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Leadership Hierarchy-based Formation Control via Adaptive Chaotic Pigeon-inspired Optimization

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Abstract: Formation control of multi-agent systems (MASs) is a significant research subject in the field of cooperative control. In this paper, we propose a novel consensus-based formation control approach with minimal resource cost and excellent adaptability for second-order nonlinear multi-agent systems. Specifically, an improved constrained adaptive chaotic pigeon-inspired optimization algorithm (ACPIO) is proposed for tuning parameters, which promotes the automation of controller design and alleviates the workload of conventional designer. Moreover, a variant of pinning control method integrating with hierarchical leadership model of pigeon flocks is introduced, which achieves excellent adaptability and reduces computational complexity simultaneously. Additionally, sufficient conditions are derived for achieving the desired formation pattern based on Lyapunov stability theory and matrix theory. Numerical simulation results demonstrate the feasibility and effectiveness of the proposed method for formation control of second-order nonlinear MASs.

Keywords: multi-agent systems, formation control, leadership hierarchy, pinning control, improved pigeon-inspired optimization

1. INTRODUCTION

The multi-agent systems (MASs) (Li et al. (2004); Zhou et al. (2019)) are composed of multiple interacting intelligent agents, generally used to conduct complex tasks cooperatively within various environments, such as surveillance (Nigam et al. (2011)), source seek (Han and Chen (2014)) and military combat (Cil and Mala (2010)). Formation control is one of the most actively studied topics within the realm of MASs, aiming to drive multiple agents to achieve prescribed constraints on their states. However, this is no easy task due to its sensitivity to external interference and system uncertainty.

In order to establish and maintain a certain spatial configuration for MASs, a variety of formation control methods have been proposed. Common methods of formation control falls into three strategies: leader-follower, virtual structure and behavior-based strategy. It has been indicated these approaches have their own disadvantages though they are applied widely in formation problem (Beard et al. (2001)). For instance, the leader-follower strategy lacks robustness because that the failure of the leader may destroy the whole formation. The virtual structure strategy is not fully distributed. To improve the robustness, Ren (2007) unified these control strategies within the framework of consensus protocol.

Since many pinning control methods for MASs have been developed, it would be useful to study the consensus problem. However, existing studies on pinning control achieve a limited success with failure to tie hierarchy relationship with nodes. Encouragingly, Qiu and Duan (2017) proposed a hierarchical leadership strategy that could theoretically construct a hierarchical model with satisfactory adaptability. Based on the strategy, the MASs may have the following advantages: information transfers more efficient than other types of networks (Zafeiris and Vicsek (2015)), and agents with certain hierarchical structures can improve individual navigation accuracy (Flack et al. (2015)). Therefore, it is worth proposing a variant of pinning control that integrates with hierarchical leadership strategy.

However, it is time-consuming to manually adjust parameters(Hai et al. (2019)). Therefore, establishing an effective mechanism of tuning parameters is necessary. Pigeon-inspired optimization (PIO) algorithm (Duan and Qiao (2014); Zhang and Duan (2015); Duan and Wang (2015)) has proven to be feasible and effective for optimization problems. But there are still some shortcomings. In order to improve the population diversity and promote the searching ability for global optima, many efforts have been made (Duan et al. (2018); Xu and Deng (2018); Qiu and Duan (2020)). In this paper, the weight adaptive strategy and chaos theory(Luo and Duan (2014)) are applied in PIO algorithm, namely ACPIO algorithm.

To address aforementioned issues, formation control problems of second-order nonlinear MASs are investigated in this paper. Specifically, based on hierarchical leadership, the pinning strategy with optimal control parameters is

developed to guarantee synchronization. The main contributions of the proposed approach are as follows:
(1) A novel consensus-based formation control approach is proposed. For purpose, the ACPIO algorithm is proposed as the solver of optimal control parameters.
(2) Sufficient condition for the existence of the pinning controller is derived using matrix theory and Lyapunov stability theory.
(3) A variant of pinning control method integrating with leadership hierarchy model is introduced to achieve better adaptability and reduce computational burden as well.

The remainder of this paper is organized as follows. The preliminaries and problem formulation are introduced in Section 2. Section 3 expounds the design and analysis of pinning control. Numerical simulation results are presented in Section 4 to verify the effectiveness of the proposed approach. Finally, Section 5 draws the conclusions.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Notations

Let $\mathbb{R}^{n\times n}$ be the set of real matrices of size $n\times n$, and $\mathbb{R}^n$ be the set of vectors of size $n\times 1$. I_n is the n-dimensional identity matrix and $1_n(0_n)$ means the n-dimensional column vector with each entry being 1(0). $\|\cdot\|$ indicates the Euclidean norm. $\lambda_{min}(A)$ and $\lambda_{max}(A)$ denotes the minimum and maximum eigenvalue, respectively. Matrix $A<0(A\leq 0)$ means that A is real symmetric and negative definite (semi-negative definite). The symbol $\otimes$ denotes Kronecker product. Additionally, $[\cdot]$ represents the floor function.

2.2 Graph theory

Consider a directed graph $G=(V,E,A)$, where $V=\{V_1,V_2,...,V_N\}$ and $E\subseteq\{(V_i,V_j):V_i,V_j\in V\}$ are the set of nodes and edges, respectively. $A=(a_{ij})\in\mathbb{R}^{N\times N}$ is the non-negative adjacency matrix of G with element a_{ij}. A directed edge e_{ij} in the network G is denoted by a ordered pair of (V_i,V_j), where V_i and V_j indicate the terminal and initial nodes, respectively, which means that node V_i can receive information from node V_j. $a_{ij}>0$ if and only if there is a directed edge (V_i,V_j) in G. The Laplacian matrix $L=(l_{ij})\in\mathbb{R}^{N\times N}$ of the directed network is defined as $l_{ij}=-a_{ij}(i\neq j)$; $l_{ii}=\sum_{i=1}^N a_{ij}$ and $\sum_{j=1}^N l_{ij}=0$ (Olfati-Saber and Murray (2004)).

2.3 Problem formulation

A class of second-order nonlinear multi-agent systems is considered in our work, which is composed of a leader agent and N-1 follower agents. The mathematical motion model for the following agent i is described as follows:

$$\begin{cases}\dot{x}_i(t)=v_i(t),\\ \dot{v}_i(t)=f(x_i,v_i,t)+u_i\end{cases}\tag{1}$$

where $x_i=[x_{i1},x_{i2},...,x_{in}]^T\in\mathbb{R}^n$, $v_i=[v_{i1},v_{i2},...,v_{in}]^T\in\mathbb{R}^n$ are position and velocity state vectors, respectively. $f:\mathbb{R}^n\to\mathbb{R}^n$ stands for a continuous mapping. $f(x_i,v_i,t)=[f_1(x_{i1},v_1),f_2(x_2,v_2),...,f_{in}(x_{in},v_{in})]^T\in\mathbb{R}^n$ is a smooth nonlinear function. $u_i=[u_{i1},u_{i2},...,u_{in}]^T\in\mathbb{R}^n$ is the control input. When agent i is the general leader ($i=N_L$), $u_i=0$, which means the leader agent is an isolated agent. To simplify the consensus of system, the expected position $x_i^e(t)=x_{N_L}(t)$ and expected velocity $v_i^e(t)=v_{N_L}(t)$, and the desired spatial configuration is formed by setting the position offset from general leader.

Let the state error be $\Delta x_i(t)=x_i(t)-x_i^e(t)$, $\Delta v_i(t)=v_i(t)-v_i^e(t)$, then the error variable of the system can be expressed as:

$$\begin{cases}\Delta x(t)=(\Delta x_1^T(t),\Delta x_2^T(t),\ldots,\Delta x_N^T(t)),\\ \Delta v(t)=(\Delta v_1^T(t),\Delta v_2^T(t),\ldots,\Delta v_N^T(t))\end{cases}\tag{2}$$

Assumption 1. (Yu et al. (2009)) The nonlinear function $f_i(x_i,v_i,t)$ is bounded. There exists positive constants α and β, such that for any $x_i,x_j,v_i,v_j\in\mathbb{R}^n$, the following inequality hold:

$$\|f(x_i,v_i,t)-f(x_j,v_j,t)\|\leq\alpha\|x_i-x_j\|+\beta\|v_i-v_j\|\tag{3}$$

Definition 1. (Wang and Wu (2012)) The second-order consensus in multi-agent systems (1) is said to be achieved, if the solution of (1) satisfy:

$$\begin{cases}\lim\limits_{t\to\infty}\|\Delta x(t)\|=0\\ \lim\limits_{t\to\infty}\|\Delta v(t)\|=0\ \ (i=1,2,...,N.)\end{cases}\tag{4}$$

Lemma 1. (Zhou et al. (2019)) For matrices A, B, C and D with appropriate dimensions, the following equations hold:

$$(A\otimes B)^T=A^T\otimes B^T$$
$$(A+B)\otimes C=A\otimes C+B\otimes C$$
$$(A\otimes B)(C\otimes D)=AC\otimes BD$$

Lemma 2. (Xu et al. (2004)) For a given pair of x, $y\in\mathbb{R}^n$, and a positive-define matrix $Q\in\mathbb{R}^{n\times n}$, the following inequality holds

$$2x^Ty\leq x^TQx+y^TQ^{-1}y$$

3. CONSENSUS-BASED FORMATION WITH PINNING CONTROLLERS

3.1 Stability analysis

Inspired by the pinning control scheme, the control input can be designed as follows:

$$u_i(t)=-k(t)[d_i(\Delta x_i(t)+\Delta v_i(t))+\sum_{j=1}^N l_{ij}(\Delta x_j+\Delta v_j)]\tag{5}$$

where

$$\dot{k}(t)=\frac{1}{2}e^{\tau t}\xi(t)^TK\xi(t)$$
$$D=diag\{d_1,d_2,\ldots,d_N\}$$
$$\hat{L}=L+D$$
$$\gamma=\hat{L}\otimes I_n$$
$$\xi(t)=(\Delta x^T(t),\Delta v^T(t))^T$$
$$K=\begin{bmatrix}\gamma&\gamma\\ \gamma&\gamma\end{bmatrix}$$

The symbol d_i denotes the diagonal element of matrix D, and $d_i>0$ indicates agent i is pinned.

With the developments as above, the main results are presented in the following.

Theorem. Suppose that the nonlinear function $f(\cdot)$ satisfies *Assumption 1*, the multi-agent system (1) can achieve the desired spatial configuration under control protocol (5), if there exist positive constants $\tau > 0, \alpha > 0, \beta > 0, k^* > 0$ satisfying the following

$(1)\lambda_{min}(\hat{L}) > 1$
$(2)\Gamma < 0$

where

$$\Gamma = \begin{bmatrix} (1+2\alpha)I_{Nn} + (\tau - k^*)\gamma & (1-k^*)\gamma + \tau I_{Nn} \\ (1-k^*)\gamma + \tau I_{Nn} & (3+2\beta+\tau)I_{Nn} - k^*\gamma \end{bmatrix}$$

Proof. We consider a Lyapunov function defined by

$$V(t) = \frac{1}{2}\xi^T(t)\begin{bmatrix} \gamma & I_{Nn} \\ I_{Nn} & I_{Nn} \end{bmatrix}\xi(t) + \frac{1}{2}e^{-\tau t}(k(t) - k^*)^2$$

From the condition 1 in *Theorem*, the first term is positive definite. It is obvious that the second term is non-negative. Thus, the function $V(t)$ is positive definite.

$$\begin{aligned} \dot{V}(t) =& \xi^T(t)\begin{bmatrix} \gamma & I_{Nn} \\ I_{Nn} & I_{Nn} \end{bmatrix}\dot{\xi}(t) \\ & - \frac{1}{2}\tau e^{-\tau t}(k(t)-k^*)^2 + e^{-\tau t}(k(t)-k^*)\dot{k}(t) \\ =& \xi^T(t)\begin{bmatrix} \gamma & I_{Nn} \\ I_{Nn} & I_{Nn} \end{bmatrix}\begin{bmatrix} 0_{Nn\times 1} \\ \Delta f_i \end{bmatrix} - \xi^T(t)\times \\ & \begin{bmatrix} \gamma & I_{Nn} \\ I_{Nn} & I_{Nn} \end{bmatrix}\begin{bmatrix} 0_{Nn} & -I_{Nn} \\ k(t)(\hat{L}\otimes I_n) & k(t)(\hat{L}\otimes I_n) \end{bmatrix}\xi(t) \\ & - \frac{1}{2}\tau e^{-\tau t}(k(t)-k^*)^2 + e^{-\tau t}(k(t)-k^*)\dot{k}(t) \\ =& \xi^T(t)\begin{bmatrix} \gamma & I_{Nn} \\ I_{Nn} & I_{Nn} \end{bmatrix}\begin{bmatrix} 0_{Nn\times 1} \\ \Delta f_i \end{bmatrix} \\ & + \frac{1}{2}\xi^T(t)\begin{bmatrix} 0_{Nn} & \gamma \\ \gamma & 2I_{Nn} \end{bmatrix}\xi(t) \\ & - \frac{1}{2}k^*\xi^T(t)K\xi(t) - \frac{1}{2}\tau e^{-\tau t}(k(t)-k^*)^2 \end{aligned} \tag{6}$$

Using *Assumption 1* and *Lemma 1*, the first term can be derived that

$$\begin{aligned} & \xi^T(t)\begin{bmatrix} \gamma & I_{Nn} \\ I_{Nn} & I_{Nn} \end{bmatrix}\begin{bmatrix} 0_{Nn\times 1} \\ \Delta f_i \end{bmatrix} \\ \le& \frac{1}{2}\Delta x^T(t)\Delta x(t) + \|\Delta f_i\| + \frac{1}{2}\Delta v^T(t)\Delta v(t) \\ \le& (\frac{1}{2}+\alpha)\Delta x^T(t)\Delta x(t) + (\frac{1}{2}+\beta)\Delta v^T(t)\Delta v(t) \\ \le& \frac{1}{2}\xi^T(t)\begin{bmatrix} (1+2\alpha)I_{Nn} & 0_{Nn} \\ 0_{Nn} & (1+2\beta)I_{Nn} \end{bmatrix}\xi(t) \end{aligned} \tag{7}$$

Then, substituting (7) into (6), and using condition 2 in *Theorem*, we have:

$$\begin{aligned} \dot{V}(t) \le& \frac{1}{2}\xi^T(t)\Gamma\xi(t) - \frac{1}{2}\tau\xi^T(t)\begin{bmatrix} \gamma & I_{Nn} \\ I_{Nn} & I_{Nn} \end{bmatrix}\xi(t) \\ & - \frac{1}{2}\tau e^{-\tau t}(k(t)-k^*)^2 \\ \le& -\tau V(t) \end{aligned} \tag{8}$$

Therefore, it can be derived that

$$V(t) \le V(0)e^{-\tau t} \tag{9}$$

By the condition 2 in the *Theorem*, it can be derived that $\dot{V}(t) < 0$. Hence, by Lyapunov stability theory, the second-order consensus is achieved under control protocol (5).

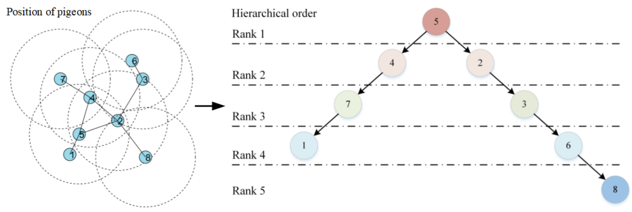


Fig. 1. Example illustration

3.2 Leadership hierarchy of pigeon flocks

The hierarchical structure in the in-flight leader-follower relationship of pigeons are discovered by analyzing the pigeon flight data (Nagy et al. (2010)). And the leadership hierarchy model of pigeon flocks (Qiu and Duan (2017)) enables agents to form a self-organizing hierarchical leadership network required in formation. In order to expound the transformation process from a random connected topology structure to the leadership hierarchy demanded for a certain formation, graph theory is applied. The pigeon flock can be expressed as a undirected graph. If the communication range R_c and distance R_{ij} between pigeons i and j satisfy the condition: $R_{ij} \le R_c$, a communication connection $e_{ij} = (V_i, V_j)$ exists between individual i and j. Therefore, a leadership edge exists between the sequential pair (V_i, V_j), where V_j is the initial vertex representing the leader j and V_i is the terminal vertex denoting the follower i.

The final leadership network is shown in Fig.1 and the specific steps are as follows:

Step 1. Establish interaction relationship network. Each agent i attempts to identify the set N_c^i of agents within interaction range. The number of agents in N_c^i is n_c^i.

Step 2. Select the leader. The N_L-th vertex V_{N_L} is selected to be the general leader, where $N_L = [(l_1+1)/2 + 1/2]$, $Rank_{N_L} = 1$, l_1 is the length of the longest path p_1. The identifier $Flag_L$ of the first to the leader vertices in p_1 are set to be 1 and $Rank_k = |k - [(l_1+1)/2 + 1/2]| + 1$. Except agent N_L, if the identifier $Flag_L^k = 1$ of the k-th vertex, it will follow the $(k+1)$-th vertex. Otherwise, it will follow the $(k-1)$-th vertex.

Step 3. Form interim leadership network. If agent i is not in p_1 and agent $j \in N_c^i$ is in p_1, agent i will be stored in set p_2 and follow agent j. Before all agents have determined their leaders, repeat the preceding procedure. If agent j satisfying the condition is not unique, agent i will follow the greatest numbered one. The hierarchical rank and identifier will change accordingly.

Step 4. Obtain final leadership network. The identifier $Flag_c^i = 1$ of agent i denotes the current leader and hierarchical rank are temporary. If $i \in p_2$, $Flag_c^i = 1$. Otherwise, agent i will keep following the current leader. If agent i is not the only follower of the current leader, it will continue to change its leader.

3.3 ACPIO for control parameters

In order to obtain the optimal control parameters combination for the formation control of MASs, the fitness function is designed as follows (Xu et al. (2004);Vega et al. (2018)):

$$J = \int_0^{t_0} (U^T(t)R_1U(t) + \xi^T(t)R_2\xi(t)). \quad (10)$$

where $U(t) = [u_1^T(t), u_2^T(t), \dots, u_N^T(t)]^T$, $\xi(t)$ is the error vector. R_1 and R_2 denote the corresponding weight matrix.

For MASs, the optimal parameters problem can be formulated as finding the control parameters d_i and τ (5) that minimize the fitness function J:

$$\begin{aligned} &argmin \quad J, \\ &s.t.\|u_i(t)\| \le u_{max}, \\ &\quad \|v_i(t)\| \le v_{max}, \forall t \in [0, t_0), i = 1, 2, \dots, N. \end{aligned} \quad (11)$$

PIO is a novel swarm intelligence optimization method based on the special pigeon behavior (Duan and Qiao (2014)). The population in PIO is set to N_p. The pigeon i is associated with a position vector $X_i = [X_{i1}, X_{i2}, \dots, X_{in}]^T$ and velocity vector $Vel_i = [Vel_{i1}, Vel_{i2}, \dots, Vel_{in}]^T$. The control parameters to be optimized are constrained.

Unique to other optimization algorithms, two operators of PIO algorithm contribute to the extension of search span(Duan and Qiao (2014)). The map and compass operator imitates the sun and the earth's magnetic field during pigeons flying. The new position and velocity of pigeon i at the t-th iteration can be calculated with the follows:

$$Vel_i(t) = Vel_i(t-1)e^{-R\cdot t} + r_1(X_g - X_i(t-1)). \quad (12)$$

$$X_i(t) = X_i(t-1) + Vel_i(t-1). \quad (13)$$

where R is the map and compass factor, r_1 is a random number in the range of $[0, 1]$ and X_g is the current global best position which can be obtained by comparing all the position among all the pigeons.

When close to the destination, they will rely on the adjacent landmarks. In the landmark operator, half of pigeons are decreased by N_p in every generation. The position updating rule for pigeon i at t-th iteration can be given by:

$$N_p(t) = N_p(t-1)/2. \quad (14)$$

$$X_c(t-1) = \frac{\sum X_i(t-1)J(X_i(t-1))}{\sum N_p(t-1)J(X_i(t-1))}. \quad (15)$$

$$X_i(t) = X_i(t-1) + r_2(X_c(t-1) - X_i(t-1)). \quad (16)$$

where r_2 is a random number in the range of $[0, 1]$.

In order to improve the population diversity and promote the searching ability for global optima, the weight adaptive strategy and chaos theory are applied in PIO algorithm. The adaptive weight R_i can be calculated as follows:

$$R_i(t) = \frac{Nc - t}{Nc} \Delta X_i(t) \frac{R_{max} - R_{min}}{max\{\Delta X_i(t)\}} + R_{min}. \quad (17)$$

$$\Delta X_i(t) = \sqrt{\sum_{k=1}^{n} (X_{i,k} - X_{g,k})^2}. \quad (18)$$

where R_{max} and R_{min} are maximum and minimum value of R_i, Nc denotes the maximum iteration.

Chaos is a highly unstable motion of deterministic systems in finite phase space which usually exists in nonlinear systems. A new group of extremum with strong ergodicity and irregularity can be obtained by using chaotic mapping to help the algorithm to jump out of the local optimum. Consider the following logistic equation.

$$x_{i+1} = \mu x_i(1 - x_i), \quad i = 0, 1, 2 \dots. \quad (19)$$

where μ represents a control parameter, when $0 \le x_0 \le 1$, $\mu = 4$, which means logistic is in a completely chaotic state. To guarantee $x_i \in [0, 1]$, we have made corresponding changes as follows:

$$Cx_i = (x_i - x_{min})/(x_{max} - x_{min}). \quad (20)$$

$$Cx_i^{t+1} = 4Cx_i^t(1 - Cx_i^t). \quad (21)$$

$$\hat{x}_i = x_{min} + Cx_i(x_{max} - x_{min}). \quad (22)$$

where Cx_i^t and $\hat{x}_i$ are the chaos variable after t iterations and new value obtained by chaos optimization, respectively.

Remark that, in the end of both operators, a greedy strategy in (23) should be performed to improve the solutions' quality.

$$X_i(t) = \begin{cases} X_i(t), & J(X_i(t)) > J(X_i(t-1)) \\ X_i(t-1), & J(X_i(t)) \le J(X_i(t-1)) \end{cases} \quad (23)$$

The process of ACPIO algorithm for solving formation control parameters of MASs can be described as:

Step 1: Initialize the position and velocity of agents and identifier;

Step 2: Obtain the hierarchical rank and identifiers by leadership hierarchy model of pigeon flocks.

Step 3: Initialize the parameters of ACPIO algorithm, such as the population Nu, iteration threshold N_{c1} and N_{c2}, the stagnation threshold S_{max} etc. Randomly generate the pigeons' positions and velocities.

Step 4: Calculate and evaluate the fitness of each pigeon according to (10).

Step 5: Update the adaptive factor R by (17) and the position and velocity of pigeons by using map and compass operator or landmark operator.

Step 6: Update the iteration number of stagnation S. If optimum is not updated, $S = S + 1$. Otherwise, $S = 0$.

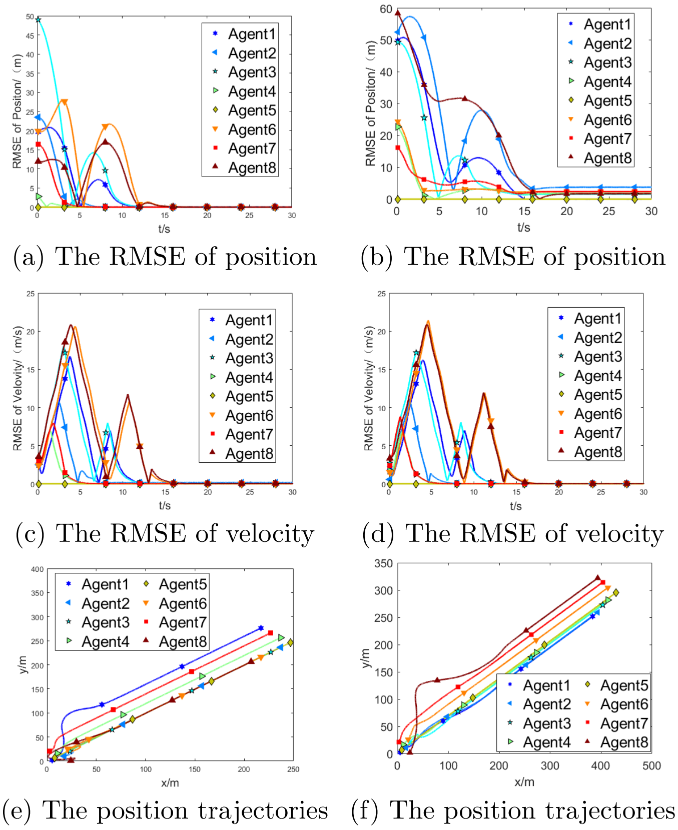
Step 7: Update the position of pigeons by using (20)- (22) when $S \ge S_{max}$.

Step 8: Output the optimal results when terminal condition is satisfied. Otherwise, go to *Step 5*.

Step 9: Apply the optimal parameters in pinning control method to establish and maintain a desired spatial configuration.

4. NUMERICAL SIMULATIONS

In this section, in order to verify the feasibility and effectiveness of our proposed method, series of numerical simulations have been conducted. The initial communication topology among the agents is shown in Fig.1(a). Let the nonlinear function be $f(x_i(t), v_i(t), t) = [cos(4t) + 0.01cos(2x_i(t)) + 0.01sin(4v_i(t))] * 1_2$ and $k^* = 2$. The hierarchical rank and control parameter are obtained by leadership hierarchy strategy and ACPIO algorithm, respectively. The comparative evolution curves shown in

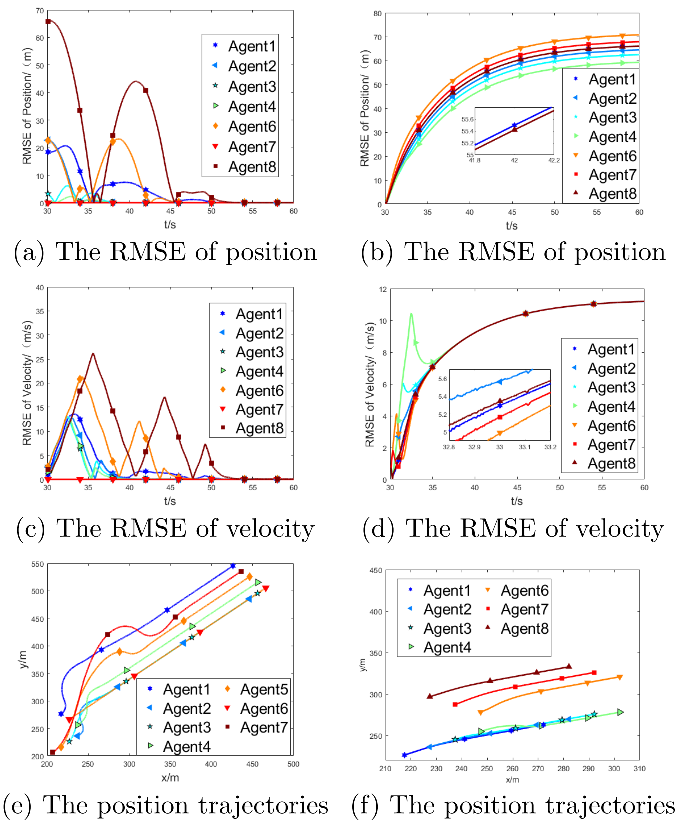


(a) The RMSE of position (b) The RMSE of position

(c) The RMSE of velocity (d) The RMSE of velocity

(e) The position trajectories (f) The position trajectories

Fig. 2. Comparative simulation results under ideal condition.



(a) The RMSE of position (b) The RMSE of position

(c) The RMSE of velocity (d) The RMSE of velocity

(e) The position trajectories (f) The position trajectories

Fig. 3. Comparative simulation results in case of agent 5 failure.

Fig. 4 illustrate that ACPIO can find a better solution than standard PIO algorithm due to its less cost.

Case 1 Ideal condition. A flock of 8 agents is disposed to move under an ideal condition. According to the leadership hierarchy network shown in Fig.1(b), the agents with high rank are chosen as the pinned ones such as agents 2, 3, 4 and 7. In addition, the optimal parameters $d_2 = 49.5, d_3 = 11.9, d_4 = 16.4, d_7 = 13.7$ and $\tau = 0.40$. Fig. 2(a) and Fig. 2(c) show the root mean square error (RMSE) of the position and velocity using the proposed method, respectively. The corresponding position trajectories and formation forming of agents are shown in Fig. 2(e). Comparative simulations are carried out without leadership hierarchy relationship using the same initial parameters. The corresponding results are shown as Figs 2(b), 2(d) and 2(f). Both methods can achieve the desired formation, but the performance of proposed approach is more superior because of its less RMSE of position and velocity.

Case 2 Agents failure. Based on case 1, the leadership hierarchy network under the circumstance that one or more agents fail at a time. Assuming that the failure time $T_f = 30s$ and the failure agent index $Num_f = 5$. Therefore, the hierarchical network changes voluntarily with current states. The index of pinned agents is 2, 3, 4 and 6. The optimal parameters are recalculated as $d_2 = 16.6, d_3 = 4.8, d_4 = 17.6, d_6 = 15.3$ and $\tau = 0.73$. Fig. 3(a) and Fig. 3(c) illustrate the RMSE of the position and velocity using the proposed method after agent failure, respectively. Fig. 3(e) describes the position transformation of agents during formation reconstruction. As shown in Fig. 3(e), after failure of agent 5, the general

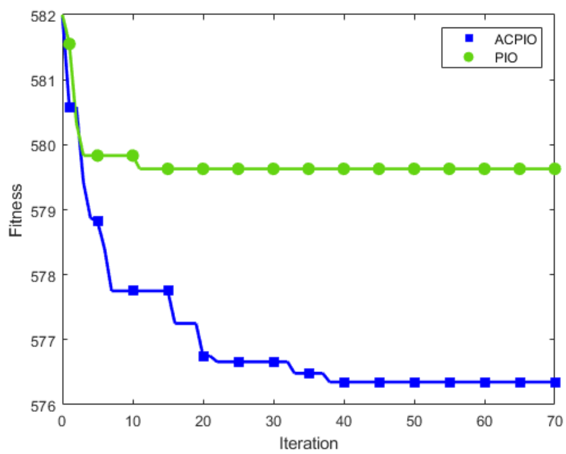


Fig. 4. The comparative evolution curves of PIO and ACPIO

leader changes from agent 5 to agent 7. Comparative simulations are also carried out using the same initial parameters. The corresponding results are shown as Figs 3(b), 3(d) and 3(f). Therefore, it is clearly that the proposed method is fault-tolerant.

5. CONCLUSION

This paper mainly provides a consensus-based formation control method for MASs. Using the constrained adaptive chaotic PIO algorithm and leadership hierarchy mechanism, the performance of the proposed pinning control approach is superior in terms of adaptability and flexibility. In addition, utilizing Lyapunov stability theory and matrix theory, the sufficient conditions are derived theoretically for achieving the desired formation pattern. Numerical simulation results verify the feasibility and effectiveness of the proposed method.

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