A Topology Identification and Impedance Estimation Method for Distribution Network with Distributed Generations

Zhenyu Zhang*, Yong Li*, Jun Zhang*, Jing Duan*, Yijia Cao*

*College of Electrical and Information Engineering, Hunan University, Changsha 410082, China; (email:yongli@hnu.edu.cn)

Abstract: This paper proposes a topology identification and impedance estimation (TIIE) method for the distribution network with distributed generation (DG) units. When the DG unit is connected to the distribution grid, the power injections between the different buses in the distribution grid are no longer independent of each other. This paper demonstrates that the topology identification result has to be corrected when the existing voltage measured based methods are applied to the distribution network involving DG units. To solve the problem, we develop the TIIE method to correct the topology identification error by using the estimation of line impedance. The proposed method does not require any prior knowledge of the network. The case results show a high accuracy on the connectivity identification as well as the estimation of line parameter.

Keywords: Topology identification; Distribution network; Impedance estimation; Distributed generations; Data-driven method.

1. INTRODUCTION

The topology of the distribution network is a key parameter for modeling the distribution network. With the development of smart grids, distributed power generation has gradually increased, and the operating state of the system has become more and more complicated. In order to achieve complete control of the state of the distribution network, the topology information of the distribution network must be required.

The proposed topological identification methods are mainly divided into the following types: regression methods (Liao et al., 2016), maximum likelihood classification methods (Alam et al., 2014; Erseghe et al., 2013), correlation analysis methods (Luan et al., 2015; Weng et al., 2017) , mixed integer quadratic programming methods (Tian et al., 2016), and matrix decomposition methods (Pappu et al., 2018). In these methods, matrix decomposition and mixed integer quadratic planning methods (Pappu et al., 2018; Tian et al., 2016) not only require a large amount of historical measurement data (voltage, active power, and reactive power) , but also require a lot of known topological information, which makes it difficult to apply in practice.

The method based on regression and correlation analysis (Liao et al., 2016; Luan et al., 2015; Weng et al., 2017) does not require any a priori knowledge of topology and solves the problem that a priori information is too difficult to obtain. This type of method uses only voltage measurement data, and the data is relatively easy to obtain. However, such methods often have a premise: the power injection in the distribution grid should be independent so that the voltage is topology related.

With the penetration of distributed generation (DG), DG units with voltage control has been applied for the voltage support of the distribution network (Kraiczy et al., 2017). Through exploits the reactive power capability of DG units (Kryonidis et al., 2019) or electric vehicles (Sousa et al., 2019), voltage control of the distribution network can be achieved. But meanwhile, support for voltage means that the injection of power is controlled by the voltage, which undermines the power injection independence assumption of the distribution network, which in turn leads to failure of algorithms that rely on this assumption.

To solve this problem, we note that the topology identification problem can be considered in conjunction with the parameter estimation problem, and the sensitivity matrix can serve as a suitable link between the two. The values in the sensitivity matrix are related not only to the parameters of the line but also to the topology in the network. Therefore, we use a sensitivity matrix to correct for the effect of DG's voltage control on topology identification.

In this regard, we propose a topology identification and impedance estimation (TIIE) method. With the impedance and topology are mutually verified, the effects of voltage control caused by DG units can be eliminated. Besides, the proposed method has a higher accuracy of impedance estimation by modifying the traditional sensitivity formula.

2. BASIC PROBLEM FORMULATION

This section describes some of the concepts in the method before the formal introduction of the method used. This section has three main parts, the concept of sensitivity matrix, the concept of mutual information, and the extent to which both are affected by voltage-containing DG.

2.1 Sensitivity Matrix

The typical power flow equations are as follows:

$$P_i = \sum_{k=1}^{N} |V_i||V_k|(G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}) \qquad (1)$$

$$Q_i = \sum_{k=1}^{N} |V_i||V_k|(G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}) \qquad (2)$$

where $i = 1,\ldots,N$, P_i is the active power injected at bus i, Q_i is the reactive power injected at bus i, G and B are the real and imaginary parts of the admittance matrix, V_i and V_k are the voltage magnitude at bus i and k, θ_{ik} is the phase angle difference between bus i and k.

Based on the power flow equations (1), (2) and Jacobian Matrix, the relationships between the voltages and power injections are represented by the following matrix expression:

$$\begin{bmatrix} \Delta E \\ \Delta \theta \end{bmatrix} = \begin{bmatrix} \dfrac{\partial P}{\partial V} & \dfrac{\partial P}{\partial \theta} \\ \dfrac{\partial Q}{\partial V} & \dfrac{\partial Q}{\partial \theta} \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \qquad (3)$$

where ΔE and $\Delta \theta$ are the voltage magnitudes (rms) and phase variations corresponding to the active or reactive power injections ΔP and ΔQ.

Brenna et al. based on the (3) proposed a more easier theory than classical theory and it is suitable for radial middle voltage distribution networks (Brenna et al., 2010). Without considering phase angle, the proposed new theory can be expressed simply as follow:

$$\Delta E = S_P \Delta P + S_Q \Delta Q \qquad (4)$$

where S_p and S_Q are the active and reactive sensitivity matrix. Further, taking S_Q as an example, the sensitivity matrix can be expressed as:

$$S_Q = -\frac{1}{E_N} X_Q \qquad (5)$$

where E_N is the rated voltage of network, X_Q is a coefficient matrix and the value of its elements can be uniquely determined by the following rules:

1. Define the reachable path between the two nodes as T_{ij}.

2. Determine the shortest path L between the reference node and T_{ij}.

3. The value of $X_Q[i,j]$ is obtained by arithmetically summing the reactance on the path L.
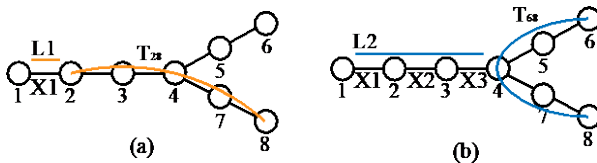

(a)   (b)

Fig. 1. A schematic diagram of the calculation. (a) Calculation of $X_Q[2,8]$. (b) Calculation of $X_Q[6,8]$.

As shown in Fig.1 (a), to calculate $X_Q[2,8]$, first you need to find the T28 shown by the orange line, and then sum the reactance on the shortest path L1. That is $X_Q[2,8] = X1$. Similarly, in Fig.2(b), $X_Q[6,8] = X1 + X2 + X3$. This rule is equally valid for solving S_P, just replace the reactance with resistor.

It can be found that the sensitivity matrix is a matrix related to line impedance and network topology. In other words, if the sensitivity matrix can be obtained in another way, it is possible to recover the impedance of the line and the network topology therefrom. Our method achieves this and will introduce in the Section 3.

2.2 Mutual information

The mutual information of two random variables is a measure of how independent the two random variables are. In the topology identification problem, the mutual information can be used to measure the correlation between the voltages of the two nodes, and further, the topology can be reconstructed according to the mutual information(Weng et al., 2017) .

For two continuous random variables X and Y are whose joint probability distribution is $P_{XY}(x,y)$, and the mutual information between them denoted $I(X;Y)$, the calculation formula of $I(X;Y)$ is as follow:

$$I(X;Y) = \int_y \int_x P_{XY}(x,y) \log \frac{P_{XY}(x,y)}{P_X(x)P_Y(y)} \qquad (6)$$

where P_{XY} is the joint probability density function of X and Y, and P_X and P_Y are the marginal probability density functions of X and Y respectively.

However, in practice, the estimation of continuous probability mutual information is still a difficult problem. It is difficult to get the probability density distribution of the variables. And the specific estimation method of mutual information is not the focus of this paper, so in this paper, we use the mutual information estimation method proposed by Kraskov et al (Kraskov et al., 2004).

2.3 The effect of voltage-controlled DG

In this section, we will discuss the impact of DG on sensitivity matrix estimation and mutual information estimation. Since the voltage control of the DG destroys the power injection independence condition of the DG access node, it can be expected that the mutual information at the DG access node will increasing, and this increased trend will propagate with the line.

As shown in Fig.2, The effect of DG on data-driven methods for mutual information and sensitivity matrix estimation is demonstrated where two DGs with voltage control function are deployed at the positions corresponding to 16,31 in the matrix. The mutual information is estimated using the Kraskov method, and the data-driven method for estimating the

Table 1. Nomenclature

Variables and Parameters	
$\overline{E_n}$	Average RMS voltage of bus n
E_N	Rated voltage of network
ΔE_n^m	Difference between the mth and $(m-1)^{th}$ voltage measurements at bus n
$\Delta P_n^m, \Delta Q_n^m$	Difference between the mth and $(m-1)^{th}$ real/reactive power measurements at bus n
R_{ij}, X_{ij}	Resistance/reactance of the line between bus i and j
P_n^o, Q_n^o	Origin real/reactive power at bus n.
P_n^t, Q_n^t	Real/reactive power at bus n at time t
Matrix	
S_P, S_Q	Matrix of real/reactive sensitivity
X_Q	Matrix of reactance, $X_Q = -E_N S_Q$
M	Matrix of voltage mutual information
Indices	
N_s	Index of substation outlet bus
N_n	Index of bus which has minimum electrical distance to N_s in Ω_n
i, j, n	Indices of buses
Set	
Ω_n	Set for lines with negative impedance

sensitivity matrix will be described in the next section. The red box indicates the part that was negatively affected.

It can be seen that the mutual information matrix M has strongly interfered, leading to the degradation of the ability that the mutual information evaluates the topology. In the absence of DG with voltage control, the connection between mutual information and topology has a strong correlation. However, when DG is introduced, the influence caused by voltage control is also increased, and the value of the mutual information matrix is generally increased. When using such metrics for topology identification, there is an inevitable error. But, there is almost no effect to X_Q calculated from the sensitivity matrix which shows that the estimation of the sensitivity matrix is more robust. According to the above analysis, we propose a method to correct the influence to M by cross-validating the topology and its corresponding impedance.

3. METHOD

Based on the previous analysis, we propose a TIIE method that combines the advantages of sensitivity analysis and mutual information analysis. The idea of the algorithm is to first use the measurement information (including voltage, active and reactive power) to estimate the sensitivity matrix and mutual information matrix of the system. Mutual information is then used to estimate an initial topology. Then using the initial topology, the line impedance parameters corresponding to the topology can be estimated from the sensitivity matrix. By checking the rationality of the impedance parameters, the bad values in the mutual information estimation can be eliminated, and then a new topology is re-estimated based on the modified mutual information matrix. By repeating this process, you can eliminate the effects of voltage-controlled DG and get the right topology.
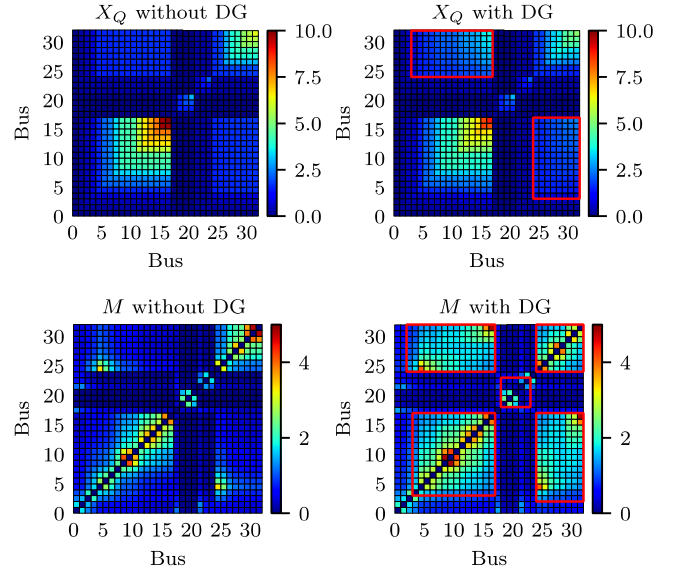

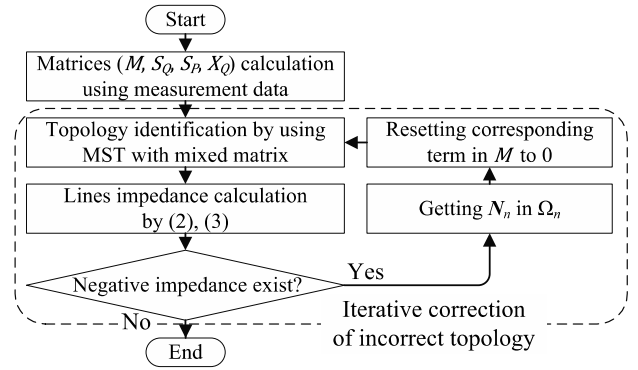Fig. 2. Heat map of X_Q and M with and without DG.

Fig. 3. Flowchart of the proposed TIIE method.

The flowchart of the proposed TIIE method is shown in Fig. 3. Table 1 shows the notations used in the proposed method. In step 1, the sensitivity matrices S_Q and S_P are obtained by the least-squares solution of the following overdetermined equation:

$$\begin{bmatrix} \Delta E_1^1 & \cdots & \Delta E_n^1 \\ \vdots & \ddots & \vdots \\ \Delta E_1^m & \cdots & \Delta E_n^m \end{bmatrix} = \begin{bmatrix} \Delta Q_1^1 & \cdots & \Delta Q_n^1 & \Delta P_1^1 & \dots & \Delta P_n^1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \Delta Q_1^m & \dots & \Delta Q_n^m & \Delta P_1^m & \dots & \Delta P_n^m \end{bmatrix} \begin{bmatrix} S_Q \\ S_P \end{bmatrix} \quad (7)$$

In step 2, generate a network topology by using the maximum spanning tree (MST) algorithm. Here, we choose the mixed matrix (which equals to the sum of M and X_Q) as its weight matrix.

Lines impedance calculation in step 3 are shown in (8) and (9). The bus i and j are a pair of adjacent buses, and i is closer to N_s. Note that the formulas have been optimized and are different from (5). Particularly, when S_Q and S_P have been obtained by solving (7), $\overline{E_J}$ is used to replace E_N. Experimentally, we show that this method can reduce the error in impedance estimation.

$$X_{ij} = -\left(S_Q(j,j) - S_Q(i,i)\right)\overline{E_J} \quad (8)$$

$$R_{ij} = -\big(S_P(j,j) - S_P(i,i)\big)\overline{E}_J \qquad (9)$$

Step 4 is to check whether to exist negative values in the solved impedance. The negative value indicates that there is an error existing in the identified topology. Meanwhile, we get the set Ω_n. In step 5 we get N_n.

In step 6, the terms corresponding to N_s and the neighbor bus of N_s in Ω_n are reset to 0 in M. It is the key to correct the misidentification caused by similar voltage changes. Line impedance cannot be negative, so the connection corresponding to lines with negative impedance must be wrong. By this method, topology recognition errors due to voltage control of the DG can be partially corrected.

Finally, by repeating steps 2-6, the erroneous connection is gradually eliminated until there is no negative impedance. At last, we get the topological relationship and the line impedances.

4. CASE RESULT

The IEEE 33-bus system is used to validate the proposed TIIE method. The system data is given in the pandapower software (Thurner et al., 2018). To obtain the effect of DG units voltage control by simulation, two buses in the network are modeled as voltage-controlled DG unit buses. We set the target for busbar voltage control to float within 1% of the voltage rating and assume that the controller will always complete the control task. So, we get two buses with similar voltage profiles. For all PQ buses, we vary the power according to a random uniform distribution according to the following equations:

$$P_n^t \sim \text{Unif}(0.5P_n^o, 1.5P_n^o), t = 1, \dots, T \qquad (4)$$

$$Q_n^t \sim \text{Unif}(0.5Q_n^o, 1.5Q_n^o), t = 1, \dots, T \qquad (5)$$

where the letters' meanings in equations are shown in table 1.

By verifying our method on the simulated data, we can obtain the accuracy of topology identification and the accuracy is defined as the number of correctly identified lines divided by the total number of lines (since the voltage of the bus 0 is fixed, so the total number of lines is 31 not 32). We test the performance of the method when DG units are connected to different buses, and two DG units are set up in each case, i.e. Case 2: 16, 29. Case 2: 17, 21. Case 3: 21, 32. Case 4: 21,24. Fig. 4 shows a schematic diagram of DG deployment on the IEEE33 bus system. The proposed method uses the voltage, active power and reactive power measurements on each bus bar.
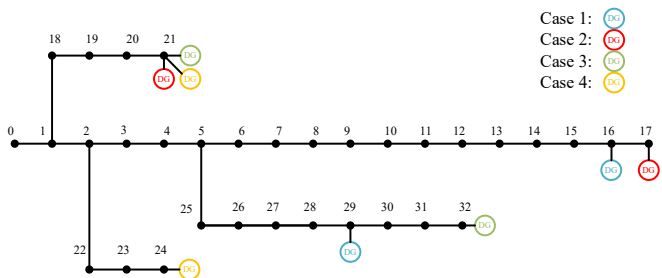
We conducted 100 independent replicate trials at different sample sizes and the results obtained are presented in Fig. 5 and 6. To avoid the effect of extreme values on the results, we show the mean values with their 95% confidence intervals. As can be seen from Fig. 5, our method performs poorly with a small sample size. This is because the sensitivity matrix estimated at this time is not accurate. With the increased sample size, the accuracy and stability of the proposed method are better than those based on mutual information. Fig. 6 shows the mean square error of impedance estimation compared with traditional sensitivity theory. This proves that we use the average value of the voltage on the local node instead of the rated voltage value in equation (8)(9), which plays a positive role in the accuracy of impedance estimation.
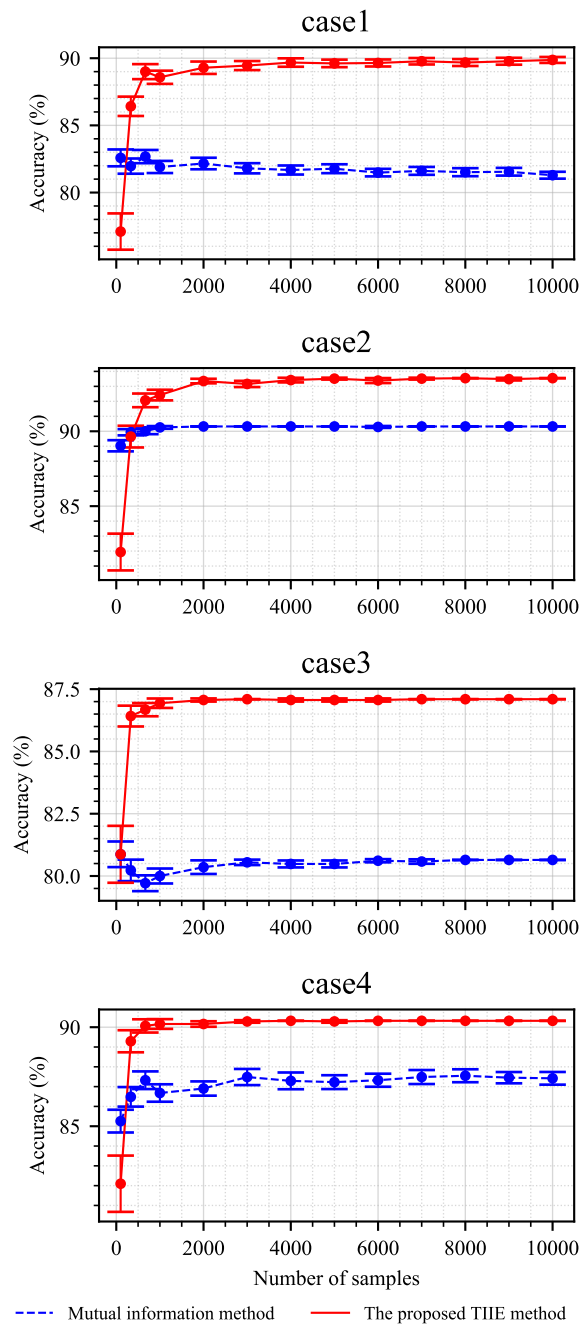

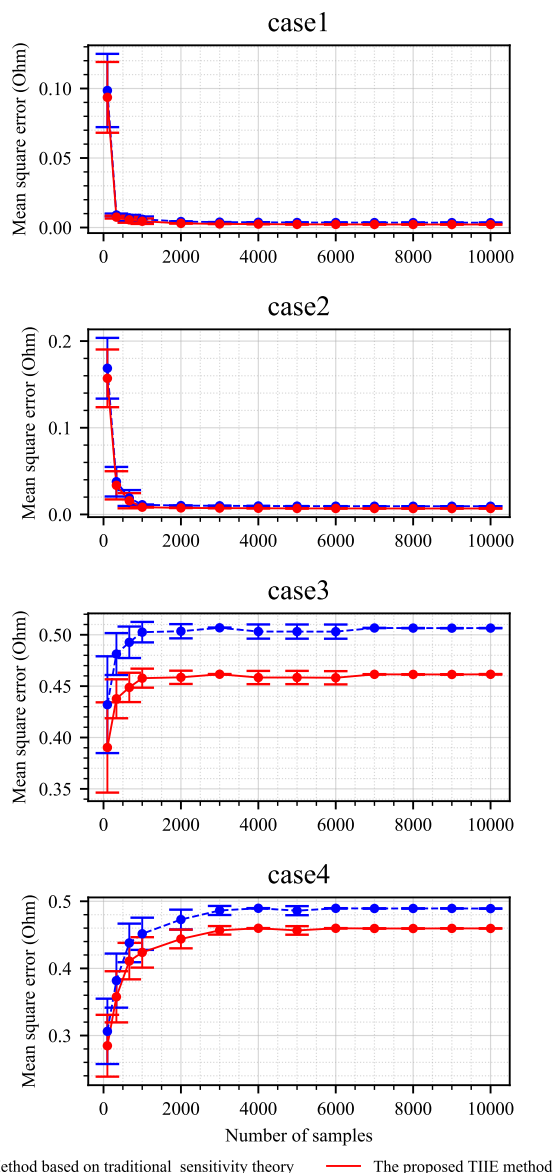
Fig. 5. Topology identification accuracy.

Fig. 4. Schematic diagram of DG deployment.

Fig. 6. Mean square error of the impedance estimation.

5. CONCLUSION

In this paper, a TIIE method without considering the prior information is proposed to solve the problem of topology identification errors caused by the DG units. In this method, we estimate the sensitivity matrix by solving the least-squares problem. Further, the proposed method consists of iteratively impedance estimation and topology recognition and corrects the topological errors by judging the rationality of impedance estimation results. Case results indicate that, compared with the traditional method, the proposed TIIE method exhibits a better performance in identifying topology and estimating the impedance of the distribution network with DG units.

ACKNOWLEDGEMENTS

REFERENCES

Alam, S. M. S., Natarajan, B., & Pahwa, A. (2014). Distribution Grid State Estimation from Compressed Measurements. *IEEE Transactions on Smart Grid*, *5*(4), 1631–1642. https://doi.org/10.1109/TSG.2013.2296534

Bolognani, S., Bof, N., Michelotti, D., Muraro, R., & Schenato, L. (2013). Identification of power distribution network topology via voltage correlation analysis. *52nd IEEE Conference on Decision and Control*, 1659–1664. https://doi.org/10.1109/CDC.2013.6760120

Brenna, M., Berardinis, E. D., Foiadelli, F., Sapienza, G., & Zaninelli, D. (2010). Voltage Control in Smart Grids: An Approach Based on Sensitivity Theory. *Journal of Electromagnetic Analysis and Applications*, *02*(08), 467–474. https://doi.org/10.4236/jemaa.2010.28062

Erseghe, T., Tomasin, S., & Vigato, A. (2013). Topology Estimation for Smart Micro Grids via Powerline Communications. *IEEE Transactions on Signal Processing*, *61*(13), 3368–3377. https://doi.org/10.1109/TSP.2013.2259826

Kraiczy, M., Fakhri, L. A., Stetz, T., & Braun, M. (2017). Do It Locally: Local Voltage Support by Distributed Generation—A Management Summary: Management Summary of IEA Task 14 Subtask 2—Recommendations Based on Research and Field Experience. *IEA Internation Energy Agency: Fribourg, Switzerland*.

Kraskov, A., Stoegbauer, H., & Grassberger, P. (2004). Estimating Mutual Information. *Physical Review E*, *69*(6). https://doi.org/10.1103/PhysRevE.69.066138

Kryonidis, G. C., Demoulias, C. S., & Papagiannis, G. K. (2019). A new voltage control scheme for active medium-voltage (MV) networks. *Electric Power Systems Research*, *169*, 53–64. https://doi.org/10.1016/j.epsr.2018.12.014

Liao, Y., Weng, Y., & Rajagopal, R. (2016). Urban distribution grid topology reconstruction via Lasso. *2016 IEEE Power and Energy Society General Meeting (PESGM)*, 1–5. https://doi.org/10.1109/PESGM.2016.7741545

Luan, W., Peng, J., Maras, M., Lo, J., & Harapnuk, B. (2015). Smart Meter Data Analytics for Distribution Network Connectivity Verification. *IEEE Transactions on Smart Grid*, *6*(4), 1964–1971. https://doi.org/10.1109/TSG.2015.2421304

Pappu, S. J., Bhatt, N., Pasumarthy, R., & Rajeswaran, A. (2018). Identifying Topology of Low Voltage Distribution Networks Based on Smart Meter Data. *IEEE Transactions on Smart Grid*, *9*(5), 5113–5122. https://doi.org/10.1109/TSG.2017.2680542

Sousa, T., Hashemi, S., & Andersen, P. B. (2019). Raising the potential of a local market for the reactive power provision by electric vehicles in distribution grids. *IET Generation, Transmission & Distribution*, *13*(12), 2446–2454. https://doi.org/10.1049/iet-gtd.2018.5947

Thurner, L., Scheidler, A., Schafer, F., Menke, J.-H., Dollichon, J., Meier, F., Meinecke, S., & Braun, M.

(2018). Pandapower—An Open-Source Python Tool for Convenient Modeling, Analysis, and Optimization of Electric Power Systems. *IEEE Transactions on Power Systems*, *33*(6), 6510–6521. https://doi.org/10.1109/TPWRS.2018.2829021

Tian, Z., Wu, W., & Zhang, B. (2016). A Mixed Integer Quadratic Programming Model for Topology Identification in Distribution Network. *IEEE Transactions on Power Systems*, *31*(1), 823–824. https://doi.org/10.1109/TPWRS.2015.2394454

Weng, Y., Liao, Y., & Rajagopal, R. (2017). Distributed Energy Resources Topology Identification via Graphical Modeling. *IEEE Transactions on Power Systems*, *32*(4), 2682–2694. https://doi.org/10.1109/TPWRS.2016.2628876