Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Probabilistic Bounds on Vehicle Trajectory Prediction Using Scenario Approach

Xun Shen * **Xingguo Zhang** * **Pongsathorn Raksincharoensak** *

* *Department of Mechanical Systems Engineering, Tokyo University of Agriculture and Technology, Tokyo, 184-8588, Japan, (e-mail: shenxun@go.tuat.ac.jp; xgzhang@go.tuat.ac.jp; pong@cc.tuat.ac.jp).*

Abstract: The automotive industry concerns about improving road safety. One of the major challenges is to assess road risk and react accordingly in order to avoid accidents. This requires predicting the evolution of the surrounding vehicle trajectories. However, the prediction involves uncertainties from driver operations and ground situations. It is critical to obtain the vehicle trajectory prediction with probabilistic-guarantee bounds. This contribution paper proposes a novel approach to obtain probabilistic ellipsoidal bounds for vehicle trajectory prediction. The vehicle dynamics model adopts a classical bicycle model. The uncertainty of the future trajectory is from the driver's intend and road condition which can be simplified by setting some parameters of the vehicle dynamics model as a stochastic model. Then, a stochastic optimization problem is formulated to obtain the probabilistic ellipsoidal bounds on the future vehicle trajectories. The proposed approach is validated in a numerical simulation which shows the relationship between the computation complexity and the conservatism of the probabilistic ellipsoidal bounds. The proposed method can be generally used for a physics-based motion method, maneuver-based motion method, and interaction-aware motion method by defining the probability distribution of uncertain variables differently.

Keywords: Vehicles, motion prediction, motion model, probabilistic bound, scenario approach.

1. INTRODUCTION

For both Advanced Driver Assistance Systems (ADAS) and Autonomous Vehicles, safety is the cornerstone and types of research have been focused on assuring a safe, comfortable and cooperative experience for drivers and passengers. For realization of road safety, the ADAS or Autonomous Vehicles should be capable of assessing the road risk to avoid potential accidents. An intuitive explanation of risk is the severity and likelihood of the damage that a vehicle may potentially suffer in the future. Since the major traffic participants are vehicles, it is necessary to predict the evolution of the vehicle trajectory under various scenarios.

The current vehicle trajectory prediction method can be roughly summarized into three classifications: physics-based motion method, maneuver-based motion method, and interaction-aware motion method (S. Lefevre et al. , 2014). The physical-based motion method gives the vehicle trajectory prediction considering only the laws of physics (C-F Lin et al. , 2000; A. Eidehall et al. , 2008). The maneuver-based motion method takes driver intends into account, for instance, turning left or right, lane changing et al (G. S. Aoude et al. , 2012). The interaction-aware motion method is a refined edition of the maneuver-based motion method by adding the dependencies between vehicles maneuvers into consideration (G. Agamennoni et al. , 2012). Recently, Deep Learning (DL) and Neural Networks (NNs) become popular in the vehicle trajectory prediction (C. Ju et al. , 2019; Y. Hu et al. , 2018; X. Huang et al. , 2019). In (Y. Hu et al. , 2018), a semantic-based intention and motion prediction structure were presented for probabilistic vehicle trajectory prediction. The uncertainty of driver intention was used to estimate prediction confidence which was added to the physics-based predictor to improve prediction performance in (X. Huang et al. , 2019). Especially, (C. Ju et al. , 2019) proposed two-layer prediction methodology with interaction for acceleration prediction and motion layer for motion prediction. The interaction layer gives the interaction-aware acceleration of a vehicle based on NNs model. The motion layer uses a simple two-order primary kinematic equation as a vehicle dynamics model and Kalman Filter is applied to give recursive prediction revision. The above researches are capable to give a single trajectory or multiple possible vehicle trajectories. While they are not able to calculate the probabilistic bounds on the vehicle trajectories which are more clear to reflect the potential risk. Especially, fitting an ellipsoid to predicted samples is not done before for the vehicle trajectory predictions. In (T. Campbell et al. , 2015), a basic uncertainty set is constructed from a union of posterior predictive ellipsoids for the Dirichlet process Gaussian mixture.

This study addresses the problem of calculating the probabilistic ellipsoidal bounds on the vehicle trajectory which is inspired by the work in (C. Ju et al. , 2019) and (T. Campbell et al. , 2015). A classical bicycle model is used to describe vehicle dynamics. The uncertainty of the future trajectory are from the driver's intend and road condition which can be simplified by setting some parameters of the vehicle dynamics model as a stochastic variable. Then, a

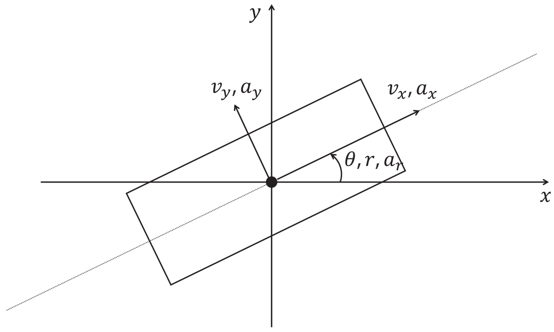

Fig. 1. Brief illustration of the vehicle model.

stochastic optimization problem is formulated to obtain the probabilistic ellipsoidal bounds on the future vehicle trajectories. A numerical simulation is conducted to validate the proposed approach which shows the relationship between the computation complexity and the conservatism of the probabilistic ellipsoidal bounds. Generally, the proposed method for probabilistic bounds computation is not constrained by the complexity of the model and can be used for a physics-based motion method, maneuver-based motion method, and interaction-aware motion method by changing the probability distribution of uncertain variables.

The paper is organized as follows. Section 2 formulates the problem after introducing the vehicle dynamics model. Then, the proposed method for computing the probabilistic ellipsoidal bounds on vehicle trajectory is presented in Section 3. In Section 4, the proposed method is validated through a numerical simulation. Finally, Section 5 concludes the contribution paper.

2. PROBLEM DESCRIPTION

2.1 Vehicle Dynamics Model

The vehicle dynamic model is implemented as a classical bicycle model as shown in Fig. 1 by following the contents of (S. Taheri, 1990). Specifically, denote the state variable as $s := \{x, y, \theta, v_x, v_y, r\}$ where x and y are the coordinates of position, θ is the orientation, v_x and v_y are velocities, and r is the yaw rate. The discrete bicycle model is written as follows:

$$\begin{aligned} &v_{x,k+1} = v_{x,k} + a_{x,k}dt, \\ &v_{y,k+1} = v_{y,k} + a_{y,k}dt, \\ &r_{k+1} = r_k + a_{r,k}dt, \\ &\theta_{k+1} = \theta_k + r_k dt, \\ &x_{k+1} = x_k + \{v_{x,k}cos(\theta_k) - v_{y,k}sin(\theta_k)\}dt, \\ &y_{k+1} = y_k + \{v_{x,k}sin(\theta_k) + v_{y,k}cos(\theta_k)\}dt \end{aligned} \tag{1}$$

where $k = 0, 1, ...$ denotes the time index, and dt is a constant sample time, and $a_{x,k}$, $a_{y,k}$ and $a_{r,k}$ denotes the accelerations of coordinates and yaw rate at time step k respectively. Moreover, $a_{x,k}$, $a_{y,k}$ and $a_{r,k}$ depends on the driver operation and the road condition which are uncertain. The sample spaces of them can be denoted as $\Delta_{a_{x,k}} \subset \mathbb{R}$, $\Delta_{a_{y,k}} \subset \mathbb{R}$ and $\Delta_{a_{r,k}} \subset \mathbb{R}$ respectively. The probability measures, $\mathrm{Pr}_{a_{x,k}}$, $\mathrm{Pr}_{a_{y,k}}$ and $\mathrm{Pr}_{a_{r,k}}$, are known but not limited as Gaussian distribution. This study does not concerns about how to get the probability measure which can be obtained as the study in (C. Ju et al. , 2019). For simplicity, $\{a_{x,k}, a_{y,k}, a_{r,k}\}$ is denoted as stochastic variable vector ω_k. Accordingly, the sample spaces is as $\Delta_{\omega_k} \subset \mathbb{R}^3$ and the probability measure is as Pr_{ω_k}. Moreover, the initial state s_0 is also uncertain with sample space as $\Delta_{s_0} \subset \mathbb{R}^6$ and probability measure as Pr_{s_0}. Then, Eq. 1 is simplified as

$$\begin{aligned} &s_{k+1} = f(s_k, \omega_k), \\ &\omega_k \in \Delta_{\omega_k} \subset \mathbb{R}^3 \\ &s_0 \in \Delta_{s_0} \subset \mathbb{R}^6 \end{aligned} \tag{2}$$

where $f : \mathbb{R}^6 \times \mathbb{R}^3 \to \mathbb{R}^6$ is the function expressed in Eq. 1 which is convex, continuous and differentiable on $\mathbb{R}^6 \times \mathbb{R}^3$.

2.2 Problem Formulation

Due to the uncertain initial state and stochastic variables, the state vector at $k-$step is uncertain state. The purpose is to give the probabilistic ellipsoidal bounds on the vehicle trajectory on time step $k = 1, 2, 3, ...$ which are the probabilistic ellipsoidal bounds on x_k and y_k. For convenience, denoting $\{x_k, y_k\}$ as p_k. Obviously, $p_k \subset s_k$. Thus, the problem can be defined well.

Problem 1. Given a vehicle dynamics system of the form described in (2), find the tightest possible outer bounds on the trajectory $p_k \subset s_k$ for any $k \in \{1, 2, 3, ...\}$ to make sure that the probability that p_k locates in the bounds is larger than a given probability level. The problem can be formulated as

$$\begin{aligned} &\forall k = 0, 1, 2, 3, ... \\ &\min_{U_k} \det M_{k+1}^{-1} \\ &s.t. \quad s_{k+1} = f(s_k, \omega_k), \\ &\qquad s_0 \in \Delta_{s_0} \subset \mathbb{R}^6, \\ &\qquad \omega_k \in \Delta_{\omega_k} \subset \mathbb{R}^3, \\ &\qquad \mathrm{Pr}\{(p_{k+1} - C_{k+1})^T M_{k+1}(p_{k+1} - C_{k+1}) \le 1\} \ge 1 - \alpha \end{aligned} \tag{3}$$

where M_{k+1} is a matrix with 2 columns and 2 raws, C_{k+1} is a 2-dimension vector, and $U_{k+1} = \{C_{k+1}, M_{k+1}\}$ is the input variable for the problem.

Problem 1 is NP-hard problem due to the existence of probabilistic constraint in which the uncertainty is with continuous distribution (Y. Wu et al. , 2018). This work addresses Problem 1 with the scenario approach (G. C. Calafiore et al. , 2006).

3. PROPOSED METHOD

3.1 Scenario Approach

A typical uncertain convex optimization problem with probabilistic constraint expressed in (R. Jiang et al. , 2016) is as

$$\begin{aligned} &\min_{u \in \mathscr{U}} J(u) \\ &s.t. \quad \mathrm{Pr}\{h(u, \delta) \le 0\} \ge 1 - \alpha, \ \delta \in \Delta, \ \alpha \in (0, 1) \end{aligned} \tag{4}$$

where $u \in \mathscr{U} \subset \mathbb{R}^{n_u}$ denotes the decision variable whose set $\mathscr{U}$ is convex and closed, $\delta \in \Delta \subset \mathbb{R}^{n_\delta}$ is an uncertain parameter, the set Δ is the sample space of δ and Pr is a probability measure on Δ, α is a given probability level, moreover, for any fixed value $\delta \in \Delta$, both $J(u,\delta) : \mathscr{U} \times \Delta \to \mathbb{R}$ and $h(u,\delta) : \mathscr{U} \times \Delta \to \mathbb{R}$ are continuous and convex in u. The constraint of program (4) is a probabilistic constraint (A. Nemirovski et al. , 2006).

Definition 1. For a given u, the probability measure of violating the constraint in (4) is defined as

$$V(u) \doteq \Pr\{\delta \in \Delta : h(u,\delta) > 0\}. \tag{5}$$

Then, the probabilistic constraint can also be written as

$$V(u) \le \alpha. \tag{6}$$

The program (4) is NP hard due to probabilistic constraint (K. G. Murty et al. , 2009).

If samples $\delta^{(i)}, i = 1, ..., N$ is identically extracted from Δ according to probability measure Pr, a deterministic convex optimization problem can be formed as

$$\begin{aligned} &\min_{u \in \mathscr{U}} \ J(u) \\ &s.t. \quad h(u, \delta^{(i)}) \le 0, \ i = 1, ..., N \end{aligned} \tag{7}$$

which is a standard convex finite optimization problem and a solution can be found at low computational cost by available solvers when N is not too large (M. C. Campi et al. , 2011). The optimal solution $\hat{u}_N$ of the program (7) is called as the scenario solution for program (4) generally. Moreover, since the extractions $\delta^{(i)}, i = 1, ..., N$ is randomly extracted, the optimal solution $\hat{u}_N$ is random variable. On the other hand, the degree of robustness of $\hat{u}_N$ should be investigated, precisely, the measure that $\hat{u}_N$ satisfies the probabilistic constraint described by (6). The following theorem shows that $V(\hat{u}_N)$ tends rapidly to zero as N increases.

Theorem 1. Let $\alpha \in (0,1), \beta \in (0,1)$ be two given probability levels. If

$$N \ge \frac{2}{\alpha} \ln \frac{1}{\beta} + 2n_u + \frac{2n_u}{\alpha} \ln \frac{2}{\alpha}, \tag{8}$$

then, it holds that

$$\Pr{}^N\{(\delta^{(1)}, ..., \delta^{(N)} \in \Delta^N : V(\hat{u}_N) \le \alpha\} \ge 1 - \beta. \tag{9}$$

The proof of Theorem 1 is omitted here which is introduced in (G. C. Calafiore et al. , 2006). Theorem 1 indicates that the scenario approach cannot be robust against all situations while the level of robustness is able to be retained. Note that β is an important factor and choosing $\beta = 0$ makes $N = \infty$. However, for practical purposes, β has very limited importance due to its appearance under the sign of logarithm: although β is selected as 10^{-10} which is near zero in practical purpose, N does not grow significantly.

3.2 Proposed Algorithm

The problem of deciding probabilistic ellipsoidal bound of p_1, vehicle position at time index $k = 1$, can be expressed as

$$\begin{aligned} &\min_{U_1} \ \det \ M_1^{-1} \\ &s.t. \quad s_1 = f(s_0, \omega_0), \\ &\qquad s_0 \in \Delta_{s_0} \subset \mathbb{R}^6, \\ &\qquad \omega_0 \in \Delta_\omega \subset \mathbb{R}^3, \\ &\qquad \Pr\{(p_1 - C_1)^T M_1 (p_1 - C_1) \le 1\} \ge 1 - \alpha \end{aligned} \tag{10}$$

Where $U_1 = \{C_1, M_1\}$ stands for the decision variable for time index $k = 1$. Then, the independent samples $s_0^{(i)}, \omega_0^{(i)}, i = 1, ..., N_s$ are randomly extracted from Δ_{s_0} and Δ_{ω_0} and the deterministic problem is formed consequently as

$$\begin{aligned} &\min_{U_1} \ \det \ M_1^{-1} \\ &s.t. \quad \forall i \in \{1, ..., N_s\}, \\ &\qquad s_1^{(i)} = f(s_0^{(i)}, \omega_0^{(i)}), \\ &\qquad s_0^{(i)} \in \Delta_{s_0} \subset \mathbb{R}^6, \\ &\qquad \omega_0^{(i)} \in \Delta_\omega \subset \mathbb{R}^3, \\ &\qquad (p_1^{(i)} - C_1)^T M_1 (p_1^{(i)} - C_1) \le 1. \end{aligned} \tag{11}$$

The solution to problem expressed in (11) is denoted as $\hat{U}_1(N_s) = \{\hat{M}_1(N_s), \hat{C}_1(N_s)\}$.

Corollary 1. If

$$N_s \ge \frac{2}{\alpha} \ln \frac{1}{\beta} + 12 + \frac{12}{\alpha} \ln \frac{2}{\alpha}, \tag{12}$$

$$\Pr{}^{N_s} \left\{ \begin{matrix} (s_0^{(1)}, ..., s_0^{(N_s)}) \in \Delta_{s_0}^{N_s} \\ (\omega_0^{(1)}, ..., \omega_0^{(N_s)}) \in \Delta_{\omega_0}^{N_s} \end{matrix} : V(\hat{U}_1(N_s)) \le \alpha \right\} \ge 1 - \beta \tag{13}$$

holds.

Proof. The input variables for both (10) and (11) are U_1 which consists of C_1 and M_1. The dimension for C_1 is equal to the dimension of p_1 which equals 2. Since M_1 is a 2×2 matrix, the number of parameters to be decided is also 4. Thus, the dimension of the input variables is 6. Thus, Corollary 1 is proved according to Theorem 1.

Lemma 1. If $s_0^{(i)} \in \Delta_{s_0}$ and $\omega_j^{(i)} \in \Delta_{\omega_j}$ holds for any $i \in \{1, ..., N_s\}$ and $j \in \{0, ..., k-1\}$, then, $s_{k-1}^{(i)}$ is randomly extracted from $\Delta_{s_{k-1}}$.

Proof. For $k = 1$, although both Δ_{s_1} and $\Pr_{s_1}$ are unknown, since, for any $i \in 1, .., N_s$, $s_1^{(i)}$ are calculated from

$$s_1^{(i)} = f(s_0^{(i)}, \omega_0^{(i)}) \tag{14}$$

where $s_0^{(i)} \in \Delta_{s_0}, \omega_0^{(i)} \in \Delta_{\omega_0}$ holds. Thus, $s_1^{(i)} \in \Delta_{s_1}$ holds consequently. Moreover, $s_0^{(i)} \in \Delta_{s_0}, \omega_0^{(i)} \in \Delta_{\omega_0}$ are randomly selected and $s_1^{(i)}$ is consequently determined, which implies that $s_1^{(i)}$ is randomly selected from Δ_{s_1}. Therefore, Lemma 1 stands for $k = 1$. Suppose that, Lemma 1 stands for $k = l > 1$. Then, as $k = l + 1$, $s_l^{(i)}$ can be calculated as

$$s_l^{(i)} = f(s_{l-1}^{(i)}, \omega_{l-1}^{(i)}), \ \omega_{l-1}^{(i)} \in \Delta_{\omega_{l-1}}. \tag{15}$$

Since $s_{l-1}^{(i)} \in \Delta_{s_{l-1}}, \forall i \in \{1, ..., N_s\}$, $s_l^{(i)} \in \Delta_{s_l}$ holds consequently. Moreover, $s_{l-1}^{(i)} \in \Delta_{s_{l-1}}, \omega_{l-1}^{(i)} \in \Delta_{\omega_{l-1}}$ are randomly selected and $s_{l-1}^{(i)}$ is consequently determined, which implies that $s_l^{(i)}$ is randomly selected from Δ_{s_l}. Then, the proof is completed.

For the case $k \in \{1, 2, 3, ...\}$, the original problem expressed as

$$\begin{aligned}\min_{U_k}\ & \det\ M_k^{-1}\\ s.t.\ \ & s_k = f(s_{k-1}, \omega_{k-1}),\\ & s_{k-1} \in \Delta_{s_{k-1}} \subset \mathbb{R}^6,\\ & \omega_{k-1} \in \Delta_{\omega_{k-1}} \subset \mathbb{R}^3,\\ & \Pr\{(p_k - C_k)^T M_k (p_k - C_k) \le 1\} \ge 1 - \alpha.\end{aligned} \tag{16}$$

where U_k stands for the input variable $\{C_k, M_k\}$, Δ_{x_k} is the sample space of s_k for any $k \in \{1, 2, 3, ...\}$. Absolutely, both $\Pr_{s_k}$ and Δ_{s_k} are unknown.

Theorem 2. When $k \in 1, 2, 3, ...$, for any $i \in \{1, ..., N_s\}$, calculate the samples of $s_{k-1}^{(i)}$ recursively from $s_0^{(i)}$ and $\omega_0^{(i)}, \omega_1^{(i)}, ...$ through

$$\begin{aligned}& \forall i \in 1, ..., N_x\\ & s_1^{(i)} = f(s_0^{(i)}, \omega_0^{(i)}),\\ & ......\\ & s_{j+1}^{(i)} = f(s_j^{(i)}, \omega_j^{(i)}),\\ & ......\\ & s_{k-1}^{(i)} = f(s_{k-2}^{(i)}, \omega_{k-2}^{(i)})\end{aligned} \tag{17}$$

where the samples $s_0^{(1)}, ..., s_0^{(N_s)}$ and $\omega_j^{(1),...,\omega_j^{(N_s)}}, \forall j \in \{0, ..., k-2\}$ are randomly selected from Δ_{s_0} and Δ_{ω_j}. Then, the optimal solution $\hat{U}_k(N_s)$ of the deterministic problem

$$\begin{aligned}\min_{U_k}\ & \det\ M_k^{-1}\\ s.t.\ \ & \forall i \in \{1, ..., N_s\},\\ & s_k^{(i)} = f(s_{k-1}^{(i)}, \omega_{k-1}^{(i)}),\\ & s_{k-1}^{(i)} \in \Delta_{s_{k-1}} \subset \mathbb{R}^6,\\ & \omega_{k-1}^{(i)} \in \Delta_{\omega_{k-1}} \subset \mathbb{R}^3,\\ & (p_k^{(i)} - C_k)^T M_k (p_k^{(i)} - C_k) \le 1.\end{aligned} \tag{18}$$

satisfies

$$\Pr{}^N \left\{ \begin{matrix} (s_{k-1}^{(1)}, ..., s_{k-1}^{(N_s)}) \in \Delta_{s_{k-1}}^{N_s} \\ (\omega_{k-1}^{(1)}, ..., \omega_{k-1}^{(N_s)}) \in \Delta_{\omega_{k-1}}^{N_s} \end{matrix} : V(\hat{U}_k(N_s)) \le \alpha \right\} \ge 1-\beta \tag{19}$$

if (12) holds.

Proof. The case for $k = 1$ is proved by Corollary 1. When $k > 1$, since $s_{k-1}^{(i)}$ is randomly extracted from $\Delta_{s_{k-1}}\}, \forall i \in \{1, .., N_s\}$ according to Lemma 1 and $\omega_{k-1}^{(i)}, \forall i \in \{1, ..., N_s\}$ is randomly extracted from Δ_ω, $\hat{U}_k(N_s)$ also satisfies (19) if (12) holds according to Corollary 1 by replacing 0 and 1 with $k-1$ and k. Thus, Theorem 1 is completely proved.

The proposed algorithm is summarized as in Algorithm 1.

4. SIMULATION VALIDATION

In this section, an example of the implementation of the probabilistic ellipsoidal bounds calculation for vehicle trajectory prediction is presented as validation. The vehicle dynamics model is expressed in Eq. 1.The system starts with the initial state as

Algorithm 1 Compute probabilistic ellipsoidal bounds on the vehicle trajectory prediction from $k = 1$ to n

Input: $s_0^{(i)}, \omega_0^{(i)}, i = 1, .., N_s$
Output: $C_1, M_1, ..., C_n, M_n$
1: **For** $k = 1$ to n **do**
2:   **For** $i = 1$ to N_s **do**
3:     Step 1: Calculate $s_k^{(i)}$ through $s_k^i = f(s_{k-1}^{(i)}, \omega_{k-1}^{(i)})$
4:   **End for**
5:   Step 2: solve the problem $\min_{\{C_k, M_k\}} \det\ M_k^{-1}$ with constraint $(p_k^{(i)} - C_k)^T M_1 (p_k^{(i)} - C_k) \le 1$
6: **End for**

$$\begin{bmatrix} v_{x,0} \\ v_{y,0} \\ r_0 \\ \theta_0 \\ x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{20}$$

For the uncertain parameters, the probability distribution is Gaussian distribution and $\forall k = 0, 1, ...$ the mean vector and covariance matrix are written as

$$\begin{bmatrix} \bar{a}_{x,k} \\ \bar{a}_{y,k} \\ \bar{a}_{r,k} \end{bmatrix} = \begin{bmatrix} 0.15 \\ 0.1 \\ 0.1 \end{bmatrix} \tag{21}$$

and

$$\delta_{\omega,k} = \begin{bmatrix} 0.25 & 0.0001 & 0.000016 \\ 0.0001 & 0.0025 & 0.000025 \\ 0.000016 & 0.000025 & 0.0025 \end{bmatrix}. \tag{22}$$

The numerical simulation were performed for the algorithm proposed in previous section. The state variable in this problem contains 6 variables. While, only 2 variables were considered for the bound calculation. Hence, fixing a priori probabilistic levels $\alpha = 0.1$ and $\beta = 0.1$, and using Eq. 12, the sample size should satisfy $N_s \ge 417$. The numerical solution results about time evolutions of the bound by different N_s for steps $k = 101, 111, ..., 491, 501$ are shown in Fig. 2. The red points are the realization from posteriori Monte-Carlo analysis based on the initial state and uncertain parameters' probability distribution. The posterior test generates 501 trajectories. The blue lines are the 0.9−probability ellipsoidal bounds for every step. The red points at every step are supposed to locates inside the ellipsoidal with probability 0.9. As shown in Fig. 3 which indicates that larger N_s brings larger bounds for the same time step. This implies that more conservative bounds are obtained with more samples. The result makes senses since the constraints become more when extracting more samples. Intuitively speaking, the ellipse should be larger to encircle more points. The constraint failure probability α is calculated in every case and comprehensive statistical analysis results are shown in Fig. 4. As N_s gets larger, the probability of constraints failure decreases statistically, Moreover, the probability that $\alpha > 0.1$ denotes as $\Pr\{\alpha > 0.1\}$ is also calculated for every N_s and listed in Tab. 1. When $N_s \ge 150$, $\Pr\{\alpha > 0.1\}$ is already smaller than 0.1 which means that it is potential to use less scenarios to ensure the probabilistic level which can reduce the computation burden.

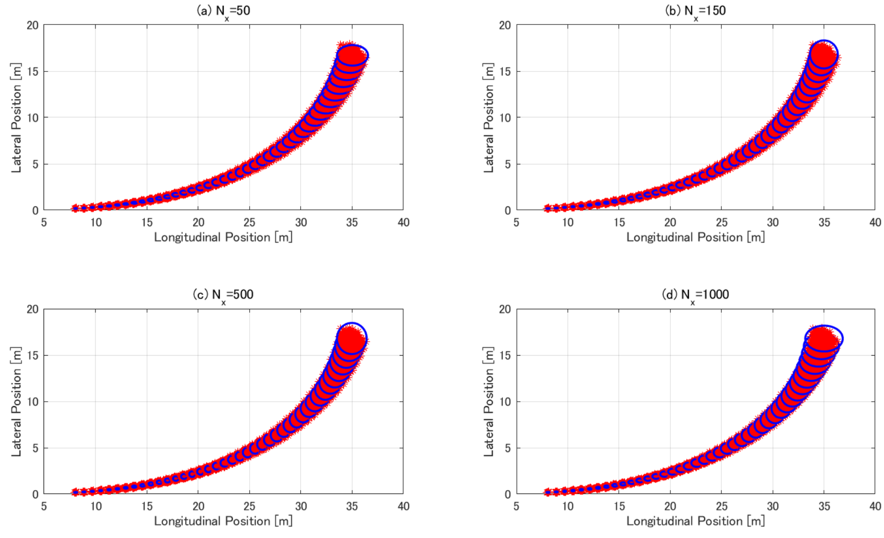

Fig. 2. Time evolution of bounds computed by proposed algorithm of time steps $k = 101, 111, ..., 501$ using different sample numbers: (a) Sample number N_x is 50; (b) Sample number N_x is 150; (c) Sample number N_x is 500; (d) Sample number N_x is 1000.

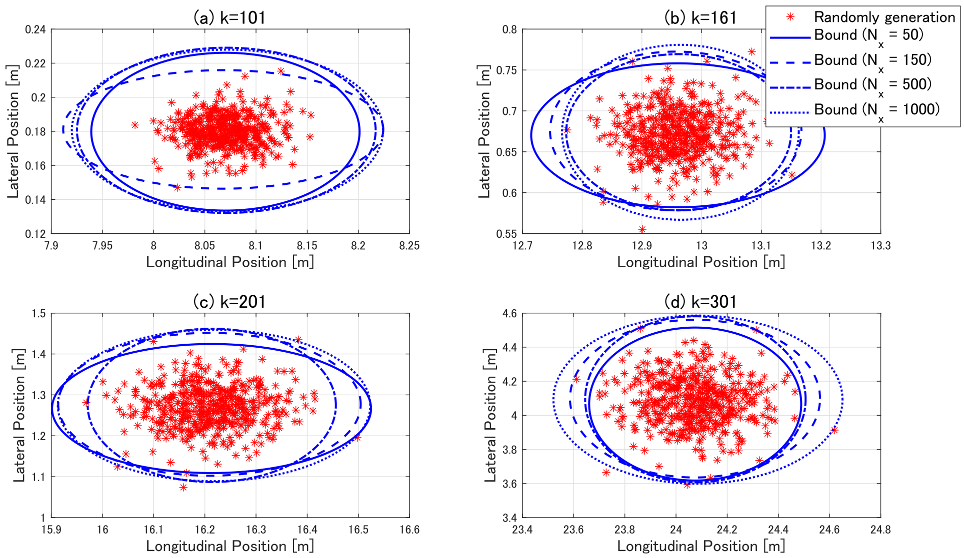

Fig. 3. Bounds computed by proposed algorithm at different time steps: (a) time step $k = 101$; b) time step $k = 161$; c) time step $k = 201$; d) time step $k = 301$. The bounds are different types of blue lines. The red "*" are the randomly generated points at the certain time steps.

Table 1. The probability that constraint failure probability $\alpha > 0.1$

N_x	50	150	300	500	1000
$\Pr\{\alpha > 0.1\}$	0.1419	0.0639	0.0559	0.0259	0.02

5. CONCLUSION AND FUTURE WORK

Scenario approach-based algorithm is proposed for calculating the probabilistic ellipsoidal bounds of the vehicle trajectory prediction. The probabilistic ellipsoidal bounds are essentially approximate ones. For given probabilistic level, the least number of samples for calculating the bound can be determined. Then, after sample extraction, the ellipsoidal bounds parameters is able to be calculated through solving a deterministic optimization problem. The obtained solution can be used for obtaining the probabilistic ellipsoidal bounds. Through the simulation validation, the proposed algorithm can achieve the goal for computing the probabilistic bounds for future trajectory. However, there are also future work to be done for improvement.

(1) The lower bound of the sample number for certain probabilistic level of violation should be decreased for saving computation resource;

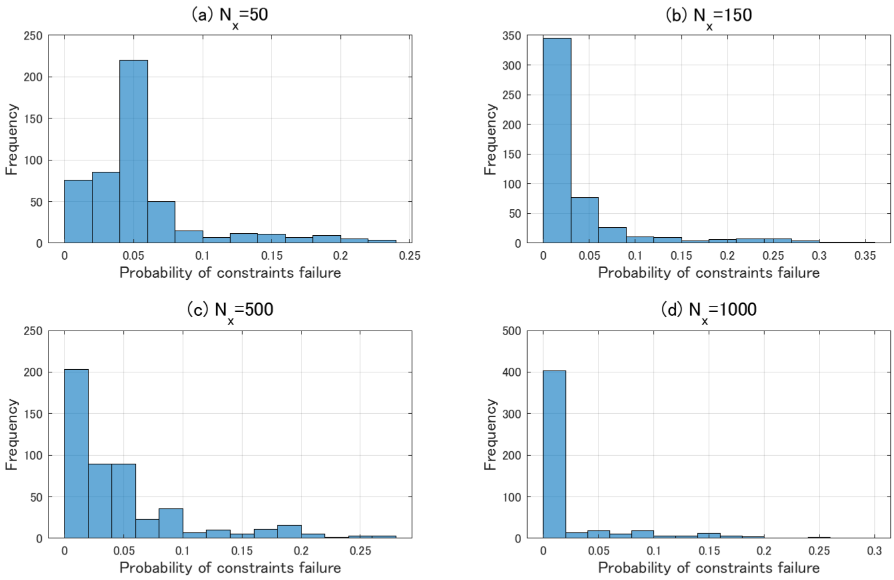

Fig. 4. Statistical analysis on probability of constraints failure for cases with different sample numbers: (a) Sample number N_x is 50; (b) Sample number N_x is 150; (c) Sample number N_x is 500; (d) Sample number N_x is 1000.

(2) In the current stage, the probability distribution of uncertain parameters is supposed to be arbitrary. Essentially, for more efficiently extracting the samples, the information probability distribution of uncertain parameters should be known. Then, the resulted probabilistic bounds would be more reliable and practical.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of Tokyo University of Agriculture and Technology. Any opinions, findings recommendations, and conclusions in this paper are those of the authors, and do not necessarily reflect the views of Tokyo University of Agriculture and Technology.

REFERENCES

S. Lefevre, D. Vasquez and C. Laugier. A survey on motion prediction and risk assessment for intelligent vehicles. *ROBOMECH Journal*, volume 1, no. 1, pages 1–14. 2014.

C-F Lin, A. G. Ulsoy, and D. J. LeBlanc. Vehicle dynamics and external disturbance estimation for vehicle path prediction. *IEEE Transactions on Control System Technology*, volume 8, no. 3, pages 508–518. 2014.

A. Eidehall, and L. Petersson. Statistical threat assessment for general road scenes using Monte Carlo sampling. *IEEE Transactions on Intelligent Transportation Systems*, volume 9, no. 1, pages 137–147. 2008.

G. S. Aoude, V. R. Desaraju, L. H. Stephens, and J. P. How. Driver behavior classification at intersections and validation on large naturalistic dataset. *IEEE Transactions on Intelligent Transportation Systems*, volume 13, no. 2, pages 724–736. 2012.

G. Agamennoni, J. I. Nieto and E. M. Nebot. Estimation of multivehicle dynamics by considering contextual information. *IEEE Transactions on Robotics*, volume 28, no. 5, pages 855–870. 2012.

C. Ju, Z. Wang, C. Long, X. Zhang, G. Cong and D. E. Chang. Interaction-aware kalman neural networks for trajectory prediction. *arXir.org*, arXiv:1902.10928.

T. Campbell and P. H. Jonathan. Bayesian nonparametric set construction for robust optimization. *IEEE 2015 American Control Conference*, 2015.

Y. Hu, W. Zhan and M. Tomizuka. Probabilistic prediction of vehicle semantic intention and motion. *arXir.org*, arXiv:1804.03629.

X. Huang, S. G. McGill, B. C. Williams, L Fletcher and G. Rosman. Uncertainty-aware driver trajectory prediction at urban intersections. *arXir.org*, arXiv:1901.05105.

S. Taheri. *An investigation and design of slip control braking systems integrated with four-wheel steering*. Ph. D. Dissertation, Clemson University, 1990.

G. C. Calafiore and M. C. Campi. The scenario approach to robust control design. *IEEE Transactions on Automatic Control*, volume 51, no. 5, pages 742–753. 2006.

GR. Jiang and Y. Guan. Data-driven chance constrained stochastic program. *Math. Program., Ser. A*, volume 158, no. 5, pages 291-327, 2016.

A. Nemirovski, and A. Shapiro. Convex Approximations of Chance Constrained Programs. *SIAM Journal on Optimization*, volume 17, no. 4, pages 969-996, 2006.

K. G. Murty and S. N. Kabadi. Some NP-complete problems in quadratic and nonlinear programming. *Mathematical Programming*, volume 39, no. 4, pages 117-129, 2009.

M. C. Campi, and S. Garatti. A sampling-and-discarding approach to chance-constrained optimization: feasibility and optimality. *Journal of Optimization Theory and Applications*, volume 148, no. 2, pages 257-280, 2011.

Y. Wu, and T. Shen. Policy iteration algorithm for optimal control of stochastic logical dynamical systems. *IEEE Transactions on Neural Networks and Learning Systems*, volume 29, no. 5, pages 2031-2036, 2018.