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Event-triggered Consensus Control of Multi-agent Systems with Nonuniform Communication Delays via Reduced-Order Observers ⋆

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Abstract: This paper studies a consensus problem for linear multi-agent systems (MASs) over directed communication networks with nonuniform time-varying delays. To overcome the limited computing and storage resources, a distributed control scheme is designed for each agent by using the event-triggered strategy. At the same time, a reduced-order observer is put forward in the controller design when only the relative output measurement is available. The communication network model with nonuniform time-varying delays is more challenging than the fixed delays or non-delays in the literature. Theoretical analysis is provided to show that the proposed control scheme can guarantee the consensus of MASs, with Zeno-behavior excluded and the upper bound of time delay obtained. A numerical example is provided to illustrate the feasibility and effectiveness of the theoretical results.

Keywords: Consensus control, multi-agent system, distributed event-triggered strategy, reduced-order observer, nonuniform time-varying delay.

1. INTRODUCTION

Consensus problems of MASs have been investigated extensively since Degroot (1974) developed a consensus algorithm in an opinion pooling problem from the perspective of the probability theory. Up to now, consensus control strategies have been widely applied in many fields, such as formation control, containment control, and distributed optimization and learning (see Hu and Feng (2011); Hu et al. (2013); Chen et al. (2018); Peng et al. (2019); Yuan et al. (2019)), etc.

Event-triggered control strategies seem more applicable for cooperative control of MASs when the communication, computation and storage resources are limited. Generally, the event-triggered strategies can be divided into two categories: state-dependent and state-independent strategies. The early works mainly adopted state-dependent strategies. For example, a distributed event-triggered control strategy was firstly proposed for a first-order MAS in Dimarogonas and Johansson (2009), in which centralized and decentralized schemes were studied. Hu et al. (2011) made the first attempt to propose a distributed event-triggered control for leader-follower MASs. An extension was further presented to consider a tracking problem of second-order leader-follower MASs in Hu et al. (2015). Another class of event-triggered control schemes often adopt state-independent strategies, which can be found in Seyboth et al. (2013); Yang et al. (2016). The key point is that the threshold function is independent of the state information of neighbors. Very recently, event-triggered strategy was further extended to some practical scenarios such as quasi-containment control in Yuan et al. (2019), consensus control with input saturation in Yi et al. (2019).

It is noted that the aforementioned works related to event-triggered consensus control strategies seldom concerned observer-type protocols. However, the state information of agents may not be fully measured in practice, so the protocols based on observer type seem to have more practical significance. As far as we know, observer based control protocols can be divided into full-order and reduced-order state observers. In the case of full-order observer protocols, some observer-based consensus controls were developed in Li, Duan, et al. (2010); Li, Soh, et al. (2017). If the agent dynamics are of high order or the number of agents is large, the full-order observer design

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may result in computational redundancy and need large storage capacity. To this end, Li et al. (2011) presented a new algorithm to design reduced-order observers in consensus control. Li et al. (2019) proposed two kinds of reduced-order output-feedback consensus controls with adaptive gain laws based on edge and node respectively. A new Kx-functional observer-based output feedback event-based control was proposed in Jian et al. (2019a,b).

In this paper, we present the first attempt to address the event-based consensus problem for MASs under directed graph with reduced-order observer and nonuniform time-varying delays, a topic that remains challenging. The main contributions of this paper are threefold. First, in order to effectively reduce the storage space, a dimension reduction method is introduced in the reduced-order observer design. Second, an event-triggered strategy based on state-independent threshold is proposed and thus the strategy avoids to compute the threshold with the information from the neighbors. Third, nonuniform time-varying delays are considered in the event-triggered consensus of MASs.

The rest of paper is organized as follows. Section 2 gives some preliminaries and the consensus control problem is formulated. Section 3 presents the main results. Some simulation results are given in Section 4. Conclusions are given in Section 5.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Some preliminaries

By convention, $\mathbf{R}^{m\times n}$ and $\mathbf{C}^{m\times n}$ are the set of $m \times n$ real and complex matrices, respectively. $Re(s)$ denotes the real part of $s \in \mathbf{C}$. $\mathbf{I}_n$ is the $n \times n$ identity matrix. $\mathbf{1}_n = (1,\ldots,1)^T \in \mathbf{R}^n$. x^T denotes the transpose of vector x. The conjugate transpose of matrix A is represented by A^H. $\lambda_{min}(A)$ and $\lambda_{max}(A)$ represent the minimum and maximum eigenvalues of the matrix A, respectively. $\| \bullet \|$ represents the Euclidean norm. $\otimes$ denotes the Kronecker product. A matrix is said to be Hurwitz stable if all of its eigenvalues have negative real parts.

A directed communication network can be modeled as a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{v_1,\ldots,v_N\}$ represents the set of N agents, $\mathcal{E} = \{e_{ij} \mid (v_i, v_j) \in \mathcal{V}\times\mathcal{V}\}$ is the set of edges, and $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{N\times N}$ is the adjacency matrix of $\mathcal{G}$ defined by $a_{ii} = 0$, $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The set of all neighbors of node v_i can be defined by $\mathcal{N}_i = \{v_j \in \mathcal{V} \mid e_{ji} \in \varepsilon\}$. The degree matrix $\mathcal{D} = \{d_1,\ldots,d_N\} \in \mathbf{R}^{N\times N}$ of $\mathcal{G}$ is a diagonal matrix with diagonal elements $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$. The Laplacian matrix of $\mathcal{G}$ is defined as $L = \mathcal{D} - \mathcal{A}$, which satisfies $L\mathbf{1}_N = 0$. The Laplacian matrix has the following property.

Lemma 1. (Ren and Beard (2005)) Zero is an eigenvalue of L with $\mathbf{1}_N$ and a nonnegative vector $r \in \mathbf{R}^N$ as the corresponding right and left eigenvectors, respectively, that is, $r^T L = 0, r^T\mathbf{1}_N = 1$. Moreover, all other nonzero eigenvalues have positive real parts. Furthermore, zero is a simple eigenvalue of L if and only if the graph $\mathcal{G}$ has a directed spanning tree.

2.2 Problem formulation

Consider a general linear MAS, where the agent dynamics is given by

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t), \\ y_i(t) = Cx_i(t), \end{cases} \tag{1}$$

where $x_i(t) \in \mathbf{R}^n$ is the state, $u_i(t) \in \mathbf{R}^p$ is the control input, $y_i(t) \in \mathbf{R}^q$ is the measured output, and A, B, C are constant matrices with compatible dimensions.

The objective of this study is to design a suitable event-triggered consensus control scheme such that all the agents can achieve consensus, i.e., $\lim_{t\to\infty}(x_i(t) - x_j(t)) = \mathbf{0}$ for all $i, j = 1,\cdots,N$, when nonuniform delays exist in the communication links, and at the same time, Zeno behavior is excluded.

Throughout this study, the following assumptions are adopted.

Assumption 2. For the high-order MAS (1), (A, B) is controllable, (A, C) is observable, and C is of full row rank.

Assumption 3. The directed graph $\mathcal{G}$ has a spanning tree.

The following lemma will be used in the consensus analysis of the MAS (1).

Lemma 4. (Yang et al. (2016)) Suppose that $A \in \mathbf{R}^{n\times n}$ is Hurwitz. Then there exists a nonsingular matrix P_A such that $P_A^{-1}AP_A = J_A$ with J_A being the Jordan canonical form of A and $\left\|e^{At}\right\| \le \|P_A\| \left\|P_A^{-1}\right\| c_A e^{-a_A t}$, where c_A is a positive constant determined by A, and $0 < a_A < -\max \mathrm{Re}(\lambda_i(A))$.

3. MAIN RESULTS

3.1 Reduced-order observer based consensus control

For each agent i, there exists a series of event time instants t_k^i $(k = 0, 1, \ldots)$ determined by an event-triggered threshold function. To reach the consensus of MAS (1) without using any global information, a distributed event-triggered control scheme together with a reduced-order observer is proposed for $t \in [t_k^i, t_{k+1}^i)$:

$$\begin{aligned} \dot{v}_i(t) &= Fv_i(t) + Gy_i(t) + TBu_i(t), \\ u_i(t) &= cKQ_1 \sum_{j\in\mathcal{N}_i} a_{ij}[y_i(t_k^i - \tau_{ij}(t)) - y_j(t_{k'(t)}^j - \tau_{ij}(t))] \\ &\quad + cKQ_2 \sum_{j\in\mathcal{N}_i} a_{ij}[v_i(t_k^i - \tau_{ij}(t)) - v_j(t_{k'(t)}^j - \tau_{ij}(t))]. \end{aligned} \tag{2}$$

where $v_i(t) \in \mathbf{R}^{n-q}$ is the observer state, $c > 0$ is the coupling strength, $\tau_{ij}(t)$ represents the communication delay from agent j to agent i, $K \in \mathbf{R}^{p\times n}$ is a control gain matrix, and $t_{k'(t)}^j$ denotes the last event instant of agent j. $F \in \mathbf{R}^{(n-q)\times(n-q)}$ is Hurwitz and has no common eigenvalues with matrix A, $G \in \mathbf{R}^{(n-q)\times q}$, $T \in \mathbf{R}^{(n-q)\times n}$ is the only solution to the Sylvester equation $TA–FT = GC$ and $\begin{bmatrix}C^T & T^T\end{bmatrix}^T$ is nonsingular, $Q_1 \in \mathbf{R}^{n\times q}$ and $Q_2 \in \mathbf{R}^{n\times(n-q)}$ are given by $[Q_1\ Q_2] = \left\{\begin{bmatrix}C^T & T^T\end{bmatrix}^T\right\}^{-1}$.

3.2 Network model with nonuniform time-varying delays

Assume that the nonuniform time-varying delays are asymmetrical and uniformly bounded. For convenience, let $\Psi = \{\tau_{ij}(t) = \tau_\sigma : \sigma \in \{1, \dots, m\}\}$ $(m \le N(N-1))$ be the collection of independent time-varying delays affecting the communication links. There exists a set of communication topologies $\{\mathcal{G}_1, \cdots, \mathcal{G}_m\}$ such that the network $\mathcal{G}_\sigma$ contains only a delay τ_σ.

To illustrate the topology decomposition technique related to nonuniform delays, we consider a MAS having six agents, with the communication topology $\mathcal{G}$ shown in Fig. 1. The MAS has four different delays, i.e., $\tau_1 = \tau_{12} = \tau_{14} = \tau_{61}$, $\tau_2 = \tau_{13} = \tau_{41} = \tau_{56}$, $\tau_3 = \tau_{51} = \tau_{45}$, $\tau_4 = \tau_{23}$, and thus $\mathcal{G}$ is decomposed to four subgraphs $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3$ and $\mathcal{G}_4$, as shown in Fig. 2.

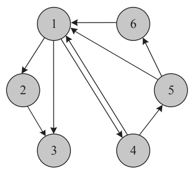

Fig. 1. A communication network with nonuniform delays

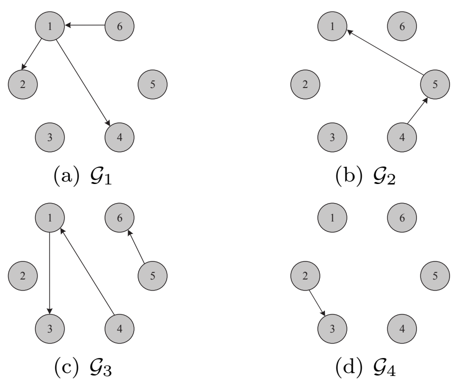

Fig. 2. The subgraphs of $\mathcal{G}$ associated with the delays τ_1, τ_2, τ_3 and τ_4.

Suppose that the Laplacian matrix corresponding to $\mathcal{G}_\sigma$ is L_σ. It is clear that $L_\sigma \mathbf{1}_N = 0, \forall\sigma \in \{1, \cdots, m\}$ and $\sum_{\sigma=1}^{m} L_\sigma = L$.

Lemma 5. L_σ is a Laplacian matrix of the subgraph of $\mathcal{G}$. Then there exists a non-singular matrix S such that

$$S^{-1}LS = J = \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & J_1 \end{bmatrix}, \qquad S^{-1}L_\sigma S = J_\sigma = \begin{bmatrix} 0 & l_\sigma^T \\ \mathbf{0} & J_{1\sigma} \end{bmatrix},$$

where $l_\sigma \in \mathbf{C}^{(N-1)\times 1}, J_{1\sigma} \in \mathbf{C}^{(N-1)\times(N-1)}$.

Proof. According to Assumption 2 that $\mathcal{G}$ contains a directed spanning tree, assuming that there exists a constant $\alpha \neq 0$ and non-singular matrices $S = [\alpha\mathbf{1}_N \; S_1]$, $S^{-1} = \begin{bmatrix} \frac{1}{\alpha} r^T \\ S_2 \end{bmatrix}$, where $r \in R^N$ is a nonnegative vector such that $r^T L = 0$ and $r^T \mathbf{1}_N = 1$. From the Jordan decomposition of L, we have $S^{-1}LS = J = \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & J_1 \end{bmatrix}$, of which the Jordan matrix $J_1 \in \mathbf{C}^{(N-1)\times(N-1)}$ is an upper triangular matrix with diagonal line consist with the nonzero eigenvalue of L (see **?**). From the foreshadowing above $L_\sigma \mathbf{1}_N = 0, \sigma \in \{1, \dots, m\}$. Hence,

$$\begin{aligned} S^{-1}L_\sigma S &= \begin{bmatrix} \frac{1}{\alpha} r^T \\ S_2 \end{bmatrix} L_\sigma [\alpha\mathbf{1}_N \; S_1] \\ &= \begin{bmatrix} r^T L_\sigma \mathbf{1}_N & \frac{1}{\alpha} r^T L_\sigma S_1 \\ \alpha S_2 L_\sigma \mathbf{1}_N & S_2 L_\sigma S_1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & l_\sigma^T \\ \mathbf{0} & J_{1\sigma} \end{bmatrix}. \end{aligned}$$

The proof is thus completed.

3.3 Consensus analysis

For each agent i, we define two measurement error vectors as $e_i^x(t) = x_i(t_k^i) - x_i(t), e_i^v(t) = v_i(t_k^i) - v_i(t)$. Let $\eta_i(t) = [x_i^T(t), v_i^T(t)]^T, \eta(t) = [\eta_1^T(t), \dots, \eta_N^T(t)]^T$ and $\xi_i(t) = [e_i^{xT}(t), e_i^{vT}(t)]^T, \xi(t) = [\xi_1^T(t), \dots, \xi_N^T(t)]^T$. Then, from (1) and (2), the closed-loop system is given by

$$\dot{\eta}(t) = (I_N \otimes M)\eta(t) + \sum_{\sigma=1}^{m} (cL_\sigma \otimes R)[\eta(t-\tau_\sigma) + \xi(t-\tau_\sigma)], \quad (3)$$

where $M = \begin{bmatrix} A & \mathbf{0} \\ GC & F \end{bmatrix}, R = \begin{bmatrix} BKQ_1C & BKQ_2 \\ TBKQ_1C & TBKQ_2 \end{bmatrix}$.

Next, we will show that the event-triggered control (2) can guarantee consensus of the MAS under a threshold function given by

$$f_i(t, \xi_i(t)) = \|\xi_i(t)\| - c_1 e^{-\alpha(t-t_0)}, \quad (4)$$

for some $c_1 > 0$, and α is a positive constant to be determined. Thus, the event triggered times are given by $t_{k+1}^i = \inf\{t : t > t_k^i, f_i(t) > 0\}$.

Theorem 6. Under Assumptions 2 and 3, consensus of the MAS can be achieved under the event-triggered control scheme (2).

Proof. From Lemma 1, zero is a simple eigenvalue of L and all other eigenvalues have positive real parts. By Lemma 5, there exists a coordinate transformation $\varepsilon(t) = (S^{-1} \otimes I_{2n-q})\eta(t)$. Let $\varepsilon(t) = [\varepsilon_1^T(t), \dots, \varepsilon_N^T(t)]^T$, where $\varepsilon_i(t) = [\bar{x}_i^T(t), \bar{v}_i^T(t)]^T$. Then, system (3) can be rewritten as follow:

$$\begin{aligned} \dot{\varepsilon}(t) &= (S^{-1} \otimes I_{2n-q})\dot{\eta}(t) \\ &= (I_N \otimes M)\varepsilon(t) + \sum_{\sigma=1}^{m} c(\begin{bmatrix} 0 & l_\sigma^T \\ \mathbf{0} & J_{1\sigma} \end{bmatrix} \otimes R)\varepsilon(t-\tau_\sigma) \\ &\quad + \sum_{\sigma=1}^{m} c(\begin{bmatrix} l_\sigma^T S_2 \\ J_{1\sigma} S_2 \end{bmatrix} \otimes R)\xi(t-\tau_\sigma), \end{aligned} \quad (5)$$

Define $\varepsilon_{2-N}(t) = [\varepsilon_2^T(t), \dots, \varepsilon_N^T(t)]^T$, then system (5) can be divided into the following two subsystems:

$$\begin{aligned} \dot{\varepsilon}_1(t) &= M\varepsilon_1(t) + \sum_{\sigma=1}^{m} (cl_\sigma^T \otimes R)\varepsilon_{2-N}(t-\tau_\sigma(t)) \\ &\quad + \sum_{\sigma=1}^{m} (cl_\sigma^T S_2 \otimes R)\xi_1(t-\tau_\sigma(t)), \end{aligned} \quad (6)$$

and

$$\dot{\varepsilon}_{2\text{–}N}(t)=(I_{N\text{–}1}\otimes M)\varepsilon_{2\text{–}N}(t)+\sum_{\sigma=1}^{m}(cJ_{1\sigma}\otimes R)\times \\ \varepsilon_{2\text{–}N}(t-\tau_\sigma)+\sum_{\sigma=1}^{m}(cJ_{1\sigma}S_2\otimes R)\,\xi_{2\text{–}N}(t-\tau_\sigma), \tag{7}$$

By Newton-Leibniz formula, one has $\varepsilon_{2-N}(t-\tau_\sigma)=\varepsilon_{2-N}(t)-\int_{t-\tau_\sigma}^{t}\dot{\varepsilon}_{2-N}(s)ds$, then the subsystem (7) can be rewritten as

$$\begin{aligned}\dot{\varepsilon}_{2\text{–}N}(t)&=(I_{N-1}\otimes M)\varepsilon_{2\text{–}N}(t)+\sum_{\sigma=1}^{m}(cJ_{1\sigma}\otimes R)\left\{\varepsilon_{2\text{–}N}(t)-\int_{t-\tau_\sigma}^{t}\dot{\varepsilon}_{2\text{–}N}(s)ds\right\}+\sum_{\sigma=1}^{m}(cJ_{1\sigma}S_2\otimes R)\,\xi_{2\text{–}N}(t-\tau_\sigma)\\ &=\bar{M}\varepsilon_{2-N}(t)-\sum_{\sigma=1}^{m}C_{1\sigma}\int_{t-\tau_\sigma}^{t}\dot{\varepsilon}_{2-N}(s)ds\\ &\quad+\sum_{\sigma=1}^{m}C_{2\sigma}\xi_{2-N}(t-\tau_\sigma),\end{aligned} \tag{8}$$

where $\bar{M}=I_{N-1}\otimes M+cJ_1\otimes R, C_{1\sigma}=cJ_{1\sigma}\otimes R$, and $C_{2\sigma}=cJ_{1\sigma}S_2\otimes R$. The solution of (8) is given by

$$\varepsilon_{2-N}(t)=e^{\bar{M}(t-t_0)}\varepsilon_{2-N}(t_0)+\int_{t_0}^{t}e^{\bar{M}(t-\theta)}\left\{\sum_{\sigma=1}^{m}(-C_{1\sigma})\int_{\theta-\tau_\sigma}^{\theta}\dot{\varepsilon}_{2-N}(s)ds+\sum_{\sigma=1}^{m}C_{2\sigma}\xi_{2-N}(\theta-\tau_\sigma)\right\}d\theta, \tag{9}$$

In order to check whether $\bar{M}$ is Hurwitz, it is equivalent to analyze the stability of

$$\bar{F}=\begin{bmatrix}A+c\lambda_iBKQ_1C & c\lambda_iBKQ_2\\ GC+c\lambda_iTBKQ_1C & F+c\lambda_iTBKQ_2\end{bmatrix},$$

Let matrix $\bar{F}$ be multiplied by $\bar{T}=\begin{bmatrix}I_n & 0\\ -T & I_{n-q}\end{bmatrix}$ and $\bar{T}^{-1}=\begin{bmatrix}I_n & 0\\ T & I_{n-q}\end{bmatrix}$, then we have

$$\begin{aligned}\bar{T}\bar{F}\bar{T}^{-1}&=\bar{T}\begin{bmatrix}A+c\lambda_iBKQ_1C & c\lambda_iBKQ_2\\ GC+c\lambda_iTBKQ_1C & F+c\lambda_iTBKQ_2\end{bmatrix}\bar{T}^{-1}\\ &=\begin{bmatrix}A+c\lambda_iBK & c\lambda_iBKQ_2\\ \mathbf{0} & F\end{bmatrix},\end{aligned}$$

Select the coupling coefficient $c\geq\frac{1}{2\min_{\lambda_i\neq0}\{Re(\lambda_i(L))\}}$, then there exists a $P>0$ which satisfies the following algebraic Riccati equation:

$$\begin{aligned}&(A+c\lambda_iBK)^TP+P(A+c\lambda_iBK)\\ &=A^TP+PA-2cRe(\lambda_i(L))PBB^TP\\ &=-Q+(1-2cRe(\lambda_i(L)))PBB^TP\leq -Q,\end{aligned}$$

Hence, $A+c\lambda_iBK(i=2,\ldots,N)$ are stable matrices. Since F is Hurwitz, $\bar{F}$ is Hurwitz as well. Then,from Lemma 4, one has, for $t\geq t_0$,

$$\left\|e^{\bar{M}(t-t_0)}\right\|\leq k_1e^{-\gamma(t-t_0)}, \tag{10}$$

where $k_1=\|P_{\bar{M}}\|\left\|P_{\bar{M}}^{-1}\right\|c_{\bar{M}}$, $P_{\bar{M}}$ is a nonsingular matrix such that $P_{\bar{M}}^{-1}\bar{M}P_{\bar{M}}=J_{\bar{M}}$, $J_{\bar{M}}$ is the Jordan canonical form of $\bar{M}$, $c_{\bar{M}}>0$ is a positive constant determined by $\bar{M}$, and $0<\gamma<-max\{Re(\lambda_i(\bar{M}))\}$.

Assume that there exist $\alpha,\lambda\in(0,\gamma)$ such that

$$\frac{k_1(m\alpha_1+m\alpha_2\sum_{\sigma=1}^{m}e^{\lambda\tau_\sigma})(e^{\lambda\tau_{max}}-1)}{\lambda(\gamma-\lambda)}<1, \tag{11}$$

$$\chi=\frac{\sqrt{N-1}\alpha_3k_1mc_1\left(e^{\lambda\tau_{max}-1}\right)+\alpha k_1k_2}{\alpha(\gamma-\alpha)-k_1(m\alpha_1+m\alpha_2\sum_{\sigma=1}^{m}e^{\alpha\tau_\sigma})\left(e^{\lambda\tau_{max}-1}\right)}, \tag{12}$$

where $\alpha_1=\|C_{1max}\|\,\|I_{N-1}\otimes M\|$, $\alpha_2=\|C_{1max}\|^2$, and $\alpha_3=\|C_{1max}\|\,\|C_{2max}\|$ with $C_{1max}=max\{C_{1\sigma}\},C_{2max}=max\{C_{2\sigma}\}$, $k_2=\|C_{2max}\|\sqrt{N-1}c_1\sum_{\sigma=1}^{m}e^{\lambda\tau_\sigma}$, then the following inequality holds for $t\geq t_0$.

$$\|\varepsilon_{2-N}(t)\|<k_1\|\varepsilon_{2-N}(t_0)\|e^{-\lambda(t-t_0)}+\chi e^{-\alpha(t-t_0)}. \tag{13}$$

First, we show that λ exists. Define $f(\lambda)=k_1(m\alpha_1+m\alpha_2\sum_{\sigma=1}^{m}e^{\lambda\tau_\sigma})(e^{\lambda\tau_{max}}-1)-\lambda(\gamma-\lambda)$. Obviously, $f(0)=0$ and $f'(0)=m\alpha_1\tau_{max}+m^2\alpha_2\tau_{max}-\gamma<0$ when $\tau_{max}<\frac{\gamma}{k_1(m\alpha_1+m^2\alpha_2)}$. Thus the constant λ satisfies (11) can be set up.

Next, we prove that inequality (13) is true. Define $k_1\|\varepsilon_{2-N}(t_0)\|e^{-\lambda(t-t_0)}+\chi e^{-\alpha(t-t_0)}=\omega(t)$. If equation (13) does not hold for any $t\in(t_0-\tau_{max},t^*)$, then there must exist a $t^*>t_0$ such that $\|\varepsilon_{2-N}(t^*)\|=\omega(t^*)$, and $\|\varepsilon_{2-N}(t)\|<\omega(t)$. Define $D=\frac{k_1}{\lambda}\|\varepsilon_{2-N}(t_0)\|(m\alpha_1+m\alpha_2\sum_{\sigma=1}^{m}e^{\lambda\tau_\sigma})\left(e^{\lambda\tau_{max}-1}\right)$, and $E=\frac{1}{\alpha}[\,(m\alpha_1+m\alpha_2\sum_{\sigma=1}^{m}e^{\alpha\tau_\sigma})\chi+\sqrt{N-1}m\alpha_3c_1\,]\left(e^{\alpha\tau_{max}-1}\right)$, Then we have

$$\begin{aligned}&\left\|\sum_{\sigma=1}^{m}(-C_{1\sigma})\int_{\theta-\tau_\sigma}^{\theta}\dot{\varepsilon}_{2-N}(s)ds\right\|\\ &\leq\int_{\theta-\tau_{max}}^{\theta}\left\{m\alpha_1\|\varepsilon_{2-N}(s)\|+m\alpha_2\sum_{\sigma=1}^{m}\|\varepsilon_{2-N}(s-\tau_\sigma)\|+m\alpha_3\sum_{\sigma=1}^{m}\|\xi_{2-N}(s-\tau_\sigma)\|\right\}ds\\ &\leq De^{-\lambda(\theta-t_0)}+Ee^{-\alpha(\theta-t_0)},\end{aligned}$$

and

$$\left\|\sum_{\sigma=1}^{m}C_{2\sigma}\xi_{2-N}(\theta-\tau_\sigma)\right\|\leq\|C_{2max}\|\sum_{\sigma=1}^{m}\|\xi_{2-N}(\theta-\tau_\sigma)\|\leq k_2e^{-\alpha(\theta-t_0)}.$$

From above and (9), (13), we have

$$\begin{aligned}\omega(t^*)&=\|\varepsilon_{2-N}(t^*)\|<k_1e^{-\gamma(t^*-t_0)}\|\varepsilon_{2-N}(t_0)\|\\ &\quad+\int_{t_0}^{t^*}k_1e^{-\gamma(t^*-\theta)}\left[De^{-\lambda(\theta-t_0)}+(E+k_2)e^{-\alpha(\theta-t_0)}\right]d\theta\\ &<k_1\|\varepsilon_{2-N}(t_0)\|e^{-\lambda(t^*-t_0)}+\frac{k_1(E+k_2)}{\gamma-\alpha}\\ &\quad\times(e^{-\alpha(t^*-t_0)}-e^{-\gamma(t^*-t_0)})\\ &<k_1\|\varepsilon_{2-N}(t_0)\|e^{-\lambda(t^*-t_0)}+\chi e^{-\alpha(t^*-t_0)}\triangleq\omega(t^*).\end{aligned}$$

The contradictory result shows that (13) holds under the conditions (11) and (12). Thus, (13) implies $\lim_{t\to\infty} \varepsilon_{2-N}(t) = \mathbf{0}$, that is, $\lim_{t\to\infty} \bar{x}_i(t) = \mathbf{0}$, and $\lim_{t\to\infty} \bar{v}_i(t) = \mathbf{0}, i = 2, \cdots, N$.

On the other hand, since $\eta(t) = (S \otimes I_{2n-q})\varepsilon(t) = \mathbf{1}_N \otimes \varepsilon_1(t) + (S_1 \otimes I_{2n-q})\varepsilon_{2-N}(t)$, thus we have $\eta(t) - \mathbf{1}_N \otimes \varepsilon_1(t) \to \mathbf{0}$ as $t \to \infty$. Moreover, the variable $\varepsilon_1(t)$ evolves according to system (6). Then, from the definitions of $\eta_i(t)$ and $\varepsilon_1(t)$, we have $x_i(t) - \bar{x}_1(t) \to 0, v_i(t) - \bar{v}_1(t) \to \mathbf{0}, i = 1, \cdots, N$, as $t \to \infty$, which shows that consensus is reached for the MAS. The proof is thus completed.

Theorem 7. Zeno behavior can be avoided in the close-loop system (3) under event-triggered control scheme (2).

Proof. Since $\xi_i(t) = \eta_i(t_k^i) - \eta_i(t)$, thus the upper right-hand Dini derivative of $\xi_i(t)$ over interval $[t_k^i, t_{k+1}^i)$ is given by

$$\begin{aligned} D^+ \|\xi_i(t)\| &\le \left\|\dot{\xi}_i(t)\right\| \le \|\dot{\eta}_i(t)\| \le \|\dot{\eta}(t)\| \\ &= \left\|(I_N \otimes M)\eta(t) + \sum_{\sigma=1}^{m} (cL_\sigma \otimes R)[\eta(t-\tau_\sigma) + \xi(t-\tau_\sigma)]\right\| \\ &\le \|I_N \otimes M\| \|\eta(t)\| + \sum_{\sigma=1}^{m} \|cL_\sigma \otimes R\| [\|\eta(t-\tau_\sigma)\| + \|\xi(t-\tau_\sigma)\|]. \end{aligned}$$

Since $\|\eta(t)\| \le \alpha_4 e^{-\lambda(t-t_0)} + \alpha_5 e^{-\alpha(t-t_0)}$, where $\alpha_4 = k_1 \|S \otimes I_{2n-q}\| \varepsilon(t_0)$, $\alpha_5 = \chi \|S \otimes I_{2n-q}\|$. Thus, we have

$$\begin{aligned} \left\|\dot{\xi}_i(t)\right\| \le{}& \|I_N \otimes M\| \left[\alpha_4 e^{-\lambda(t-t_0)} + \alpha_5 e^{-\alpha(t-t_0)}\right] \\ &+ \sum_{\sigma=1}^{m} \|cL_\sigma \otimes R\| \Big[\alpha_4 e^{-\lambda(t-t_0-\tau_\sigma)} \\ &+ \alpha_5 e^{-\alpha(t-t_0-\tau_\sigma)} + \sqrt{N} c_1 e^{-\alpha(t-t_0-\tau_\sigma)}\Big] \\ ={}& \alpha_6 e^{-\lambda(t-t_0)} + \alpha_7 e^{-\alpha(t-t_0)} \triangleq \varphi(t), \end{aligned}$$

where $\alpha_6 = \alpha_4 \left(\|I_N \otimes M\| + mc \|L_{max} \otimes R\| \sum_{\sigma=1}^{m} e^{\lambda\tau_\sigma}\right)$, $\alpha_7 = \|I_N \otimes M\| \alpha_5 + mc \sum_{\sigma=1}^{m} e^{\lambda\tau_\sigma} \|L_{max} \otimes R\| [\alpha_5 + \sqrt{N} c_1]$.

During the interval $[t_k^i, t_{k+1}^i)$, it is not difficult to obtain that $\|\xi_i(t)\| = \|\int_{t_k^i}^t \dot{\xi}_i(s)ds\| \le \int_{t_k^i}^t \varphi(s)ds$. From the threshold function given by (4), the next event time of agent i will not be triggered before $f_i(t, \xi_i(t)) \ge 0$ or equivalently $\|\xi_i(t)\| = c_1 e^{-\alpha(t-t_0)}$. Hence, the next event is not triggered before $\int_{t_k^i}^t \varphi(s)ds = c_1 e^{-\alpha(t-t_0)}$. Let $\tau = t - t_k^i$ be the time length between the two triggered events. Thus, τ is greater than or equal to the solution to the implicit equation $\left(\alpha_6 e^{-\lambda(t_k^i - t_0)} + \alpha_7 e^{-\alpha(t_k^i - t_0)}\right)\tau = c_1 e^{-\alpha(t_k^i + \tau - t_0)}$, or equivalently, $\alpha_6 e^{-(\lambda-\alpha)(t_k^i - t_0)} + \alpha_7 = c_1 e^{-\alpha\tau}$. Since $0 < \alpha < \lambda < \gamma$, we know that $\alpha_6 e^{-(\lambda-\alpha)(t_k^i - t_0)} + \alpha_7$ is bounded by $\alpha_6 + \alpha_7$. Thus, the solution to the implicit equation is greater than or equal to the solution to $(\alpha_6 + \alpha_7)\tau = c_1 e^{-\alpha\tau}$. Thus, if there exist $c_1 > 0$ and $0 < \alpha < \lambda < \gamma$, there is a positive lower bound τ on the inter-event time for agent i. Therefore, Zeno behavior is avoided. The proof is completed.

4. SIMULATION RESULTS

Consider a MAS with six agents, which is shown in Fig. 1. It is not difficult to find that the nonzero eigenvalues of L are $1, 1.3376 \pm 0.5623i,\ 2, 3.3247$, respectively. The state matrices are given by $A = [0, 1, 0; 0, 0, 1; 0, 0, -4]$, $B = [0, 0; 1, 0; 0, 1]$, and $C = [1, 0, 0; 0, -1, 0]$. Select $F = -1, G = [-2 \ \ -1]$ in the reduced-order observer, and we solve the Sylvester equation to obtain that $T = [-2 \ 3 \ 1]$. Additionally, $Q_1 = [1, 0; 0, -1; 2, 3]$ and $Q_2 = [0, 0, 1]$ can be calculated by $[Q_1 \ Q_2] = \{[C^T \ T^T]^T\}^{-1}$. We use the LMI toolbox in MATLAB to solve the Riccati equation to obtain a solution P, and then have $K = -B^T P = \begin{bmatrix} -0.9869 & -1.6953 & -0.3156 \\ -0.1613 & -0.3156 & -0.1871 \end{bmatrix}$. Choose $c = 0.6$, $c_1 = 0.66$, and $\alpha = 0.34$. According to Theorem 6, the upper bound of the communication delay is given by $\tau_{max} = 0.3$. The initial conditions of the agents are randomly selected in $[0, 1]$. Nonuniform delays are selected as $\tau_1 = 0.29 |sin(t)|, \tau_2 = 0.27 |cos(t)|, \tau_3 = \tau_{14}(t) = 0.14 |sin(t) + cos(t)|$, and $\tau_4 = 0.2 + 0.05 |sin(t)|$.

Fig. 3 shows the evolution of the 3-th state variables $x_i(t)$ of the six agents. From Fig. 3, consensus is achieved for the six agents under the proposed event-triggered strategy when the communication links suffer from nonuniform time-varying delays. Fig. 4 shows the evolution of the observer state v_i. Additionally, the evolution of the measurement errors are shown in Fig. 5. It can be found that all the measurement errors are bounded by $c_1 e^{-\alpha t}$.

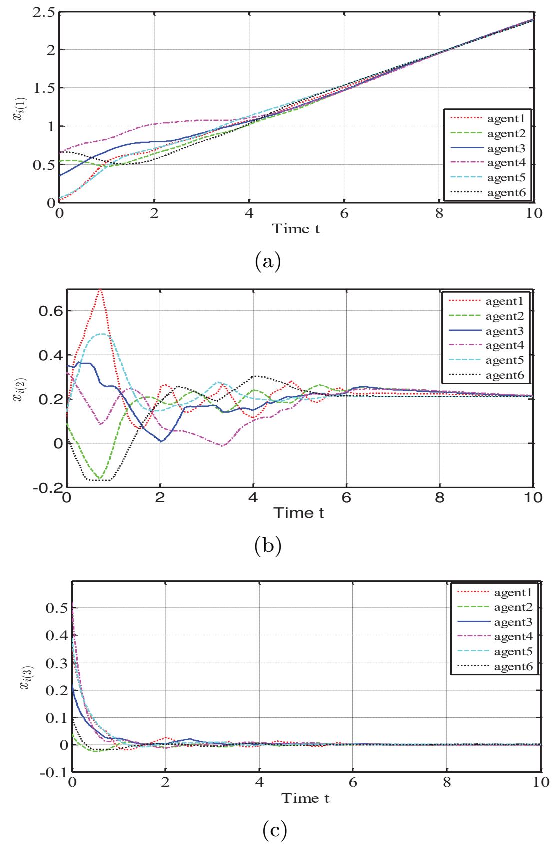

Fig. 3. The state evolution of the six agents.

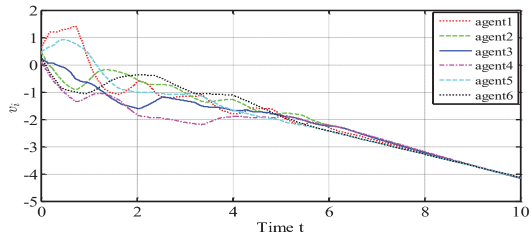

Fig. 4. The state evolution of the reduced observers.

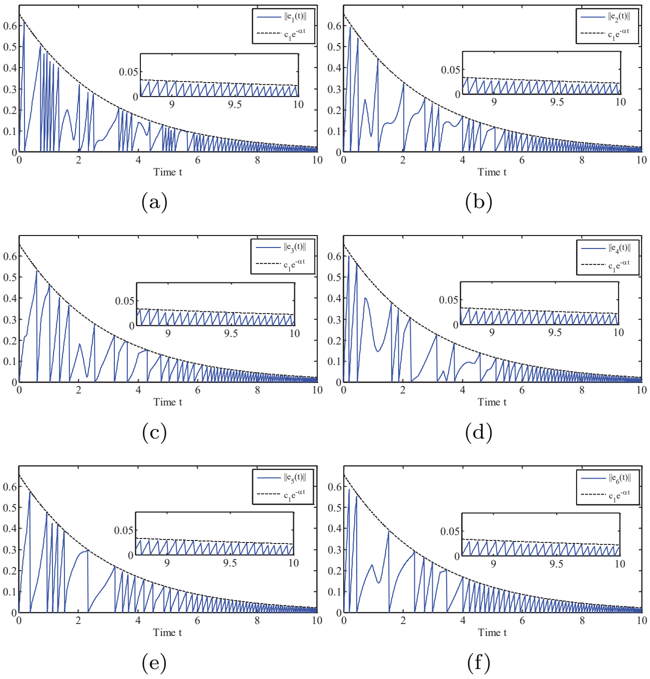

Fig. 5. The evolution of the measurement errors of the six agents.

5. CONCLUSION

This paper addressed an event-triggered consensus problem of a general linear MAS over directed communication network. A distributed consensus control scheme has been proposed by using reduced-order observer. Moreover, state-independent threshold function has been presented for each agent to achieve consensus under the proposed control scheme. Some sufficient conditions have been established for the consensus of MAS, with the upper bound of nonuniform time-varying delays obtained. Additionally, it has been shown that Zeno behavior can be avoided.

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