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Relay Feedback Identification with Shifting Filter for PID Control

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Abstract: The paper describes a recently introduced relay shifting method for process identification using a single relay feedback test. The aim is to obtain a process model for automatic tuning of PID controllers. This method is applicable for open-loop stable, unstable and integrating systems if there is sustained oscillation in a biased-relay feedback test. For this purpose the identification method uses a filter called a "shifting filter" which enables to estimate the next point on a process frequency characteristic. Furthermore, this additional point can be used to estimate the parameters of the process transfer function with multiple parameters, including static gain, even under a static load disturbance. In the paper, a new more robust algorithm for fitting a second-order time delayed model is introduced. It can be used for the PID controller design of the most processes describable by linear models. For the first time the shifting filter is also applied for the relay feedback identification of unstable systems. The method is demonstrated on examples of stable, unstable and integrating systems.

Keywords: System identification, relay control, parameter estimation, frequency characteristic, static gain, time-delay, auto-tuning.

1. INTRODUCTION

There are many methods for automatic tuning of PID controllers. The relay method belongs among those successfully applied in practice. Rotac (1961) originally used this approach for process identification. Åström and Hägglund (1984) proposed the relay identification method for controller auto-tuning as an alternative to Ziegler-Nichols continuous cycling method (Ziegler and Nichols, 1943). The relay method enables to find the ultimate gain and the ultimate frequency like the Ziegler-Nichols method but in a short experimental time and in a controllable manner.

Successful results obtained by using the relay feedback to process identification have generated interest in this approach, leading to the design of new relay identification methods. These methods can be categorized into three groups for single-input-single-output (SISO) systems: Describing function method, curve fitting approach and frequency response estimation for model fitting (Liu, Wang and Huang, 2013). Currently, there are several review publications focused on relay feedback identification, e.g. Yu (1999), Liu and Gao (2012), Liu, Wang and Huang (2013), Chidambaram and Sathe (2014), Kalpana and Thyagarajan (2018), Ruderman (2019). The presented methods mostly assume linear low-order time delayed models with low number of parameters, which are sufficient for modelling of many industrial processes. But only a few presented relay methods are able to obtain all model parameters using one relay test without a priori information. In addition, some methods of the relay identification do not take into account problems with the effects from load disturbance, measurement noise and nonzero initial process conditions that are in practical applications often encountered.

This paper aims to describe in summary the relay identification method which enables one to estimate up to three points on a process frequency characteristic from a single relay feedback test using a filter called a "shifting filter" without any assumption about a model (Hofreiter, 2015). These points can then be used for estimation up to five parameters of a process transfer function from a single relay feedback experiment. For this purpose the paper introduces a new more robust algorithm for fitting the second order time delayed model (called the SOTD model) which can be used for automatic tuning of PID controllers. For the first time the shifting filter is also applied for the relay feedback identification of unstable systems.

2. RELAY SHIFTING METHOD

2.1 Specifications

Consider a process that can be described by a time invariant linear dynamical model around its operating point. The process is under a two-position biased relay control, see Fig. 1, where w denotes the desired variable, y is the controlled variable, u is the manipulated variable and e is the control error. The process is described by the frequency transfer function $G_P(j\omega)$, where ω is the angular frequency and j is the imaginary unit. The relay shifting method uses a biased relay. The steady state characteristic of the two-position biased relay with hysteresis is depicted in Fig. 2. The steady state characteristic is selected so that there is stable oscillation with the period Tp ($T_p=T_1+T_2$, $T_1\neq T_2$) during the relay feedback test, see Fig. 3. The biased relay with a hysteresis reduces the influence of noisy environment and enables application of the shifting filter for the fitting model.

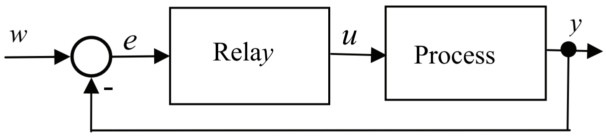

Fig. 1. Closed-loop system with two-position biased relay.

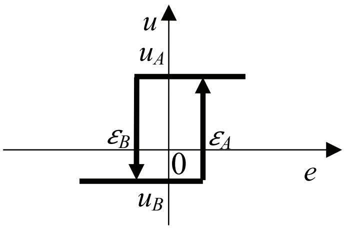

Fig. 2. The steady state characteristic of the two-position biased relay with hysteresis.

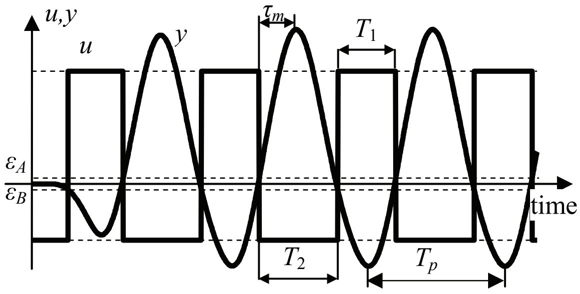

Fig. 3. The time courses of u and y during the relay feedback test.

2.2 Estimation of Frequency Response Points

The basic idea of the shifting method consists in determining the time courses of the auxiliary variables $u_a(t)$ and $y_a(t)$ using the shifting filter with the frequency transfer function filter

$$G_F\left(j\omega\right)=1+e^{-j\omega\frac{T_p}{2}} \tag{1}$$

The filter filters out all odd harmonic frequencies, including the fundamental harmonic frequency ω_1, and amplifies twice the even harmonic frequencies, including ω_2, where

$$\omega_1=\frac{2\pi}{T_p}, \tag{2}$$

$$\omega_2=2\cdot\omega_1 . \tag{3}$$

The block diagram of this filter is shown in Fig. 4.

The auxiliary variables u_a and y_a can be easily calculated from the time courses u and y by

$$u_a\left(t\right)=u\left(t\right)+u\left(t-\frac{T_p}{2}\right), \tag{4}$$

$$y_a\left(t\right)=y\left(t\right)+y\left(t-\frac{T_p}{2}\right). \tag{5}$$

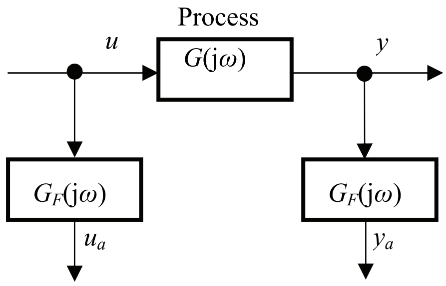

Fig. 4. The block diagram with the shifting filter (1)

The time courses of u and y during the relay feedback test, together with calculated u_a and y_a, are shown in Fig. 5.

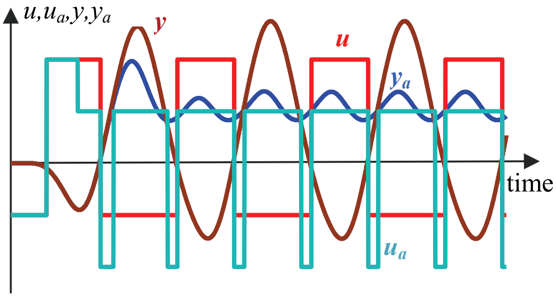

Fig. 5. The time courses of u and y during the relay feedback test together with calculated u_a and y_a.

The frequency response points $G(j\omega_1)$ and $G(j\omega_2)$ can be determined according to Hofreiter (2015) or by

$$G\left(j\omega_1\right)=\frac{\int_t^{t+T_p} y\left(\tau\right)e^{-j\omega_1\tau}d\tau}{\int_t^{t+T_p} u\left(\tau\right)e^{-j\omega_1\tau}d\tau},\ t>t_L, \tag{6}$$

$$G\left(j\omega_2\right)=\frac{\int_t^{t+T_p} y_a\left(\tau\right)e^{-j\omega_2\tau}d\tau}{\int_t^{t+T_p} u_a\left(\tau\right)e^{-j\omega_2\tau}d\tau},\ t>t_L, \tag{7}$$

where t_L is the time when the stable oscillation was achieved, and the integrals in the numerators are computed by numerical integration.

The newly obtained point $G(j\omega_2)$ determined by the shifting method permits the estimation of two other model parameters from a single relay feedback test. The position of the points $G(j\omega_1)$ and $G(j\omega_2)$ in the Nyquist frequency characteristic of a proportional system is shown in Fig. 6. In Fig. 6, the point $G(0)$ corresponding to the static gain K of a proportional system is also depicted. The value K is often assumed to be known a priory, e.g. Luyben (1987) or more relay tests are necessary, e.g. Li, Eskimat and Luyben (1991). The static gain can be also determined by the following formula (computed by numerical integration) if the asymmetrical relay is used and the working point (u_0,y_0) is known exactly, see Shen, Wu and Yu (1996) or Berner, Hägglund, and Åström (2016).

$$K = G(0) = \frac{\int_t^{t+T_p} \left(y(\tau) - y_0 \right) d\tau}{\int_t^{t+T_p} \left(u(\tau) - u_0 \right) d\tau}, t > t_L \,. \tag{8}$$

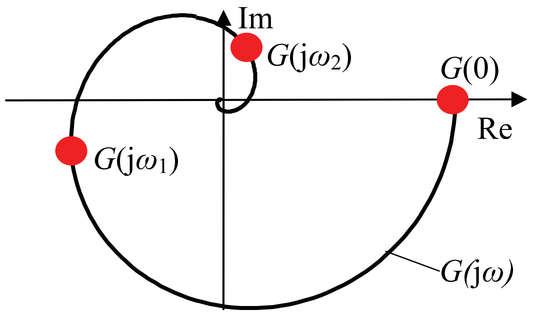

Fig. 6. The Nyquist frequency characteristic of a proportional system with the points G(jω_1), G(jω_2) and the point G(0) corresponding to the static gain K.

Remark 1

A great advantage of the above procedure is that the location of the points $G(j\omega_1)$ and $G(j\omega_2)$ is based on the relay experiment without assuming any model structure. The newly obtained point $G(j\omega_2)$ determined by the shifting method allows the estimation of two other parameters of the model from a single relay test. It is possible due to the use of the second-order harmonic of the relay oscillations. Therefore, this approach can be applied to models with more parameters and different structures.

Remark 2

The next advantage of this approach is that the presence of a static load disturbance with a magnitude of d_A does not have any influence on the calculation $G(j\omega_1)$ and $G(j\omega_2)$ as it holds

$$\int_t^{t+T_p} d_A \cdot e^{-j\omega_i} d\tau = d_A \int_t^{t+T_p} e^{-j\omega_i} d\tau = 0, \; i = 1,2 \tag{9}$$

The vector θ of model parameters can be determined by minimizing the criteria

$$Kr(\theta) = \sum_{i=0}^{2} \left(G(j\omega_i) - M(j\omega_i, \theta) \right)^2 , \tag{10}$$

where $M(j\omega_i,\theta)$ is the frequency response point of a linear model at the frequency ω_i and with parameters given by the vector θ.

The value of the parameter vector θ that minimises the criterion (10) can be determined by

$$\theta = \arg\min_{\theta \in \Theta} Kr(\theta) , \tag{11}$$

where Θ is a set of possible values of θ.

3. MODIFICATIONS OF RELAY SHIFTING METHOD FOR PRACTICE

3.1 Moving the positions of $G(j\omega_1)$ and $G(j\omega_2)$

In some cases the position of the frequency response point G(jω_2) is not very convenient for model fitting. A better position of this point can be achieved by a transport delay D or by an integrator additionally inserted into the closed loop (Hofreiter, 2018), see Fig. 7, where s is the complex variable in Laplace transform. This solution allows to place the points G(jω_2) and G(jω_1) to the 3rd and 4th quadrant (see Fig. 8).

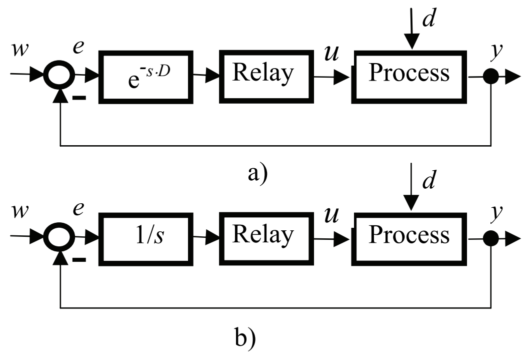

Fig. 7. The relay feedback test with an additional a) transport delay D, b) integrator.

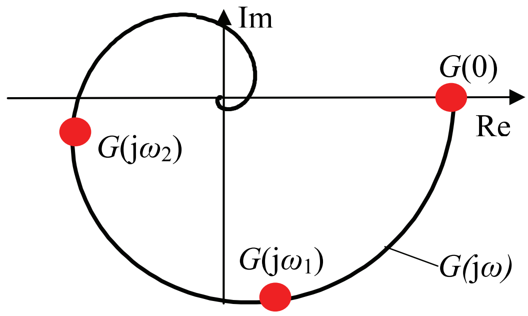

Fig. 8. The Nyquist frequency characteristic of a proportional system with the points G(jω_1), G(jω_2) and G(0) obtained from the relay feedback test with additional integrator/delay in a closed loop.

3.2 Estimation of the static gain K

The static gain K cannot be calculated according to formula (8) if we do not know *a priory* the values u_0, y_0, e.g., due to a static load disturbance d with a magnitude of d_A. But if we use the relay shifting method and the SOTD model with 4 parameters, we can roughly estimate the static gain K from a proportional model estimated only from the points G(jω_1), G(jω_2).

4. SOTD MODEL FITTING

Most industrial processes can be described near the operating point using the SOTD model with the transfer function

$$M(s)=\frac{K\cdot e^{-s\cdot\tau}}{a_2 s^2+a_1 s+1}. \qquad (12)$$

where K, a_2, a_1, τ are estimated parameters, and s is the complex variable in the Laplace transform. This model is very versatile and can be used to describe both oscillating and non-oscillating systems, as well as stable and unstable systems. As this model has only four parameters they can be estimated only from points $G(j\omega_1)$, $G(j\omega_2)$. It means including the static gain K without any further information.

The parameters K, a_2, a_1 and τ of the model (11) can be determined based on the knowledge of the values ω_1, ω_2, $G(j\omega_1)$ and $G(j\omega_2)$ obtained by the shifting method from a single relay feedback test. For this purpose we can use the criterion (10) where the vector of model parameters

$$\theta=[K\ a_2\ a_1\ \tau]^T, \qquad (13)$$

where "T" denotes the transpose of a matrix.

For a stable system the value of the vector θ that minimises the criterion (10) can be determined by

$$\hat{\theta}=\arg\min_{\theta\in\Theta} Kr(\theta), \qquad (14)$$

where $\Theta=\{(K, a_2, a_1, \tau): K>0, a_2>0, a_1>0, \tau\in\langle 0,\tau_m\rangle\}$ and τ_m see Fig. 3.

By denoting the real and imaginary part of the complex values $G(j\omega_1)$ and $G(j\omega_2)$

$$G(j\omega_i)=R_i+I_i\cdot j,\ \text{for } i=1,2 \qquad (15)$$

then

$$\hat{\theta}=\arg\min_{\substack{\tau=0,\Delta\tau,\cdots,\tau_m\\ K,a_2,a_1>0}} Kr\left(\begin{bmatrix}\left(Z^T Z\right)^{-1}\cdot Z^T p\\ \tau\end{bmatrix}\right) \qquad (16)$$

where $\Delta\tau$ is the chosen precision of the estimation τ and

$$Z=\begin{bmatrix}\cos\omega_1\tau & R_1\omega_1^2 & I_1\omega_1\\ -\sin\omega_1\tau & I_1\omega_1^2 & -R_1\omega_1\\ \cos\omega_2\tau & R_2\omega_2^2 & I_2\omega_2\\ -\sin\omega_2\tau & I_2\omega_2^2 & -R_2\omega_2\end{bmatrix}, p=\begin{bmatrix}R_1\\ I_1\\ R_2\\ I_2\end{bmatrix}. \qquad (17)$$

Remark 3

If we know, for example, the static gain K or the transport delay τ, the number of estimated parameters will be reduced and a similar procedure may be used. A similar procedure can be also used for fitting the SOTD model of an unstable process or for an integrating process described by a model with the transfer function

$$M_I(s)=\frac{1}{s(a_2 s+a_1)}e^{-s\cdot\tau}. \qquad (18)$$

5. EXAMPLES

The relay shifting method can be demonstrated on different types of processes (stable /unstable /integrating, oscillating/non-oscillating, with/without transport delay). In all simulated examples, the model parameters are estimated only from the points $G(j\omega_1)$, $G(j\omega_2)$ obtained by the shifting method. The biased relay with a hysteresis has, for the first two examples, the following parameters, see Fig. 2:

$$u_A=2,\ u_B=-1,\ \varepsilon_A=0.1,\ \varepsilon_B=-0.1 \qquad (19)$$

5.1 *Example #1- Stable Process*

A process with the transfer function

$$P_1(s)=\frac{1}{(s+1)^5} \qquad (20)$$

is controlled by the biased relay with the additional integrator, see Fig. 7b. The time course of the relay output u and the output y of the process $P_1(s)$ are depicted in Fig. 9, provided that the process was initially in a steady state. From the time courses u and y it follows that the period of stable oscillation

$$T_P=21.9\ \text{s}. \qquad (21)$$

From (2)÷(7)

$$\omega_1=\frac{2\pi}{T_p}=0.287\ \text{rad}\cdot\text{s}^{-1},\ \omega_2=\frac{4\pi}{T_p}=0.574\ \text{rad}\cdot\text{s}^{-1} \qquad (22)$$

$$G(j\omega_1)=0.13-0.81j,\ G(j\omega_2)=-0.43-0.24j \qquad (23)$$

$$\tau_m=0.2\,\text{s} \qquad (24)$$

The model transfer function $M_1(s)$ obtained by minimizing the criterion (16), is

$$M_1(s)=\frac{0.96}{4.1s^2+3.36s+1}e^{-1.54s}. \qquad (25)$$

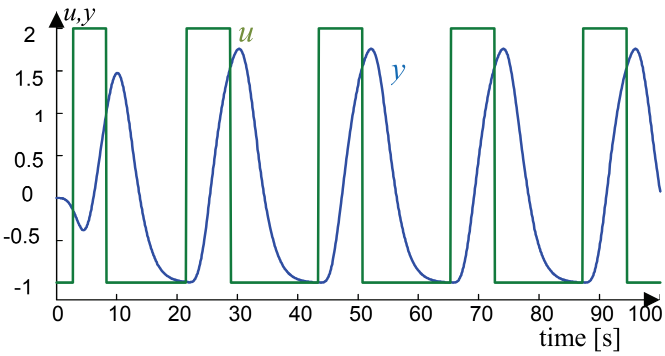

Fig. 9 The time courses of the relay output u and the process output y obtained from the relay feedback experiment with integrator.

The Nyquist frequency characteristics of the transfer functions $P_1(s)$ and $M_1(s)$ are shown in Fig. 10. In the same figure, the points $G(j\omega_1)$ and $G(j\omega_2)$ are depicted as well. The step response h_{P1} of the process P_1 and the step response h_{M1} of the model M_1 are shown in Fig. 11.

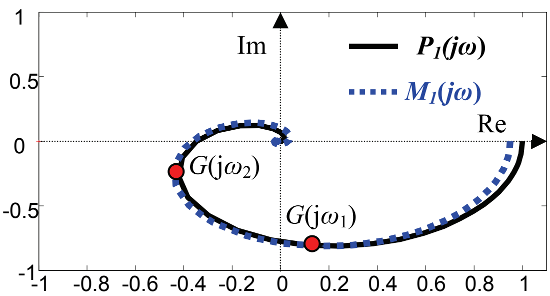

Fig. 10 The Nyquist frequency characteristics of the transfer functions $P_1(s)$ and $M_1(s)$.

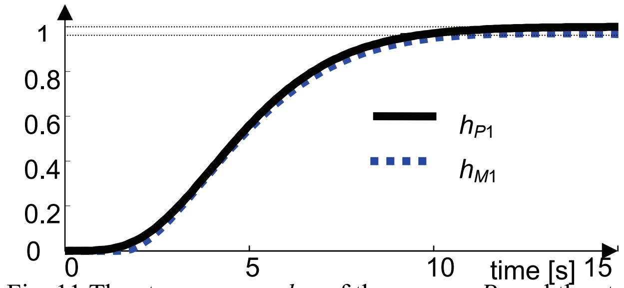

Fig. 11 The step response h_{P1} of the process P_1 and the step response h_{M1} of the model M_1.

5.2 *Example #2- Integrating Process with Delay*

An integrating process with the transfer function

$$P_2(s) = \frac{-s+1}{s\cdot(s+1)^3}e^{-5s} \tag{26}$$

is controlled by the biased relay, see Fig. 1. The time course of the relay output u and the output y of the process $P_2(s)$ are depicted in Fig. 12. From the time courses u and y it follows that the period of stable oscillation

$$T_P = 41.1 \text{ s.} \tag{27}$$

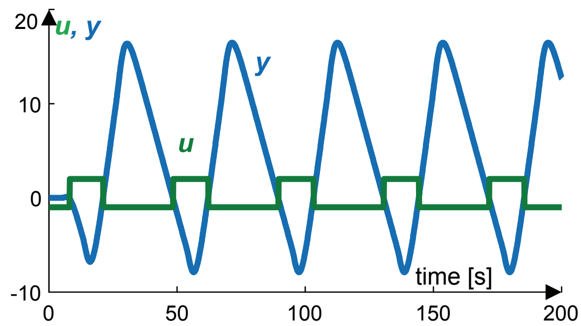

Fig. 12 The time courses of the relay output u and the integrating process output y.

From (2), (3), (4), (5), (6) and (7)

$$\omega_1 = \frac{2\pi}{T_p} = 0.15 \text{ rad}\cdot\text{s}^{-1},\ \omega_2 = \frac{4\pi}{T_p} = 0.30 \text{ rad}\cdot\text{s}^{-1} \tag{28}$$

$$G(j\omega_1) = -6.27 - 1.22j,\ G(j\omega_2) = -1.19 - 2.74j \tag{29}$$

$$\tau_m = 11.3 \text{ s} \tag{30}$$

The model transfer function $M_2(s)$, obtained by the shifting method is

$$M_2(s) = \frac{1}{1.45s^2 + s}e^{-7.6s} \tag{31}$$

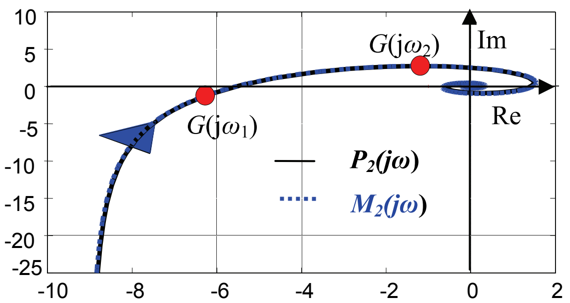

Fig.13 The Nyquist frequency characteristics of P_2 and M_2.

The Nyquist frequency characteristics of the transfer functions $P_2(s)$ and $M_2(s)$ are shown in Fig. 13. The step response h_{P2} of the process P_2 and the step response h_{M2} of the model M_2 are shown in Fig. 14.

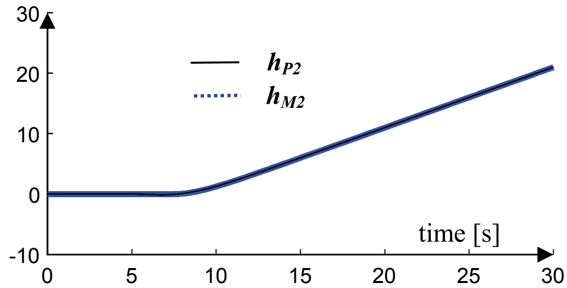

Fig. 14 The step response h_{P2} of the process P_2 and the step response h_{M2} of the model M_2.

5.3 *Example #3- Unstable Process with Delay*

An unstable process with the transfer function

$$P_3(s) = \frac{1}{(5s-1)(2s+1)(0.5s+1)}e^{-0.5s} \tag{32}$$

is controlled by the biased relay, see Fig. 1. In this case, the parameters of the biased relay are

$$u_A=2,\ u_B=-1.7,\ \varepsilon_A=0.1,\ \varepsilon_B=-0.1. \tag{33}$$

The time course of the relay output u and the output y of the process $P_3(s)$ are depicted in Fig. 15. From the time courses u and y it follows that the period of stable oscillation

$$T_P = 24.5 \text{ s} \tag{34}$$

From (2)÷(7)

$$\omega_1 = \frac{2\pi}{T_p} = 0.26 \text{ rad}\cdot\text{s}^{-1},\ \omega_2 = \frac{4\pi}{T_p} = 0.52 \text{ rad}\cdot\text{s}^{-1} \tag{35}$$

$$G(j\omega_1) = -0.54 - 0.09j,\ G(j\omega_2) = -0.25 + 0.03j, \tag{36}$$

$$\tau_m = 5.6 \text{ s.} \tag{37}$$

The model transfer function $M_3(s)$, obtained by the shifting method, is

$$M_3(s) = \frac{1.03}{10.82s^2 + 3.04s - 1}. \tag{38}$$

The Nyquist frequency characteristics of the transfer functions $P_3(s)$ and $M_3(s)$ are shown in Fig. 16. The step response h_{P3} of the process P_3 and the step response h_{M3} of the model M_3 are shown in Fig. 17.

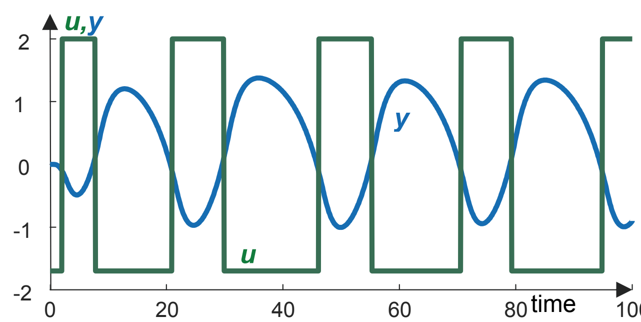

Fig. 15 The time courses of the relay output u and the unstable process output y.

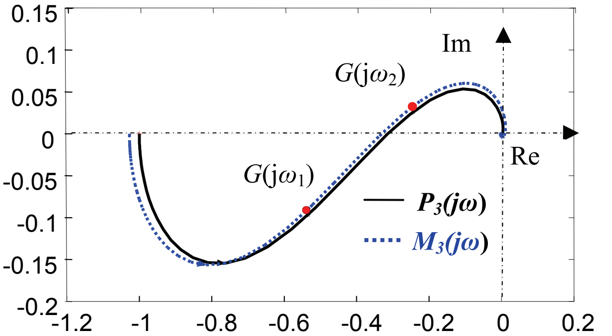

Fig. 16 The Nyquist frequency characteristics of the transfer functions $P_3(s)$ and $M_3(s)$.

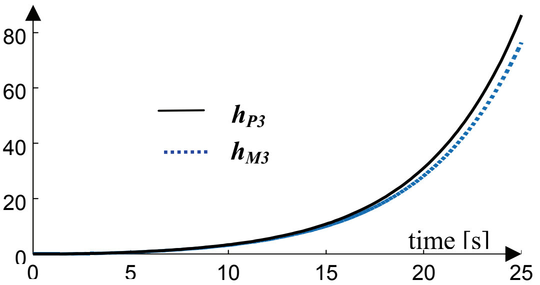

Fig. 17 The step response h_{P3} of the process P_3 and the step response h_{M3} of the model M_3.

6. CONCLUSION

The relay shifting method has the following properties:

- The obtained frequency points $G(j\omega_1)$ and $G(j\omega_2)$ are determined from a single relay test without any prior knowledge of the model.
- The static load disturbance has no effect on the positions of the frequency points $G(j\omega_1)$ and $G(j\omega_2)$.
- The method enables to estimate all the parameters of the SOTD model from a single relay feedback test.
- By using the SOTD model, it is possible to estimate the static gain even in the presence of a constant load disturbance.
- The shifting method can be used for overdamped/underdamped systems, for time-delayed systems and noisy environment.
- The relay shifting method is primarily proposed for the automatic tuning of controllers.
- The shifting method can also be used for integrating and unstable systems if there are stable oscillations for the relay feedback test.
- The shifting method is appropriate only for systems describable by linear and time invariant models.

In addition to the simulation examples, the relay shifting method was also successfully used for PLC Tecomat Foxtrot automatic tuning. This PLC was used to control the laboratory apparatuses called "Air Aggregate", "Water Levitation" and "Air Levitation". For this purpose, an integrated development package for PLC Tecomat, known as Mosaic, was used (Hornychová and Hofreiter, 2019).

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