

# Closed-loop real-time supply chain management for perishable products <sup>★</sup>

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**Abstract:** Supply chain networks are dynamical systems with particular control challenges that stem from inventory deterioration and external disturbances (i.e., unanticipated consumer demand, time delays, etc.). For industries handling highly perishable inventory (e.g, fresh produce, vaccines, biologics) controlling product quality throughout the multiple echelons of the supply chain is critical to minimize inventory waste and satisfy consumer quality requirements. However, quality, as a function of time and environmental conditions (i.e., temperature, humidity, light, etc.), is difficult to model accurately resulting in unpredicted inventory spoilage. In this paper we demonstrate a novel closed-loop, feedback-based control framework that employs real-time product quality measurements for optimal supply chain management. A moving horizon approach is used to periodically update decisions (i.e., production, transportation, storage, and respective environmental conditions) based on feedback information. We demonstrate that the postulated feedback controller effectively stabilizes the supply chain dynamics, while minimizing costs. An illustrative case study is provided.

*Keywords:* Supply chain management, Perishable inventory, Quality control, Feedback control

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## 1. INTRODUCTION

Supply chains are complex, time-sensitive networks of facilities (i.e., suppliers, manufacturers, distribution centers, and retailers) experiencing constant exchange of different products and information. For enterprises to remain competitive, efficient supply chain management (SCM) can reduce production, inventory, and transportation costs while simultaneously satisfying, oftentimes uncertain, consumer demand (Shah, 2005; Papageorgiou, 2009). Control-theoretic approaches have been extensively applied to SCM problems (Perea et al., 2000; Sarimveis et al., 2008). Similar to the systems studied under control theory, supply chain models are often comprised of multiple types of inventory balances, demand fluctuations, lead-time delays, sales forecasting, etc. Therefore, for improved performance and stability, SCM calls upon frameworks conventionally thought to belong to the realm of process industries.

An additional intricacy is when inventory quality evolves (typically decays) during the product's "residence time" through the supply chain, requiring optimal control of environmental conditions to guarantee that the quality and efficacy of the products meet consumer standards (Blackburn and Scudder, 2009). Failing to consider such product quality dynamics within SCM can result in substantial inventory waste, for which a stark example is the food industry, where approximately 25% of the food produced for human consumption in the United States is wasted along the supply chain (Dou et al., 2016). To address

this problem, an increasing number of publications have proposed approaches to control perishable inventory systems (Abbou et al., 2017; Ignaciuk, 2013; Hamiche et al., 2019). None of these works, however, consider the supply chain structure comprehensively (i.e., different inventory dynamics across supply chain facilities), as well the impact of environmental conditions used to regulate product quality. From an operations research perspective, some works have addressed temperature-sensitive product perishability in multi-echelon supply chains (e.g., (Rong et al., 2011; Amorim et al., 2012)). Such frameworks integrate, typically empirical, data-driven product quality models within SCM schemes for optimal production, distribution, and control of environmental conditions throughout the supply chain (Abbott, 1999; Van Boekel, 1996). The *open-loop* nature of these formulations, however, fails to account for the genetic, environmental, and handling variability experienced by products in the daily operation of the supply chain, which is expected to significantly disturb SCM control policies (Goyal and Giri, 2001). In an effort to "close the loop", Lejarza and Baldea (2019) proposed a receding horizon optimization strategy to account for fluctuations in product degradation rate and demand.

Recent technological advances (e.g., smart packaging (Kuswandi et al., 2011) and hyperspectral imaging (Gowen et al., 2007)) have enabled reliable, real-time, and non-destructive product quality measurements throughout all stages of the supply chain. Motivated by these developments, in the work herein we propose a control theoretic framework to integrate the aforementioned technologies with existing supply chain optimization models for real-time management of perishable inventory systems.

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Our main contribution is to develop a *closed-loop*, feedback-based control strategy that uses current inventory quality and quantity measurements to reduce supply chain operating costs, while satisfying consumer demand. Based on an existing perishables production-distribution model (Rong et al., 2011), we provide a detailed stability analysis, and demonstrate how different feedback strategies impact production, distribution, and environmental conditions control policies. To illustrate our framework we perform numerical simulations on a representative supply chain network, and draw conclusions on the different supply chain management and quality control strategies.

## 2. PROBLEM STATEMENT

### 2.1 Product quality dynamics

We define product quality,  $q(t)$ , as a time dependent variable, and assume there exists at least one control variable  $k(t)$  which can be independently manipulated during inventory storage and shipment, which directly influences  $q(t)$ . The evolution of product quality is therefore assumed to be of the form:

$$\frac{dq(t)}{dt} = f(t, q(t), k), \quad q(t_0) = q_0 > 0 \quad (1)$$

We further assume that quality is non-negative and that degrades monotonically in time, i.e.,  $f(t, q, k) \leq 0 \forall t, k, q$ . We use a difference equation to discretize the initial value problem in (1), and obtain:

$$q(t + \Delta t) \approx q(t) + f(t, q(t), k)\Delta t \quad (2)$$

Some additional features of the quality control problem are:

- (1) there exist upper and lower bounds on  $k(t)$ ,  $k \leq k_{\max}$  and  $k \geq k_{\min}$ , respectively, representative of the physical capabilities of the inventory storage and distribution equipment available
- (2) there exists a quality threshold  $q_{\min} > 0$ , such that if  $q(t) \leq q_{\min}$  the product is considered spoiled and needs to be discarded at a cost
- (3) the product naturally degrades in time, such that

$$\lim_{t \rightarrow \infty} q(t) = q_{\infty} < q_{\min}$$

and the quality drops below the minimum threshold in a finite amount of time such that the product cannot be used to meet consumer criteria

Generally product quality models, as (1), are obtained empirically, as opposed to being derived from first principles, and are based on (perceived) metrics such as color, firmness, and water for fresh produce (Abbott, 1999; Van Boekel, 1996). Therefore, in practical SCM applications, these empirical models are unable to accurately predict product degradation owing to the significant variability that exists between different units/batches of the same product. Such inherent modeling errors call upon closed-loop control frameworks such that corrective decisions can be implemented when realizations of product quality deviate from the model predictions.

### 2.2 Supply chain dynamics

Several previous works developed dynamic models for inventory control in the supply chain (e.g., (Sarimveis et al.,

2008) and references therein). Since each type of facility employs different manipulated variables (e.g., production, incoming shipments, and sales) to regulate inventory levels, each will naturally have different dynamic models. In the most general form, the discrete time inventory dynamics are derived from conservation principles and can be written as:

$$I_{i,t+1} = I_{i,t} + \sum_{m \in \mathcal{M}(i)} u_{m,t-\omega_m}^{\text{in}} - \sum_{n \in \mathcal{N}(i)} u_{n,t}^{\text{out}} \quad (3)$$

where

- the state variable  $I_{i,t} \geq 0$  is the amount of inventory at time  $t$  in facility  $i$
- the manipulated variables  $u_{m,t}^{\text{in}} \geq 0 \forall m \in \mathcal{M}(i)$  represent inventory inflows (inbound shipments and production) at facility  $i$
- the manipulated variables  $u_{n,t}^{\text{out}} \geq 0 \forall n \in \mathcal{N}(i)$  represent inventory outflows (outbound shipments and sales) at facility  $i$
- the time delay  $\omega_m$  is the lead-time for inventory inflows (production or transportation lead-time)

From a control-theoretic standpoint, multiple approaches exist for dealing with systems as the one in (3) (Sarimveis et al., 2008). Of particular interest is model-based control which employs the dynamic model in (3) to predict future supply chain states and compute optimal production-distribution policies. Frameworks such as model predictive control (MPC) have been extensively studied for supply chain management problems (Perea-Lopez et al., 2003; Subramanian et al., 2013, 2014) owing to their capability to handle multiple state and control variables, time delays, disturbances, as well as state and input constraints. We will elaborate further on our proposed control methodology in a subsequent section.

### 2.3 Integrated supply chain and product quality dynamics

The general dynamic models for product quality (1) and inventory (3) must be integrated such that consumer standards are met. We follow the modeling approach introduced by Rong et al. (2011), which accounts for time-varying product quality as a function of environmental conditions throughout the supply chain. The diagram in Figure 1 demonstrates the flow of perishable inventory, as a function of  $k(t)$ . The line breaks in the quality function (red line in Figure 1) convey unmodeled, random product spoilage at the different stages of the supply chain.

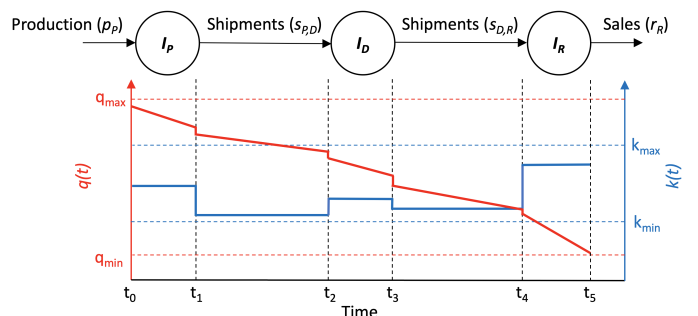


Fig. 1. Evolution of supply chain (inventories) and product (quality) dynamics over time for a producer  $P$ , distribution center  $D$ , and retailer  $R$ .

To integrate the quality model with the inventory dynamics in (3), we consider a finite, discrete number of quality levels which are merged with inventory variables  $I_{i,t}$ . Following the approach in (Rong et al., 2011), inventory variables  $I_{i,q,k,t}$ , for facility  $i$  at time  $t$ , are characterized in terms of quality  $q$  and environmental conditions  $k$ . Similarly, the aforementioned manipulated variables become  $u_{m,q,k,t-\omega_m}^{\text{in}}$  and  $u_{n,q,k,t}^{\text{out}}$ . The integrated supply chain dynamic model for facility  $i$  reflects a degradation rate of  $\Delta q_{i,k}$  quality levels per time period in storage, and  $\Delta q_{m,k}$  quality units during transportation lead time  $\omega_m$ . The inventory dynamics are given by:

$$I_{i,q,k,t+1} = I_{i,q+\Delta q_{i,k},k,t} + \sum_{m \in \mathcal{M}(i)} u_{m,q+\Delta q_{m,k},k,t-\omega_m}^{\text{in}} - \sum_{n \in \mathcal{N}(i)} u_{n,q,k,t}^{\text{out}} \quad (4)$$

for all  $q \in \{Q | q_{i,\min} \leq q \leq q_{i,\max}\}$ , where  $Q$  is the set of all quality levels, and  $q_{i,\min}$ ,  $q_{i,\max}$  are the minimum and maximum product qualities allowed in facility  $i$ , respectively.

We note that the aforementioned supply chain variables are typically subject to constraints regarding production, shipment, and storage capacities, as well demand satisfaction among others. Further details on the mathematical expressions of these constraints can be found in (Rong et al., 2011) and (Perea-Lopez et al., 2003).

### 3. FEEDBACK-BASED CONTROL

The control scheme proposed herein stems from the economic model predictive control (EMPC) paradigm (Rawlings and Mayne, 2009; Ellis et al., 2014), which was recently demonstrated for supply chain systems with demand disturbances (Subramanian et al., 2014). These schemes involve solving a finite horizon optimization problem in a rolling horizon fashion, by applying the first element in the control sequence to the system, and obtaining the next state measurement which becomes the initial state when the problem is solved again. To conduct a comparative analysis of our proposed methodology we consider the following three main feedback cases:

- *Case 1:* feedback information includes inventory levels, and environmental conditions are fixed over time to some conservative value (assumed to result in minimal product deterioration)
- *Case 2:* feedback information includes inventory levels, and environmental conditions are regulated over time to attain further cost minimization
- *Case 3:* feedback information includes *both* inventory levels *and* quality measurements, and environmental conditions are regulated over time to minimize costs counteracting any measured product quality disturbances

For notation compactness, in this section we denote the state variables  $I_{i,q,k,t}$  with the vector  $x \in \mathbb{X}$ , and manipulated variables  $u_{m,q,k,t-\omega_m}^{\text{in}}$ ,  $u_{n,q,k,t}^{\text{out}}$  with the vector  $u \in \mathbb{U}$ . The sets  $\mathbb{X}$  and  $\mathbb{U}$  capture the constraints on state and input variables, respectively, and are typically of the form  $\mathbb{X} = \{x \in \mathbb{R}^{N_x} \mid A_x x \leq b_x\}$  and  $\mathbb{U} = \{u \in \mathbb{R}^{N_u} \mid A_u u \leq$

$b_u\}$ , bounding state and input variables from above and below.

*Assumption 1.* The constraint set  $\mathbb{X}$  is convex and closed. The constraint set  $\mathbb{U}$  is convex and compact.

We consider an objective function reflecting production, transportation, holding, and disposal costs which are associated with the states and control inputs, and is denoted as the *stage cost* function  $\ell(x, u) : \mathbb{X} \times \mathbb{U} \rightarrow \mathbb{R}$ , which is a linear function of the form

$$\ell(x, u) = c_x^T x + c_u^T u$$

where  $c_x$  and  $c_u$  are vectors of cost factors for state and input variables, respectively.

*Definition 1.* (Optimal steady-state). If the integrated inventory and quality dynamic model for the entire supply chain is given in compact form by  $x^+ = h(x, u)$ , where  $h(x, u) : \mathbb{X} \times \mathbb{U} \rightarrow \mathbb{X}$ , the optimal-steady state  $(x_s, u_s)$  solves

$$\underset{x, u}{\operatorname{argmin}} \ell(x, u) \quad \text{s.t.} \quad x = h(x, u), \quad x \in \mathbb{X}, \quad u \in \mathbb{U} \quad (5)$$

and is assumed to be unique.

The MPC objective function  $V_N(x, \mathbf{u}) : \mathbb{X} \times \mathbb{U}^N \rightarrow \mathbb{R}$  minimizes the operating costs of the supply chain for a given prediction horizon  $N$ , and depends on the current system state and predictions on the future evolution of both states and inputs. The control input sequence over time,  $\mathbf{u}$ , is given by  $\mathbf{u} = \{u_0, u_1, \dots, u_{N-1}\}$ . Only the initial state  $x$  is considered since future (predicted) states are implicit functions of  $\mathbf{u}$ . The control objective can be therefore written as:

$$V_N(x, \mathbf{u}) = \sum_{t=0}^{N-1} \ell(x_t, u_t)$$

For the work herein we consider a *centralized* controller, implying that there is a global supply chain coordinator that has complete information of the dynamics, as well as current measurements of state variables at all facilities (i.e., the states are globally observable). In this way, optimal control inputs are computed for each node such that the total supply chain operating costs are minimized. Therefore, a single optimization problem is solved online and is given by:

$$\begin{aligned} \min_{\mathbf{u}} \quad & V_N(\mathbf{u}, x) \\ \text{s.t.} \quad & x_{t+1} = h(x_t, u_t) \quad \forall t \in \mathbb{I}_{0:N-1} \\ & x_t \in \mathbb{X}, \quad \forall t \in \mathbb{I}_{0:N-1} \\ & u_t \in \mathbb{U}, \quad \forall t \in \mathbb{I}_{0:N-1} \\ & x_0 = x \\ & x_N = x_s \end{aligned} \quad (6)$$

where  $\mathbb{I}_{0:N-1}$  denotes the set of integers  $\{0, \dots, N-1\}$ . The resulting implicit feedback control law

$$u = \kappa_N(x) = \mathbf{u}_0(x) \quad (7)$$

is the first element in the optimal solution of the optimization problem in (6). The admissible region,  $\mathcal{X}_N$ , is

$$\mathcal{X}_N = \{x \in \mathbb{X} \mid \exists \mathbf{u} \in \mathbb{U}^N, \text{ such that (6) is feasible}\} \quad (8)$$

*Proposition 1.* The product quality dynamics are inherently stable, and regulating environmental conditions  $k(t)$  does not affect the stability of the integrated supply chain dynamic system.

**Proof.** By definition, the quality variables are bounded from above and below so that  $q_\infty \leq q(t+1) \leq q(t) \leq q_0 \forall t \in \mathbb{I}_{0:N-1}$ . This is because  $dq(t)/dt = f(q(t), k(t)) < 0 \forall k(t) \in S \forall t \in \mathbb{I}_{0:N-1}$ , which implies that eventually the product spoils such that  $\lim_{t \rightarrow \infty} q(t) = q_\infty \leq q_{i,\min}$  and inventory losses occur. Clearly,  $q_\infty$  is an asymptotically stable equilibrium for the quality dynamics, irrespective of the manipulated environmental conditions  $k(t)$ . While an inappropriate choice of input variable  $k(t)$  may result in inventory waste, ultimately it will never make the integrated supply chain dynamics unstable (stability here is implied in the bounded-input, bounded-state sense, meaning that inventory will be non-negative and will not exceed storage capacity at all time periods).

*Proposition 2.* Demonstrating the stability of the inventory balances as in (3) (i.e., ignoring product perishability) is sufficient to guarantee the stability of the integrated supply chain model under the proposed feedback-based algorithm.

**Proof.** If the dynamical system in (3) is closed-loop stable under the feedback control law  $\kappa_N(x)$ , the stability of the integrated supply chain dynamical system in (4) is implied by Proposition 1.

In order to demonstrate the closed-loop stability of (3) we adopt the same approach as Subramanian et al. (2014).

*Remark 1.* Without considering inventory quality dynamics (i.e., assuming that product quality is constant over time) the supply chain inventory dynamics are linear and are amenable to a state-space representation of the form  $x^+ = Ax + Bu$  (e.g., (Wang and Rivera, 2008; Subramanian et al., 2014)).

*Assumption 2.* There exists a multiplier  $\lambda_s$  such that  $(x_s, u_s)$  is the unique solution to

$$\operatorname{argmin}_{x,u} \ell(x,u) + \lambda_s^T [x - (Ax + Bu)] \quad \text{s.t. } x \in \mathbb{X}, u \in \mathbb{U} \quad (9)$$

*Assumption 3.* The system  $x^+ = Ax + Bu$  is strictly dissipative with respect to the supply rate  $s(x,u) = \ell(x,u) - \ell(x_s, u_s)$  and storage function  $\lambda(x) = \lambda_s^T x$ . That is, there exists a positive definite function  $\rho(\cdot)$  such that

$$\lambda_s^T (Ax + Bu - x_s) \leq \rho(x - x_s) + s(x,u) \quad \forall (x,u) \in \mathbb{X} \times \mathbb{U}$$

The following theorem is from (Subramanian et al., 2014, Theorem 4).

*Theorem 1.* (Lyapunov function with terminal constraint). Let the the system  $(A, B)$  be stabilizable and Assumptions 1, 2, and 3 hold. Then the steady-state solution of the closed-loop system  $x^+ = Ax + B\kappa_N(x)$  is asymptotically stable with  $\mathcal{X}_N$  as the region of attraction. The Lyapunov function is

$$\tilde{V}(x) = V_N^0(x) + \lambda_s^T [x - x_s] - N\ell(x_s, u_s) \quad (10)$$

where  $V_N^0(x)$  is the optimal cost of (6).

**Proof.** The proof is the same as the one presented in (Rawlings et al., 2012, Theorem 2).

*Remark 2.* Since different feedback strategies and approaches for controlling environmental conditions stem from Cases 1, 2, and 3, each case will have a different control law  $\kappa_N(x)$ . Therefore, as long as the optimization problem defined in (6) is feasible in each case, the previous stability analysis holds.

## 4. NUMERICAL EXAMPLES

To illustrate the advantages of the proposed supply chain and product quality control scheme, we consider an illustrative case study consisting of a network of two producers ( $P_1$  and  $P_2$ ) that supply inventory directly to retailers ( $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$ ), as shown in Figure 2.

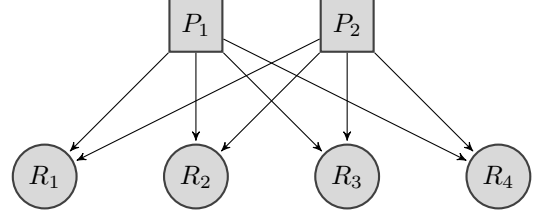


Fig. 2. Supply chain network topology considered for the subsequent numerical experiments.

The time scale discretization interval is in days, and decisions are made and revised on a daily basis. For the controller, the prediction horizon of choice is  $N = 10$  days. The transportation lead times ( $\omega_{i,j}$  in days) and costs ( $f_{i,j}$  in monetary units per unit shipped) for shipments between producer  $i$  and retailer  $j$  are:

$$\omega_{i,j} = \begin{bmatrix} 2 & 3 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \quad f_{i,j} = \begin{bmatrix} 0.44 & 0.66 & 0.44 & 0.66 \\ 7.5 & 7.5 & 7.5 & 7.5 \end{bmatrix}$$

Inventory storage costs are 2 monetary units per day per inventory unit, and are assumed to be the same at all facilities. Transportation and storage costs are dependent on temperature, and obtained by multiplying them by a factor  $n_k$  which accounts for the coefficient of performance for cooling at different temperatures. As discussed earlier, we consider a finite number of possible storage and shipment temperatures, which are in this case  $k = \{2, 4, 6, 8, 10\}$  °C, and as in (Rong et al., 2011) we consider  $n_k = [1.00 \ 0.88 \ 0.77 \ 0.65 \ 0.54]$ , reflecting the fact that lower temperatures are costly. Further, production costs are lower at  $P_1$  and  $P_2$ , and are 0.165 and 1.65 monetary units per unit manufactured, respectively.

As for the product quality dynamics, we consider an exponential Arrhenius temperature-dependent, zeroth-order model of the form:

$$\frac{dq}{dt} = -k_0 \exp(-E_a/RT), \quad q(t=0) = q_0 \quad (11)$$

where  $k_0$  is the pre-exponential factor,  $E_a$  is the activation energy,  $R$  is the universal gas constant, and  $T$  is temperature in Kelvin. Such models have been frequently used to capture the degradation of food products over time (Blackburn and Scudder, 2009). Following the modeling approach in (Rong et al., 2011), we consider a discretization of 750 quality levels which provides enough resolution to accurately represent the quality dynamics as a function of temperature. Using this scale, the maximum initial quality would be a level of 750. For example, a consumer quality requirement of 80%, would correspond to a quality level of 600 (Rong et al., 2011). We note that this discretization is subject to change depending on the product under consideration. The resulting daily product degradation for the different predefined temperatures was obtained directly from (Rong et al., 2011) and is  $\Delta q_k = [11 \ 13 \ 16 \ 20 \ 27]$ .

In this case study, demand (number of inventory items requested at time period  $t$ ) is assumed to be deterministic and to have a constant value of 650 units per day at each retailer facility. While this framework is naturally suited to also account for demand forecasting errors, as has been previously demonstrated (Perea-Lopez et al., 2003; Subramanian et al., 2013, 2014; Lejarza and Baldea, 2019), the novelty of our proposed work is to address measurable “disturbances” in product quality. To simulate such effects, quality disturbances, denoted  $\delta q$ , are random such that  $\delta q \in [-10, 10]$ , and are injected to the model at each sampling time after solving (6). For the two-echelon supply chain example herein, quality disturbances occur during shipment from producers to retailers such that  $s_{i,j,q,k,t} \rightarrow s_{i,j,q+\delta q,k,t}$  for  $q_\infty \leq q(t) + \delta q(t) \leq q(t+1)$ .

#### 4.1 Results

Numerical simulations were conducted over a time period of 30 days, decisions and feedback being implemented daily. Figure 3 shows the average disturbance on inventory quality during shipments, over the entire time horizon. The resulting inventory levels and storage temperatures are displayed in Figure 4.

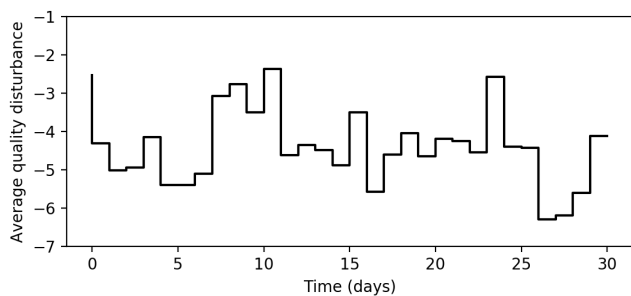


Fig. 3. Average quality disturbance during shipments.

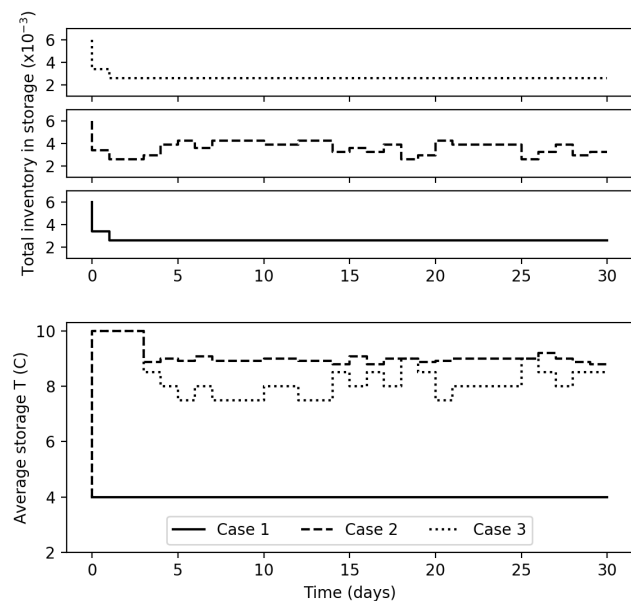


Fig. 4. Total inventory (top), and average storage temperature in °C (bottom) over time.

From the top plot in Figure 4, we note that each case is stabilized by their respective feedback control law. All cases are started at a non steady-state amount of

inventory, but we observe that the steady-state is rapidly attained. The steady-state total inventory level is 2600 for all cases, which corresponds to the total daily demand ( $4 \times 650$  units). For Case 2, we note that periodic inventory disturbances occur resulting from product waste, due to the unmodeled quality fluctuations, which cannot be prevented without decreasing storage temperatures.

From the bottom plot in Figure 4, per definition of Case 1, storage temperature is fixed at 4 °C and does not vary neither in time nor per facility. To potentially minimize operating costs, Cases 2 and 3 allow for flexible and higher storage temperatures, relative to Case 1. In Figure 4 we note that Case 3 deviates from the storage temperatures obtained for Case 2 as a consequence of the unmodeled product quality disturbances that accounted for via the proposed feedback strategy. The lower temperatures for Case 3 decrease the rate of inventory degradation, counteracting the random product spoilage (i.e.,  $\delta q < 0$ ) that occurred during transportation. These observations confirm that different types of feedback (i.e., the different cases considered) yield vastly different optimal control policies on the environmental conditions during storage. In the (likely) presence of product quality forecasting errors, it is important to consider the appropriate feedback mechanism, namely Case 3, not only for improved consumer satisfaction, but also for optimal supply chain performance.

Next, in Table 1 we consider how the different feedback control laws affect inventory production policies for each of the cases. Since producing inventory at  $P_1$  is inexpensive, relative to  $P_2$ , in all cases producer  $P_1$  operates at maximum capacity to meet incoming orders. In Case 1, the lower storage temperatures obtained by fixing  $k$  to a conservatively low, but expensive value, result in zero inventory waste despite the random spoilage shown in Figure 3. Further, implementing product quality feedback in Case 3, allows for storage temperature adjustments also preventing inventory waste. Therefore, for Cases 1 and 3, shipments from  $P_1$  to retailers are, in general, of sufficient quality and quantity to meet demand such that few orders are placed to  $P_2$ . In Case 2, lacking inventory quality feedback, a substantial amount of the inventory shipped from  $P_1$  to retailers is spoiled and must be disposed at a cost. In order to fulfill demand, an increased amount of (more expensive) orders must be placed to  $P_2$ , as displayed in Table 1.

Table 1. Daily average production and inventory waste (in number of product units) with respective standard deviation

	Production at $P_1$	Production at $P_2$	Waste
Case 1	$2300 \pm 0$	$381 \pm 319$	$0 \pm 0$
Case 2	$2300 \pm 0$	$1398 \pm 513$	$576 \pm 130$
Case 3	$2300 \pm 0$	$380 \pm 317$	$0 \pm 0$

Last, Table 2 shows the average daily operating cost and standard deviation over the 30 day time horizon. Table 2, as well as the previously discussed results, emphasize that the choice of control policy has a significant impact on supply chain operating costs. While Case 1 results in inefficient temperature control (i.e., higher temperatures are admissible to reduce costs), relaxing the fixed temperature constraint can result in substantially worse economic performance when product quality feedback is not imple-

mented (i.e., Case 2). The proposed feedback strategy in Case 3 attains the lowest operating costs, indicating that measuring product quality throughout the supply chain is critical to improve operations by reducing inventory waste, and energy consumption.

Table 2. Daily average operating costs (in monetary units) and respective standard deviations

	Average	Standard deviation
Case 1	46,703	21,561
Case 2	89,884	26,732
Case 3	35,681	17,589

## 5. CONCLUSION

In this paper, we introduced a novel *closed-loop* feedback-based SCM optimization framework for perishable inventory. On one hand, we demonstrated that while conservatively fixing environmental conditions in Case 1 prevents inventory spoilage, it can substantially increase the operating costs. On the other hand, optimal control of environmental conditions can result in significant amounts of wasted inventory when: (i) product degradation is not modeled accurately or (ii) no inventory quality feedback is implemented. We showed that implementing recent technological developments in product quality monitoring via feedback results in improved decision-making, reducing energy consumption, inventory waste, and thereby minimizing the total supply chain operating costs.

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