Command-Filtering-Based Adaptive Finite-Time Tracking Control for Ball and Plate System

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Abstract: The finite-time tracking control problem is studied for non-strict feedback ball and plate systems considering friction, coupling term and external disturbance comprehensively. An adaptive finite-time tracking control strategy based on command filtering is proposed, and the adaptive neural finite-time tracking controller is given for the ball and plate system with unknown input saturation. The designed controller can guarantee that the tracking error of the system can converge to a small neighborhood of the origin within finite time, and all signals in the closed loop system are bounded in the finite time. Finally, a simulation for the designed controller is given, and the simulation results verify the effectiveness and superiority of the proposed control scheme.

Keywords: Ball and plate systems, Unknown input saturation, Command filtering, Adaptive control, Finite-time control

1. INTRODUCTION

A ball and plate system is a typically underactuated, multivariable and strongly coupled nonlinear dynamic system. Not only is it utilized to check the effectiveness of control algorithm, but also the research results for it possesses important reference valuable in vessel, aerospace and other engineering fields.

Up to now, there have been many achievements for ball and plate systems. However, these results in (Bang et al. 2018 and Kassem et al. 2015) ignored the coupling, friction and other factors in the system so that the control accuracy of the system is difficult to be guaranteed. (Wang et al., 2010a and Wang et al., 2010b) considered the effect of friction, but the coupling term was treated as a known term, the system was divided into two unique second-order systems when the controller was designed, the underactuated characteristic of ball and plate system is not embodied. In (Ali et al., 2019, and Han et al., 2014), the friction and coupling of system were considered as the bounded disturbance, and the system was divided into two single-input single-output subsystems to design the controller respectively.

In the existing research method for ball and plate systems, backstepping technique was utilized in some papers, such as in (Ker et al., 2007 and Kazim et al., 2017). However, for the non-strict feedback ball and plate system, there are still some difficulties in controller design using backstepping technique. Hence, when the system is studied, the model of the system is simplified or some assumptions are adopted so that the precise control for the system was difficult to be guaranteed. In recent years, there have been papers in which the variable separation technique, fuzzy and neural network approximation method are adopted to deal with the non-strict feedback terms in order to carry on backstepping control design (Chen et al., 2015, Sun et al., 2016 and Tong et al., 2016). Meanwhile, the command filtered technique is utilized to solve the problem of "explosion of complexity" (Dong et al., 2012 and Wang et al., 2016). However, the disposing result like this paper is not seen in the current literature for the ball and plate system.

In recent years, the finite-time control method has received widespread attentions by many scholars with the advantages of fast convergence and strong robustness. At present, there are two main design ideas for finite-time control method. One is that the designed controller is required to make Lyapunov function of the closed-loop system to satisfy a finite-time stability condition, such as (Wang et al., 2016 and Sun et al., 2017). The other is that the controller is designed by transforming the system with a time-vary function (Song et al., 2017) or by constraining the state with a finite-time prescribed performance function (Liu et al., 2019). Up to now, in the researches on ball and plate systems, the relevant research results for finite-time control have not been found. Because there is controller saturation phenomenon in the actual system, it is necessary to dispose the phenomenon in controller design (Zhou et al., 2014).

Hence, in this paper, the tracking controller of the ball and plate system with external disturbance, friction, coupling and saturation input is designed by combining command filtering technique, practically finite-time control theory with the backstepping design method. Different from (Wang et al., 2016), a finite-time controller is designed for non-strict feedback system in this paper, and a different filter is chosen as well as a different method is taken to handle input saturation. The designed controller can guarantee that the tracking error converges to a small neighborhood of the origin within a finite time. Meanwhile, all the signals of the closed-loop system are practically finite-time stable.

The main contribution of this paper can be summarized as follows: 1) In the controller design process, the complete ball and plate system model is considered. Namely, the effects of disturbance, friction and coupling are considered at the same time, and the system is regarded as an interconnected system rather than two divided unique systems. 2) The adaptive neural network finite-time tracking control problem is firstly studied for the above-mentioned ball and plate system with unknown input saturation, and the finite- time tracking controller design method is given. 3) By means of command filtering technique, the complexity of finite-time control design of the ball and plate system is reduced so that the design process becomes more concise.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1 Problem formulation

The mathematical model of the ball and plate system can be established in the following form (Wang et al., 2010b). It is regarded as the coupling of the two subsystems.

$$S_X \begin{cases} \dot{x}_{1,1} = x_{1,2}, \\ \dot{x}_{1,2} = A(x_{1,1}x_{1,4}^2 - g\sin(x_{1,3}) + x_{1,4}x_{2,1}x_{2,4} + \dfrac{f_x}{m}) + d_{1,1}, \\ \dot{x}_{1,3} = x_{1,4}, \\ \dot{x}_{1,4} = u_x + d_{1,2}, \\ y_x = x_{1,1}. \end{cases}$$

$$S_Y \begin{cases} \dot{x}_{2,1} = x_{2,2}, \\ \dot{x}_{2,2} = A(x_{2,1}x_{2,4}^2 - g\sin(x_{2,3}) + x_{1,1}x_{1,4}x_{2,4} + \dfrac{f_y}{m}) + d_{2,1}, \\ \dot{x}_{2,3} = x_{2,4}, \\ \dot{x}_{2,4} = u_y + d_{2,2}, \\ y_y = x_{2,1}. \end{cases} \quad (1)$$

where $x_{1,1}$ and $x_{2,1}$ denote the displacement of ball in X-direction and Y-direction, respectively. $x_{1,3}$ represents the angle between X-direction of plate and horizon, $x_{2,3}$ is the angle between Y-direction of plate and horizon. $A = m/(m + I_b/r^2)$, I_b, m, r mean the rotary inertia, the quality and the radius of the ball, respectively. g indicates the gravitational acceleration, $f_x = -\mu_x mg\cos(x_3)$ and $f_y = -\mu_y mg\cos(x_7)$ are the frictions in X-direction and Y-direction, the rolling friction coefficients μ_x, μ_y are considered to be unknown (Han et al., 2014). $x_{1,4}x_{2,1}x_{2,4}$ and $x_{1,4}x_{2,1}x_{2,4}$ are the coupled terms between the subsystem S_X in X-direction and the subsystem S_Y in Y-direction. $d_{1,1}, d_{1,2}, d_{2,1}, d_{2,2}$ are bounded external disturbances, and $|d_{i,j}| \le d, i = 1,2, j = 1,2$. d is an unknown positive constant. y_x, y_y denote the outputs of the system in two directions, u_x, u_y mean the control inputs. The saturation nonlinearities are described by

$$u_x = sat(v_x) = \begin{cases} sign(v_x)u_{xm}, & |v_x| \ge u_{xm}, \\ v_x, & |v_x| < u_{xm}. \end{cases}$$

$$u_y = sat(v_y) = \begin{cases} sign(v_y)u_{ym}, & |v_y| \ge u_{ym}, \\ v_y, & |v_y| < u_{ym}. \end{cases}$$

where u_{xm}, u_{ym} denote the boundaries of unknown saturations u_x, u_y . Because u_x, u_y are not smooth functions, according to (Zhou et al., 2014), the state space description of the ball and plate system can be transformed as

$$S_X \begin{cases} \dot{x}_{1,1} = x_{1,2}, \\ \dot{x}_{1,2} = A(x_{1,1}x_{1,4}^2 - g\sin(x_{1,3}) + x_{1,4}x_{2,1}x_{2,4} + \dfrac{f_x}{m}) + d_{1,1}, \\ \dot{x}_{1,3} = x_{1,4}, \\ \dot{x}_{1,4} = g_{v_{xy}} v_x + \rho(v_x) + d_{1,2}, \\ y_x = x_{1,1}. \end{cases}$$

$$S_Y \begin{cases} \dot{x}_{2,1} = x_{2,2}, \\ \dot{x}_{2,2} = A(x_{2,1}x_{2,4}^2 - g\sin(x_{2,3}) + x_{1,1}x_{1,4}x_{2,4} + \dfrac{f_y}{m}) + d_{2,1}, \\ \dot{x}_{2,3} = x_{2,4}, \\ \dot{x}_{2,4} = g_{v_{y\kappa}} v_y + \rho(v_y) + d_{2,2}, \\ y_y = x_{2,1}. \end{cases} \quad (2)$$

where $0 < g_{xm} \le g_{v_{xy}} \le 1$, $0 < g_{ym} \le g_{v_{y\kappa}} \le 1$ with g_{xm} and g_{ym} are unknown constant, $|\rho(v_x)| \le D_x, |\rho(v_y)| \le D_y$ with D_x and D_y are bounded constant. The control objective of this paper is as follows: Design the controllers v_x, v_y for the ball and plate system so that the outputs y_x, y_y of the system can track precisely the given reference signals y_{xd}, y_{yd} within a finite time, and the controllers can guarantee all the signals in the closed-loop system being practically finite-time stable.

2.2 Preliminaries

In order to acquire our main result, the following Assumption and Lemmas will be introduced.

Assumption 1 The reference signals y_{xd}, y_{yd} and their first derivatives are continuous and bounded functions.

Definition 1 (Sun et al., 2017) Let $\zeta = 0$ be the equilibrium point of a nonlinear system $\dot{\zeta} = f(\zeta)$. It is called to be semiglobal practical finite-time stable if for all initial condition $\zeta(t_0) = \zeta_0$, there exist a constant $\nu > 0$ and a settling time $T(\nu, \zeta_0) < +\infty$ such that

$$\|\zeta(t)\| < \nu, \ \forall t > t_0 + T.$$

In this paper, the radial basis function neural network (RBFNN) will be used to approximate a continuous function $f(Z)$ defined in a compact set Ω (Li et al., 2017).

$$f(Z)=W^{\mathrm{T}}S(Z)+\delta(Z).$$

where $Z\in\Omega_z\in R^q$ is the input vector of the RBFNN, q means the input dimension of the RBFNN. $W=[w_1,w_2,\ldots,w_n]^{\mathrm{T}}$ is an ideal constant weight vector, $n>1$ denotes the node number of the RBFNN, $\delta(Z)$ is the approximate error of the RBFNN and it satisfies $|\delta(Z)|\le\varepsilon$, where ε is an arbitrary positive constant. $S(Z)=[s_1(Z),\ s_2(Z),\ldots,s_n(Z)]^{\mathrm{T}}$ represents the basis function vector, $s_i(Z)$ is chosen as Gaussian-like function, that is

$$s_i(Z)=\exp\left[\frac{-(Z-\mu_i)^{\mathrm{T}}(Z-\mu_i)}{r_i^2}\right],\quad i=1,2,\ldots,n,$$

where $\mu_i=[\mu_{i1},\mu_{i2},\ldots,\mu_{in}]^{\mathrm{T}}$ is the center of the basis function, r_i is called the width of the basis function.

Lemma 1 (Sun et al., 2016) If there are any given positive integers l and n, then

$$\|S(Z)\|^2\le\|S(Z_l)\|^2,$$

where $l\le n$, $S(Z)$ and $S(Z_l)$ are the basis function vectors of the RBFNN, $Z=[z_1,\cdots,z_n]^{\mathrm{T}}$ and $Z_l=[z_1,\cdots,z_l]^{\mathrm{T}}$ are the input vectors.

Lemma 2 (Li et al., 2017) For any given $\zeta>0$, the following inequality holds.

$$xy\le\frac{\zeta^p}{p}|x|^p+\frac{1}{q\zeta^q}|y|^q,\ \forall(x,y)\in R^2.$$

where $p>1,q>1$, and $(p-1)(q-1)=1$.

Lemma 3 (Sun et al., 2017) For any $x_r\in R, r=1,\cdots,m,$ and a real number p with $0<p\le1$, the following inequality holds.

$$\left(\sum_{r=1}^{m}|x_r|\right)^p\le\sum_{r=1}^{m}|x_r|^p\le m^{(1-p)}\left(\sum_{r=1}^{m}|x_r|\right)^p.$$

Lemma 4 (Sun et al., 2017) For the positive number λ,η,ϑ and real variables χ,ς, the following inequality holds.

$$|\chi|^\lambda|\varsigma|^\eta\le\frac{\lambda}{\lambda+\eta}\vartheta|\chi|^{\lambda+\eta}+\frac{\eta}{\lambda+\eta}\vartheta^{-\frac{\lambda}{\eta}}|\varsigma|^{\lambda+\eta}.$$

Lemma 5 (Sun et al., 2017) Consider the nonlinear system $\dot{\zeta}=f(\zeta)$, there are the positive function $V(\zeta)$, the constant $c>0$, $0<\beta<1$ and $b>0$, such that

$$\dot{V}(\zeta)\le-cV^\beta(\zeta)+b,t\ge0.$$

Then the system $\dot{\zeta}=f(\zeta)$ is semi-globally practically finite-time stable.

3. COMMAND-FILTERING-BASED ADAPTIVE FINITE-TIME TRACKING CONTROLLER DESIGN

In this section, the adaptive finite-time tracking controller will be designed for the system (2). In order to avoid repeating derivation of virtual control in the process of controller design, the following first-order low-pass filter is introduced in this paper.

$$\tau_{i,j}\dot{s}_{i,j}+s_{i,j}=\alpha_{i,j},s_{i,j}(0)=\alpha_{i,j}(0).$$

where $\tau_{i,j}(i=1,2,\ j=1,2,3)$ is a filtering time constant, $\alpha_{i,j}$ denotes the virtual control laws in the process of design, $s_{i,j}$ means the outputs of the low-pass filter. In this paper, the following coordinate transformation is utilized.

$$z_{i,j}=x_{i,j}-s_{i,j-1},\ i=1,2,\ j=1,\cdots,4, \tag{3}$$

where $s_{1,0}=y_{xd},s_{2,0}=y_{yd}$.

Next, the controller design is performed for the subsystem in X-direction.

Step 1 Based on (2) and (3), it produces that

$$\dot{z}_{1,1}=\dot{x}_{1,1}-\dot{y}_{xd}=z_{1,2}+\alpha_{1,1}+(s_{1,1}-\alpha_{1,1})-\dot{y}_{xd}. \tag{4}$$

In order to deal with the effect of the filter error $s_{1,1}-\alpha_{1,1}$, a compensating signal $e_{1,1}$ needs to be designed. Therefore, the compensating tracking error signal is defined in the following form.

$$v_{1,1}=z_{1,1}-e_{1,1},v_{1,2}=z_{1,2}-e_{1,2}, \tag{5}$$

where $e_{1,2}$ will be given later.

Consider a Lyapunov candidate function as follows.

$$V_{1,1}=\frac{1}{2}v_{1,1}^2. \tag{6}$$

Combining (5) and (6), one has

$$\dot{V}_{1,1}=v_{1,1}\dot{v}_{1,1}=v_{1,1}(z_{1,2}+\alpha_{1,1}+(s_{1,1}-\alpha_{1,1})-\dot{y}_{xd}-\dot{e}_{1,1}). \tag{7}$$

The virtual control $\alpha_{1,1}$ and the compensating signal $e_{1,1}$ are designed in the following form, respectively.

$$\alpha_{1,1}=-c_{1,1}v_{1,1}^{2\beta-1}-e_{1,1}+\dot{y}_{xd}, \tag{8}$$

$$\dot{e}_{1,1}=-e_{1,1}+(s_{1,1}-\alpha_{1,1})+e_{1,2}, \tag{9}$$

where $e_{1,1}(0)=0$, $c_{1,1}>0$ and $1/2<\beta<1$ are design parameters. Substituting (8) and (9) into (7) results in

$$\dot{V}_{1,1}=-c_{1,1}v_{1,1}^{2\beta}+v_{1,1}v_{1,2}. \tag{10}$$

Step 2 According to (2) and (3), one has

$$\begin{aligned}\dot{z}_{1,2}&=\dot{x}_{1,2}-\dot{s}_{1,1}\\&=A(x_{1,1}x_{1,4}^2-g\sin(x_{1,3})+x_{1,4}x_{2,1}x_{2,4}+f_x/m\\&\quad+gx_{1,3})-Agz_{1,3}-Ag(s_{1,2}-\alpha_{1,2})+d_{1,1}-\dot{s}_{1,1}.\end{aligned} \tag{11}$$

Set

$$f_1(Z_1)=A(x_{1,1}x_{1,4}^2-g\sin(x_{1,3})+x_{1,4}x_{2,1}x_{2,4}+f_x/m+gx_{1,3}),$$

where $Z_1=[x_{1,1},x_{1,3},x_{1,4},x_{2,1},x_{2,4}]^{\mathrm{T}}$. Obviously, the nonlinear function $f_1(Z_1)$ involves friction and coupling. A RBFNN $W_1^{\mathrm{T}}S_1(Z_1)$ is used to approximate the unknown function $f_1(Z_1)$, such that

$$f_1(Z_1)=W_1^{\mathrm{T}}S_1(Z_1)+\delta_1(Z_1),\quad|\delta_1(Z_1)|\le\varepsilon_1. \tag{12}$$

Substituting (12) into (11), (13) can be achieved.

$$\begin{aligned}\dot{z}_{1,2}&=W_1^{\mathrm{T}}S_1(Z_1)+\delta_1(Z_1)-Agz_{1,3}-Ag\alpha_{1,2}\\&\quad-Ag(s_{1,2}-\alpha_{1,2})+d_{1,1}-\dot{s}_{1,1}.\end{aligned} \tag{13}$$

In order to deal with the filter error $s_{1,2}-\alpha_{1,2}$, the compensating signal $e_{1,2}$ needs to be designed, and the following compensating tracking error signal is defined.

$$v_{1,3} = z_{1,3} - e_{1,3}, \tag{14}$$

where $e_{1,3}$ will be given later. Based on (5) and (13), one has

$$\dot{v}_{12} = W_1^{\mathrm{T}} S_1(Z_1) + \delta_1(Z_1) - Agz_{1,3} - Ag\alpha_{1,2} - Ag(s_{1,2} - \alpha_{1,2}) + d_{1,1} - \dot{s}_{1,1} - \dot{e}_{1,2}.$$

By using Lemma 2 and $|d_{1,1}| \le d$, the following result can be obtained.

$$v_{1,2}\dot{v}_{12} \le v_{1,2}\Big(\frac{v_{1,2}\|W_1\|^2 \|S_1(Z_1)\|^2}{2a_1} + v_{1,2} - Agz_{1,3} - Ag\alpha_{1,2} - Ag(s_{1,2} - \alpha_{1,2}) - \dot{s}_{1,1} - \dot{e}_{1,2}\Big) + \frac{a_1}{2} + \frac{\varepsilon_1^2}{2} + \frac{d^2}{2},$$

where a_1 is an arbitrarily positive constant. Defining $\theta_1 = \|W_1\|^2$, $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$, $\hat{\theta}_1$ is the estimation of the unknown constant θ_1. According to Lemma 1, it produces that

$$v_{1,2}\dot{v}_{12} \le v_{1,2}(v_{1,2}\theta_1 \|S_1(Z_1^*)\|^2 / 2a_1 + v_{1,2} - Agz_{1,3} - Ag\alpha_{1,2} - Ag(s_{1,2} - \alpha_{1,2}) - \dot{s}_{1,1} - \dot{e}_{1,2}) + \frac{a_1}{2} + \frac{\varepsilon_1^2}{2} + \frac{d^2}{2}, \tag{15}$$

where $Z_1^* = [x_{1,1}]$. Choose a Lyapunov function candidate as

$$V_{1,2} = \frac{1}{2}v_{1,2}^2 + \frac{1}{2}\tilde{\theta}_1^2. \tag{16}$$

The virtual control $\alpha_{1,2}$, compensating signal $e_{1,2}$ and adaptive law can be designed as:

$$\alpha_{1,2} = \frac{1}{Ag}(c_{1,2}v_{1,2}^{2\beta-1} + e_{1,2} + z_{1,1} - \dot{s}_{1,1} + v_{1,2}(\hat{\theta}_1 \|S_1(Z_1^*)\|^2 / 2a_1 + 1), \tag{17}$$

$$\dot{e}_{1,2} = -e_{1,1} - e_{1,2} - Age_{1,3} - Ag(s_{1,2} - \alpha_{1,2}), \tag{18}$$

$$\dot{\hat{\theta}}_1 = v_{1,2}^2 \|S_1(Z_1^*)\|^2 / 2a_1 - \sigma\hat{\theta}_1, \tag{19}$$

where $e_{1,2}(0) = 0$, $c_{1,2} > 0$ and $\sigma > 0$ are design parameters. Combining (16) ~ (19), it produces that

$$\dot{V}_{1,2} \le -c_{1,2}v_{1,2}^{2\beta} - v_{1,1}v_{1,2} - Agv_{1,2}v_{1,3} + \sigma\tilde{\theta}_1\hat{\theta}_1 + \frac{a_1}{2} + \frac{\varepsilon_1^2}{2} + \frac{d^2}{2}. \tag{20}$$

Step 3 According to the calculation of (2) and (3), the following result is attained.

$$\dot{z}_{1,3} = \dot{x}_{1,3} - \dot{s}_{1,2} = z_{1,4} + \alpha_{1,3} + (s_{1,3} - \alpha_{1,3}) - \dot{s}_{1,2}. \tag{21}$$

In order to handle the filter error $s_{1,3} - \alpha_{1,3}$, the compensating signal $e_{1,3}$ needs to be designed. The compensating tracking error signal is defined as follows.

$$v_{1,4} = z_{1,4} - e_{1,4}, \tag{22}$$

where $e_{1,4}$ will be given later.

Select the following Lyapunov function candidate

$$V_{1,3} = \frac{1}{2}v_{1,3}^2. \tag{23}$$

Then

$$\dot{V}_{1,3} = v_{1,3}\dot{v}_{1,3} = v_{1,3}(z_{1,4} + \alpha_{1,3} + (s_{1,3} - \alpha_{1,3}) - \dot{s}_{1,2} - \dot{e}_{1,3}) \tag{24}$$

The virtual control $\alpha_{1,3}$ and the compensating signal $e_{1,3}$ can be designed as follows.

$$\alpha_{1,3} = -c_{1,3}v_{1,3}^{2\beta-1} - e_{1,3} + Agz_{1,2} + \dot{s}_{1,2}, \tag{25}$$

$$\dot{e}_{1,3} = Age_{1,2} - e_{1,3} + e_{1,4} + (s_{1,3} - \alpha_{1,3}), \tag{26}$$

where $e_{1,3}(0) = 0$, $c_{1,3} > 0$ is a design parameter. Substituting (25) and (26) into (24) results in

$$\dot{V}_{1,3} = -c_{1,3}v_{1,3}^{2\beta} + Agv_{1,2}v_{1,3} + v_{1,3}v_{1,4}. \tag{27}$$

Step 4 Based on (2) and (3), the following relationship can be obtained.

$$\dot{z}_{1,4} = \dot{x}_{1,4} - \dot{s}_{1,3} = g_{v_{xy}}v_x + \rho(v_x) + d_{1,2} - \dot{s}_{1,3}. \tag{28}$$

Construct a Lyapunov function candidate as follows

$$V_{1,4} = \frac{1}{2}v_{1,4}^2.$$

Then, the following result can be obtained.

$$\dot{V}_{1,4} = v_{1,4}\dot{v}_{1,4} = v_{1,4}(g_{v_{xy}}v_x + \rho(v_x) + d_{1,2} - \dot{s}_{1,3} - \dot{e}_{1,4}).$$

The actual control v_x and compensating signal $e_{1,4}$ are designed in the following form

$$v_x = -c_{1,4}v_{1,4}^{2\beta-1} - e_{1,4} - a_2v_{1,4} - z_{1,3} + \dot{s}_{1,3}, \tag{29}$$

$$\dot{e}_{1,4} = -e_{1,3} - e_{1,4}, \tag{30}$$

where $e_{1,4}(0) = 0$, $a_2 > 0$, $c_{1,4} > 0$ are two design parameters. With the help of Lemma 2, one has

$$\dot{V}_{1,4} \le -c_{1,4}g_{xm}v_{1,4}^{2\beta} - v_{1,3}v_{1,4} + (d^2 + D_x^2)/2a_2g_{xm}. \tag{31}$$

Because the subsystem in Y-direction has the similar structure to the subsystem in X-direction, its four design steps are similar to the above process. Therefore, the virtual control laws, actual control law and adaptive laws of the subsystem S_Y are similarly designed as follows.

$$\alpha_{2,1} = -c_{2,1}v_{2,1}^{2\beta-1} - e_{2,1} + \dot{y}_{yd}, \tag{32}$$

$$\alpha_{2,2} = \frac{1}{Ag}(c_{2,2}v_{2,2}^{2\beta-1} + e_{2,2} + z_{2,1} - \dot{s}_{2,1} + v_{2,2}(\hat{\theta}_2 \|S_2(Z_2^*)\|^2 / 2a_1 + 1), \tag{33}$$

$$\alpha_{2,3} = -c_{2,3}v_{2,3}^{2\beta-1} - e_{2,3} + Agz_{2,2} + \dot{s}_{2,2}, \tag{34}$$

$$v_y = -c_{2,4}v_{2,4}^{2\beta-1} - e_{2,4} - a_2v_{2,4} - z_{2,3} + \dot{s}_{2,3}, \tag{35}$$

$$\dot{\hat{\theta}}_2 = v_{2,2}^2 \|S_2(Z_2^*)\|^2 / 2a_1 - \sigma\hat{\theta}_2, \tag{36}$$

where $c_{2,j}\,(j = 1,\cdots,4)$ are all positive design parameters. $Z_2^* = [x_{21}]$, $\theta_2 = \|W_2\|^2$. The compensating signals can be designed as follows

$$\dot{e}_{2,1} = -e_{2,1} + (s_{2,1} - \alpha_{2,1}) + e_{2,2}, \tag{37}$$

$$\dot{e}_{2,2} = -e_{2,1} - e_{2,2} - Age_{2,3} - Ag(s_{2,2} - \alpha_{2,2}), \tag{38}$$

$$\dot{e}_{2,3} = Age_{2,2} - e_{2,3} + e_{2,4} + (s_{2,3} - \alpha_{2,3}), \tag{39}$$

$$\dot{e}_{2,4} = -e_{2,3} - e_{2,4}, \tag{40}$$

where $e_{2,1}(0) = 0$, $e_{2,2}(0) = 0$, $e_{2,3}(0) = 0$, $e_{2,4}(0) = 0$. And $\dot{V}_{2,1}, \dot{V}_{2,2}, \dot{V}_{2,3}, \dot{V}_{2,4}$ can be obtained similarly.

Hence, the main results of this paper can be stated as follows.

Theorem 1 Consider the ball and plate system (2) satisfying Assumption 1. If the compensating signals are designed according to (9), (18), (26), (30), (37)~(40), the virtual control laws are designed according to (8), (17), (25), (32) ~(34), the actual control laws and adaptive laws are designed as (29), (35), (19), (36), then the tracking error of the closed-loop system can converge to the neighborhood of the origin within finite-time. Meanwhile, all the signals of the closed-loop system are practically finite-time stable.

Proof: Construct the Lyapunov function candidate of the whole system as

$$V=\sum_{i=1}^{2}\sum_{j=1}^{4}\frac{1}{2}v_{i,j}^2+\sum_{i=1}^{2}\frac{1}{2}\tilde{\theta}_i^2.$$

According to Lemma 2, it follows that

$$\tilde{\theta}_1\hat{\theta}_1\le\frac{\theta_1^2}{2}-\frac{\tilde{\theta}_1^2}{2}, \tag{41}$$

$$\tilde{\theta}_2\hat{\theta}_2\le\frac{\theta_2^2}{2}-\frac{\tilde{\theta}_2^2}{2}. \tag{42}$$

By the above design results, it produces that

$$\begin{aligned}\dot{V}\le&-c\sum_{i=1}^{2}\sum_{j=1}^{4}(\frac{v_{1,j}^2}{2})^{\beta}+c\sum_{i=1}^{2}((\frac{\tilde{\theta}_i^2}{2})^{\beta}-(\frac{\tilde{\theta}_i^2}{2})^{\beta}-\frac{\tilde{\theta}_i^2}{2})\\&+2d^2+a_1+\frac{\sigma\theta_1^2+\sigma\theta_2^2+\varepsilon_1^2+\varepsilon_2^2+D_x^2+D_y^2}{2}\\&+\frac{d^2+D_x^2}{2a_2g_{xm}}+\frac{d^2+D_x^2}{2a_2g_{ym}},\end{aligned} \tag{43}$$

where $c=\min\left\{2^{\beta}c_{i,j},2^{\beta}c_{i,4}g_{xm},\sigma\right\}, i=1,2,\ j=1,2,3$. Based on Lemma 4, let $\varsigma=(\tilde{\theta}_1^2/2)^{\beta}$, $\chi=1$, $\lambda=1-\beta$, $\eta=\beta$, $\vartheta=\beta^{\beta/(1-\beta)}$, then

$$(\frac{\tilde{\theta}_1^2}{2})^{\beta}\le(1-\beta)\vartheta+\frac{\tilde{\theta}_1^2}{2}. \tag{44}$$

Similarly, we have

$$(\frac{\tilde{\theta}_2^2}{2})^{\beta}\le(1-\beta)\vartheta+\frac{\tilde{\theta}_2^2}{2}. \tag{45}$$

Substituting (44) and (45) into (43) results in

$$\dot{V}\le-c\sum_{i=1}^{2}\sum_{j=1}^{4}(\frac{v_{1,j}^2}{2})^{\beta}+c\sum_{i=1}^{2}(\frac{\tilde{\theta}_i^2}{2})^{\beta}+b, \tag{46}$$

where b is a bounded constant, and

$$\begin{aligned}b=&2(1-\beta)\vartheta+2d^2+a_1+\frac{d^2+D_x^2}{2a_2g_{xm}}+\frac{d^2+D_x^2}{2a_2g_{ym}}\\&+\frac{\sigma\theta_1^2+\sigma\theta_2^2+\varepsilon_1^2+\varepsilon_2^2+D_x^2+D_y^2}{2}.\end{aligned}$$

With the help of Lemma 3, (46) can be transformed as

$$\dot{V}\le-c(\sum_{i=1}^{2}\sum_{j=1}^{4}\frac{v_{1,j}^2}{2}-\sum_{i=1}^{2}\frac{\tilde{\theta}_i^2}{2})^{\beta}+b=-cV^{\beta}+b.$$

According to Lemma 5, the closed-loop system is semi-globally practically finite-time stable, namely, $v_{i,j},\tilde{\theta}_i, i=1,2,\ j=1,\cdots,4$ are all practically finite-time stable. From (Sun et al., 2017), we can know the bound of tracking error is $\left|z_{i,1}-e_{i,1}\right|\le 2(b/(1-\omega)c)^{1/2\beta}$ for the settling time $T_r=(1/(1-\beta)a\omega)[V^{1-\beta}(0)-((b/(1-\omega)a))^{[(1-\beta)/\beta]}]$ with any $0<\omega<1$. Obviously, the bigger the c is, the smaller the bound is. From Lemma 3 in (Dong et al., 2012), we can know that $e_{i,j}$ are bounded, then $z_{i,j}$ are also bounded, namely, all the signals in the closed-loop system are bounded. Meanwhile, these signals are practically finite-time stable.

4. SYSTEM SIMULATION STUDY

In this section, a circular trajectory tracking simulation experiment is performed for a ball and plate system by MATLAB. The physical parameters of the ball and plate system are given as follows (Han et al., 2014): $m=0.263kg$, $r=0.02m$, $I_b=4.2\times10^{-5}kg\cdot m^2$, $g=9.81m/s^2$, $\mu_x=\mu_y=0.004$. In the trajectory tracking experiment, the tracking signals of X-direction and Y-direction are all chosen as $y_{xd}=0.5\sin(0.1\pi t)$, $y_{yd}=0.5\cos(0.1\pi t)$. The disturbances are selected as $d_{1,1}=d_{2,1}=0.5$, $d_{2,1}=d_{2,2}=0.3$. The initial states of the system are chosen as $[x_{1,1},x_{1,2},x_{1,3},x_{1,4},x_{2,1},x_{2,2},x_{2,3},x_{2,4}]^{\mathrm{T}}=[0.14,0,-0.05,0,0.35,0,0.05,0]^{\mathrm{T}}$ .The controller design parameters are selected as $[c_{1,1},c_{1,2},c_{1,3},c_{1,4},c_{2,1},c_{2,2},c_{2,3},c_{2,4}]^{\mathrm{T}}=[5,1,2,2,2,1,1,1]^{\mathrm{T}}$, $\beta=97/101$, $a_1=1$, $a_2=0.1$, σ=0.5, $u_{xm}=u_{ym}=15$. And the filter time constants are all chosen as 0.001, the initial value of adaptive laws θ_1,θ_2 are all selected as 0. Two RBF neural networks are chosen as follows. They all contain 7 nodes, the centers are evenly distributed in interval $[-3,3]$, and the width are all selected as 2. The basis function is chosen as Gaussian-like function. The compensating signals, virtual control laws, actual control law and adaptive laws can be calculated according to Theorem 1. The adaptive finite-time tracking controller is obtained. By means of the experiment for the ball and plate systems in MATLAB, the simulation results can be shown in Figs.1-3. Fig.1 shows tracking effect in an actual circular orbit. In order to compare with existing research results, the simulation result of the controller in (Ker et al., 2007) is also given simultaneously in Fig.1. In the simulation results of (Ker et al., 2007), the parameters of the ball and plate system are the same as this paper. The controller design parameters are the same as the original article, namely, $[c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8]^{\mathrm{T}}=[3.5,9.5,35,20,2.5,6.5,25,11]^{\mathrm{T}}$. The change rule of adaptive parameters are indicated in Fig.2. Fig.3 depicts the trajectories of the control inputs of the ball and plate system in X-direction and Y-direction.

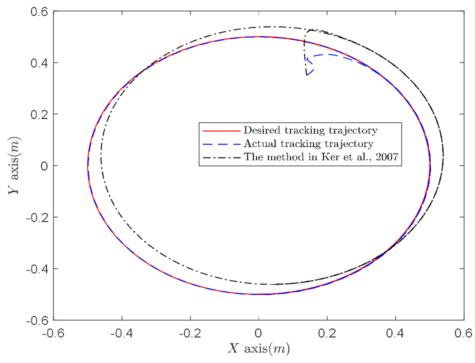

Fig. 1 Tracking curves of the ball on the plate

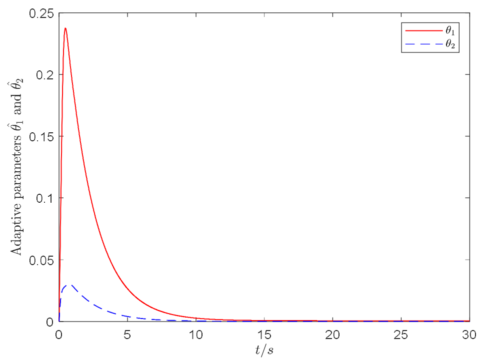

Fig. 2 Adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$

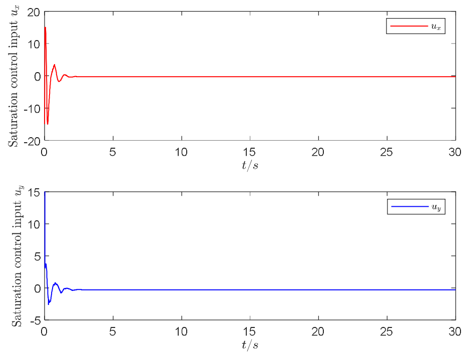

Fig. 3 Control inputs

From Figs.1-3, it can be seen that the ball on the plate tracks the desired trajectory accurately under the condition of controller saturation by using the proposed method in this paper. However, the control method in (Ker et al., 2007) is difficult to achieve the ideal control effect when the system exists friction, coupling and external disturbance. Therefore, the control method in this paper is more superior.

5. CONCLUSION

In this paper, the adaptive finite-time trajectory tracking control problem is studied for the ball and plate system with external disturbance, frictional force and coupling influence. Meanwhile, the controller saturation phenomenon of system is considered in the control design. By means of backstepping technique, command filtering method, Adaptive neural network technique and practically finite-time control theory, the tracking controller of the ball and plate system is given. The designed controller can guarantee that the tracking error of the system converges to a small neighborhood of the origin. And all the signals in the closed-loop system are practically finite-time stable. The simulation results demonstrate the effectiveness and superiority of the proposed method.

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