Model Following Quasi-Sliding Mode Control Strategy

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Abstract: Our study introduces a new model reference based approach to the design of sliding mode controller for discrete-time dynamical systems subject to external disturbances. We propose to begin the control design with generation of the reference trajectory for the system using its mathematical model and a hyperbolic tangent based sliding mode reaching law. Next, for the real disturbed plant, we propose a reaching function, which follows the reference trajectory in each step. Further, we prove that this approach ensures existence of quasi-sliding motion according to the definition of Gao *et al*. Moreover, the proposed controller offers a significant reduction of the width of the achieved quasi-sliding mode band in comparison to other sliding mode methodologies, which results in an improvement of the system's robustness. The properties of our control scheme are finally illustrated with a simulation example.

Keywords: bounded disturbances, discrete-time dynamical systems, model reference control, reaching law based control, sliding-mode control.

1. INTRODUCTION

Model reference control has been present in the control design literature for decades. It is known as one of the most intuitive control methods commonly used in adaptive controllers. Over the years, the advantages of model following control have been thoroughly studied by Landau (1979), Butler (1992), Nguyen (2018) and many others. A few authors also proposed using reference models in continuous-time sliding mode controllers, e.g. Zinober *et al*. (1982), Bartolini *et al*. (1988), Cunha *et al*. (2003), Muniandi *et al*. (2019). However, despite the recent rapid development of discrete-time systems theory, the application of reference models in discrete-time sliding mode controllers has not been investigated widely.

The discrete-time sliding mode control was for the first time considered in 1985 by Milosavljević, who developed the necessary condition for the occurrence of the quasi-sliding motion. The sufficient conditions were later presented by Sarpturk *et al*. (1987) and Kotta *et al*. (1989). The stability and control design for the discrete-time sliding mode was also studied by Furuta (1990), who proposed a Lyapunov function approach. However, not until the seminal work of Gao *et al*. (1995) had the attributes of the quasi-sliding mode been clearly described. According to the paper of Gao, for the quasi-sliding mode to exist the following three conditions must be satisfied:

- The system's representative point, starting from any initial position, moves monotonically towards the sliding surface in the so-called reaching phase and crosses it in finite time.
- After the first crossing of the sliding plane, the representative point moves along it with a zig-zagging motion, recrossing it in each successive time step in the so-called sliding phase.
- Once the representative point of the system enters an *a priori* known band around the sliding plane it will never leave it again.

Gao *et al*. (1995) also proposed an innovative reaching law based control design. Afterwards, numerous researchers have followed their design path and several new reaching laws have been designed, e.g. Golo *et al*. (2000), Veselić *et al*. (2010), Qu *et al*. (2014), Leśniewski *et al*. (2015), Chakrabarty *et al*. (2016), Ma *et al*. (2019a, b).

In this work we adopt the quasi-sliding mode definition of Gao *et al*. (1995) and introduce a model reference based approach to the control design. Our idea is to use the system's mathematical model to obtain the desired profile of the sliding variable. Therefore, we apply the hyperbolic tangent reaching law of Leśniewski *et al*. (2015). In the next step, we develop a new model following reaching law to control the real disturbed system. Finally, we prove that this method not only ensures all three attributes of the quasi-sliding mode but also guarantees a significant improvement of the system's robustness. A similar approach has been recently presented by Bartoszewicz *et al*. (2019). However, that work utilizes a different method of the desired trajectory generation. Therefore, the resulting properties of the closed-loop system are different than presented further in this study.

2. REFERENCE TRAJECTORY BASED SMC STRATEGY

2.1 Problem statement

Our aim is to develop a quasi-sliding mode control strategy for a disturbed discrete-time plant, represented with:

$$\boldsymbol{x}(k+1)=\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{b}u(k)+\boldsymbol{b}d(k), \qquad (1)$$

where $\boldsymbol{x}(k)$ is an $n \times 1$ state vector, $\boldsymbol{A}$ is the plant's state matrix, vector $\boldsymbol{b}$ represents the input distribution, $u(k)$ denotes the control signal and $d(k)$ denotes the disturbance. For any $k \geq 0$ the disturbance satisfies:

$$\left|d(k)\right| \leq d_{\max}. \qquad (2)$$

The system's initial condition is represented by $\boldsymbol{x_0} = \boldsymbol{x}(0)$ and the demand state is $\boldsymbol{x_d}$. For the design of a sliding mode controller we first choose the sliding variable as $s(k) = \boldsymbol{ce}(k)$, where $\boldsymbol{c}$ is an $1 \times n$ vector, which satisfies $(\boldsymbol{cb})^{-1} \neq 0$, and $\boldsymbol{e}(k)$ is the state error vector defined as: $\boldsymbol{e}(k) = \boldsymbol{x_d} - \boldsymbol{x}(k)$. Next, we select the sliding surface:

$$s(k)=0. \qquad (3)$$

Having chosen the sliding hyperplane, we denote the disturbance impact on the sliding variable with $D(k) = \boldsymbol{cb}d(k)$. Considering (2), for any $k \geq 0$, $D(k)$ satisfies:

$$\left|D(k)\right| \leq D_{\max} = \left|\boldsymbol{cb}d_{\max}\right|. \qquad (4)$$

As the disturbance is bounded, the control design is feasible. In the next sections we will present a new model following sliding mode controller for the system (1) and compare it with an existing control strategy using a hyperbolic tangent reaching law presented by Leśniewski *et al*. (2015).

2.2 Hyperbolic tangent reaching law

Control for system (1) may be designed according to one of many discrete-time sliding mode reaching laws developed over the years. Most of them are based on the original switching type reaching law of Gao *et al*. (1995), in the form:

$$s(k+1)=(1-q)s(k)-\varepsilon\operatorname{sgn}\left[s(k)\right]-D(k), \qquad (5)$$

where q, $\varepsilon > 0$ and $q < 1$ and the signum function is understood as:

$$\operatorname{sgn}(z)=\begin{cases} 1 \text{ for } z \geq 0 \\ -1 \text{ for } z < 0 \end{cases}. \qquad (6)$$

To fulfil the switching condition the control parameters must satisfy:

$$\varepsilon > \frac{(2-q)D_{\max}}{q}. \qquad (7)$$

This original reaching law consists of two parts: the proportional term responsible for the convergence of the system's representative point to the sliding hyperplane and the signum term, which ensures recrossing the switching plane in the sliding phase. The strategy results in the zig-zagging motion of the representative point inside the quasi-sliding mode band, whose width is $2(\varepsilon + D_{\max})$.

Although the reaching law (5) was fundamental for the later development of quasi-sliding mode control, it causes some implementation difficulties. Namely, the further from the sliding plane the representative point of the system is, the greater the values of the proportional term are. This may result in unacceptably large magnitude of the control input. This observation led Leśniewski *et al*. (2015) to the development of a new hyperbolic tangent based reaching law. Their idea was to modify the proportional term in order to restrict the pace of convergence and therefore the control signal. They proposed the following reaching function:

$$s(k+1)=s(k)-r\tanh\left[\frac{s(k)}{r}\right]-\varepsilon\operatorname{sgn}\left[s(k)\right]-D(k), \qquad (8)$$

where $\varepsilon > D_{\max}$ and $r > 0$. The reaching law may be expressed as:

$$\begin{aligned}&\operatorname{sgn}\left[s(k+1)\right]\left|s(k+1)\right| = \\ &\operatorname{sgn}\left[s(k)\right]\left\{\left|s(k)\right|-r\tanh\left[\frac{\left|s(k)\right|}{r}\right]-\varepsilon-\operatorname{sgn}\left[s(k)\right]D(k)\right\}.\end{aligned} \qquad (9)$$

One may notice that in the reaching phase, when $\operatorname{sgn}[s(k + 1)] = \operatorname{sgn}[s(k)]$, (8) becomes:

$$\left|s(k)-s(k+1)\right| = r\tanh\left[\frac{\left|s(k)\right|}{r}\right]+\varepsilon+\operatorname{sgn}\left[s(k)\right]D(k). \qquad (10)$$

Considering that ε is greater than $D_{\max}$ and (4) holds, one may easily notice that the rate of change of $s(k)$ is restricted as follows:

$$0 < \left|s(k)-s(k+1)\right| \leq r+\varepsilon+D_{\max}. \qquad (11)$$

It may be concluded from (11) that the absolute value of the sliding variable decreases in each control step. Therefore the representative point of the system reaches the sliding surface in finite time. Moreover, as opposed to the Gao's strategy, the reaching law (8) ensures limitation of the maximum rate of change of $s(k)$. Consequently, with the application of the reaching law (8), the maximum control signal becomes limited as well.

On the other hand, when the sliding plane has been crossed for the first time and $\operatorname{sgn}[s(k+1)] = -\operatorname{sgn}[s(k)]$, then the absolute value of $s(k+1)$ becomes:

$$\left|s(k+1)\right| = r\tanh\left[\frac{\left|s(k)\right|}{r}\right] - \left|s(k)\right| + \varepsilon + \operatorname{sgn}\left[s(k)\right]D(k). \quad (12)$$

As the hyperbolic tangent function satisfies:

$$r\tanh\left[\frac{\left|s(k)\right|}{r}\right] - \left|s(k)\right| \le 0, \quad (13)$$

then from (12) we conclude that the ultimate band width is expressed as $2(\varepsilon + D_{\max})$. Finally, to ensure that after the first crossing of the sliding plane occurred, it will be crossed again in each successive time step the control parameters must satisfy:

$$r\tanh\left(\frac{\varepsilon + D_{\max}}{r}\right) > 2D_{\max}. \quad (14)$$

As the requirements (7) and (14) are different, the control strategy of Leśniewski and Bartoszewicz with an appropriate choice of r and ε, enables to restrict the maximum value of the control input and at the same time reduces the width of the quasi-sliding mode band.

2.3 Reference model

In this section we present a new control strategy utilizing the hyperbolic tangent function as well. Our idea is to use the hyperbolic tangent reaching law to obtain the desired evolution of the sliding variable and then use the generated profile to control the real disturbed system (1).

We propose to use the mathematical model of the plant to generate the ideal evolution of the sliding variable. The model does not depend on disturbances and its trajectories may be generated in advance and saved in a look-up table. We propose to control the model with the nonperturbed version of the strategy of Leśniewski *et al.* (2015), which allows to control the pace of convergence of the system and guarantees a relatively small width of the ultimate band. We denote the model's sliding variable with $s_m(k)$. Moreover, the model's initial conditions satisfy:

$$s_m(0) = s(0) = \boldsymbol{c}\left[\boldsymbol{x_d} - \boldsymbol{x_0}\right]. \quad (15)$$

We propose to control the model according to the switching type reaching law presented by Leśniewski *et al.* (2015):

$$s_m(k+1) = s_m(k) - r\tanh\left[\frac{s_m(k)}{r}\right] - \varepsilon\operatorname{sgn}\left[s_m(k)\right], \quad (16)$$

where r and ε are greater than zero.

The hyperbolic tangent function is lower and upper bounded by ± 1. Therefore, its application in the reaching law (16) lets us specify the maximum and minimum rate of change of the sliding variable. When $\operatorname{sgn}[s_m(k+1)] = \operatorname{sgn}[s_m(k)]$, the reaching law becomes:

$$\left|s_m(k) - s_m(k+1)\right| = r\tanh\left[\frac{\left|s_m(k)\right|}{r}\right] + \varepsilon. \quad (17)$$

Consequently, the rate of change of the model's sliding variable satisfies:

$$\varepsilon \le \left|s_m(k) - s_m(k+1)\right| \le \varepsilon + r. \quad (18)$$

As the minimum change of the sliding variable in one control step is ε, we notice that not later than at $i \le |s_m(0)| / \varepsilon + 1$ the representative point crosses the sliding plane for the first time and the sign of the model's sliding variable $s_m(i)$ becomes opposite than the sign of $s(0)$. This proves that the sliding plane is crossed in finite time.

If, for some k, $\operatorname{sgn}[s_m(k+1] = -\operatorname{sgn}[s_m(k)]$, then the reaching law (16) becomes:

$$\begin{aligned}&\operatorname{sgn}\left[s_m(k+1)\right]\left|s_m(k+1)\right| = \\ &-\operatorname{sgn}\left[s_m(k+1)\right]\left\{\left|s_m(k)\right| - r\tanh\left[\frac{\left|s_m(k)\right|}{r}\right] - \varepsilon\right\}.\end{aligned} \quad (19)$$

After some transformations from (19) we get:

$$\left|s_m(k+1)\right| = r\tanh\left[\frac{\left|s_m(k)\right|}{r}\right] - \left|s_m(k)\right| + \varepsilon. \quad (20)$$

One may notice that for any $|s_m(k)|$:

$$r\tanh\left[\frac{\left|s_m(k)\right|}{r}\right] - \left|s_m(k)\right| \le 0 \quad (21)$$

holds. Therefore, from (20) and (21) we notice that when the sign of the model's sliding variable changes, its absolute value satisfies:

$$\left|s_m(k+1)\right| \le \varepsilon. \quad (22)$$

Next, we show that if the change of the sign occurred at step $k+1$ it occurs at step $k+2$ as well. For $s_m(k+2)$ we may write:

$$\begin{aligned}&\operatorname{sgn}\left[s_m(k+2)\right]\left|s_m(k+2)\right| = \\ &\operatorname{sgn}\left[s_m(k+1)\right]\left\{\left|s_m(k+1)\right| - r\tanh\left[\frac{\left|s_m(k+1)\right|}{r}\right] - \varepsilon\right\}.\end{aligned} \quad (23)$$

Considering (22), we may notice that, on the right hand-side of (23), the term in the curly brackets is always smaller than zero. Therefore, the sign of the model's sliding variable switches again. We conclude that for any $k \geq i$ the sign of $s_m(k)$ changes at each subsequent step and its absolute value satisfies (22). This proves that the evolution of the model's sliding variable fulfils all the requirements for the existence of quasi-sliding mode, as stated by Gao *et al*. Next, we will use the model's sliding variable as a reference for the real system subject to external disturbances.

2.4 Model following reaching law

In this part we present a model following reaching law for the system (1). We propose the following reaching function:

$$s(k+1) = s_m(k+1) - D(k). \tag{24}$$

The idea of this new reaching function is to drive the system's states, at each control step, to their desired values, determined by the model. As the model's sliding variable $s_m(k)$ does not bear any disturbance influence, the plant's sliding variable at step k is only influenced by $D(k-1)$. Meanwhile, in the previous strategies $s(k)$ bore the impact of all the disturbance values from the beginning of the control process up to $D(k-1)$. Therefore, the model following control improves the robustness of the system.

From (1), (3) and the reaching law (24) results the following control signal:

$$u(k) = (\boldsymbol{cb})^{-1}\left[\boldsymbol{cx_d} - \boldsymbol{cAx}(k) - s_m(k+1)\right]. \tag{25}$$

As the disturbance $D(k)$ may push the representative point of the system away from the sliding hyperplane, the control parameters r and ε of the model must be selected in a specific way in order to provide the existence of the quasi-sliding motion.

For the quasi-sliding mode to emerge the representative point of the system must cross the switching plane in finite time and remain inside its predefined vicinity, recrossing it at each consecutive time instant. It has already been proved in section *2.3* that the model's representative point crosses the sliding plane in finite time. For the sake of clarity, we denote the first moment k when $\text{sgn}[s_m(k)] = -\text{sgn}[s(0)]$ with i. Next, for the real plant, we denote the first moment k when $\text{sgn}[s(k)] = -\text{sgn}[s(0)]$ with k_0. Further in the paper we will demonstrate that the finite k_0 actually exists. According to Gao's definition, for the quasi-sliding mode to emerge, the sliding variable must satisfy:

$$\text{sgn}\left[s(k)\right] = -\text{sgn}\left[s(k-1)\right], \tag{26}$$

for any $k \geq k_0$.

<u>Theorem 1:</u>

If $r\tanh[\varepsilon / r] > D_{\max}$, then the sign of plant's sliding variable is opposite to the sign of $s(0)$ for the first time at step $k_0 \leq i+2$. For any $k \geq k_0$ (26) is satisfied and the quasi-sliding mode compliant with the definition of Gao *et al*. (1995) emerges. Moreover, for any $k \geq k_0 + 1$ the absolute value of $s(k)$ is bounded by $\varepsilon + D_{\max}$.

<u>Proof:</u>

To begin, we express $s(k+1)$ using the reaching laws (24) and (16):

$$\begin{aligned}&\text{sgn}\left[s(k+1)\right]\left|s(k+1)\right| = \\ &\text{sgn}\left[s_m(k)\right]\left\{\left|s_m(k)\right| - r\tanh\left[\frac{\left|s_m(k)\right|}{r}\right] - \varepsilon\right\} - D(k).\end{aligned} \tag{27}$$

Considering that for any $k \geq i$ the model's sliding variable satisfies (22), we notice that the term in the curly brackets in (27) is always negative. Moreover, from (4) we conclude that $D(k) \leq D_{\max}$. Consequently, if:

$$r\tanh\left(\frac{\varepsilon}{r}\right) > D_{\max}, \tag{28}$$

then for any $k \geq i+2$:

$$\text{sgn}\left[s(k)\right] = \text{sgn}\left[s_m(k-2)\right] = -\text{sgn}\left[s(k-1)\right]. \tag{29}$$

We conclude that the finite k_0 exists and (26) holds for any $k \geq k_0$, if (28) is satisfied.

Considering (24) we may calculate the ultimate band width. In section *2.3* we have shown that, for any $k \geq i$, $s_m(k)$ satisfies (22). Therefore, for any $k \geq i$, the absolute value of $s(k)$ satisfies:

$$\left|s(k)\right| \leq \varepsilon + D_{\max}, \tag{30}$$

which ends the proof. ■

The width of the ultimate band obtained with the reaching laws (8) and (24) is expressed by $2(\varepsilon + D_{\max})$. However, parameter ε in those control strategies is chosen differently. In the model following strategy the control parameters must satisfy (28), which after some calculations may be transformed to the form:

$$r\left[1 - \frac{2}{e^{\frac{2\varepsilon}{r}} + 1}\right] > D_{\max}. \tag{31}$$

Assuming that $r > D_{\max}$, from (31) we get:

$$\frac{2\varepsilon}{r} > \ln\frac{r + D_{\max}}{r - D_{\max}}. \tag{32}$$

On the other hand, in the original strategy of Leśniewski *et al*. (2015), the control parameters must be chosen so that:

$$r\tanh\left(\frac{\varepsilon + D_{\max}}{r}\right) > 2D_{\max}. \tag{33}$$

Following the same transformations as for the model reference strategy and assuming that $r > 2D_{\max}$, from (33) we get:

$$\frac{2\varepsilon}{r} + \frac{2D_{\max}}{r} > \ln\frac{r + 2D_{\max}}{r - 2D_{\max}}. \quad (34)$$

Comparing (32) and (34), we conclude that the greater the value of $D_{\max}$, the more beneficial the model following strategy becomes. First of all, in the model following strategy r must be greater than $D_{\max}$ instead of $2D_{\max}$, which gives a wider choice for the designer than the original strategy. Moreover, for the values of r close to $2D_{\max}$, the right hand-side of (34) reaches very high values, which results in much larger ε for the original strategy than for the new model following control. As parameter r determines the pace of the convergence, it determines the magnitudes of the control signal as well. Therefore, it is important to keep the value of r relatively small. This shows that the model following strategy offers a significant reduction of the achieved ultimate band width and an improvement of the robustness of the system, while keeping a restricted pace of convergence and a limited magnitude of the control signal.

3. SIMULATION RESULTS

To verify our results we carried out a simulation example. We considered a simple mechanical actuator represented by the transfer function $G(s) = \frac{1}{s^2 + 0.8s}$. We assume that the system is controlled through sample-and-hold devices, with the discretization period $T = 1$. Therefore, we obtain the following state space representation:

$$\boldsymbol{x}(k+1) = \begin{bmatrix} 1 & 0.688 \\ 0 & 0.449 \end{bmatrix} \boldsymbol{x}(k) + \begin{bmatrix} 0.389 \\ 0.688 \end{bmatrix} \left[u(k) + d(k) \right]. \quad (35)$$

The initial conditions are $\boldsymbol{x_0} = [-100\ 0]^{\mathrm{T}}$ and our objective is to achieve the demand state $\boldsymbol{x_d} = [0\ 0]^{\mathrm{T}}$. The disturbance $d(k)$ changes in the whole control process between its extreme values ± 10, i.e. for $k \in [0, 20]$ $d(k) = -10$ and for $k \in [21, 40]$ $d(k) = 10$. We choose vector $\boldsymbol{c} = [1\ 0.566]$, in order to guarantee stability of the closed-loop system and set the sliding hyperplane as $s(k) = \boldsymbol{ce}(k) = 0$, obtaining the initial condition $s_0 = s(0) = 100$. Therefore, the maximum disturbance impact $D_{\max}$ is 7.784. To clearly illustrate the benefits of the model following concept we compare our strategy with the hyperbolic tangent reaching law of Leśniewski *et al.* (2015). We assume that the control signal is limited with $u_{\max} = \pm 45$. Therefore, according to (33), for the hyperbolic tangent strategy we set $r = 22$ and $\varepsilon = 11.7$. On the other hand, taking into account the control limitation and theorem 1, for the model following strategy we choose $r = 27$ and $\varepsilon = 8.05$. The results obtained with our simulations are shown in figs. 1-4. For the sake of clarity, we used blue solid line to plot the results of the original strategy and red dashed line for the model reference strategy.

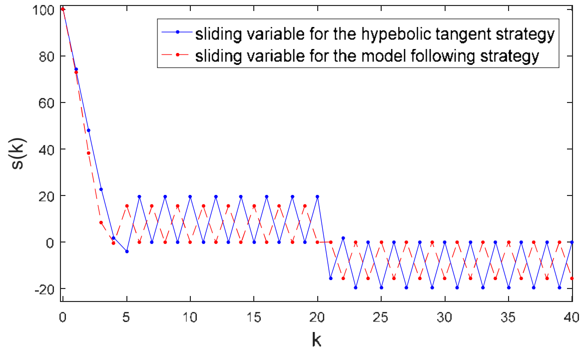

Fig. 1 Evolution of the sliding variable for both strategies.

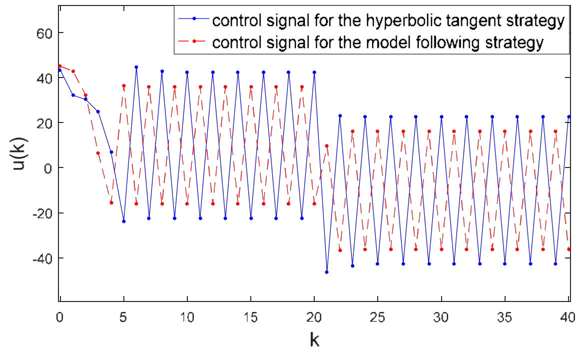

Fig. 2 The control input.

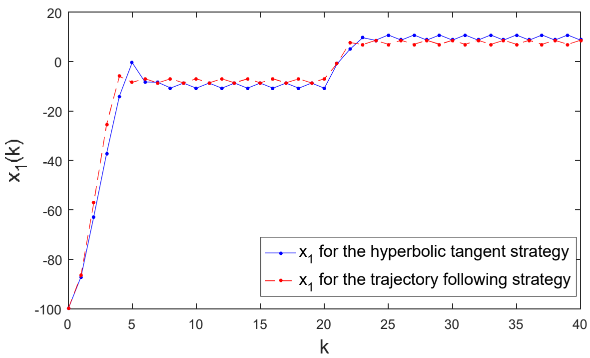

Fig. 3 The first state variable x_1.

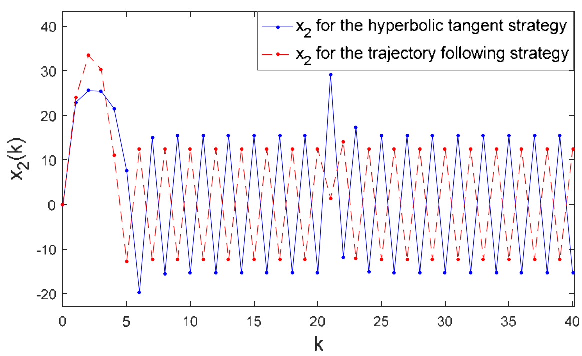

Fig. 4 The second state variable x_2.

Fig. 1 exhibits the evolution of the sliding variable in both control cases. It may be seen that both strategies ensure the convergence of the system's representative point to the switching plane and the quasi-sliding motion. The original strategy resulted in the quasi-sliding mode band width equal to 19.5, while with the model following control this band was reduced to 15.65. Fig. 2, which presents the control signal confirms that the control constraint was satisfied in both cases. Figs. 3 and 4 show the evolutions of the state variables. It may be easily noticed that the model reference control strategy ensured a reduction of the errors of both state variables in the sliding phase.

4. CONCLUSIONS

This work presents a new reference model following sliding mode controller for discrete-time disturbed dynamical systems. Our design method assumes using a mathematical model of the system to generate the demand evolution of the sliding variable. For the generation of the model's trajectory we use a hyperbolic tangent switching type reaching law, which provides relatively fast convergence of the system and imposes a limitation on the maximum rate of change of the sliding variable. Therefore, the maximum control magnitude becomes limited as well. Next, we proposed a model following reaching law for the real system subject to external disturbances. We showed that our control method provides all three attributes of the quasi-sliding mode, defined by Gao *et al*. and at the same time improves the system's robustness. It is worth mentioning that the benefits of our control strategy grow with the growing influence of disturbances. Therefore, the strategy is especially useful in strongly disturbed cases. Lastly, our results were illustrated with a simulation example.

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