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Leader-following Consensus of Linear Fractional-order Multi-agent Systems via Event-triggered Control Strategy

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Abstract: Many complex systems can be more accurately described by fractional-order models. In this paper, a leader-following consensus problem of fractional-order multi-agent systems (FOMASs) is firstly formulated and then an event-trigger consensus control is proposed for each agent. Under the assumption that the interconnection network topology has a spanning tree, consensus of the closed-loop FOMAS is analyzed with the help of the Mittag-Leffler functions and stability theory of fractional-order differential equations. It is shown that Zeno behavior can be avoided. Simulation results are presented to demonstrate the effectiveness of the theoretical results.

Keywords: Leader-following consensus, fractional-order multi-agent system, distributed event-trigger control, Zeno behavior, Mittag-Leffler function.

1. INTRODUCTION

In recent years, coordinate control of multi-agent systems (MASs) has been a hot topic due to its wide application in formation control, distributed sensor filtering, robotic cooperation, and so on. A particular objective of multi-agent coordination is to reach consensus, which requires all agents to achieve a desired common goal using only neighboring information. The consensus problem was firstly concerned in the fields of management science by Degroot (1974) and statistical physics by Vicsek et al. (1995). The consensus problem and its application have been studied in distributed decision-making system by Borkar et al.(1982) and Tsitsiklis et al.(1986). So far, a large number of the consensus problems of first-order, second-order and high-order MASs have been extensively studied by Tian et al. (2008); Li et al. (2016); Zhang et al.(2015); He et al. (2017); Wan et al.(2016).

Many works about consensus problems of MASs are investigated in the framework of integer-order dynamics, however, many complex phenomena such as macromolecule fluids, porous media and electro-magnetic waves, cannot be accurately described by integer-order models. It has been found that many complex behaviors of agents are more suitably modeled by fractional-order dynamics. Distributed coordination of MASs with fractional-order dynamics was early studied by Cao et al. (2008-2010) and Ren et al. (2011). Sufficient conditions were derived by Yin et al.(2013) to ensure the consensus of heterogeneous fractional-order MASs in terms of linear matrix inequalities. Consensus of FOMASs with heterogeneous input delays and communication delays were studied by Shen et al. (2012), which showed that consensus conditions do not rely on communication delays, but depend on input delays when the fractional order belongs to (0, 1). A leader-following consensus problem of FOMASs with nonlinear dynamics was considered by Yu et al. (2015), and some sufficient conditions on consensus were presented. A consensus problem of FOMASs was addressed using adaptive pinning control method by Chai et al. (2012), and then was extended to the case with delay by Liang et al. (2016). The observer-based strategy for the consensus of FOMASs with input delay was studied by Zhu et al. (2017).

In many physical systems, due to the limited on-board resources and capabilities of computation, communication and actuation, event-triggered control strategies guarantee that agents update their control protocols only at some event time instants and thus draw enthusiastic research interest. Guinaldo et al. (2011) considered a distributed event-based control strategy for a networked dynamical system consisting of N linear time-invariant subsystems with perfect decoupling conditions. The event-triggered conditions were given by some state-independent functions by Seyboth et al. (2013) and Guinaldo et al. (2011). However, up to now, there are rare literature considering consensus of FOMASs with event-triggered control. Furthermore, some techniques employed in integer-order MASs cannot be straightly used to deal with consensus control of FOMASs. Even though a consensus problem of a FOMAS was addressed using sampled-data control by Yu et al. (2017), the sampling instants have fixed period, which is not flexible. Recently, a leader-following problem of FOMASs with single input was investigated by Wang et al. (2017). An event-triggered leader-following consensus problem of general linear FOMASs and the system with input delay has been investigated by Ye et al.(2018).

Motivated by the above discussion,this paper studies a leader-following consensus problem of a general linear FOMAS by employing a novel event-triggered control strategy and

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function. The main contributions of this paper are summarized as follows: First, a novel event-triggered control strategy is developed for each agent without using real-time relative information from its neighboring agents. Second, a theoretical analysis is provided for both the consensus analysis of the FOMAS and Zeno behavior by using the Mittag-Leffler function method, which is more challenging for the consensus analysis of integer-order MASs.

The rest of this paper is organized as follows. Section 2 presents some preliminaries and formulates the leader-following problem of a FOMAS. A distributed event-triggered consensus control is proposed and some theoretical results are presented in Sections 3. Simulation results are given in Section 4 to illustrate the effectiveness of the theoretical results. Conclusions are given in Section 5.

2. PRELIMINARIES AND PROBLEM FORMULATION

This section presents some basic notations about the algebraic graph theory, the Caputo fractional-order derivatives and the fractional integral, and then formulates the leader-following consensus problem of a linear FOMAS.

2.1 Graph Theory

Let $V=\{1,2,\cdots,N\}$ be a set of nodes and $E=\{(j,i)\,|\,i,j\in V\}$ be a set of edges. A directed graph $G=(V,E)$ is used to model the communication network topology among a group of autonomous agents. The i th node represents the i th agent. The set of in-neighbors of agent i is denoted by $N_i=\{j\in V\,|\,(j,i)\in E\}$. Thus $j\in N_i$ means that agent i can receive the information of agent j . A path in a digraph is a sequence $i_0,i_1,\cdots,i_l$ of distinct nodes such that $(i_{l_1-1},i_{l_1})\in E$, $l_1=1,2,\cdots,l$. A directed tree is a digraph, where every node has exactly one parent except for the root. A directed spanning tree of a digraph is a directed tree formed by graph edges that connects all the nodes of the graph.

Let $\mathcal{A}=(a_{ij})_{N\times N}$ be the adjacency matrix, where $a_{ij}>0$ if $(j,i)\in E$ and $a_{ij}=0$, otherwise. $D=diag\{d_1,d_2,\cdots,d_n\}$ is the degree matrix whose diagonal elements are defined by $d_i=\sum_{j\in N_i}a_{ij}$. The Laplacian matrix of the weighted digraph G is defined as $L=D-\mathcal{A}$. It is well-known that L has exactly one zero eigenvalue and all the other eigenvalues have positive real parts if and only if the digraph G has a directed spanning tree. Furthermore, when the N agents interact with a leader, we use a diagonal matrix $\bar{\mathrm{B}}=diag(b_1,b_2,\cdots,b_n)$ to describe the interaction relationships among the agents and the leader. Let $H=L+\bar{\mathrm{B}}$. Then we have

Lemma2.1. (Ren et al. 2011) All the eigenvalues of the matrix H have positive real parts if and only if the interconnection network of the leader-follower system has a spanning tree with the leader being the root node.

2.2 Caputo Fractional-order Operator

Caputo and Riemann-Liouville (R-L) fractional-order derivatives are commonly used in fractional-order dynamical systems. Since the initial conditions for fractional-order differential equations with Caputo fractional-order derivative have the same form with the traditional integer-order differential equations, we will adopt the Caputo fractional-order derivative to model the FOMAS in this paper .

Definition2.1. The Caputo fractional-order derivative of a function $f(t)$ with order q is defined as follows:

$$D^q f(t)=\frac{1}{\Gamma(n-q)}\int_{t_0}^{t}\frac{f^{(n)}(\theta)}{(t-\theta)^{q-n+1}}d\theta\ ,$$

where n is an integer, q satisfies $n-1<q<n$, and $\Gamma(\cdot)$ is the Gamma function defined as

$$\Gamma(z)=\int_0^\infty e^{-t}t^{z-1}dt\ .$$

Particularly, when $0<q<1$,

$$D^q f(t)=\frac{1}{\Gamma(1-q)}\int_{t_0}^{t}\frac{f'(\theta)}{(t-\theta)^{q}}d\theta\ .$$

Definition2.2. The fractional integral of order α for a function $f(x)$ is defined as

$$I^\alpha f(t)=\frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\theta)^{\alpha-1}f(\theta)d\theta\ .$$

2.3 Problem Formulation

In this paper, we consider a consensus problem of a FOMAS consisting of one leader and N following agents. The dynamics of the leader labeled by 0 and the followers are respectively described by the following fractional-order differential equations:

$$\begin{cases}D^q x_0(t)=Ax_0(t)\\ D^q x_i(t)=Ax_i(t)+Bu_i(t),\quad i=1,2,\cdots,N\end{cases}\tag{1}$$

where $D^q x_i(t)$ is the Caputo derivative of $x_i(t)$, $q\in(0,1]$ is the fractional order, $x_i(t)\in R^n$ and $u_i(t)\in R^m$ denote the state and control input of the i th agent, respectively, $A\in R^{n\times n}$ and $B\in R^{n\times m}$ are constant matrices with appropriate dimensions.

Definition2.3. The FOMAS (1) is said to achieve leader-following consensus if there is a state feedback $u_i(t)$ such that the closed-loop system satisfies

$$\lim_{t\to+\infty}\|x_i(t)-x_0(t)\|=0\ ,\tag{2}$$

for any initial condition $x_i(t_0),i=1,2,\cdots N$.

In sequel, we need the following useful lemmas related with Mittag-Leffler functions.

Lemma2.2. (De et al. 2011) For any matrix A and a constant $q\in(0,1]$, there exist constants M_1,M_2 such that

$$\left\|E_{q,1}(At^q)\right\|\le M_1\left\|e^{At}\right\|,\ \left\|E_{q,q}(At^q)\right\|\le M_2\left\|e^{At}\right\|,$$

where $E_{\alpha,\beta}(z)$ is the Mittag-Leffler function defined by

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)},\alpha>0,\beta>0\ .$$

Lemma2.3. (Ye et al. 2007) Suppose $a(t)$ is a nonnegative locally integrable function during the time-interval $[0,T]$

$(T \le +\infty)$, and $g(t)$ is a nonnegative, non-decreasing continuous function defined in $[0, T]$, $g(t) \le M$ (constant), and $v(t)$ is nonnegative and locally integrable during the interval $[0, T]$, satisfying $v(t) \le a(t) + g(t)\int_{t_0}^{t}(t-s)^{q-1}v(s)ds$ for a positive constant q. Then we have

$$v(t) \le a(t) + g(t)\int_{t_0}^{t}\left[\sum_{n=1}^{\infty}\frac{(g(t)\Gamma(q))^n}{\Gamma(nq)}(t-s)^{nq-1}a(s)\right]ds, \quad t_0 \le t < T.$$

Furthermore, if $a(t)$ is a non-decreasing function during the interval $[0,T)$, then we have

$$v(t) \le a(t)E_{q,1}(g(t)\Gamma(q)(t-t_0)^q).$$

Lemma2.4. (Podlubny I. 1998) Suppose $0 < q \le 1$ and β is a constant. If there is a ω such that $\frac{\pi q}{2} < \omega < \pi q$, then we have

$$|E_{q,\beta}| \le C_1(1+|z|)^{1-\beta/q}\exp(\mathrm{Re}(z^{1/q})) + \frac{C_2}{1+|z|},$$

where C_1, C_2 are positive constants, and $|\arg(z)| \le \omega$.

Lemma2.5. (Kilbas et al. 2006) Let $\Omega = [0,b]$ be an interval on the real axis R, let $n = [q]+1$ with $[q] = \max\{m \in \mathbb{Z} \mid m \le q\}$ for $q \notin \mathbb{N}$ or $n = q$ for $q \in \mathbb{N}$. If $y(t) \in C^n[0,b]$, then we have

$$I^q D^q y(t) = y(t) - \sum_{k=0}^{n-1}\frac{y^{(n)}(0)}{k!}t^k.$$

Particularly, if $0 < q \le 1$ and $y(t) \in C^1[0,b]$, then

$$I^q D^q y(t) = y(t) - y(0).$$

3. EVENT-BASED LEADER-FOLLOWING CONSENSUS OF FOMAS

In this section, the design and analysis of a consensus control is considered for a general linear FOMAS and the rigorous analysis is also given for Zeno behavior.

3.1 Event-triggered Consensus Control

Let us consider the latest broadcasted state of agent i given by $x_i(t_k^i)$ at the event-triggered time instant t_k^i. Define a relative state information from the neighboring agents for agent i as follows:

$$p_i(t) = \sum_{j=1}^{N}a_{ij}\left(x_i(t) - x_j(t)\right) + b_i\left(x_i(t) - x_0(t)\right).$$

Then an event-triggered control is proposed for agent i as follows:

$$u_i(t) = -K\,p_i(t_k^i),\ t \in [t_k^i, t_{k+1}^i) \quad (3)$$

where $K \in R^{m\times n}$ is the control gain matrix to be determined. Define the relative measurement error for agent i as $e_i(t) = p_i(t_k^i) - p_i(t)$. The triggering time sequence $\{t_k^i\}$ for agent i is defined by

$$t_{k+1}^i = \inf\left(t \mid t > t_k^i, f_i(t) > 0\right), \quad (4)$$

where the event-triggered function is given by

$$f_i(t) = \|e_i(t)\| - \beta_1\|p_i(t_k^i)\| - \beta_2 e^{-\gamma(t-t_0)}, \quad (5)$$

for some parameters $\beta_1 > 0, \beta_2 > 0$, and $\gamma > 0$. When $f_i(t) = 0$ during , we can have the next triggered time instant t_{k+1}^i. In this distributed control strategy, we assume that each agent can obtain its neighbors' information at its own triggered time t_k^i.

3.2 Consensus Analysis

Let $\varepsilon_i(t) = x_i(t) - x_0(t), i = 1,2,\cdots,N$. From (1) and (3), we have

$$D^q\varepsilon_i(t) = A\varepsilon_i(t) - BK\left(\sum_{j=1}^{N}a_{ij}\left(x_i(t)-x_j(t)\right) + b_i\left(x_i(t)-x_0(t)\right) + e_i(t)\right). \quad (6)$$

Denote $\varepsilon(t) = \left(\varepsilon_1^T(t),\cdots,\varepsilon_N^T(t)\right)^T$, $e(t) = \left(e_1^T(t),\cdots,e_N^T(t)\right)^T$, and $\bar{A} = I_N \otimes A - H \otimes BK$, $H = L + \bar{B}$, then (6) can be rewritten in the following compact form

$$D^q\varepsilon(t) = \bar{A}\varepsilon(t) - \left(I_N \otimes BK\right)e(t). \quad (7)$$

Now we give a main result for the consensus analysis of the FOMAS (1) under the controller (3).

Theorem 3.1. Assume that (A,B) is stabilizable and the communication network topology of the leader-follower system has a directed spanning tree with the leader as the root. Then the leader-following consensus can be achieved for the FOMAS under the controller (3) with the gain matrix $K = hB^TP$, P is positive definite solution of the following algebraic Riccati equation (ARE): $A^TP + PA - PBB^TP + Q = 0$, Q is a positive definite matrix.

Proof. First, we show the existence of the control gain matrix K. Since (A,B) is stabilizable, then the algebraic Riccati equation has a unique nonnegative definite solution $P = P^T$ for any given positive definite matrix $Q = Q^T$. Moreover, all the eigenvalues of $A - BB^TP$ have negative real parts. On the other hand, by Lemma 2.1, all the eigenvalues of H have positive real parts, which implies that there is a positive constant h such that $h\mathrm{Re}\lambda_i(H) > 1$. Thus, the gain matrix $K = hB^TP$ can ensure that $\bar{A}$ is a Hurwitz matrix.

Second, we can have the solution of (7) expressed by

$$\begin{aligned}\varepsilon(t) &= E_{q,1}\left(\bar{A}(t-t_0)^q\right)\varepsilon(t_0) \\ &+ \int_{t_0}^{t}(t-s)^{q-1}E_{q,q}\left(\bar{A}(t-s)^q\right)\left(I_N \otimes BK\right)\left(-e(s)\right)ds.\end{aligned} \quad (8)$$

Define a positive constant as $\lambda = -\max_i\left\{\mathrm{Re}\lambda_i(\bar{A})\right\}$. Thus λ is a positive constant. Furthermore, there exists a positive constant M_3 such that $\|e^{\bar{A}(t-t_0)}\| \le M_3e^{-\lambda(t-t_0)}$, $t > t_0$. Then, by Lemma 2.2 and (8) we have

$$\begin{aligned}\|\varepsilon(t)\| &\le \|E_{q,1}(\bar{A}(t-t_0)^q)\|\cdot\|\varepsilon(t_0)\| \\ &+ \int_{t_0}^{t}(t-s)^{q-1}\|E_{q,q}(\bar{A}(t-s)^q)\|\cdot\|I_N \otimes BK\|\cdot\|e(s)\|ds \\ \|\varepsilon(t)\| &\le Me^{-\lambda(t-t_0)}\|\varepsilon(t_0)\| \\ &+ M\int_{t_0}^{t}(t-s)^{q-1}e^{-\lambda(t-s)}\|I_N \otimes BK\|\cdot\|e(s)\|ds\end{aligned} \quad (9)$$

where $M = \max\{M_1M_2, M_1M_3\}$.

Now we consider the norm of $e(t)$. The event-triggered function guarantees that

$$\begin{aligned}\| e_i(t)\| &\le \beta_1 \| p_i(t_k^i)\| + \beta_2 e^{-\gamma(t-t_0)} \\ &= \beta_1 \| p_i(t) + e_i(t)\| + \beta_2 e^{-\gamma(t-t_0)} \\ &\le \beta_1(d_i + b_i)\| \varepsilon_i(t)\| + \beta_1 \sum_{j=1}^{N} a_{ij} \| \varepsilon_j(t)\| + \beta_1 \| e_i(t)\| + \beta_2 e^{-\gamma(t-t_0)} \end{aligned} \tag{10}$$

Then we have

$$\| e(t)\| \le \frac{\beta_1(d+Na)}{1-\beta_1}\| \varepsilon(t)\| + \frac{N\beta_2}{1-\beta_1} e^{-\gamma(t-t_0)} \quad , \tag{11}$$

where $d = \max_i \{d_i + b_i\}, a = \max_{i,j}\{a_{ij}\}$. From (9) and (11), we have

$$\begin{aligned}\| \varepsilon(t)\| &\le M e^{-\lambda(t-t_0)} \| \varepsilon(t_0)\| \\ &+ M\int_{t_0}^{t} (t-s)^{q-1} e^{-\lambda(t-s)} \| I_N \otimes BK\| \cdot \frac{N\beta_2}{1-\beta_1} e^{-\gamma(s-t_0)} ds \\ &+ M\int_{t_0}^{t} (t-s)^{q-1} e^{-\lambda(t-s)} \| I_N \otimes BK\| \cdot \frac{\beta_1(d+Na)}{1-\beta_1} \| \varepsilon(s)\| ds \\ &\le M\left[e^{-\lambda(t-t_0)} \| \varepsilon(t_0)\| + \| I_N \otimes BK\| \cdot \frac{N\beta_2}{1-\beta_1} e^{-\gamma(t-t_0)} \frac{\Gamma(q)}{(\lambda-\gamma)^q}\right] \\ &+ M\int_{t_0}^{t} (t-s)^{q-1} e^{-\lambda(t-s)} \| I_N \otimes BK\| \cdot \frac{\beta_1(d+Na)}{1-\beta_1} \| \varepsilon(s)\| ds. \end{aligned} \tag{12}$$

Multiplying $e^{\lambda(t-t_0)}$ on both sides of (12) leads to

$$\begin{aligned}\| \varepsilon(t)\| e^{\lambda(t-t_0)} &\le M\left[\| \varepsilon(t_0)\| + \| I_N \otimes BK\| \cdot \frac{N\beta_2}{1-\beta_1} e^{(\lambda-\gamma)(t-t_0)} \frac{\Gamma(q)}{(\lambda-\gamma)^q}\right] \\ &+ M \| I_N \otimes BK\| \cdot \frac{\beta_1(d+Na)}{1-\beta_1} \int_{t_0}^{t} (t-s)^{q-1} e^{\lambda(s-t_0)} \| \varepsilon(s)\| ds. \end{aligned} \tag{13}$$

Let $a(t) = M\left[\| \varepsilon(t_0)\| + \| I_N \otimes BK\| \cdot \frac{N\beta_2}{1-\beta_1} e^{(\lambda-\gamma)(t-t_0)} \frac{\Gamma(q)}{(\lambda-\gamma)^q}\right], g(t) = M \| I_N \otimes BK\| \cdot \frac{\beta_1(d+Na)}{1-\beta_1}$, $v(t) = e^{-\lambda(t-t_0)} \| \varepsilon(t)\|$, by Lemma 2.3, we have

$$\begin{aligned}\| \varepsilon(t)\| e^{\lambda(t-t_0)} &\le M\left[\| \varepsilon(t_0)\| + \| I_N \otimes BK\| \cdot \frac{N\beta_2}{1-\beta_1} e^{(\lambda-\gamma)(t-t_0)} \frac{\Gamma(q)}{(\lambda-\gamma)^q}\right] \\ &\cdot E_{q,1}\left[M \| I_N \otimes BK\| \cdot \frac{\beta_1(d+Na)}{1-\beta_1} \Gamma(q)(t-t_0)^q \right] \end{aligned} \tag{14}$$

For any $\frac{\pi q}{2} < \omega < \pi q$, we have $\arg\left(M \| I_N \otimes BK\| \cdot \frac{\beta_1(d+Na)}{1-\beta_1} \Gamma(q)(t-t_0)^q \right) < \omega$. Invoking Lemma2.4, there exist constants C_1, C_2 such that

$$\begin{aligned}\| \varepsilon(t)\| e^{\lambda(t-t_0)} &\le M\left[\| \varepsilon(t_0)\| + \| I_N \otimes BK\| \cdot \frac{N\beta_2}{1-\beta_1} e^{(\lambda-\gamma)(t-t_0)} \frac{\Gamma(q)}{(\lambda-\gamma)^q}\right] \\ &\cdot \left[C_1 e^{\left[M\| I_N \otimes BK\| \cdot \frac{\beta_1(d+Na)}{1-\beta_1}\Gamma(q)\right]^{1/q}(t-t_0)} + \frac{C_2}{\theta_1} \right] \end{aligned} \tag{15}$$

where $\theta_1 = 1 + M \| I_N \otimes BK\| \cdot \frac{\beta_1(d+Na)}{1-\beta_1} \Gamma(q)(t-t_0)^q$, which implies that

$$\begin{aligned}\| \varepsilon(t)\| &\le M\left[\| \varepsilon(t_0)\| + \| I_N \otimes BK\| \cdot \frac{N\beta_2}{1-\beta_1} e^{(\lambda-\gamma)(t-t_0)} \frac{\Gamma(q)}{(\lambda-\gamma)^q}\right] \\ &\cdot \left\{ C_1 e^{\left\{\left[M\| I_N \otimes BK\| \cdot \frac{\beta_1(d+Na)}{1-\beta_1}\Gamma(q)\right]^{1/q} - \lambda\right\}(t-t_0)} + C_2\theta_2 \right\}, \end{aligned} \tag{16}$$

where $\theta_2 = e^{-\lambda(t-t_0)}\left[1 + M \| I_N \otimes BK\| \cdot \frac{\beta_1(d+Na)}{1-\beta_1} \Gamma(q)(t-t_0)^q \right]^{-1}$.

When we select $\beta_1 < \frac{\lambda^q}{M \| I_N \otimes BK\| \cdot (d+Na)\Gamma(q) + \lambda^q}$ and $\gamma > \lambda$, then we have $\| \varepsilon(t)\| \to 0, as\ t \to \infty$, and thus the leader-following consensus can be achieved asymptotically.

Next, the rigorous analysis of Zeno behavior is presented to guarantee the feasibility of the proposed distributed event-triggered strategy.

Theorem 3.2. The linear FOMAS (1) with the control strategy (3) will not exhibit Zeno behavior.

Proof. Zeno behavior is excluded if the length of the inter-event times $t_{k+1}^i - t_k^i$ is strictly positive. From (1) and (3), we have

$$D^q p_i(t) = A p_i(t) - hBB^T P\left\{ \sum_{j=1}^{N} a_{ij}\left[p_i(t_k^i) - p_j(t_k^j)\right] + d_i p_i(t_k^i) \right\}, \tag{17}$$

Let $\omega_i^k = \max_{t\in[t_k^i, t_{k+1}^i]} \| D^q P_i(t)\|$ and $\sigma_i^k = \max_{1\le l\le k} \| \omega_i^l \|$. It is evident that ω_i^k and σ_i^k exist from the proof of Theorem 3.1. Combing with Lemma 2.5, we have

$$\begin{aligned}\| e_i(t)\| &= \| p_i(t_k^i) - p_i(t)\| \\ &= \frac{1}{\Gamma(q)}\left\| \int_{t_0}^{t_k^i} (t_k^i - s)^{q-1} D^q p_i(s) ds - \int_{t_0}^{t} (t-s)^{q-1} D^q p_i(s) ds \right\| \\ &= \frac{1}{\Gamma(q)}\left\| \int_{t_0}^{t_k^i} \left[(t_k^i - s)^{q-1} - (t-s)^{q-1}\right] D^q p_i(s) ds \right\| \\ &+ \frac{1}{\Gamma(q)}\left\| \int_{t_k^i}^{t} (t-s)^{q-1} D^q p_i(s) ds \right\| \\ &\le \frac{\sigma_i^{k-1}}{\Gamma(q)} \int_{t_0}^{t_k^i} \left[(t_k^i - s)^{q-1} - (t-s)^{q-1}\right] ds + \frac{\omega_i^k}{\Gamma(q)} \int_{t_k^i}^{t} (t-s)^{q-1} ds \\ &= \frac{\sigma_i^{k-1}}{\Gamma(q+1)}\left[(t_k^i - t_0)^q + (t - t_k^i)^q - (t-t_0)^q \right] + \frac{\omega_i^k}{\Gamma(q+1)}(t-t_k^i)^q \\ &\le \frac{2\sigma_i^k}{\Gamma(q+1)}(t-t_k^i)^q \end{aligned} \tag{18}$$

Further, we have

$$\frac{2\sigma_i^k}{\Gamma(q+1)}(t_{k+1}^i - t_k^i)^q \ge \| e_i(t_{k+1}^i)\| > \beta_2 e^{-\gamma(t_{k+1}^i - t_0)}. \tag{19}$$

From (19), we have

$$t_{k+1}^i - t_k^i > \left(\frac{\Gamma(q+1)\beta_2}{2\sigma_i^k} e^{-\gamma(t_{k+1}^i - t_0)} \right)^{1/q}.$$

The proof is thus completed.

4. SIMULATION EXAMPLE

In this section, a numerical example will be given to demonstrate the effectiveness of the theoretical results. Consider a linear FOMAS with a leader and four followers. The interconnection topology graph of all agents is illustrated in Fig.1.

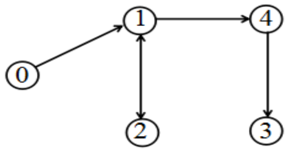

Fig.1 Interconnection network topology

The matrices A and B are given as follows:

$$A=\begin{pmatrix}0&1&0\\0&-1&1\\-1&0&0\end{pmatrix},\quad B=\begin{pmatrix}1\\0\\1\end{pmatrix}.$$

For simplicity, we assume that $a_{ij}=1$ if $j\in N_i$, otherwise, $a_{ij}=0$. There is a directed spanning tree with leader rooted in the node 0. Thus the Laplacian matrix L and $H=L+\bar{B}$ are given as follows:

$$L=\begin{pmatrix}1&-1&0&0\\-1&1&0&0\\0&0&1&-1\\-1&0&0&1\end{pmatrix},\quad H=\begin{pmatrix}2&-1&0&0\\-1&1&0&0\\0&0&1&-1\\-1&0&0&1\end{pmatrix}.$$

By simple computation, we have $\lambda_i(H)=1,1,2.618,0.32$. It is not difficult to see that (A,B) is stabilizable. Select Q as the identity matrix. Then we solve the ARE and have

$$P=\begin{pmatrix}4.2841&1.8156&-2.0332\\1.8156&1.4473&-0.4978\\-2.0332&-0.4978&1.9663\end{pmatrix}.$$

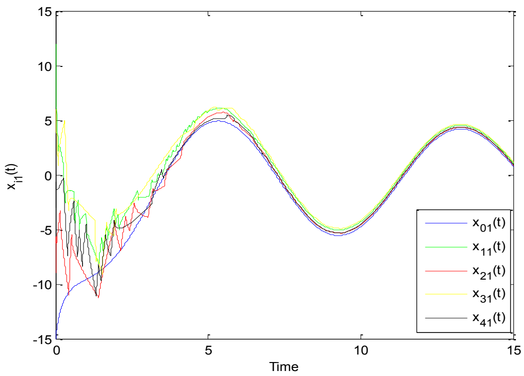

Fig. 2 State evolution of the 1th component

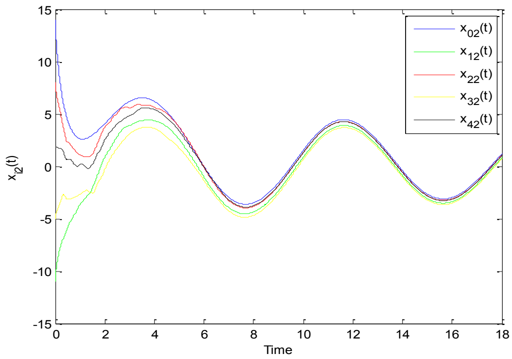

Fig.3 State evolution of the 2th component

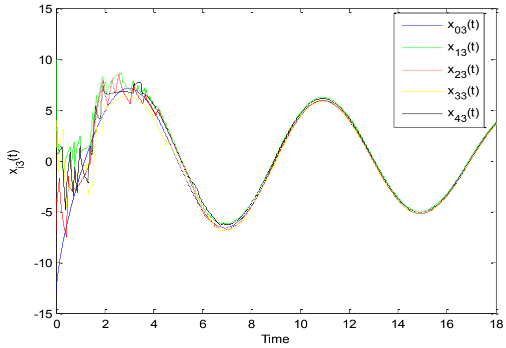

Fig.4 State evolution of the 3th component

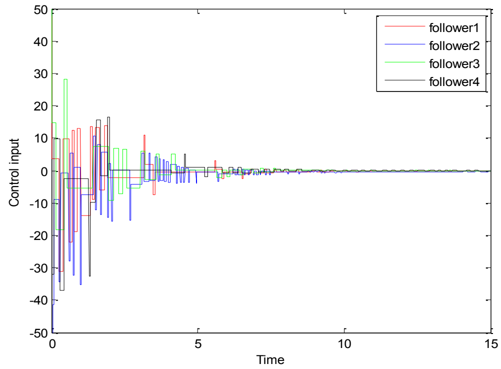

Fig.5 Control input $u_i(t)$ of all followers

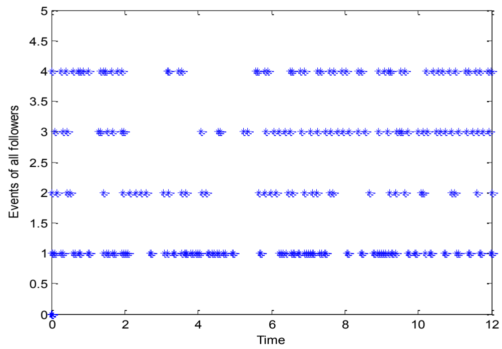

Fig.6 Triggering instances of all agents

The gain matrix is given by $K=hB^TP$ with $h=3$. Let $q=0.8,\gamma=0.5361,\beta_1=0.0066,\beta_2=10$ and the initial state is given by $x_0(0)=[-15;14;-13], x_1(0)=[12;-11;10], x_2(0)=[-9;8;-7]$, $x_3(0)=[6;-5;4]$, and $x_4(0)=[-3;2;-1]$. Let $x_{ij}(t)$ denote the j th component of the i th agent. Fig. 2-4 illustrate the state trajectories of the leader and the four followers, which shows that the followers can follow the leader under the proposed event-triggered control strategy (3). Fig. 5 gives the evolution of the control inputs while Fig. 6 gives the event-triggered time instants of the four followers, which shows that Zeno behavior is avoided.

6. CONCLUSIONS

This paper has made a first attempt to study a consensus control of a general linear FOMAS over a directed interaction network. A distributed event-triggered state-feedback control strategy has been proposed to guarantee the consensus. With the help of the Mittag-Leffler function method, the leader-following consensus of the FOMAS has been analyzed.

Additionally, a rigorous proof has also been provided for Zeno behavior.

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