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Decentralized Predictor Output Feedback for Large-scale Systems with Large Delays ⋆

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Abstract: A majority of existing literature on time-delay systems focus on the robust stability of a single plant with respect to a "small" delay. This paper proposes a decentralized predictor-based feedback to compensate large delays for large-scale interconnected systems. The full-state of each subsystem is assumed to be unmeasurable and the observer-based output feedback is employed. Two methods are used to tackle the large delays: the backstepping-based partial differential equation (PDE) method is employed for continuous-time control, which derives simpler linear matrix inequality (LMI) conditions and manages with larger delays, whereas the reduction-based ordinary differential equation (ODE) method is applied to sampled-data implementation under continuous-time measurement. Instead of treating the large-scale systems as a whole, a decentralized Lyapunov-Krasovskii method is presented to guarantee the exponential stability of the large-scale systems under decentralized predictors.

Keywords: Decentralized, Predictor, Output Feedback, Large-scale Systems, Delay.

1. INTRODUCTION

By virtue of rapidly-developed communication and digital technologies, networked control systems (NCSs) show great potential in modern control. However, the development of NCSs is also full of challenges. Among many technical difficulties, an important and popular topic is the time-delay, which render the controlled system unstable when disregarded. A large body of existing literature on NCSs concentrate on the robust stability analysis with respect to "small" delays in the feedback loop via communication network. In other words, the delays are not compensated and the largest values of the delays that preserve the performance are investigated in terms of LMI condition Fridman (2014); Freirich (2016); Liu et al. (2012).

To compensate large delays, a key tool is the predictor feedback, which has found a widespread application in practice since it was developed 60 years ago Smith (1959). However, most results assume a single plant with a centralized controller Artstein (1982); Selivanov et al. (2016a,b). The recent paper Liu et al. (2018) considers predictor-based stabilization for two interconnected systems, but the results are based on state feedback and restricted to continuous-time control.

This paper extends the predictor feedback to decentralized control for large-scale interconnected systems with large input delays. Here the large delays denote such delays that do not preserve the stability of the control system (which is stable without the delays), and need compensation. Otherwise, the delays are called small. Different from our preliminary work Zhu et al. (2020) where the state-feedback is considered, this paper addresses a more challenging problem where the full-state of each subsystem is assumed to be unmeasured. The observer-based output feedback is important in implementation. Accordingly, the closed-loop system is more complicated, the proposed Lyapunov-Krasovskii functional (LKF) and the resulting LMI are more sophisticated than those of state feedback. We propose two approaches for the delay compensation: the backstepping-based PDE method and the reduction-based ODE method. The PDE-based predictor is capable to derive simpler LMI conditions and withstand larger delays, whereas the ODE-based method is applicable to both continuous-time and sampled-data stabilization.

Instead of analyzing the large-scale systems as a global system, a decentralized Lyapunov-Krasovskii method is presented for the exponential stability analysis of the large-scale systems under decentralized predictors, in which the delays and sampling instants of each subsystem may be distinct from each other. One of the main challenges of decentralized analysis is to deal with the distributed delay terms from the neighbors. To address the distinct delay terms in the closed-loop system, various inequality techniques like Jensen, Wirtinger, Halanay and S-procedure Fridman (2014) are employed.

2. CONTINUOUS-TIME FEEDBACK

2.1 PDE Framework

Consider large-scale interconnected linear systems with input delays as follows:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t-r_i) + \sum_{i\neq j} F_{ij} x_j(t) \tag{1}$$

$$y_i(t) = C_i x_i(t) \tag{2}$$

where $i = 1,2,\cdots,M$ is the subsystem index, $x_i(t) \in \mathbb{R}^{n_i}$, $y_i(t) \in \mathbb{R}^{q_i}$ and $u_i(t) \in \mathbb{R}^{m_i}$ are the state, output and local control input of the ith plant, respectively, $x_j(t) \in \mathbb{R}^{n_j}$ are coupling terms. The control input is subject to a large constant and known input delay $r_i > 0$. We assume that the plant state $x_i(t)$ is unmeasurable, the pair (A_i, B_i) is stabilizable and (A_i, C_i) is detectable, which means there exist matrices of appropriate

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dimensions K_i and L_i such that $A_i + B_iK_i$ and $A_i - L_iC_i$ are Hurwitz.

In this section, we deal with the case of continuous-time feedback by the PDE-based framework Krstic (2009).

We introduce a multi-variable function

$$v_i(\sigma,t) = u_i(t+\sigma-r_i), \quad \sigma \in [0,r_i] \tag{3}$$

to represent the control input $u_i(\theta)$ over the time interval $\theta \in [t-r_i,t]$. With (3), the system (1)-(2) is represented by the ODE-PDE cascade as follows:

$$\dot{x}_i(t) = A_ix_i(t) + B_iv_i(0,t) + \sum_{i\neq j} F_{ij}x_j(t) \tag{4}$$

$$y_i(t) = C_ix_i(t) \tag{5}$$

$$\partial_t v_i(\sigma,t) = \partial_\sigma v_i(\sigma,t), \quad \sigma \in [0,r_i] \tag{6}$$

$$v_i(r_i,t) = u_i(t) \tag{7}$$

It is apparent that (3) is a solution of the transport PDE (6)-(7). We denote $\hat{x}_i(t)$ to be an estimate of the unmeasured state $x_i(t)$ and the observer is designed as

$$\dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_iv_i(0,t) + L_i\left(y_i(t) - C_i\hat{x}_i(t)\right) \tag{8}$$

with the estimation error $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$ satisfying

$$\dot{\tilde{x}}_i(t) = (A_i - L_iC_i)\tilde{x}_i(t) + \sum_{i\neq j} F_{ij}x_j(t) \tag{9}$$

The predictor-based boundary controller is designed as

$$u_i(t) = v_i(r_i,t) = K_i\left(e^{A_ir_i}\hat{x}_i(t) + \int_0^{r_i} e^{A_i(r_i-\delta)}B_iv_i(\delta,t)d\delta\right) \tag{10}$$

For convenience of stability analysis, we bring in the invertible backstepping transformation

$$\begin{aligned} w_i(\sigma,t) = {} & v_i(\sigma,t) - K_ie^{A_i\sigma}\hat{x}_i(t) \\ & - K_i\int_0^{\sigma} e^{A_i(\sigma-\delta)}B_iv_i(\delta,t)d\delta \end{aligned} \tag{11}$$

$$\begin{aligned} v_i(\sigma,t) = {} & w_i(\sigma,t) + K_ie^{(A_i+B_iK_i)\sigma}\hat{x}_i(t) \\ & + K_i\int_0^{\sigma} e^{(A_i+B_iK_i)(\sigma-\delta)}B_iw_i(\delta,t)d\delta \end{aligned} \tag{12}$$

through which the transport PDE (6)-(7), the observer and its error (8)-(9) are converted into the closed-loop target system as follows:

$$\dot{\hat{x}}_i(t) = (A_i + B_iK_i)\hat{x}_i(t) + B_iw_i(0,t) + L_iC_i\tilde{x}_i(t) \tag{13}$$

$$\dot{\tilde{x}}_i(t) = (A_i - L_iC_i)\tilde{x}_i(t) + \sum_{i\neq j} F_{ij}\left(\hat{x}_j(t) + \tilde{x}_j(t)\right) \tag{14}$$

$$\partial_t w_i(\sigma,t) = \partial_\sigma w_i(\sigma,t) - K_ie^{A_i\sigma}L_iC_i\tilde{x}_i(t), \quad \sigma \in [0,r_i] \tag{15}$$

$$w_i(r_i,t) = 0 \tag{16}$$

Remark 1: Substituting $\sigma = r_i$ into (11), the boundary condition (16) for stabilization is guaranteed by the observer-based feedback law (10). However, in the case of sampled in time inputs, the continuous-time control law (10) is replaced by the sampled-data control $u_i(t) = v_i(r_i,t) = K_i\left(e^{A_ir_i}\hat{x}_i(t_k^i) + \int_0^{r_i} e^{A_i(r_i-\delta)}B_iv_i(\delta,t_k^i)d\delta\right), t \in [t_k^i, t_{k+1}^i), k \in \mathbb{Z}_0^+$, where t_k^i is the sampling instant of the ith subsystem, $\mathbb{Z}_0^+$ stands for the set of non-negative integrals. Accordingly, the boundary condition (16) becomes non-homogeneous such that $w_i(r_i,t) = K_i\left(e^{A_ir_i}\left(\hat{x}_i(t_k^i) - \hat{x}_i(t)\right) + \int_0^{r_i} e^{A_i(r_i-\delta)}B_i\left(v_i(\delta,t_k^i) - v_i(\delta,t)\right)d\delta\right) \neq 0$. Thus it is difficult to apply the PDE-based method to sampled-data control. ■

Theorem 1. Consider the closed-loop system consisting of the plant (1)-(2), observer (8) and controller (10). Given tuning parameters $0 < \varepsilon < \alpha$, let a parameter $\lambda_i > 0$, matrices $P_i, R_i \in \mathbb{R}^{n_i\times n_i} > 0$, $U_i \in \mathbb{R}^{m_i\times m_i} > 0$, $P_j, R_j \in \mathbb{R}^{n_j\times n_j} > 0$, for $j = 1,\cdots,M$ and $j \neq i$, satisfy the LMIs:

$$\Phi_i = \begin{bmatrix} \phi_{11}^i & P_iL_iC_i & P_iB_i & 0 & 0 & 0 \\ * & \phi_{22}^i & 0 & -\lambda_iC_i^TL_i^T & R_i\mathscr{F}_i & R_i\mathscr{F}_i \\ * & * & -U_i & 0 & 0 & 0 \\ * & * & * & -\lambda_iI_{n_i} & 0 & 0 \\ * & * & * & * & \phi_{55}^i & 0 \\ * & * & * & * & * & \phi_{66}^i \end{bmatrix} < 0 \tag{17}$$

$$M_i = \begin{bmatrix} U_i & U_iK_i \\ K_i^TU_i^T & \frac{\lambda_i}{r_i}e^{-2(1+2\alpha)r_i-2|A_i|r_i}I_{n_i} \end{bmatrix} > 0 \tag{18}$$

where Φ_i is a symmetric matrix, $I_{n_i} \in \mathbb{R}^{n_i\times n_i}$ is a unit matrix, $|A_i| = \sqrt{\lambda_{\max}\left(A_i^TA_i\right)}$ and

$$\phi_{11}^i = (A_i + B_iK_i)^TP_i + P_i(A_i + B_iK_i) + 2\alpha P_i,$$

$$\phi_{22}^i = (A_i - L_iC_i)^TR_i + R_i(A_i - L_iC_i) + 2\alpha R_i,$$

$$\phi_{55}^i = \mathrm{diag}_{j=1,\cdots,M}\left\{-\frac{2\varepsilon}{M-1}P_j, j \neq i\right\},$$

$$\phi_{66}^i = \mathrm{diag}_{j=1,\cdots,M}\left\{-\frac{2\varepsilon}{M-1}R_j, j \neq i\right\},$$

$$\mathscr{F}_i = \mathrm{row}_{j=1,\cdots,M}\left\{F_{ij}, j \neq i\right\}.$$

Then the closed-loop large-scale system is exponentially stable with a decay rate $\rho = \alpha - \varepsilon$. ■

proof: The Lyapunov-Krasovskii functional (LKF) is selected as $V_i(t) = V_{P_i}(t) + V_{R_i}(t) + V_{U_i}(t)$ where

$$V_{P_i}(t) = \hat{x}_i^T(t)P_i\hat{x}_i(t), \quad P_i > 0 \tag{19}$$

$$V_{R_i}(t) = \tilde{x}_i^T(t)R_i\tilde{x}_i(t), \quad R_i > 0 \tag{20}$$

$$V_{U_i}(t) = \int_0^{r_i} e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_iw_i(\sigma,t)d\sigma, \quad U_i > 0 \tag{21}$$

Taking the time derivative of (19) along (13), we have

$$\begin{aligned} & \dot{V}_{P_i}(t) + 2\alpha V_{P_i}(t) \\ & = \hat{x}_i^T(t)\left(2P_i(A_i + B_iK_i) + 2\alpha P_i\right)\hat{x}_i(t) \\ & \quad + 2\hat{x}_i^T(t)P_iB_iw_i(0,t) + 2\hat{x}_i^T(t)P_iL_iC_i\tilde{x}_i(t) \end{aligned} \tag{22}$$

Taking the time derivative of (20) along (14), we get

$$\begin{aligned} & \dot{V}_{R_i}(t) + 2\alpha V_{R_i}(t) \\ & = \tilde{x}_i^T(t)\left(2R_i(A_i - L_iC_i) + 2\alpha R_i\right)\tilde{x}_i(t) \\ & \quad + 2\tilde{x}_i^T(t)R_i\sum_{i\neq j} F_{ij}\left(\hat{x}_j(t) + \tilde{x}_j(t)\right) \end{aligned} \tag{23}$$

Taking the time derivative of (21) along (15)-(16) and using the integration by parts in σ, we obtain

$$\begin{aligned} & \dot{V}_{U_i}(t) + 2\alpha V_{U_i}(t) \\ & = 2\int_0^{r_i} e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_i\partial_\sigma w_i(\sigma,t)d\sigma \\ & \quad - 2\int_0^{r_i} e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_iK_ie^{A_i\sigma}d\sigma L_iC_i\tilde{x}_i(t) \\ & \quad + 2\alpha\int_0^{r_i} e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_iw_i(\sigma,t)d\sigma \\ & = -w_i^T(0,t)U_iw_i(0,t) - 2\xi_i^T(t)L_iC_i\tilde{x}_i(t) \\ & \quad - \int_0^{r_i} e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_iw_i(\sigma,t)d\sigma \end{aligned} \tag{24}$$

where $\xi_i^T(t) = \int_0^{r_i} e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_iK_ie^{A_i\sigma}d\sigma$.

Utilizing Jensen's inequality, $\xi_i^T(t)$ satisfies

$$\begin{aligned}\left|\xi_i^T(t)\right|^2 &= \left|\int_0^{r_i} e^{(1+2\alpha)\sigma} w_i^T(\sigma,t)U_iK_ie^{A_i\sigma}d\sigma\right|^2 \\ &\le r_i\int_0^{r_i}\left|e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_iK_ie^{A_i\sigma}\right|^2 d\sigma \\ &\le \underbrace{r_ie^{2(1+2\alpha)r_i+2|A_i|r_i}}_{\mu_i}\int_0^{r_i}\left|w_i^T(\sigma,t)U_iK_i\right|^2 d\sigma \end{aligned} \quad (25)$$

From (22)-(25), we have

$$\begin{aligned}&\dot{V}_i(t)+2\alpha V_i(t)-\frac{2\varepsilon}{M-1}\sum_{i\neq j}V_j(t) \\ &\quad+\frac{1}{\lambda_i}\left(\mu_i\int_0^{r_i}\left|w_i^T(\sigma,t)U_iK_i\right|^2 d\sigma-\left|\xi_i^T(t)\right|^2\right) \\ &\le -\int_0^{r_i}w_i^T(\sigma,t)\left(U_i-\frac{\mu_i}{\lambda_i}U_iK_iK_i^TU_i^T\right)w_i(\sigma,t)d\sigma \\ &\quad+\eta_i^T(t)\mathrm{diag}\left\{I,I,\frac{1}{\lambda_i}I,I\right\}\Phi_i\mathrm{diag}\left\{I,I,\frac{1}{\lambda_i}I,I\right\}\eta_i(t)\le 0 \end{aligned} \quad (26)$$

where $\lambda_i > 0$ and $\eta_i(t) = \mathrm{col}\{\hat{x}_i(t), \tilde{x}_i(t), w_i(0,t), \xi_i(t), \mathrm{col}_{j=1,\cdots,M}\{\hat{x}_j(t), j\neq i\}, \mathrm{col}_{j=1,\cdots,M}\{\tilde{x}_j(t), j\neq i\}\}$ and I is a unit matrix of appropriate dimension.

Applying Schur complement lemma in Section 3.2.3 of Fridman (2014), the inequality (26) is implied by LMI-condition (17)-(18). From (26), we conclude the LKF candidate along the solution of closed-loop system (13)-(16) satisfies $\dot{V}_i(t)+2\alpha V_i(t)\le\frac{2\varepsilon}{M-1}\sum_{j\neq i}V_j(t)$, then we have $\dot{V}(t)+2(\alpha-\varepsilon)V(t)\le 0$ where $V(t)=\sum_{i=1}^M V_i(t)$, which implies the exponential stability of the closed-loop system by the comparison principle. ■

Remark 2: In our preliminary work Zhu et al. (2020) where the full-state of each subsystem is assumed to be measurable, we compare the conventionally centralized analysis with the decentralized analysis which is similar to Theorem 1. In the centralized analysis, we treat the large-scale system as a global system and apply a full-order LKF to stability analysis. It is revealed that the LMIs via the decentralized method have essentially less decision variables and are of smaller order comparatively to the LMI resulting from the centralized one. In the sampled-data case with asynchronous sampling, the decentralized analysis leads to essentially simpler results than the centralized one, where multiple integral terms should be inserted into LKF to take care of multiple samplings. This advantage should be more apparent in the observer-based output feedback considered in this paper since the closed-loop system (13)-(16) is higher-order. ■

2.2 ODE Framework

In this section, as shown in Fig. 1, we still address the case of continuous-time control. To lay a foundation for the sampled-data implementation in later sections, we employ the ODE scheme here.

Concentrating on the system (1)-(2), the variable $\hat{x}_i(t)$ is used to denote the estimate of the unmeasured state $x_i(t)$ with the estimation error $\tilde{x}_i(t) = x_i(t)-\hat{x}_i(t)$. The observer is designed as

$$\dot{\hat{x}}_i(t) = A_i\hat{x}_i(t)+B_iu_i(t-r_i)+L_i\left(y_i(t)-C_i\hat{x}_i(t)\right) \quad (27)$$

We introduce the change of variable

$$\hat{z}_i(t) = e^{A_ir_i}\hat{x}_i(t)+\int_{t-r_i}^t e^{A_i(t-s)}B_iu_i(s)ds \quad (28)$$

If the term of estimation error $L_i\left(y_i(t)-C_i\hat{x}_i(t)\right)=0$ in (27), it is evident that (28) is an exact prediction of the future state $\hat{z}_i(t)=\hat{x}_i(t+r_i)$.

The predictor control law is selected as

$$\begin{aligned} u_i(t) &= K_i\hat{z}_i(t) && (29)\\ &= K_i\left(e^{A_ir_i}\hat{x}_i(t)+\int_{t-r_i}^t e^{A_i(t-s)}B_iu_i(s)ds\right) && (30)\end{aligned}$$

For stability analysis, taking the time derivative of (28) along (27), the dynamics of $\hat{z}_i(t)$ is calculated as

$$\begin{aligned}\dot{\hat{z}}_i(t) &= A_i\hat{z}_i(t)+B_iu_i(t)+e^{A_ir_i}L_i\left(y_i(t)-C_i\hat{x}_i(t)\right) \\ &= (A_i+B_iK_i)\hat{z}_i(t)+e^{A_ir_i}L_iC_i\tilde{x}_i(t) \end{aligned} \quad (31)$$

Making use of (29), the inverse conversion of (28) is brought in as

$$\begin{aligned}\hat{x}_j(t) &= e^{-A_jr_j}\hat{z}_j(t)-\int_{t-r_j}^t e^{A_j(t-r_j-s)}B_ju_j(s)ds \\ &= e^{-A_jr_j}\hat{z}_j(t)-\int_{t-r_j}^t e^{A_j(t-r_j-s)}B_jK_j\hat{z}_j(s)ds \\ &= e^{-A_jr_j}\hat{z}_j(t)-\underbrace{\int_{-r_j}^0 e^{-A_j(\theta+r_j)}B_jK_j\hat{z}_j(t+\theta)d\theta}_{\xi_j(t)} \end{aligned} \quad (32)$$

Subtracting (27) from (1) and substituting (32), the estimation error $\tilde{x}_i(t)=x_i(t)-\hat{x}_i(t)$ satisfies

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= A_i\tilde{x}_i(t)-L_i\left(y_i(t)-C_i\hat{x}_i(t)\right)+\sum_{i\neq j}F_{ij}x_j(t) \\ &= (A_i-L_iC_i)\tilde{x}_i(t)+\sum_{i\neq j}F_{ij}\left(\tilde{x}_j(t)+\hat{x}_j(t)\right) \\ &= (A_i-L_iC_i)\tilde{x}_i(t)+\sum_{i\neq j}F_{ij}\left(\tilde{x}_j(t)+e^{-A_jr_j}\hat{z}_j(t)-\xi_j(t)\right) \end{aligned} \quad (33)$$

Theorem 2. Consider the closed-loop system consisting of the plant (1)-(2), observer (27) and controller (30). Given tuning parameters $0<\varepsilon<\alpha$, let matrices $P_i, R_i, W_i \in \mathbb{R}^{n_i\times n_i} > 0$, $P_j, R_j, W_j \in \mathbb{R}^{n_j\times n_j} > 0$, for $j = 1,\cdots,M$ and $j\neq i$, satisfy the LMIs:

$$\Phi_i = \begin{bmatrix}\phi_{11}^i & P_ie^{A_ir_i}L_iC_i & 0 & 0 & 0 \\ * & \phi_{22}^i & R_i\mathscr{F}_i & R_i\mathscr{F}_i^{\hat{z}} & -R_i\mathscr{F}_i \\ * & * & \phi_{33}^i & 0 & 0 \\ * & * & * & \phi_{44}^i & 0 \\ * & * & * & * & \phi_{55}^i\end{bmatrix} < 0 \quad (34)$$

where Φ_i is a symmetric matrix, and

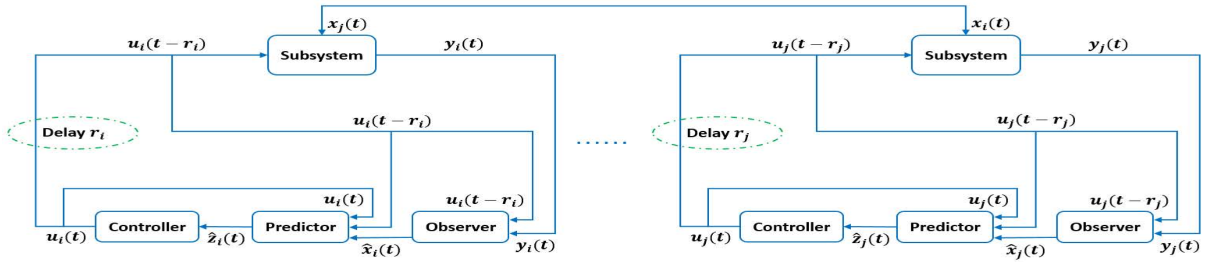

Fig. 1. Continuous-time Control for Large-scale Systems with Input Delays

$$
\begin{aligned}
\phi_{11}^i &= (A_i + B_iK_i)^T P_i + P_i(A_i + B_iK_i) + 2\alpha P_i + \bar{W}_i, \\
\bar{W}_i &= r_i K_i^T B_i^T \left(\int_{-r_i}^{0} e^{-A_i^T(\theta+r_i)} W_i e^{-A_i(\theta+r_i)} d\theta \right) B_i K_i \\
\phi_{22}^i &= (A_i - L_iC_i)^T R_i + R_i(A_i - L_iC_i) + 2\alpha R_i, \\
\phi_{33}^i &= \mathrm{diag}_{j=1,\cdots,M} \left\{ -\frac{2\varepsilon}{M-1} R_j, j \neq i \right\}, \\
\phi_{44}^i &= \mathrm{diag}_{j=1,\cdots,M} \left\{ -\frac{2\varepsilon}{M-1} P_j, j \neq i \right\}, \\
\phi_{55}^i &= \mathrm{diag}_{j=1,\cdots,M} \left\{ -\frac{1}{M-1} e^{-2\alpha r_j} W_j, j \neq i \right\}, \\
\mathscr{F}_i &= \mathrm{row}_{j=1,\cdots,M} \left\{ F_{ij}, j \neq i \right\}, \\
\mathscr{F}_i^{\hat{z}} &= \mathrm{row}_{j=1,\cdots,M} \left\{ F_{ij} e^{-A_j r_j}, j \neq i \right\}.
\end{aligned}
$$

Then the closed-loop system is exponentially stable with a decay rate $\rho = \alpha - \varepsilon$. ■

proof: The LKF is constructed as $V_i(t) = V_{P_i}(t) + V_{R_i}(t) + V_{W_i}(t)$ where

$$V_{P_i}(t) = \hat{z}_i^T(t) P_i \hat{z}_i(t), \quad P_i > 0 \tag{35}$$

$$V_{R_i}(t) = \tilde{x}_i^T(t) R_i \tilde{x}_i(t), \quad R_i > 0 \tag{36}$$

$$
\begin{aligned}
V_{W_i}(t) = r_i \int_{-r_i}^{0} \int_{t+\theta}^{t} & e^{2\alpha(s-t)} \hat{z}_i^T(s) K_i^T B_i^T e^{-A_i^T(\theta+r_i)} W_i \\
& \times e^{-A_i(\theta+r_i)} B_i K_i \hat{z}_i(s) ds d\theta, \quad W_i > 0
\end{aligned} \tag{37}
$$

Please note that $V_{W_i}(t)$ is used to handle the distributed delay $\xi_j(t)$ in (33).

Taking the time derivative of (35) along (31), we have

$$
\begin{aligned}
&\dot{V}_{P_i}(t) + 2\alpha V_{P_i}(t) \\
&= \hat{z}_i^T(t) \big(2P_i(A_i + B_iK_i) + 2\alpha P_i \big) \hat{z}_i(t) \\
&\quad + 2\hat{z}_i^T(t) P_i e^{A_i r_i} L_i C_i \tilde{x}_i(t)
\end{aligned} \tag{38}
$$

Taking the time derivative of (36) along (33), we get

$$
\begin{aligned}
&\dot{V}_{R_i}(t) + 2\alpha V_{R_i}(t) \\
&= \tilde{x}_i^T(t) \big(2R_i(A_i - L_iC_i) + 2\alpha R_i \big) \tilde{x}_i(t) \\
&\quad + 2\tilde{x}_i^T(t) R_i \sum_{i \neq j} F_{ij} \left(\tilde{x}_j(t) + e^{-A_j r_j} \hat{z}_j(t) - \xi_j(t) \right)
\end{aligned} \tag{39}
$$

Taking the time derivative of (37) and using Jensen's inequality, we have

$$
\begin{aligned}
&\dot{V}_{W_i}(t) + 2\alpha V_{W_i}(t) \\
&= r_i \hat{z}_i^T(t) K_i^T B_i^T \left(\int_{-r_i}^{0} e^{-A_i^T(\theta+r_i)} W_i e^{-A_i(\theta+r_i)} d\theta \right) B_i K_i \hat{z}_i(t) \\
&\quad - r_i \int_{-r_i}^{0} e^{2\alpha\theta} \hat{z}_i^T(t+\theta) K_i^T B_i^T e^{-A_i^T(\theta+r_i)} \\
&\quad \times W_i e^{-A_i(\theta+r_i)} B_i K_i \hat{z}_i(t+\theta) d\theta \\
&\leq \hat{z}_i^T(t) \bar{W}_i \hat{z}_i(t) \\
&\quad - e^{-2\alpha r_i} \left(\int_{-r_i}^{0} \hat{z}_i^T(t+\theta) K_i^T B_i^T e^{-A_i^T(\theta+r_i)} d\theta \right) \\
&\quad \times W_i \left(\int_{-r_i}^{0} e^{-A_i(\theta+r_i)} B_i K_i \hat{z}_i(t+\theta) d\theta \right) \\
&= \hat{z}_i^T(t) \bar{W}_i \hat{z}_i(t) - e^{-2\alpha r_i} \xi_i^T(t) W_i \xi_i(t)
\end{aligned} \tag{40}
$$

where $\bar{W}_i$ has been given underneath (34).

From (38)-(40), we get

$$
\begin{aligned}
&\dot{V}_i(t) + 2\alpha V_i(t) - \frac{2\varepsilon}{M-1} \sum_{j \neq i} V_j(t) + e^{-2\alpha r_i} \xi_i^T(t) W_i \xi_i(t) \\
&\quad - \frac{1}{M-1} \sum_{j \neq i} e^{-2\alpha r_j} \xi_j^T(t) W_j \xi_j(t) \\
&\leq \eta_i^T(t) \Phi_i \eta_i(t) \leq 0
\end{aligned} \tag{41}
$$

where $\eta_i(t) = \mathrm{col}\{\hat{z}_i(t), \tilde{x}_i(t), \mathrm{col}_{j=1,\cdots,M}\{\tilde{x}_j(t), j \neq i\}, \mathrm{col}_{j=1,\cdots,M}\{\hat{z}_j(t), j \neq i\}, \mathrm{col}_{j=1,\cdots,M}\{\xi_j(t), j \neq i\}\}$. It is apparent that inequality (41) is suggested by LMI-condition (34). Thus we derive $\dot{V}(t) + 2(\alpha - \varepsilon)V(t) \leq 0$ from (41) where $V(t) = \sum_{i=1}^{M} V_i(t)$, which implies the exponential stability of the closed-loop system. ■

3. SAMPLED-DATA FEEDBACK WITH CONTINUOUS-TIME MEASUREMENT

In this section, as revealed in Fig. 2, we consider a more complicated case where the system is with a controller-to-actuator network subject to a large transmission delay r_i and is able to continuously measure the plant output $y_i(t)$. The continuous-time control signal $u_i(t)$ is sampled at the time instants ζ_k^i and sent to the zero-order hold (ZOH) through the delayed network. The sampling series $\{\zeta_k^i\}$ satisfy

$$0 = \zeta_0^i < \zeta_1^i < \zeta_2^i < \cdots, \quad \lim_{k \to \infty} \zeta_k^i = \infty, \quad \zeta_{k+1}^i - \zeta_k^i \leq h_i \tag{42}$$

The ZOH is assumed to be event-driven so that it updates its output once it receives new data. Thus the updating instants of the ZOH satisfies $t_k^i = \zeta_k^i + r_i, \quad t_k^i < t_{k+1}^i, \quad k \in \mathbb{Z}_0^+$.

As analyzed in Remark 1, when the control signals are sampled, the PDE-based method is not trivially applicable to NCSs so that the ODE-based approach is employed.

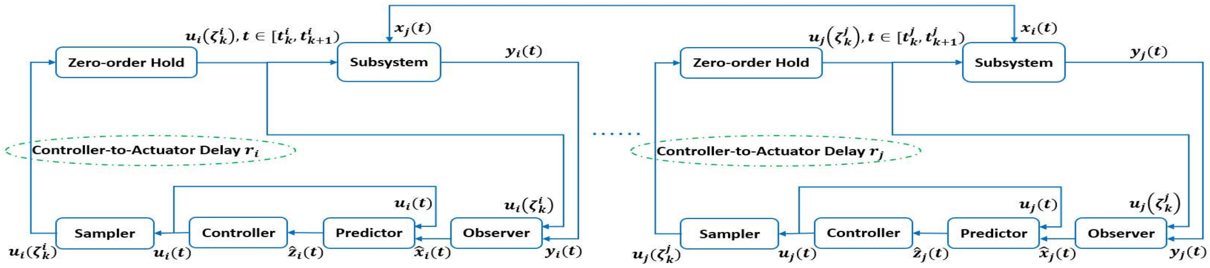

Fig. 2. Sampled-data Control with Continuous-time Measurement for Large-scale Systems with Delays

Under the controller-to-actuator network with delay, it is evident that the system (1)-(2) becomes

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(\zeta_k^i) + \sum_{i\neq j} F_{ij} x_j(t), \quad t \in [t_k^i, t_{k+1}^i) \quad (43)$$

$$y_i(t) = C_i x_i(t) \quad (44)$$

Based on the system (43)-(44), we denote $\hat{x}_i(t)$ to be the estimate of the unmeasured state $x_i(t)$, and $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$ to be the estimation error. We design the observer as

$$\dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_i u_i(\zeta_k^i) + L_i\left(y_i(t) - C_i\hat{x}_i(t)\right), \quad t \in [t_k^i, t_{k+1}^i) \quad (45)$$

We select the observer-based predictor as

$$\hat{z}_i(t) = e^{A_i r_i}\hat{x}_i(t) + \int_{t-r_i}^{t} e^{A_i(t-s)} B_i u_i(s) ds \quad (46)$$

and chose the predictor-based control law as

$$\begin{aligned} u_i(t) &= K_i \hat{z}_i(t) & (47)\\ &= K_i\left(e^{A_i r_i}\hat{x}_i(t) + \int_{t-r_i}^{t} e^{A_i(t-s)} B_i u_i(s) ds\right) & (48)\end{aligned}$$

For stability analysis, the dynamics of $\hat{z}_i(t)$ along (45) and (47) is of the form

$$\begin{aligned}\dot{\hat{z}}_i(t) &= A_i\hat{z}_i(t) + B_i u_i(t) + e^{A_i r_i} B_i\left(u_i(\zeta_k^i) - u_i(t-r_i)\right)\\ &\quad + e^{A_i r_i} L_i\left(y_i(t) - C_i\hat{x}_i(t)\right)\\ &= (A_i + B_i K_i)\hat{z}_i(t) + e^{A_i r_i} B_i K_i v_i(t) + e^{A_i r_i} L_i C_i \tilde{x}_i(t),\\ &\quad t \in [t_k^i, t_{k+1}^i) \qquad (49)\end{aligned}$$

where $v_i(t) = \hat{z}_i(\zeta_k^i) - \hat{z}_i(t-r_i) = \hat{z}_i(t_k^i - r_i) - \hat{z}_i(t-r_i)$.

Subtracting (45) from (43) and utilizing the inverse transformation of (46), the estimation error $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$ is govern by

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= A_i\tilde{x}_i(t) - L_i\left(y_i(t) - C_i\hat{x}_i(t)\right) + \sum_{i\neq j} F_{ij} x_j(t)\\ &= (A_i - L_iC_i)\tilde{x}_i(t) + \sum_{i\neq j} F_{ij}\left(\tilde{x}_j(t) + e^{-A_j r_j}\hat{z}_j(t) - \xi_j(t)\right),\\ &\quad t \in [t_k^i, t_{k+1}^i) \qquad (50)\end{aligned}$$

where $\xi_j(t) = \int_{-r_j}^{0} e^{-A_j(\theta + r_j)} B_j K_j \hat{z}_j(t+\theta) d\theta$.

Remark 3: In Selivanov et al. (2016b) where a single plant is considered, besides the observer predictor (46), the plant predictor is also introduced such that $z_i(t) = e^{A_i r_i} x_i(t) + \int_{t-r_i}^{t} e^{A_i(t-s)} B_i u_i(s) ds$. If the method of Selivanov et al. (2016b) is applied to large-scale systems, an alternative version of the closed-loop system (49)-(50) is of the form: $\dot{\hat{z}}_i(t) = (A_i + B_iK_i)\hat{z}_i(t) + e^{A_i r_i} L_i C_i e^{-A_i r_i}\tilde{z}_i(t) + e^{A_i r_i} B_i K_i v_i(t)$, $\dot{\tilde{z}}_i(t) = (A_i - e^{A_i r_i} L_i C_i e^{-A_i r_i})\tilde{z}_i(t) + e^{A_i r_i}\sum_{i\neq j} F_{ij}\left(e^{-A_j r_j}\tilde{z}_j(t) + e^{-A_j r_j}\hat{z}_j(t) - \xi_j(t)\right)$, $t \in [t_k^i, t_{k+1}^i)$, where $\tilde{z}_i(t) = z_i(t) - \hat{z}_i(t)$. It is apparent that (49)-(50) proposed in the present paper is simpler and the redundant change of variable $z_i(t)$ is avoided.

Theorem 3. Consider the closed-loop system consisting of the plant (43)-(44), observer (45) and controller (48). Given tuning parameters $0 < \varepsilon < \alpha$, let matrices $P_i, R_i, W_i, U_i \in \mathbb{R}^{n_i \times n_i} > 0$ and $P_j, R_j, W_j \in \mathbb{R}^{n_j \times n_j} > 0$ for $j = 1, \cdots, M$ and $j \neq i$, satisfy the LMI:

$$\begin{bmatrix}\Phi_i & \Psi_i\\ * & -H_i\end{bmatrix} < 0 \quad (51)$$

where

$$\Psi_i = \begin{bmatrix}(A_i + B_iK_i)^T H_i\\ C_i^T L_i^T e^{A_i^T r_i} H_i\\ K_i^T B_i^T e^{A_i^T r_i} H_i\\ 0\\ 0\\ 0\end{bmatrix}, \quad H_i = h_i^2 e^{2\alpha h_i} U_i \quad (52)$$

and Φ_i is a symmetric matrix such that

$$\Phi_i = \begin{bmatrix}\phi_{11}^i & P_i e^{A_i r_i} L_i C_i & P_i e^{A_i r_i} B_i K_i & 0 & 0 & 0\\ * & \phi_{22}^i & 0 & R_i\mathscr{F}_i & R_i\mathscr{F}_i^{\hat{z}} & -R_i\mathscr{F}_i\\ * & * & \phi_{33}^i & 0 & 0 & 0\\ * & * & * & \phi_{44}^i & 0 & 0\\ * & * & * & * & \phi_{55}^i & 0\\ * & * & * & * & * & \phi_{66}^i\end{bmatrix} < 0 \quad (53)$$

where Φ_i is a symmetric matrix, and $\phi_{11}^i, \phi_{22}^i, \mathscr{F}_i, \mathscr{F}_i^{\hat{z}}$ in (53) are exactly the same with $\phi_{11}^i, \phi_{22}^i, \mathscr{F}_i, \mathscr{F}_i^{\hat{z}}$ in (34), and $\phi_{33}^i = -\frac{\pi^2}{4} e^{-2\alpha r_i} U_i$, and $\phi_{44}^i, \phi_{55}^i, \phi_{66}^i$ in (53) are respectively identical with $\phi_{33}^i, \phi_{44}^i, \phi_{55}^i$ in (34). Then the closed-loop system is exponentially stable with a decay rate $\rho = \alpha - \varepsilon$. ■

proof: The LKF is built as $V_i(t) = V_{P_i}(t) + V_{R_i}(t) + V_{W_i}(t) + V_{U_i}(t)$ where $V_{P_i}(t), V_{R_i}(t), V_{W_i}(t)$ are exactly the same as (35)-(37), and

$$\begin{aligned}V_{U_i}(t) &= h_i^2 e^{2\alpha h_i}\int_{t_k^i - r_i}^{t} e^{2\alpha(s-t)}\dot{\hat{z}}_i^T(s) U_i \dot{\hat{z}}_i(s) ds\\ &\quad - \frac{\pi^2}{4}\int_{t_k^i - r_i}^{t - r_i} e^{2\alpha(s-t)}[\hat{z}_i(s) - \hat{z}_i(t_k^i - r_i)]^T U_i\\ &\quad \times [\hat{z}_i(s) - \hat{z}_i(t_k^i - r_i)] ds, \quad U_i > 0, \qquad (54)\\ &\quad t \in [t_k^i, t_{k+1}^i), \quad k \in \mathbb{Z}_0^+\end{aligned}$$

Please note that $V_{U_i}(t) \geq 0$ and $\lim_{t\to(t_k^i)^-} V_{U_i}(t) \geq V_{U_i}(t_k^i)$ by Wirtinger's inequality in Liu et al. (2012), Selivanov et al. (2016b) and Section 7.4 of Fridman (2014). The term $V_{W_i}(t)$ is employed to compensate $\xi_j(t)$ in (50), whereas $V_{U_i}(t)$ is utilized to compensate $v_i(t)$ in (49). The remaining is similar to the step of proof of Theorem 2.

$\blacksquare$

4. SIMULATION

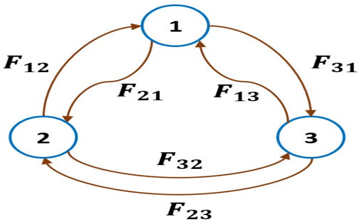

Fig. 3. Three Interconnected Subsystems

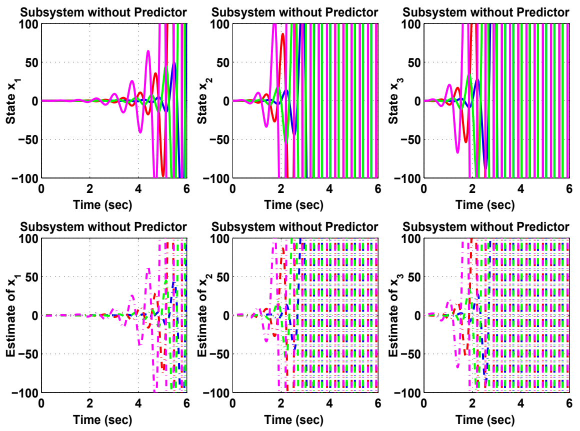

Fig. 4. Predictor-free Feedback with Small Delays $r_1 = r_2 = r_3 = 0.1s$

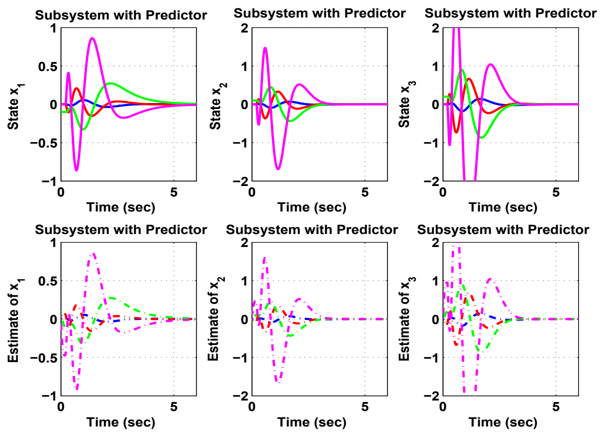

Fig. 5. Predictor-based Feedback with Large Delays $r_1 = r_2 = r_3 = 0.13s$

In this section, we use an example of two coupled inverted pendulums on two carts from Borgers et al. (2014) under the decentralized control scheme.

The system matrices are $A_1 = A_2 = A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2.9156 & 0 & -0.0005 & 0 \\ 0 & 0 & 0 & 1 \\ -1.6663 & 0 & 0.0002 & 0 \end{bmatrix}$, $B_1 = B_2 = B_3 = \begin{bmatrix} 0 \\ -0.0042 \\ 0 \\ 0.0167 \end{bmatrix}$, $C_1 = C_2 = C_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $F_{21} = F_{12} = F_{23} = F_{13} = F_{31} = F_{13} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0011 & 0 & 0.0005 & 0 \\ 0 & 0 & 0 & 0 \\ -0.0003 & 0 & -0.0002 & 0 \end{bmatrix}$. The control gains are selected as $K_1 = \begin{bmatrix} 11396 & 7196.2 & 573.96 & 1199.0 \end{bmatrix}$, $K_2 = K_3 = \begin{bmatrix} 29241 & 18135 & 2875.3 & 3693.9 \end{bmatrix}$. The observer gains are selected as $L_1 = L_2 = L_3 = \begin{bmatrix} 11.7 & -1.2 \\ 37 & -8.9 \\ -1.2 & 11 \\ -7.9 & 36 \end{bmatrix}$.

The simulation results are shown in Figs. 3-5. It is evident that the predictor-based controller promises a larger delay than the predictor-free controller.

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