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Fault-Tolerant Fully Distributed Leader-Following Consensus for Linear Multi-agent Systems with Non-cooperative Leader

Shahram Hajshirmohamadi * **Farid Sheikholeslam** ** **Nader Meskin** *** **Jawhar Ghommam** ****

* *Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan, Iran, (e-mail: shahramhajshir@gmail.com)*
** *Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan, Iran, (e-mail: sheikh@cc.iut.ac.ir)*
*** *Department of Electrical Engineering, Qatar University, Doha, Qatar, (e-mail: nader.meskin@qu.edu.qa)*
**** *Department of Electrical and Computer Engineering, Sultan Qaboos University, Muscat, Oman, (e-mail: jawher@squ.edu.om)*

Abstract: In this paper, the problem of distributed fault-tolerant control for multi-agent systems is studied. In the proposed method, the communication graph of the network is not needed to be known by the agents. The leader agent is active with a bounded non-zero control input. The control input of the leader and its bound is not known by any follower. Moreover, it is assumed that the leader is non-cooperative and does not communicate with any other agent. By considering all these limitations, the control input of the followers are designed such that the followers can track the leader in the presence of bounded additive actuator faults. A numerical simulation is provided to illustrate the effectiveness of the proposed approach.

Keywords: Multi-agent systems, Fully distributed control, Consensus, Fault-tolerant, Leader-following.

1. INTRODUCTION

During the past decade, the study of cooperative control algorithms for multi-agent systems has received a considerable attention (Davoodi et al., 2016). Leader-following consensus is a cooperative behaviour in multi-agent systems. In this behaviour, the states of a group of follower agents track the states of a leader. In the leader-following consensus control, the objective is to design a control law that guarantees this behaviour among the agents. Leader-following consensus is used for different applications such as multiple unmanned underwater vehicles (Yan et al., 2019), areal vehicles (Xuan-Mung and Hong, 2019) and power systems (Hu et al., 2014).

Leader-following control algorithms have been proposed by considering different agent models including single integrator, double integrator, linear and non-linear models. Linear multi-agent systems have gained considerable attention recently due to their generality. In fact, single integrator and double integrator systems can be modelled as linear systems. The leader-following problem for linear multi-agent systems has been extensively studied in recent years(Cheng and Ugrinovskii (2016); Xu et al. (2015); Cheng and Li (2018); Wu et al. (2017)).

Since multi-agent systems usually perform their tasks in unknown and sometimes harsh environments, the agents are subject to the occurrence of faults (Meskin and Khorasani, 2009; Meskin and Khorasani, 2009; Meskin et al., 2010; Meskin and Khorasani, 2011; Chadli et al., 2017). Due to the communication and cooperation among the agents, a fault in an agent not only affects the behaviour of that agent, but also degrades the performance of the whole team such that the consensus objective can not be achieved. Therefore, designing fault-tolerant control laws for the multi-agent system is of great importance. Different fault-tolerant control protocols have been proposed for linear multi-agent systems. In Gallehdari et al. (2017b), a fault-tolerant leader-following consensus control algorithm is proposed for the case of multiplicative faults and in Gallehdari et al. (2017a), distributed control reconfiguration strategies for directed switching topology networked multi-agent systems are developed. In Hajshirmohamadi et al. (2019b,a); Davoodi et al. (2016), the problem of simultaneous fault detection and control for multi-agent systems is considered. In these papers, an H_∞ performance is considered to attenuate the effect of fault on the consensus error. In Zhu et al. (2016), a fault-tolerant tracking control method is proposed based on intermediate estima-

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tor. In Chen et al. (2015); Zou et al. (2015); Yadegar et al. (2017); Yadegar and Meskin (2020), adaptive protocols are used to eliminate the effect of fault on the tracking error.

In most of the fault-tolerant leader-following protocols such as Gallehdari et al. (2017b); Zou et al. (2015); Chen et al. (2015), the full state measurement is needed. It should be noted that in many applications, the full state can not be measured and only an output signal is available. Moreover, in most of the proposed approaches like Gallehdari et al. (2017b); Hajshirmohamadi et al. (2019b,a); Davoodi et al. (2016), the control input of the leader should be transmitted to a group of followers while n many cases, this is not possible. An example of this situation is the case that the leader is a non-cooperative target. The proposed method in Zhu et al. (2016) is based on output feedback and does not need communication with the leader. However, the Laplacian matrix of the network is needed to be known for controller design. Laplacian matrix is based on global information and therefore this approach is not fully distributed.

In this paper, a fully distributed approach is proposed to achieve leader-following consensus among linear agents. The proposed method can also eliminate the effect of additive actuator faults and it is based on output feedback and full state measurement is not needed. Moreover, the proposed approach is fully distributed and a global information of the network is not needed for designing the distributed controllers. Based on the authors' knowledge, a leader-following method with all these features does not exist in the literature.

The outline of this paper is organized as follows. Problem formulation and preliminaries are given in Section 2. Section 3 includes the main results of the paper. A simulation example is presented in Section 4 and finally, Section 5 concludes the paper.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a team of $N+1$ homogeneous agents including N followers and one leader. Without loss of generality, it is assumed that the leader is indexed by 0 and the followers are indexed by $1,\dots,N$. The dynamics of the i-th follower is described as

$$\begin{aligned}\dot{x}_i(t) &= Ax_i(t) + B\big(u_i(t)+f_i(t)\big),\\ z_{ij}(t) &= C\big(x_i(t)-x_j(t)\big), \quad j\in\mathcal{N}_i,\end{aligned} \tag{1}$$

for $i=1,\dots,N$, where $\mathcal{N}_i$ is defined based on the communication graph of the followers. The dynamics of the leader is described by

$$\dot{x}_0(t) = Ax_0(t) + Bu_0(t), \tag{2}$$

where $x_i(t)\in\mathbb{R}^{n_\mathrm{x}}, t\geq 0, i=0,\dots,N$, is the state of the i-th agent and $z_{ij}(t)\in\mathbb{R}^{n_\mathrm{z}}, t\geq 0$, is the measured relative output of agent $i, i=1,\dots,N$, with respect to agent $j\in\mathcal{N}_i\cup 0$ if the leader is a neighbour of agent i and $j\in\mathcal{N}_i$, otherwise. The inputs $u_i(t)\in\mathbb{R}^{n_\mathrm{u}}$ and $f_i(t)\in\mathbb{R}^{n_\mathrm{u}}$ are the control and actuator fault inputs of the agent i. The matrices A, B and C are known constant matrices of appropriate dimensions. We assume that only a *non-empty subset* of the followers can measure their outputs relative to the leader. However, there is no communication link between the followers and the leader to exchange information. The objective is to design distributed controllers for the followers such that the followers can track the leader in the presence of bounded actuator faults, i.e., $\|x_i(t)-x_0(t)\|\to 0$ for $t\to 0$.

The following assumptions are used throughout this paper.

Assumption 1. It is assumed that the pair (A,B) is stabilizable and the pair (A,C) is detectable.

Assumption 2. It is assumed that $\mathrm{rank}(CB)=\mathrm{rank}(B)=n_\mathrm{u}$.

Assumption 3. For every complex number s with non-negative real part, we have:

$$\mathrm{rank}\left(\begin{bmatrix} sI_n - A & B\\ C & 0\end{bmatrix}\right) = n + n_\mathrm{u}. \tag{3}$$

Assumption 4. The control input of the leader $u_0(t)$ and the fault inputs $f_i(t)$, satisfy $\|u_0\|\leq u_\mathrm{M}$ and $\|f_i\|\leq f_\mathrm{M}$, where u_M and f_M are unknown positive constants.

Assumption 5. The communication and measurement graphs of the follower agents are equal, un-directed and connected. Moreover, at least one of the followers can measure its output relative to the leader.

Remark 1. Assumptions 2 and 3 are common in studies that consider sliding-mode observer design (Raoufi et al., 2010; Lee et al., 2014; Menon and Edwards, 2014). Assumption 3 suggests that all invariant zeros of the triple (A,B,C) are in the left half plane or equivalently the triple (A,B,C) is minimum phase. Assumption 4 requires that the control input of the leader is bounded that is a general assumption in leader-following consensus problems (Li et al., 2013; Zhu et al., 2016). Finally, Assumption 5 is common among multi-agent systems with undirected communication graphs.

Lemma 1. (Liu et al. (2007)). Let $L=[l_{ij}]_{N\times N}$ denote a symmetric and irreducible matrix satisfying $l_{ij}\leq 0, i\neq j$, and $\sum_{j=1}^{N} l_{ij}=0$, for $i=1,\dots,N$, and $G=\mathrm{diag}(g_1,g_2,\dots,g_N)$ be a non-zero matrix with $g_i\geq 0, i=1,\dots,N$. Then, all the eigenvalues of the matrix $L+G$ are positive.

3. MAIN RESULTS

3.1 Controller analysis

In this section, an observer-based distributed consensus protocol is proposed. Before introducing the distributed observer, we define the new states $\zeta_i(t)$ and $\xi_i(t)$, $i=1,\dots,N$, as

$$\begin{aligned}\zeta_i(t) &= \sum_{j\in\mathcal{N}_i}\big(x_i(t)-x_j(t)\big) + q_i\big(x_i(t)-x_0(t)\big)\\ \xi_i(t) &= \sum_{j\in\mathcal{N}_i} z_{ij}(t) + q_i z_{i0}(t)\end{aligned}$$

where q_i is defined as $q_i=1$ if the i-th follower has access to the leader relative measurement and $q_i=0$, otherwise. The proposed observer-based controller for the i-th agent is now proposed as

$$\begin{aligned}
\dot{\hat{\zeta}}_i(t) &= A\hat{\zeta}_i(t)+q_iBu_i(t)+B\sum_{j\in\mathcal{N}_i}\big(u_i(t)-u_j(t)\big)+H\xi_i(t)\\
&\quad+B\sum_{j\in\mathcal{N}_i}\Big(\eta_i(t)\mathrm{sgn}\big(\phi_i(t)\big)-\eta_j(t)\mathrm{sgn}\big(\phi_j(t)\big)\Big)\\
&\quad+q_i\eta_i(t)B\mathrm{sgn}\big(\phi_i(t)\big),\\
\dot{\eta}_i(t) &= \tau_i\psi_i^{\mathrm{T}}(t)\Gamma\psi_i(t)\\
&\quad+\tau_i\Big\|q_iF\xi_i(t)+F\sum_{j\in\mathcal{N}_i}\big(\xi_i(t)-\xi_j(t)\big)\Big\|_1,\\
u_i(t) &= -\eta_i(t)K\psi_i(t)-\eta_i(t)\mathrm{sgn}\big(\phi_i(t)\big),\ i=1,\dots,N, \quad (4)
\end{aligned}$$

where

$$\phi_i(t)=q_i\eta_i(t)F\xi_i(t)+\eta_i(t)F\sum_{j\in\mathcal{N}_i}\big(\xi_i(t)-\xi_j(t)\big)$$

and τ_i is a positive scalar. The matrices $F\in\mathbb{R}^{n_{\mathrm{u}}\times n_z}$, $H\in\mathbb{R}^{n_{\mathrm{x}}\times n_z}$, $\Gamma\in\mathbb{R}^{n_{\mathrm{x}}\times n_{\mathrm{x}}}$, and $K\in\mathbb{R}^{n_{\mathrm{u}}\times n_{\mathrm{x}}}$ are the parameters of the controller that will be designed later.

Let $\hat{\zeta}(t)=\big[\hat{\zeta}_1^{\mathrm{T}}(t),\dots,\hat{\zeta}_N^{\mathrm{T}}(t)\big]^{\mathrm{T}}$, $e_i(t)=\zeta_i(t)-\hat{\zeta}_i(t)$, $e(t)=\big[e_1^{\mathrm{T}}(t),\dots,e_N^{\mathrm{T}}(t)\big]^{\mathrm{T}}$, $\zeta(t)=\big[\zeta_1^{\mathrm{T}}(t),\dots,\zeta_N^{\mathrm{T}}(t)\big]^{\mathrm{T}}$, $Q=[q_1,\dots,q_N]^{\mathrm{T}}$, and $G(t)=\mathrm{diag}(\eta_1(t),\dots,\eta_N(t))$. Using (1), (2), and (4), the dynamics of the aggregated state vectors $\hat{\zeta}(t)$ and $e(t)$ can be written as

$$\begin{aligned}
\hat{\zeta}(t) &= \big(I_N\otimes A-\bar{L}G(t)\otimes BK\big)\hat{\zeta}(t)+(I_N\otimes HC)e(t),\\
\dot{e}(t) &= \big(I_N\otimes(A-HC)\big)e(t)\\
&\quad-\big(\bar{L}G(t)\otimes B\big)\mathrm{sgn}\Big(\big(G(t)\bar{L}\otimes FC\big)e(t)\Big)\\
&\quad+(\bar{L}\otimes B)f(t)-(Q\otimes B)u_0(t), \qquad (5)
\end{aligned}$$

where $\bar{L}$ is the modified Laplacian matrix defined as $\bar{L}=L+\mathrm{diag}(q_1,\dots,q_N)$.

Lemma 2. (Davoodi et al. (2016)). The modified Laplacian matrix $\bar{L}$ is positive definite.

Let $x_{\mathrm{e}_i}(t)=x_i(t)-x_0(t)$ and $x_{\mathrm{e}}(t)=[x_{\mathrm{e}_1}^{\mathrm{T}}(t),\dots,x_{\mathrm{e}_N}^{\mathrm{T}}]^{\mathrm{T}}$. It can be easily seen that $\zeta(t)=(\bar{L}\otimes I_{n_{\mathrm{x}}})x_{\mathrm{e}}(t)$. If $\hat{\zeta}(t)\to 0$ and $e(t)\to 0$, then $\zeta(t)\to 0$. Since $\bar{L}$ is non-singular, this leads to $x_{\mathrm{e}}(t)\to 0$ and the consensus will be achieved. In the following theorem, a sufficient condition is given to design the parameters of the leader-following consensus protocol (4).

Theorem 1. Suppose that Assumptions 1-4 hold. If there exist positive definite matrices $P_1\in\mathbb{R}^{n_{\mathrm{x}}\times n_{\mathrm{x}}}$ and $P_2\in\mathbb{R}^{n_{\mathrm{x}}\times n_{\mathrm{x}}}$ such that

$$\Xi<0, \qquad (6)$$

$$B^{\mathrm{T}}P_2=FC, \qquad (7)$$

where,

$$\Xi=\begin{bmatrix}AP_1+P_1A^{\mathrm{T}}-2BB^{\mathrm{T}} & HC\\ C^{\mathrm{T}}H^{\mathrm{T}} & P_2(A-HC)+(A-HC)^{\mathrm{T}}P_2\end{bmatrix}, \qquad (8)$$

then, the leader following consensus objective is achieved in presence of actuator fault by considering $K=B^{\mathrm{T}}P_1^{-1}$ and $\Gamma=P_1^{-1}BB^{\mathrm{T}}P_1^{-1}$. Moreover the gains η_i converge to a constant value.

Proof. Define $\tilde{\eta}_i(t)=\eta_i(t)-\alpha$, $\tilde{\eta}(t)=[\tilde{\eta}_1(t),\dots,\tilde{\eta}_N(t)]^{\mathrm{T}}$, where α is a positive constant. Consider the positive definite Lyapunov candidate function

$$\begin{aligned}
&V\big(\hat{\zeta}(t),e(t),\tilde{\eta}(t)\big)\\
&\quad=\hat{\zeta}^{\mathrm{T}}(t)(I_N\otimes P_1^{-1})\hat{\zeta}(t)+e^{\mathrm{T}}(t)(I_N\otimes P_2)e(t)\\
&\quad+\sum_{i=1}^{N}\frac{1}{\tau_i}\tilde{\eta}_i^2(t), \qquad (9)
\end{aligned}$$

The time derivative of (9) along the trajectory of (5) is obtained as

$$\begin{aligned}
&\dot{V}\big(\hat{\zeta}(t),e(t),\tilde{\eta}(t)\big)\\
&=2\hat{\zeta}^{\mathrm{T}}(t)(I_N\otimes P_1^{-1}A-\bar{\mathcal{L}}G(t)\bar{\mathcal{L}}\otimes P_1^{-1}BB^{\mathrm{T}}P_1^{-1})\hat{\zeta}(t)\\
&\quad+2\hat{\zeta}^{\mathrm{T}}(t)(I_N\otimes P_1^{-1}HC)e(t)\\
&\quad+2e^{\mathrm{T}}(t)\big(I_N\otimes P_2(A-HC)\big)e(t)\\
&\quad-2e^{\mathrm{T}}(t)(\bar{\mathcal{L}}G(t)\otimes P_2B)\mathrm{sgn}\big((G(t)\bar{\mathcal{L}}\otimes FC)e(t)\big)\\
&\quad-2e^{\mathrm{T}}(t)(Q\otimes P_2B)u_0(t)+(\bar{L}\otimes P_2B)f(t)\\
&\quad+2\sum_{i=1}^{N}\eta_i(t)\psi_i^{\mathrm{T}}(t)\Gamma\psi_i(t)\\
&\quad+2\sum_{i=1}^{N}\eta_i(t)\Big\|q_iFCe_i(t)+FC\sum_{j\in\mathcal{N}_i}\big(e_i(t)-e_j(t)\big)\Big\|_1\\
&\quad-2\alpha\sum_{i=1}^{N}\psi_i^{\mathrm{T}}(t)\Gamma\psi_i(t)\\
&\quad-2\alpha\sum_{i=1}^{N}\Big\|q_iFCe_i(t)+FC\sum_{j\in\mathcal{N}_i}\big(e_i(t)-e_j(t)\big)\Big\|_1. \qquad (10)
\end{aligned}$$

Using the fact that $x^{\mathrm{T}}\mathrm{sgn}(x)=\|x\|_1$, and $B^{\mathrm{T}}P_2=FC$, it is easy to show that

$$\begin{aligned}
&e^{\mathrm{T}}(t)(\bar{\mathcal{L}}G(t)\otimes P_2B)\mathrm{sgn}\big((G(t)\bar{\mathcal{L}}\otimes FC)e(t)\big)\\
&\quad=\sum_{i=1}^{N}\eta_i(t)\Big\|q_iFCe_i(t)+FC\sum_{j\in\mathcal{N}_i}\big(e_i(t)-e_j(t)\big)\Big\|_1, \qquad (11)
\end{aligned}$$

and

$$\begin{aligned}
&2\hat{\zeta}^{\mathrm{T}}(t)(\bar{\mathcal{L}}G(t)\bar{\mathcal{L}}\otimes P_1^{-1}BB^{\mathrm{T}}P_1^{-1})\hat{\zeta}(t)\\
&\quad=2\sum_{i=1}^{N}\eta_i\psi_i^{\mathrm{T}}(t)\Gamma\psi_i(t). \qquad (12)
\end{aligned}$$

Moreover, using the Hölder's inequality we have

$$\begin{aligned}
&-e^{\mathrm{T}}(t)(Q\otimes P_2B)u_0(t)+(\bar{L}\otimes P_2B)f(t)\\
&\quad=-e^{\mathrm{T}}(t)(\bar{\mathcal{L}}\mathbf{1}_N\otimes P_2B)u_0(t)+(\bar{L}\otimes P_2B)f(t)\\
&\quad=-e^{\mathrm{T}}(t)(\bar{\mathcal{L}}\otimes P_2B)\Big((\mathbf{1}_N\otimes I_{n_{\mathrm{u}}})u_0(t)-f(t)\Big)\\
&\quad\le\|\bar{\mathcal{L}}\otimes B^{\mathrm{T}}P_2e(t)\|_1\|(\mathbf{1}_N\otimes I_{n_{\mathrm{u}}})u_0(t)-f(t)\|_\infty. \qquad (13)
\end{aligned}$$

By using (11), (12) and (13), the inequality (10) leads to

$$\begin{aligned}
&\dot{V}\big(\hat{\zeta}(t),e(t),\tilde{\eta}(t)\big)\\
&\le2\hat{\zeta}^{\mathrm{T}}(t)(I_N\otimes P_1^{-1}A-\alpha\bar{\mathcal{L}}^2\otimes P_1^{-1}BB^{\mathrm{T}}P_1^{-1})\hat{\zeta}(t)\\
&\quad+2\hat{\zeta}^{\mathrm{T}}(t)(I_N\otimes P_1^{-1}HC)e(t)\\
&\quad+2e^{\mathrm{T}}(t)\big(I_N\otimes P_2(A-HC)\big)e(t)\\
&\quad+\|\bar{\mathcal{L}}\otimes B^{\mathrm{T}}P_2e(t)\|_1(\|(\mathbf{1}_N\otimes I_{n_{\mathrm{u}}})u_0(t)-f(t)\|_\infty-\alpha). \qquad (14)
\end{aligned}$$

Since $u_0(t)$ and $f_i(t)$ are bounded according to Assumption 4 and α can be any positive scalar, for a large enough α we have

$$\begin{aligned}
&\dot{V}\big(\hat{\zeta}(t), e(t), \tilde{\eta}(t)\big)\\
&\leq 2\hat{\zeta}^{\mathrm{T}}(t)(I_N \otimes P_1^{-1}A - \alpha\bar{\mathcal{L}}^2 \otimes P_1^{-1}BB^{\mathrm{T}}P_1^{-1})\hat{\zeta}(t)\\
&\quad + 2\hat{\zeta}^{\mathrm{T}}(t)(I_N \otimes P_1^{-1}HC)e(t)\\
&\quad + 2e^{\mathrm{T}}(t)\big(I_N \otimes P_2(A-HC)\big)e(t). \qquad (15)
\end{aligned}$$

Based on Lemma 2, the modified Laplacian $\bar{L}$ is positive definite. Hence, there exists a unitary matrix U such that $U^{\mathrm{T}}\bar{L}U = \mathrm{diag}(\lambda_1, \ldots, \lambda_N)$, where $\lambda_N > \ldots > \lambda_1 > 0$. Define the transformation $\tilde{\zeta} = (U^{\mathrm{T}} \otimes P_1^{-1})\hat{\zeta}$. Then, we have

$$\begin{aligned}
&\dot{V}\big(\tilde{\zeta}(t), \tilde{e}(t), \tilde{\eta}(t)\big)\\
&\leq 2\tilde{\zeta}^{\mathrm{T}}(t)(I_N \otimes AP_1 - \alpha\Lambda^2 \otimes BB^{\mathrm{T}})\tilde{\zeta}(t)\\
&\quad + 2\tilde{\zeta}^{\mathrm{T}}(t)(I_N \otimes HC)\tilde{e}(t)\\
&\quad + 2\tilde{e}^{\mathrm{T}}(t)\big(I_N \otimes P_2(A-HC)\big)\tilde{e}(t)\\
&= 2\sum_{i=1}^{N}\big(\tilde{\zeta}_i^{\mathrm{T}}(t)(AP_1 - \alpha\lambda_i^2 BB^{\mathrm{T}})\tilde{\zeta}_i(t)\\
&\quad + \tilde{\zeta}_i^{\mathrm{T}}(t)HC\tilde{e}_i(t) + \tilde{e}_i^{\mathrm{T}}(t)P_2(A-HC)e_i(t)\big)\\
&\leq 2\sum_{i=1}^{N}\big(\tilde{\zeta}_i^{\mathrm{T}}(t)(AP_1 - BB^{\mathrm{T}})\tilde{\zeta}_i(t)\\
&\quad + \tilde{\zeta}_i^{\mathrm{T}}(t)HC\tilde{e}_i(t) + \tilde{e}_i^{\mathrm{T}}(t)P_2(A-HC)\tilde{e}_i(t)\big)\\
&= \begin{bmatrix}\tilde{\zeta}(t)\\ \tilde{e}(t)\end{bmatrix}^{\mathrm{T}} \Xi \begin{bmatrix}\tilde{\zeta}(t)\\ \tilde{e}(t)\end{bmatrix} \triangleq -W\big(\tilde{\zeta}(t), \tilde{e}(t)\big), \qquad (16)
\end{aligned}$$

where the last inequality is obtained if α is large enough such that $\alpha\lambda_i \geq 1$. Since $\Xi < 0$, we have $V\big(\tilde{\zeta}(t), e(t)\big) \leq 0$ and the system (5) is stable. This means that $\tilde{\zeta}(t)$, $e(t)$ and $\zeta(t)$ are bounded. $\int_{t=0}^{\infty} W\big(\tilde{\zeta}(t), e(t)\big)\mathrm{d}t = V\big(\tilde{\zeta}(0), e(0)\big) - V_\infty$ exists and is bounded. Since $\dot{\tilde{\zeta}}(t)$ and $\dot{e}(t)$ are bounded, then $\dot{W}(\tilde{\zeta}(t), e(t))$ is bounded and $W(\tilde{\zeta}, e)$ is uniformly continuous. By using the Barbalat Lemma, $W(\tilde{\zeta}(t), e(t))$ converges to zero which means that $\tilde{\zeta}(t)$, $\zeta(t)$ converge to zero and as a result $x_{\mathrm{e}}(t)$ converges to zero. On the other hand, $\eta(t)$ is nondecreasing and bounded which means it converges to a constant.

Remark 2. The controller (4) and the conditions of Theorem 1 illustrate the advantage of the proposed method compared with the existing approaches. It can be seen that different from Gallehdari et al. (2017b), relative output measurements are used instead of relative states. Compared to Gallehdari et al. (2017b); Khalili et al. (2020), in the proposed method none of the followers need communication of the control input or any variable with the leader. Moreover, different from Zhu et al. (2016), in the proposed method, no global information such as the eigenvalues of the Laplacian matrix is needed.

3.2 Controller synthesis

In this subsection, we propose a method to design the parameters of the controller (4). Algorithm 1 is proposed for this purpose. In the following lines we explain how this algorithm works. According to Schur complement lemma, the inequality (6) holds if and only if the following inequalities

Algorithm 1. Obtaining the parameters of the controller (4)

1: Solve LMI $AP_1 + P_1A^{\mathrm{T}} - 2BB^{\mathrm{T}} < 0$ for P_1.
2: Solve $\mathrm{Her}\big(\bar{P}_2(A-HC)\big) < -\beta I_{n_{\mathrm{x}}}$ and $B^{\mathrm{T}}\bar{P}_2 = \bar{F}C$ for H and $\bar{F}$ by using Theorem 3 in Hui and Zak (2005).
3: Find γ such that $-(HC)^{\mathrm{T}}(AP_1 + P_1A^{\mathrm{T}} - 2BB^{\mathrm{T}})^{-1}HC < \gamma\beta I_{n_{\mathrm{x}}}$.
4: Set $F = \gamma\bar{F}$ and $K = B^{\mathrm{T}}P_1^{-1}$.

$$\mathrm{Her}(AP_1 + PA^{\mathrm{T}} - 2BB^{\mathrm{T}}) < 0, \qquad (17)$$

$$\begin{aligned}&\mathrm{Her}\big(P_2(A-HC)\big)\\ &\quad - (HC)^{\mathrm{T}}(AP_1 + PA^{\mathrm{T}} - 2BB^{\mathrm{T}})^{-1}HC < 0, \qquad (18)\end{aligned}$$

hold. Since (A, B) is stabilizable according to Assumption 1, the linear matrix inequality (17) has a positive definite solution for P_1 (Li et al., 2013). Moreover, since based on Assumption 1, (A, C) is detectable and Assumptions 2 and 3 hold, based on Theorem 3 in Hui and Żak (2005), there exist H, F, and $P_2 > 0$ such that the Lyapunov inequality $\mathrm{Her}\big(\bar{P}_2(A-HC)\big) < -\beta I_{n_{\mathrm{x}}}$ and the equality $B^{\mathrm{T}}\bar{P}_2 = \bar{F}C$ hold. On the other hand, the scalar γ can be found such that

$$-(HC)^{\mathrm{T}}(AP_1 + P_1A^{\mathrm{T}} - 2BB^{\mathrm{T}})^{-1}HC < \gamma\beta I_{n_{\mathrm{x}}}$$

and consequently we have

$$\begin{aligned}&\gamma\mathrm{Her}\big(\bar{P}_2(A-HC)\big)\\ &\quad - (HC)^{\mathrm{T}}(AP_1 + P_1A^{\mathrm{T}} - 2BB^{\mathrm{T}})^{-1}HC < 0\end{aligned}$$

By defining $P_2 = \gamma\bar{P}_2$, the inequality (18) is satisfied and hence, the inequality (6) holds. Moreover, by setting $F = \gamma\bar{F}$, the equality constraint (7) holds. Algorithm 1 can be used to find the parameters.

4. SIMULATION

In this section, a simulation example is provided to demonstrate the effectiveness of the proposed method. Consider a network of unmanned underwater vehicles (UUVs) including four followers and a leader. The linearized dynamics of the diving system is described by (1) with $x_i = [\omega_i\ \theta_i\ z_i]^{\mathrm{T}}$ where ω_i is the pitch rate, θ_i is the pitch angle, and z_i is the depth of the i-th UUV (Healey and Lienard (1993)). The parameters of the model are given as

$$A = \begin{bmatrix}-0.7 & -0.3 & 0\\ 1 & 0 & 0\\ 0 & -0.3 & 0\end{bmatrix}, B = \begin{bmatrix}0.35\\ 0\\ 0\end{bmatrix}, C = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}. \qquad (19)$$

The control input $u_i(t)$ is the deflection of the control surface from the stern plane. Each UUV can measure its depth and pitch rate relative to its neighbours. The communication graph of the network is given in Fig. 1.

The control input of the leader is selected as $u_0(t) = K_r x_0(t) + \sin(0.5t), t \geq 0$, where $K_{\mathrm{r}} = [12.85\ 24.02\ -44.68]$. The parameters of the controller is designed by using Algorithm 1 and setting $\tau = 100$. It is assumed that at $t = 50$s, a bias fault $f_2(t) = 1$ occurs in the second agent. The trajectories of the states of the agents and the adaptive gains η_i are shown in Fig. 4 and 5, respectively. It can be seen that the consensus is achieved in presence of actuator fault in agent 2.

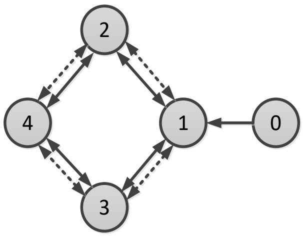

Fig. 1. The communication/measurement graph of the agents where the dashed arrows denote the communication among the agents and the solid arrows denote the relative measurements available for each agent.

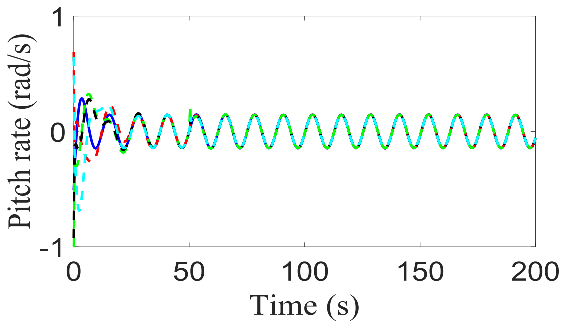

Fig. 2. The pitch rate trajectories of agents.

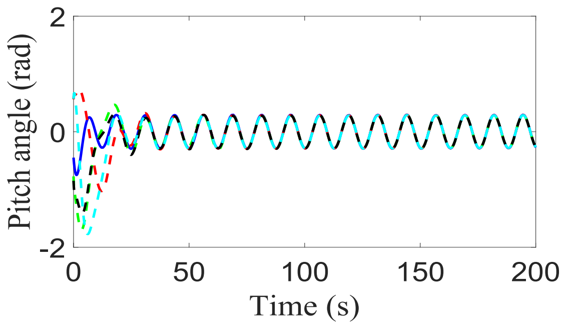

Fig. 3. The pitch angle trajectories of agents.

5. CONCLUSION

In this paper, a fault-tolerant control protocol has been proposed for linear multi-agent systems. The proposed method does not need full-state measurement of the agents. The leader is considered to be non-cooperative and its control input is assumed to be unknown to all followers. Moreover, the global information of the network is not used in controller design. By considering all these limitations, an algorithm is proposed to design the parameters of the controller by using linear matrix inequalities. A numerical example has been presented to illustrate the effectiveness of the proposed method. The case of multiplicative faults and switching network topology will be considered in our future work.

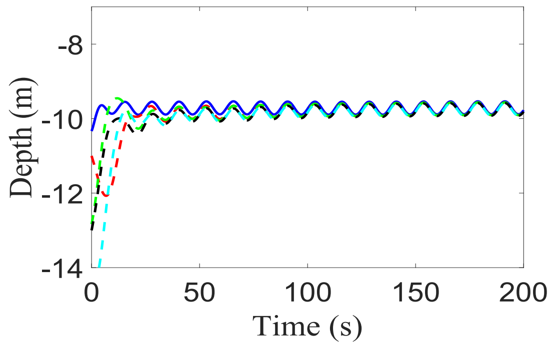

Fig. 4. The depth trajectories of agents.

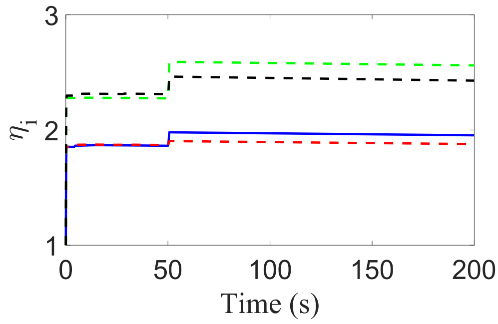

Fig. 5. Adaptive gains $\eta_i, i = 1, \ldots, 4$.

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