such high frequency tracking/rejection problems are indeed solvable, even for the case that involve multiple tacking and rejection signals simultaneously. It is also possible to give a robustness result that also exhibits an interesting relationship between robustness and delay length introduced for tracking and rejection. The paper gives a brief overview of the results obtained thus far, and also provides a new result on robustness. Some simulation results are presented to show that the method can work in various practical situations.

Keywords: State space theory, Kalman filtering, realization theory, sampled-data systems

1. INTRODUCTION

Shannon's sampling theory has had a great impact not only on signal processing, but also on digital control theory. According to this theory it is universally believed that the so-called Nyquist frequency is the absolute upper bound for any control objectives. For example, tracking or rejection of signals that reside beyond the Nyquist frequency has been regarded impossible.

On the other hand, there certainly exist many practical situations where such control objectives are demanded. For example, rejection of wind disturbances of hard-disc drives (Atsumi (2010); Zheng et al. (2016)), noise rejection of robotic arms, tracking/regulation of an electric power supply curve, where the demanded sampling frequency is not high enough to allow for these signals to be below the Nyquist frequency. Such a limitation on the sampling frequency occurs due to varied physical constraints. For example, in the case of hard-disc drives, the sampling frequency is inherently limited due to the limited number of markings on the rotating discs (and the maximum rotation speed of the discs) whereas the rotation of those discs often generate winds that have high-frequency components higher than the Nyquist frequency.

If we faithfully follow the dogma demanded by the sampling theorem, there seems to be no way out. However, this is based on somewhat loose thinking. There are two important elements either ignored or overlooked.

- The band-limiting condition is only a sufficient condition, not necessary. Hence there is certainly a room for improvement if we remove this condition.
- We do not necessarily need perfect signal recovery.

While the authors have successfully derived a new framework in digital signal processing Yamamoto et al. (2012) and shown that signal recovery beyond the Nyquist frequency is indeed

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possible, there has not been much progress in the above control problems involving such high frequency signals.

The above observations, however, also lead to a new solution in the sampled-data control context. In view of this new paradigm, we call the tracking and rejection schemes *hypertracking* and *hyperrejection* to disinguish them from the classical sampled-data control thinking.

2. PROBLEM FORMULATION

Consider the sampled-data system depicted in Fig. 1.

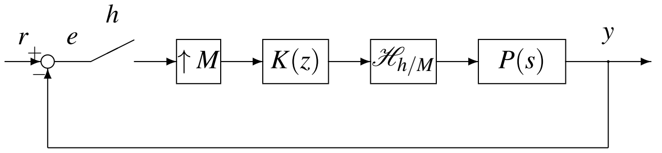

Fig. 1. Sampled-data feedback system

$P(s)$ is a linear, time-invariant, continuous-time plant, and $K(z)$ is a linear, time-invariant, discrete-time controller. The error e is sampled with sampling period h, and after sampled, it is upsampled by factor M to allow for a faster control processing. The action of $\uparrow M$ is given as follows:

$$(\uparrow M)(e)[kh+\ell] = \begin{cases} e[kh] & \text{if } \ell = 0 \\ 0 & \ell = h/M, \ldots (M-1)h/M. \end{cases} \tag{1}$$

$\mathscr{H}_{h/M}$ is the zero-order hold that holds the output as constant for the period of h/M.

We now consider the following problem:

Problem 1: *In the block diagram Fig. 1, consider the reference input* $\sin \omega t$ *where* ω *is greater than the Nyquist frequency* π/h. *Find a discrete-time controller* $K(z)$ *such that the output* $y(t)$ *nearly tracks the reference* $r(t) = \sin \omega t$ *or its delayed signal* $r(t-L) = \sin \omega(t-L)$.

We have given some solutions for this problem in Yamamoto et al. (2016, 2017, 2018). The basic scenario is the following:

- Introduce a weighting function $F(s)$ to the input so that the input has a peak at the desired tracking frequency ω.
- Invoke sampled-data H^∞ control to optimally design $K(z)$.

3. DESIGN METHOD

Observe first that our system Fig. 1 cannot be used as it is for a design block diagram for H^∞ sampled-data control. Sampling is not a bounded operator on L^2, and hence system Fig. 1 as it is cannot be used as a design model. To remedy this, we place a strictly proper anti-alias filter $F(s)$ in front of the adding point of the error. In other words, the reference signal is pre-filtered by $F(s)$. This is advantageous in that we can control frequency weighting in the input reference signals. We here emphasize that *unlike the usual case of* $F(s)$ *where we put more emphasis on the low-frequency range, we attempt to place more emphasis on the frequency that we wish to track.* This is a rather non-standard idea different from usual sampled-data control, and the objective here is to show that *this does indeed work for the tracking purpose of this paper.*

Another attempt we devise here is that we allow some delays in tracking. That is, instead of taking the error $e(t) = r(t) - y(t)$, we try to minimize the delayed error $\tilde{e}(t) := r(t-L) - y(t)$ for some positive L as stated in Problem 1. This is under analogy from the case of delayed signal construction in Yamamoto et al. (2012) where we can achieve better performance by allowing certain delays in signal reconstruction. While its real advantage in performance is yet to be investigated in the future, this will give us at least more freedom in controller design.

Incorporating these changes into Fig. 1, we obtain the following generalized plant Fig. 2 for design. Here L is a design parameter; we usually take L to be an integer multiple of h, with some small number such as 4 – 10.

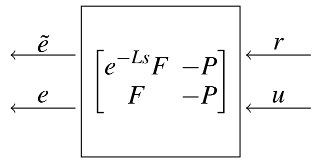

Fig. 2. Generalized plant

We skip the details in the analysis of the design stated above. Details can be found in Yamamoto et al. (2016, 2017, 2018), and we here summarize

- the results obtained in the references above, and
- give a theorem on robustness in this new method.

4. EXAMPLE, DESIGN, AND SIMULATION

Example 4.1. Consider the plant

$$P(s) := \frac{1}{s^2+2s+1} \tag{2}$$

with (normalized) sampling period $h = 1$ in Fig. 1. The Nyquist frequency is then π [rad/sec] which is just equal to 0.5 [Hz]. Suppose that we are given the tracking signal $r = \sin \omega t$, where $\omega = 3\pi/2$ [rad/sec], which is equal to 0.75 [Hz]. This is clearly above the Nyquist frequency, and a normal signal-processing intuition or a digital control thinking may tell us that it is impossible to track.

The basic idea is that we place more weight on this high frequency signal rather than the low frequency range below the Nyquist frequency. In fact, we take the weighting function

$$F(s) := \frac{s}{s^2+0.1s+(3\pi/2)^2}. \tag{3}$$

which has a clear peak at $3\pi/2$ [rad/sec] and also deemphasizes low-frequency.

The response against the sinusoid $r(t) = \sin(3\pi/2)t$ is shown in Fig. 3 with $M = 8$ and $L = 4h$. This figure clearly shows that the output tracks the reference input $\sin(3\pi/2)t$, which has the natural frequency greater than the Nyquist frequency π, and the output matches the given frequency $3\pi/2$. Note also that the output shows the delay of 4 steps which is specified by the design specification.

Fig. 4 and Fig. 5 show the Bode plot of the controller as well as its output. This output shows that the discrete-time output indeed gives a discrete-time approximation of the sinusoid $\sin(3\pi/2)t$, that is, the discrete-time controller contains an approximate internal model of the reference input. If we increase the upsampling factor M, it is expected that the designed controller produces more accurate sinusoids.

The following example shows a tracking to a yet higher frequency $5\pi/2$:

Example 4.2. Take the same plant $P(s) := 1/(s^2+2s+1)$, but with the objective of tracking the sinusoid $\sin(5\pi/2)t$, i.e., sinusoid at 1.25 [Hz]. We take a new weight

$$F(s) := \frac{s}{s^2+0.1s+(5\pi/2)^2},$$

which now has a peak at $5\pi/2$ [rad/sec].

The following Fig. 6 shows the result with $M = 16$ and $L = 4$. We skip to show the Bode plot of the controller which looks fairly similar to Fig. 4.

5. SIMULTANEOUS TRACKING AND REJECTION

Now consider the sampled-data system depicted in Fig. 7.

We take the following generalized plant Fig. 8 for the design:

the objective of tracking the sinusoid $\sin(5\pi/2)t$, i.e., sinusoid at 1.25 [Hz].

Example 5.1. Take the same plant and design specifications as above with the objective of tracking $\sin(\pi/6)t$ and rejecting the

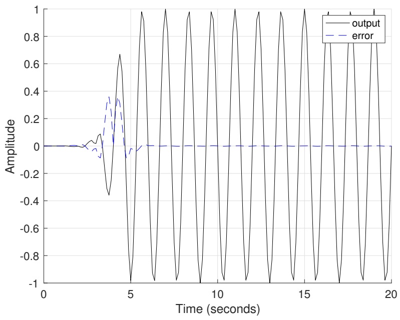

Fig. 3. System output tracking $\sin(3\pi/2)t$ along with the delayed error

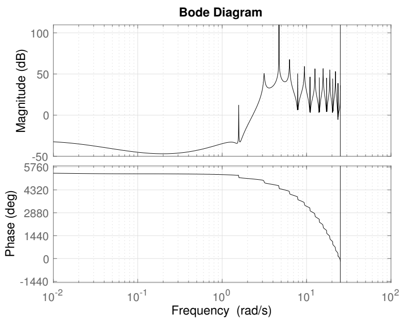

Fig. 4. Discrete-time controller

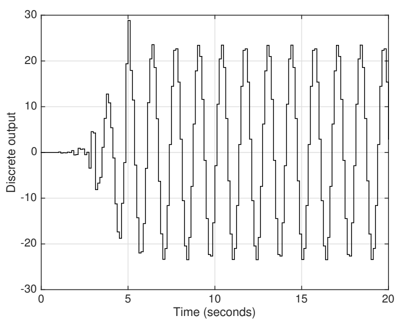

Fig. 5. Discrete-time controller output

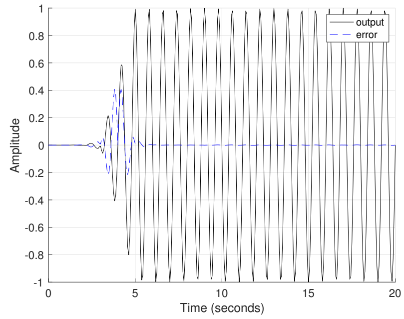

Fig. 6. System output tracking $\sin(5\pi/2)t$ along with the delayed error

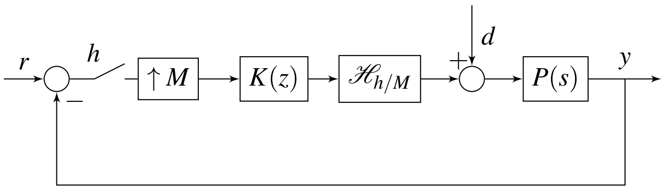

Fig. 7. Sampled-data feedback system with input disturbance

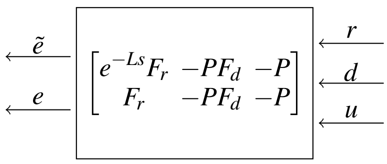

Fig. 8. Generalized plant in the presence of input disturbance

disturbance $\sin(3\pi/2)t$. That is, the tracking frequency is below the Nyquist frequency while the disturbance above the Nyquist frequency enters into the plant input. We take the following weights:

$$F_r(s) = \frac{s}{s^2+0.01s+\omega_1^2}, \quad F_d(s) = \frac{s}{s^2+0.01s+\omega_2^2}, \tag{4}$$

Fig. 9 shows that the error has been very well attenuated.

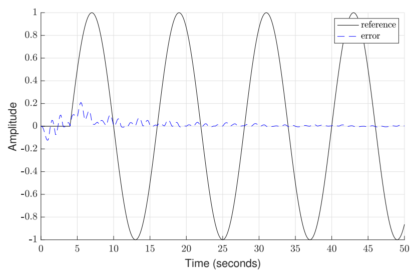

Fig. 9. Delayed error against $\sin(\pi/6)t$ in the presence of the input disturbance $\sin(3\pi/2)t$.

6. ROBUSTNESS

In order that the proposed framework of hypertracking and hyperrejection be practical, we clearly need robustness under various plant perturbations. We raise the following question:

> *Is it possible to guarantee hypertracking/hyperrejection even under the presence of plant fluctuations or reference/disturbance frequency variations?*

Let us start with the following example.

Example 6.1. We take the same plant (2) as in Example 4.1 for tracking $\sin\omega t$ with $\omega = \pi/3$. with weighting function

$$F_r(s) = \frac{s}{s^2 + 0.01s + \omega^2}. \tag{5}$$

We set the upsampling factor $M = 8$, the delay length $L = 4$. While Fig. 10 shows a fine tracking property, it produces a

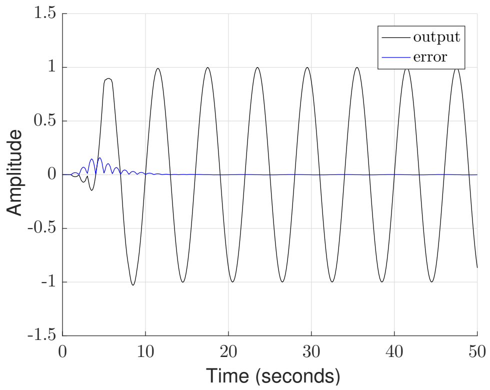

Fig. 10. System output tracking $\sin(\pi/3)t$ without disturbance

fairly large error when we variate the plant P to $P \mapsto P+\Delta$, $\Delta(s) = 0.05/(s+1)$. See Fig. 11 for comparison.

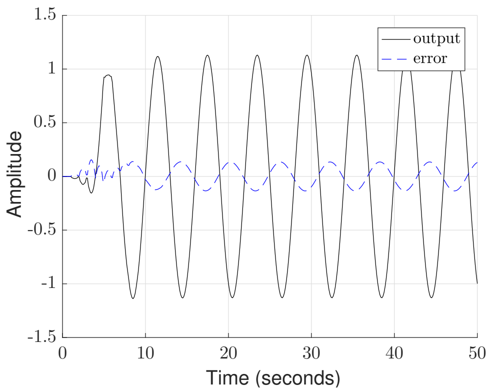

Fig. 11. System output tracking $\sin(\pi/3)t$ under the additive perturbation $0.05/(s+1)$ to the plant

In contrast, consider the following example.

Example 6.2. We take the same plant and simulation condition as in Example 6.1, only with the difference that the tracking signal r is replaced by $r(t) = \sin(\pi/2)t$ and the weighting function (5) with $\omega = \pi/2$. The result of robustness test for even with a larger plant perturbation $P \mapsto P+\Delta$, $\Delta(s) = 0.3/(s+1)$ shows a fine tracking; Fig. 12.

The difference of these two examples becomes clear by the following theorem:

Theorem 6.3. Consider the hypertracking problem Fig. 1 with tracking/rejection signal $\sin\omega t$ and tracking delay $L = mh$. If L is an integer multiple of $2\pi/\omega$, then the resulting system possesses an (approximate) internal model of those sinusoidal signals.

Proof is omitted here.

It is interesting to observe that this theorem shows that the overall (robustness) performance does not necessarily improve as the delay length becomes larger.

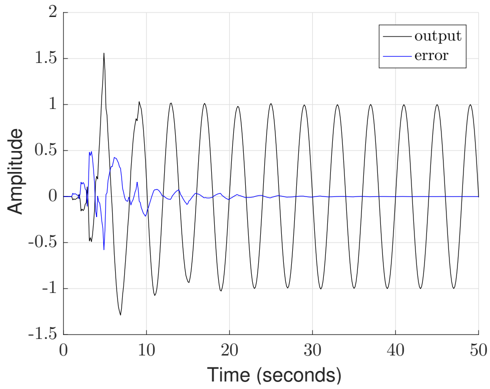

Fig. 12. System output tracking $\sin(\pi/2)t$ under the additive perturbation $0.3/(s+1)$ to the plant

7. CONCLUSION

We have summarized the results on hypertracking/hyperrejection obtained thus far, and also given a new theorem on robustness (the proof will be given elsewhere). It is highly interesting to note that such a tracking and rejection problem involving signals beyond the Nyquist frequency is indeed possible provided we give a sufficient attention on the choice of delays in tracking to guarantee necessary robustness. Further outcomes of this new scheme will be investigated in the future.

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