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Robust Fractional Order Control of a Pool of a Main Irrigation Canal in Submerged Flow Condition ⋆

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Abstract: This work addresses the robust control of a pool of a main irrigation canal working in a submerged flow condition. A laboratory prototype of hydraulic canal at the University of Castilla-La Mancha is used in this study. A series connection of a non-linear static block and a linear first order plus time delay system is proposed to model the dynamics of such process for all the considered operating regimes. The gain variations in function of the operating regimes are counted and corrected by using a gain scheduling block that inverts the before nonlinearity. However, a residual gain variation remains, whose effect is corrected by a fractional-order proportional integral PI controller that is robust to process gain changes. Such controller is tuned to make the closed-loop system fulfill two temporal specifications: (a) a desired overshoot, obtained defining an equivalent phase margin frequency specification and (b) a desired settling time, obtained defining an equivalent gain crossover frequency specification. Moreover, a third specification is defined: the isophase margin condition, which accounts for the changes in the gain. The simulated results of our canal show the adequate performance of this control system.

Keywords: Hydraulic canal control, fractional-order control, robust control, gain scheduling, system identification

1. INTRODUCTION

Automatic control of water distribution is an efficient way of solving some of the problems of water management, reduction of the huge losses of water in irrigation main canals, and promotion of sustainable development of irrigation areas (Litrico and Fromion (2009)). However, designing these controllers is not simple since irrigation main canals are complex systems with distributed parameters over long distances, significant time-delays, strong nonlinearities and dynamics that change with the operation conditions (Rivas-Perez et al. (2014)).

For an hydraulic canal system prototype, and around several operating regimes, linear first and fractional order models with time delays have been adjusted using a linearized form of the Saint Venant differential equations (Feliu-Batlle et al. (2018)). Around a chosen operating regime, it was used the Wiener-Hopf control and the Pade approximation of time delay to develop a new methodology of control for the adjusted fractional order plus time delay model. This same canal was also configured as a double pools system and identified using $MIMO$ model (Multi input Multi output) adjusting the water levels in both pools to the pump frequencies and the upstream gate positions respectively. Using a fractional order derivative term has clearly outperformed the model accuracy compared to first and second order delayed models (Feliu-Batlle et al. (2017)). In order to reproduce a real-word conditions where the functioning of the first pool is independent on the opening and closing of the upstream gate, an inner-loop control is applied to the pump frequencies based on the adjusted fractional order model. By decreasing the error between the real and simulated data with a percentage of 50%, such fractional order model clearly prove its efficiency on capturing the canal dynamics compared to the classical first order system (San-Millan et al. (2017)).

In order to generalize the functioning regime for an hydraulic canal system configured as single pool system, Linear delayed first and fractional order models, non linear delayed first and fractional order models around submerged and free flow water regimes are adjusted with the downstream end water levels when upstream pool is configured as constant water level storage. The nonlinearity caused by the gate movements and the sensor's resolution was corrected using analytical models to improve the accuracy of the global adjusted models (Gharab et al. (2019)). Global non linear fractional order model composed by the combination of two submodes of respectively submerged and free flow regimes was concluded and it precisely reproduce the experimental data around all the regimes that contains such canal dynamics. Revising the literature, there are many works that consider hydraulic structures control models using different techniques. Bolea

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proposed a gain-scheduled Smith Predictor PID controller to control an open loop canal system that deals with large variation in operating conditions. That work proposed linear parameter varying PID control system H_∞ and linear matrix inequalities pole placement where the results was validated using a single real reach canal (Bolea et al. (2013)). Reinforcement learning upstream control system was developed and applied with canal structures to maintain water depths upstream of an automatic structure inside a dead band with acceptable stability (Shahverdi and Monem (2015)). In (Zheng et al. (2019)), Zheng formulated a model predictive control for a cascaded irrigation canal system, it consists of an advanced algorithm embedded with feedforward process and with the ability to incorporate constraints in optimization.

Otherwise, the generalized $PI^\alpha D^\beta$ fractional order controller and its simplified versions like PI^α or PD^β are proposed in several works to improve the performance of the classical PID controllers in terms of robustness, e.g. (Monje et al. (2004)), and output response, e.g. (Luo et al. (2010)). Regions of feasible frequency specifications ($RFFS$) of pairs (phase margin: ϕ and gain crossover frequency: ω_c): are defined and yield stable closed-loop systems that verify these frequency specifications without ambiguity. The RFFSs of first order plus time delay systems controlled by some fractional order controllers were obtained in (Castillo-Garcia et al. (2013)). A $KRFFS$ was defined in (Gharab and Feliu-Batlle (2019)) for a fractional order model with time delay controlled by a PI^α structure. The $KRFFS$ is a subset of $RFFS$ that verifies the additional property that the closed-loop system is not fragile to gain variations (ϕ and ω_{cg} do not change sharply as the process gain changes).

We consider in this paper the series connection of a nonlinear static block and a first order plus time delay system developed modeling the dynamics of a pool of a main irrigation canal system operating in submerged flow (Gharab et al. (2019)). Based on that model, a fractional order PI^α controller in series with a term that inverts the previous nonlinear static block is designed. All these techniques have been applied to control the main pool of the laboratory hydraulic canal at the University of Castilla La Mancha.

This paper is organized as follows. Section 2 presents the mentioned experimental platform and carries out the identification of the canal dynamics. Section 3 proposes a new robust fractional order controller. Section 4 carries out some simulations of the controlled system. Finally, Section 5 is reserved for the conclusion.

2. HYDRAULIC CANAL SYSTEM MODEL

2.1 Platform Presentation

The laboratory hydraulic canal system is located in the School of Industrial Engineering at the University of Castilla-La Mancha. It is a variable slope rectangular canal with glass walls. Its dimensions are 5m long, 0.08m wide and 0.25m walls high. The canal has been divided into two pools: an upstream pool, characterized by its small dimensions (a length of about 0.50m), that acts as an upstream reservoir, and a downstream pool characterized by more large dimensions compared to the previous one (a length of about 4.50m), that plays the role of the main pool to be automated. Fig. 1 shows this setup where upstream and downstream pools are separated by an upstream slide gate translating vertically in the range [0, 50cm] by the fact of a DC motor.

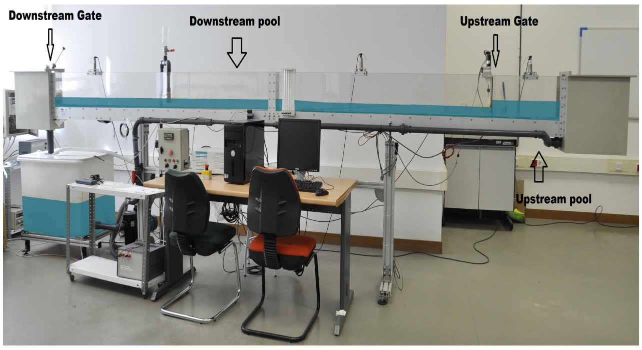

Fig. 1. Schematic representation of the laboratory prototype of the hydraulic canal system.

2.2 Saint-Venant Equations

Saint-Venant equations are often used by hydraulic engineers as an efficient tool to characterize the dynamics of hydraulic canal systems. In order to apply these non linear equations, some hypotheses have to be fulfilled: (a) the flow is one-dimensional and the velocity is uniform; (b) vertical accelerations are negligible and the pressure is hydrostatic; (c) the average canal bed slope is small and (d) the variation of canal width along the horizontal axis is small.

Saint-Venant equations are often linearized around a defined flow regime. In its actual configuration, our hydraulic canal system behaves as a single input - single output ($SISO$) system in which the upstream gate opening is the input and the water level measured at the end of the downstream pool is the output. In this configuration, the canal can operate in both submerged and free water flows. The submerged flow regime is analyzed in this paper. Four operating points in this regime are defined in function of four specific gate maneuvers whose initial and final upstream gate positions x_{upinit} and x_{upfin}, as well as the step directions (positive when the gate is opening and negative when it is closing), are shown in Table 1.

Table 1. Submerged operating points

Op	x_{up} initial	x_{up} final	step sense
Op_1	10 mm	20 mm	positive
Op_2	20 mm	10 mm	negative
Op_3	20 mm	30 mm	positive
Op_4	30 mm	20 mm	negative

2.3 Submerged operating regime

The very often proposed model on modeling hydraulic canal system is the delayed first order model (Litrico and Fromion (2009)):

$$G(s) = \frac{K}{1+Ts}e^{-Ls} \tag{1}$$

where K is the static gain, T is the time constant, and L is the time delay.

A hydraulic canal system works in submerged flow regime if the flow remains subcritical at the canal. In that case, the flow is influenced by the downstream level. Our canal works in that regime if the following condition is verified (Swamee (1992)):

$$y_{up} > 0.81 \cdot y_{dwe} \cdot \left(\frac{y_{dwe}}{x_{up}} \right) \tag{2}$$

where $x_{up}(t)$ is the upstream gate opening, whose value is very close to the electrical command signal to the gate $u(t)$, $y_{up}(t)$ is the upstream water level, which is artificially maintained constant to a value $y_{up} = 60mm$ by the action of a PI that controls the water level of the upstream pool, and $y_{dwe(t)}$ is the measured downstream end water level. The discharge through the gate in submerged flow is approximated by (Litrico and Fromion (2009)):

$$Q_s(t) = c_d \cdot b \cdot x_{up}(t) \cdot (2 \cdot g \cdot (y_{up}(t) - y_{dwe}(t)))^{\gamma} \tag{3}$$

where γ represents a non-linearity factor that is usually equal to 0.5, b is the width of the upstream gate and c_d is a discharge coefficient, generally close to 0.6 (Litrico and Fromion (2009)). Variables x_{up}, y_{up} and x_{dwe} are measured and used in the control system. Moreover x_{up} will hereafter be considered as the input u.

2.4 Non-Linear Dynamic Model

The combination of the previously stated static nonlinear model with a first order plus time delay model yields the following nonlinear dynamic model (Gharab et al. (2019)):

$$T \cdot D\Delta y_{dwe}(t) + \Delta y_{dwe}(t) = K \cdot \Delta v(t - L) \tag{4}$$

$$\Delta v(t) = (y_{up}(t) - y_{dwe}(t))^{\gamma} \cdot \Delta x_{up}(t) \tag{5}$$

where D is the derivative operator, K is the static gain, T is the time constant, L is the time delay, $x_{up}(t)$ and $y_{dwe}(t)$ are respectively the input and the output of the system, $v(t)$ is an intermediate variable proportional to $Q_s(t)$, and Δ represents the increment of the variables with respect to its value in the operating regime.

2.5 Identification Technique

Previous experiments carried out in our canal showed that gain is the parameter that most varies in model (1) when the operating regime is changed. In this subsection, identification of linear model (1) and non-linear model (4), (5) are carried out. Fig. 2 shows schemes of them, in which function $f(\cdot) = (y_{up}(t) - y_{dwe}(t))^{\gamma}$ implements the time varying gain of (5).

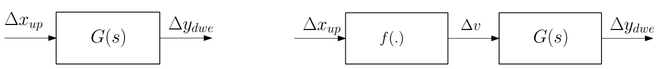

Fig. 2. (a): Linear Model (b): Non Linear Model

Let N represent the number of operating regimes (in our case 4). An identification procedure is therefore carried out of the set of the N models (1) associated to these operating regimes such that T and L are constant in the N models and only the gain K changes with the operating regime (having thus a gain K_i, for each i operating regime, $i = 1 \ldots N$). A time domain identification technique has been used based on: 1) recording the N responses obtained when pseudo-random binary signal (PRBS) are applied to the upstream gate of the main pool around the four defined operating regimes.

The amplitude of the PRBS is chosen arbitrarly $A_{PRBS} = 5\,cm$ and its period is chosen in such a way to cover all the frequency band of our considered system (taking into account the frequency ranges of the gate movement and the water fluctuation).

- The bits number : $n_{bits} = 8\ bits$.
- The number of state per signal : $N = 2^{n_{bits}} - 1 = 255$.
- The sample period is $T_s = 5\ s$.
- Each experiment last : $D = 1275\ s$.

2) subsequently executing an optimization process in which the cost:

$$C = \max_{1<i<N} \left(\max_{1<k<M} |y_{dwe,i}(k) - \hat{y}_{dwe,i}(k)| \right) \tag{6}$$

is minimized with respect to the parameters $K_1 \ldots K_N$, T and L. In this cost, M is the number of recorded samples in each test (equal for all the tests), and $y_{dwe,i}(k)$ and $\hat{y}_{dwe,i}(k)$ are the samples at instant k of, respectively, the real time response and the simulated time response of model (1) in the operating regime i. The identified models are shown in Table 2 with their corresponding costs C_i and $Nrmse_i$ indexes.

The $Nrmse$ index is the normalized root mean-squared error, which is a standard measure of the accuracy provided by the model fitted to the data. This criterion is a non-dimensional version of the root-mean-squared error $RMSE$ and is given by:

$$Nrmse_i = 100 \cdot \left(1 - \frac{\| y_{dwe,i}(k) - \hat{y}_{dwe,i}(k) \|_2}{\| y_{dwe,i}(k) - mean\,(y_{dwe,i}(k)) \|_2} \right) \tag{7}$$

Only cost values (7) over 70% are regarded as acceptable fittings.

Table 2 shows a large variation of the parameter K of model (1) in function of the operating regime: the maximum relative variation is $100 \cdot max_{1<i<N} \left| \frac{K_i - \overline{K}}{\overline{K}} \right| = 39\%$, where $\overline{K} = 0.296$ is the mean of the K_i values.

Table 2. Submerged operating regimes: linear models

regime i	K_i	T	L	C_i	$Nrmse_i$
1	0.4102	5.1249	3.3478	0.0721	92.79%
2	0.2396			0.0915	90.85%
3	0.3339			0.0803	91.97%
4	0.1996			0.0969	90.31%

The previous process is subsequently repeated fitting model (4), (5). The same experimental data as before has been used in this identification. The same cost (6) has been used, but minimized with respect to the parameters $K_1 \ldots K_N$, T, L and γ. In this case, $\hat{y}_{dwe,i}(k)$ is the output of the simulated model (4), (5). Models identified around the four regimes are presented in Table 3. Note that the best fitting to the data is provided by a value $\gamma = 0.44$ instead of the expected value 0.5.

Table 3. Submerged operating regimes: non-linear models

regime i	K_i	γ	T	L	C_i	$Nrmse_i$
1	0.0705				0.0906	90.93%
2	0.0597	0.44	5.4925	4.3005	0.0774	92.25%
3	0.0629				0.1005	89.94%
4	0.0770				0.1460	85.40%

Table 3 presents the global adjusted model which characterizes the hydraulic canal dynamics when the submerged flow condition takes place. In this case, $Nrmse_4 = 85.4\%$ is the worst fitting index. It is lower than the worst index $Nrmse_4 = 90.31\%$ (see Table 2), obtained using the linear models fittings. However, it is still a high enough index, which proves that such non-linear model highly catches the dynamics of our canal. Table 3 shows that, in this case, the maximum relative variation of the gain is 14% and $\overline{K} = 0.0675$. Then, though the static gain K varies significantly less than before, which facilitates the control of this system, it still has a noticeable variation in function of the operating regime in the submerged flow condition.

If the mean $\overline{K} = 0.0675$ were used in model (4) combined with (5), in order to reproduce the responses at the 4 operating regimes, the resulting costs of such a unique model would become significantly worst, as Fig. 3 shows. This illustrates the fact that static gain changes affect considerably the performance of the adjusted global model.

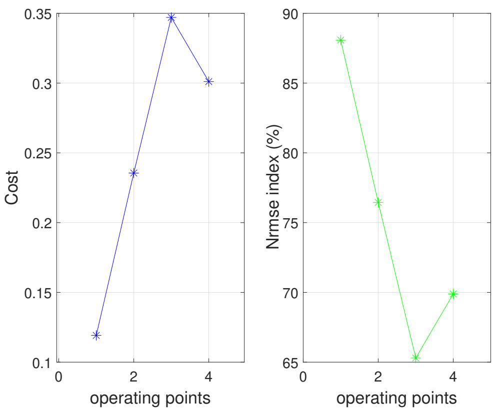

Fig. 3. Performance of the non-linear model using parameters $\overline{K}$ and T, L and γ of Table 3

3. PROPOSED CONTROLLER

Fig. 4 shows the gain scheduling control scheme proposed in this work. It is composed of the series connection of a LTI controller - whose transfer function is $C(s)$ and its output is denoted $\Delta v'(t)$ - and the inverse of the gain $f(\cdot)$. This last block is $f^{-1}(\cdot) = (y_{up}(t) - y_{dwe}(t))^{-\gamma}$, and its output is the control signal $\Delta x_{up}(t) = f^{-1}(\cdot) \cdot \Delta v'(t)$. In this figure, $\Delta y^*_{dwe}(t)$ is the reference and $e(t)$ is the error signal. f

If the inversion of the time varying component $f(\cdot)$ of the gain of the process were perfectly carried out by the gain

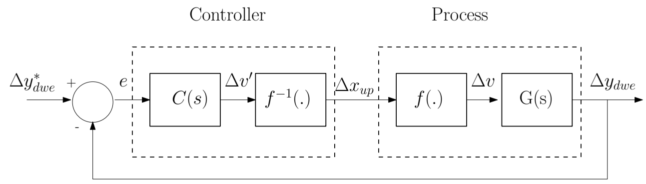

Fig. 4. Control scheme

scheduling part of the controller, an equivalent LTI closed-loop system would be obtained composed of a fictitious process transfer function $G(s)$, a fictitious linear controller $C(s)$ and a fictitious control signal $\Delta v(t) = \Delta v'(t)$. Since the gain K still experiences some variation in function of the operating regime (in accordance with Table 3) in that linearized control system, a controller $C(s)$ robust to gain changes is searched. Fractional-order controllers may have enhanced robustness features to gain changes. Then a LTI fractional-order controller is proposed as $C(s)$.

3.1 Fractional order operator

The fractional order operators have been defined in several ways, e.g, the Riemann-Liouville (RL), Caputo and Grünwald-Letnikov definitions. They generalize the standard derivative (D^n) and integral ($I^n = D^{-n}$) operators to the case D^α in which the order of the operator α is a real number (positive α means fractional-order derivative and negative α means fractional-order integral). We use in this work the Grünwald-Letnikov definition of this operator (e.g, Podlubny (1998)):

$$_aD_t^\alpha f(t) = \lim_{h \to 0} \frac{1}{h^\alpha} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t-jh) = \lim_{h \to 0} \frac{1}{h^\alpha} \Delta_h^\alpha f(t) \tag{8}$$

where $[a]$ is the integer part of a, h is the step size, D is the differential integral operator, α is the fractional order and the combinatorial function has been generalized in the following sense:

$$\binom{\alpha}{j} = \frac{\alpha(\alpha-1)....(\alpha-j+1)}{j!}$$

The fractional order integrator is then implemented considering the previous expression with a negative value of α. If zero initial conditions were considered, the Laplace transform of the integral of order α of the function $q(t)$ would be:

$$L\left[{}_aD_t^{-\alpha} q(t)\right] = \frac{1}{s^\alpha} Q(s) \tag{9}$$

where $Q(s)$ is the Laplace transform of $q(t)$.

3.2 Fractional Order PI^α Controller

We propose a fractional-order PI^α controller defined by the transfer function:

$$C(s) = K_p + \frac{K_i}{s^\alpha} \tag{10}$$

The following design specifications are proposed for the control system:

(1) An overshoot $M_p = 20\%$, which approximately corresponds to a damping $\varsigma = M_p^2/\sqrt{\pi^2 + M_p^2} = 0.45$. Then, according to the expression $\phi = 100 \cdot \varsigma$ (e.g, Ogata (1993)) an approximately equivalent frequency specification is $\phi_o = 45°$.
(2) A settling time $t_s = 110$ s. An approximately equivalent frequency specification (e.g, Ogata (1993)) can also be established: $\omega_{cgo} = 4/t_s = 0.0365$ rad/s.
(3) Since relatively small variations (though not negligible) are produced in the process gain K, a local robustness condition is searched for our controller. Then we propose that $C(s)$ fulfills the so called "local isophase margin condition".
This condition states that the phase of the frequency response of $G(s)\cdot C(s)$, which is $\angle\left(G(j\cdot\omega)\cdot C(j\cdot\omega)\right)$, must be "flat" (horizontal slope) at frequency ω_{cgo}. This condition is expressed as

$$\left.\frac{d\angle\left(G(j\cdot\omega)\cdot C(j\cdot\omega)\right)}{d\omega}\right|_{\omega=\omega_{cgo}} = 0 \qquad (11)$$

and its fulfillment implies that small changes in the gain produce negligible changes in the phase margin ϕ (e.g. Oustaloup (1991)).

Specifications 1 and 2 are achieved if the following condition in the complex plane is verified:

$$G(j\cdot\omega_{cgo})\cdot C(j\cdot\omega_{cgo}) = -e^{j\cdot\phi_o} \qquad (12)$$

which is split into two real conditions. Taking into account that $(j\cdot\omega)^{-\alpha} = e^{-j\cdot\frac{\pi}{2}\cdot\alpha}\cdot\omega^{-\alpha}$ and operating yields that:

$$K_i = \frac{\omega_{cgo}^{\alpha}}{\sin\left(\frac{\pi}{2}\cdot\alpha\right)}\cdot\Im(\chi)$$
$$K_p = -\left(\Re(\chi) + \cot\left(\frac{\pi}{2}\cdot\alpha\right)\cdot\Im(\chi)\right) \qquad (13)$$

being $\chi = e^{j\cdot\phi_o}/G(j\cdot\omega_{gco})$ and $\Re(\chi)$ and $\Im(\chi)$ the real and imaginary components respectively of χ . By making $\alpha = 1$, the rules to tune a standard PI controller that verifies the frequency specifications (ϕ_o, ω_{cgo}) are obtained.

A free parameter α is left in (13). These equations allow to obtain K_p and K_i in function of α. This extra parameter (compared to the two parameters to be tuned in a PI controller) is tuned to achieve the third specification. A simple search algorithm is implemented in which α is varied in the range $(0, 2)$. For each value of α, controller gains are obtained from (13), the frequency response of the controller is determined, and $\left.\frac{d\angle(G(j\cdot\omega)\cdot C(j\cdot\omega))}{d\omega}\right|_{\omega_{cgo}}$ is calculated. The value of the fractional order that makes zero this derivative, denoted α^*, is the one chosen for our controller. The controller gains are obtained substituting this value in (13), yielding K_p^* and K_i^*. The resulting controller is

$$C(s) = 0.419 + \frac{0.213}{s^{1.29}} \qquad (14)$$

It can be checked that these three specifications belong to the $K - RFFS$ of a first order plus time delay process controlled by a PI^{α} controller (Gharab and Feliu-Batlle (2019)).

4. SIMULATED RESULTS

4.1 PI^{α} controller

Time response of the closed-loop system are simulated using Matlab. Performances of the designed PI^{α} in the time domain are synthesized in Fig. 5. We focus on the design specifications: overshoot and settling time. For each specification, the nominal value and a maximum deviation are provided in Table (4).

Table 4. Performances of PI^{α} controller.

Controller	overshoot		Settling time(s)	
	Nominal	Maximum deviation	Nominal	Maximum deviation
$\alpha = 1.29$	20.07%	±14.7%	104.39	±8.74%

The nominal value gives the idea of the average behavior in the region defined by nominal model. The maximum deviation gives an idea of the robustness achieved for this specification (the variations in the response that can be expected from parameters changes).

The overshoot varies with a maximum deviation equal to ±14.7% which justifies that the iso-phase margin constraint around the designed gain crossover frequency is well satisfied by such controller.

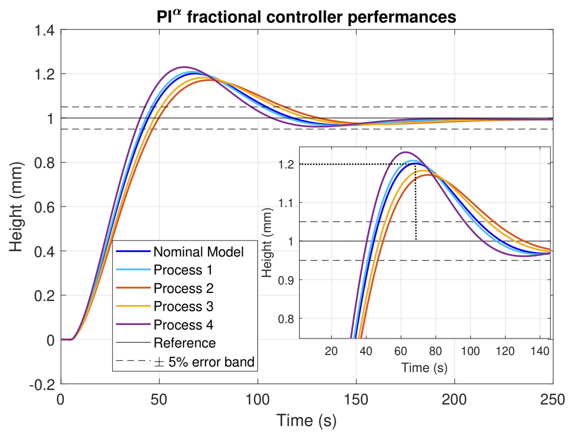

Fig. 5. Simulated closed-loop step responses.

4.2 PI controller

In this section, a standard PI controller is tuned to fulfill the two first defined time specifications: overshoot of 20% and settling time of 110 s. By defining $\alpha = 1$ in equation 13, it is possible to tune such new controller:

$$C_{PI}(s) = -6.306 + \frac{0.503}{s} \qquad (15)$$

The step responses of such controller are illustrated in Fig 6. The controlled system clearly achieves the desired performance around the nominal model, but high variances in terms of overshoot and settling time are recorded when moving away from such averaged regime, which is well detailed in Table 5.

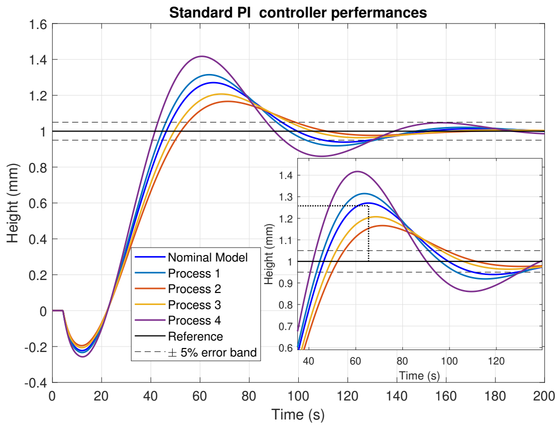

Fig. 6. Performances of a PI classic control system.

Table 5. Performances of standard PI controller.

Controller	overshoot		Settling time(s)	
	Nominal	Maximum deviation	Nominal	Maximum deviation
$\alpha = 1$	27.06%	±54.14%	128.89	±25.14%

5. CONCLUSION

The research presented in this paper has been focused on the laboratory hydraulic canal of the University of Castilla-La Mancha. The main pool of this canal operates in a submerged flow mode, in which four operating regimes have been defined. First order plus time delay models (1) were used to identify the dynamics of the canal at each of these four regimes, and it was noticed that the gains of these models changed significantly in function of the operating regime. Subsequently, an accurate global model of the canal dynamics was obtained, which combined model (1) with a variable gain function (5), in order to achieve a constant gain K in the model (1). However, in this new model, that gain K still changes slightly (in a range of ±14% of its average value) in function of the operating regime. This suggests that a controller robust to gain changes has to be designed.

Consequently, a gain scheduling control system combined with a LTI fractional-order PI controller has been proposed in this paper. PI^{α} controllers are used in this work because they have already demonstrated good robustness to gain changes if they are properly tuned. The gain scheduling block inverts (and cancels) the non-linear component of the process gain previously identified.

Simulation results have shown the adequate performance of the proposed control system. Moreover, a comparison with a standard PI controller (combined also with the same gain scheduling block) equivalent to the PI^{α} in the sense of having been tuned for the same first and second specifications (ϕ_o, ω_{cgo}) shows the superior performance of the fractional-order controller in terms of robustness.

Our next step will be testing this control system in our laboratory canal.

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