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Optimal Fast Charging Control for Lithium-ion Batteries

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Abstract: Fast charging has gained an increasing interest in the convenient use of Lithium-ion batteries. This paper develops a constrained optimization based fast charging control strategy, which is capable of meeting needs in terms of charging time, energy loss, and safety-related charging constraints. To solve it with less computational effort, a two-layer optimization strategy is proposed, where a charging time region contraction method is utilized to search the minimum expected charging time in the top layer, and the bottom layer uses the barrier method to calculate the corresponding optimal charging current with the charging time given by the top layer. Through utilizing this two-layer optimization method, the optimal charging current can be obtained that leads to the shortest charging period while guaranteeing the charging constraints with relatively low computational complexity. Extensive simulation results are provided to validate the proposed optimal fast charging control strategy, which well outperforms the constant current-constant voltage method.

Keywords: Lithium-ion battery, optimal fast charging control, two-layer optimization, state-of-charge, energy loss.

1. INTRODUCTION

In recent years, rechargeable Lithium-ion batteries play an increasingly significant role in many applications such as telecommunication and electric vehicles, due to their advantages of high energy density and low self-discharge (Ouyang et al., 2019). Several effective battery management technologies, such as state-of-charge (SOC) estimation (Wang et al., 2017) and cell equalization (Ouyang et al., 2018), have been proposed to enhance the battery's reliability and utility. Yet, how to achieve reliable charging management for batteries is still a key but challenging problem (Lu et al., 2013). Improper charging, such as overcharging or charging with an excessive current can lead to fast capacity fade of the battery and even result in safety hazards, while a low charging speed would cause inconvenience in the battery use and eventually impair the consumer satisfaction level. This hence calls for a fast charging strategy that minimizes the charging time while guaranteeing the battery's safety.

Plenty of battery charging strategies have been proposed as reviewed in (Gao et al., 2019). Among them, the most commonly utilized one is the constant current-constant voltage (CC-CV) strategy (Andrea, 2010), which charges a battery with a constant current until a threshold terminal voltage is reached and then continues to charge with the voltage kept constant until the current becomes small enough. Although it is easy for practical implementation, the charging performance depends on the empirical knowledge of the constant current value selection. To remedy this deficiency, several efforts have been devoted to study the optimal charging techniques to obtain the suitable charging current. Model predictive control (MPC) algorithms are proposed in (Yan et al., 2011) and (Zou et al., 2018) for battery charging control with both considerations of charging time and lifetime. Tian et al. (2019) develops an explicit MPC based charging strategy to reduce the computational complexity of the traditional MPC method by precomputing explicit solutions as piecewise functions, which is more efficient to be implemented in real-time. However, these strategies minimize the difference between the desired and actual SOC of the battery rather than optimizing the charging time directly, which could result in a relatively long charging time. To remedy this deficiency, the charging time is directly treated as an optimization indicator in the optimal charging control strategies (Zhang et al., 2017; H. Min et al., 2017; Liu et al., 2018). However, the formulated charging based optimization problem cannot be solved directly, since the terminal time is not fixed and the relationship between charging time and charging current cannot be explicitly expressed. Therefore, intelligent algorithms, such as genetic algorithm (Zhang et al., 2017), particle swarm optimization method (H. Min et al., 2017), and biogeography-based optimization strategy (Liu et al., 2018), are employed to solve the formulated optimization problem to search the optimal charging current. However, these intelligent algorithms bring a huge computational burden to the charging controller that makes it hard to be implemented in practical charging applications.

Considering this research gap, an optimal fast charging control strategy is proposed in this paper. Firstly, based on a battery equivalent circuit model, a constrained optimization based charging control method is formulated with taken consideration of the objectives of charging time, energy loss, and safety-related charging constraints. To address the formulated optimization problem more efficiently, a two-layer optimization strategy is proposed, where a charging time region contraction method is utilized to obtain the minimum expected charging time in the top layer, and the bottom layer uses a gradient-based algorithm, named barrier method, to calculate the suit-

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able charging current that can make difference between the actual and desired states meet the required accuracy with the charging time given by the top layer. Through this operation, the optimal charging current can be obtained that leads to the shortest charging period while guaranteeing the charging constraints. This work highlights the developed two-layer optimization strategy that can effectively solve the minimum charging time optimization with relatively low computational complexity. Extensive simulation results validate the performance of the proposed optimal fast charging control strategy.

This paper is organized as follows. In Section 2, an equivalent circuit model and charging constraints for the battery are provided. Section 3 details the process of the proposed optimal fast battery charging control strategy design. Simulation results are shown in Section 4, and conclusions are given in Section 5.

2. CHARGING MODEL AND CONSTRAINTS

2.1 Battery Model

For the model-based charging control, an accurate battery model is necessary. This paper considers the equivalent circuit model shown in Fig. 1 to describe a battery's dynamics, which strikes a balance between computational complexity and predictive accuracy as illustrated in a broad range of literature such as (Lin et al., 2015; Ouyang et al., 2018). It is composed of a voltage source to represent the open circuit voltage and a serially connected resistor R_0 to characterize the charging energy loss. The battery's SOC is defined as the ratio of the available capacity to its fully-charged capacity (Ouyang et al., 2014), which can be calculated as

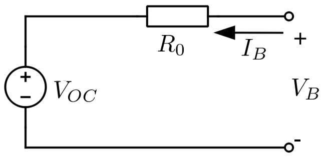

Fig. 1. Battery equivalent circuit model.

$$SOC(k+1) = SOC(k) + \frac{\eta_0 T}{Q} I_B(k) \tag{1}$$

where $SOC(k)$, $I_B(k)$, and Q denote the battery's SOC, charging current, and capacity, respectively; η_0 is the Coulomb coefficient and T is the sampling period. The battery's open circuit voltage and internal resistance are nonlinear functions of its SOC that can be described as

$$V_{OC}(k) = f(SOC(k)), \quad R_0(k) = h(SOC(k)) \tag{2}$$

where $V_{OC}(k)$ and $R_0(k)$ denote the battery's open circuit voltage and internal resistance, respectively. By using the Kirchhoff laws, the battery's terminal voltage can be formulated as follows:

$$V_B(k) = V_{OC}(k) + R_0(k) I_B(k) \tag{3}$$

where $V_B(k)$ represents the terminal voltage of the battery. To simplify the notations, let us define the system input, output and state as $u(k) \triangleq I_B(k) \in \mathbb{R}$, $y(k) \triangleq V_B(k) \in \mathbb{R}$, and $x(k) \triangleq SOC(k) \in \mathbb{R}$, respectively. Then, based on (1) - (3), the battery model can be rewritten in the following state-space representation

$$\begin{aligned} x(k+1) &= x(k) + bu(k) \\ y(k) &= f(x(k)) + h(x(k))u(k) \end{aligned} \tag{4}$$

with $b = \frac{\eta_0 T}{Q}$. Throughout this manuscript, the battery's SOC is assumed to be known since the SOC estimation methods with high accuracy have been well studied in the literature such as (Fang et al., 2014; Chen et al., 2018; Hu et al., 2018).

2.2 Charging Constraints

To ensure the battery's safety, hard constraints including the charging current, SOC and terminal voltage of the battery should be carefully guaranteed during the charging procedure.

Charging current limitation: The threshold of charging current plays an important role in the battery's safety since the excessive current would affect battery performance or even cause fire during the charging process. In light of this, the battery's charging current should be maintained below its maximum allowed value, which yields

$$0 \le u(k) \le u_M \tag{5}$$

where $u_M \in \mathbb{R}$ is the maximum allowed charging current of the battery.

SOC constraint: To avoid overcharging, the battery's SOC is not allowed to exceed its upper bound that

$$x(k+1) \le x_M \tag{6}$$

where $x_M \in \mathbb{R}$ is the the upper bound of the battery's SOC.

Terminal voltage restriction: The battery's terminal voltage at the end of each sampling interval should not exceed an allowed limit to avoid damage. Based on (4), this implies

$$f(x(k+1)) + h(x(k+1))u(k) \le y_M \tag{7}$$

where $y_M \in \mathbb{R}$ denotes the battery's maximum allowed terminal voltage.

3. CHARGING CONTROL STRATEGY DESIGN

3.1 Charging Control Formulation

For the battery charging control, the charging speed is one of the most crucial aspects, since shorter charging time can be an effective solution to alleviate the charging anxiety of the users. The charging pattern aims to minimize the time that the battery is charged from an initial SOC of $x(0) = x_0$ to the target value of x_r. The corresponding cost function can be straightforwardly expressed as

$$J_t = NT \tag{8}$$

where J_t represents the charging time and N is the sampling step number with $x(N) = x_r$.

Another important charging objective is to improve the charging efficiency by reducing the battery's energy loss during the charging process. Based upon the battery model (4), the cost function J_e with respect to the energy loss of the battery can be formulated as

$$J_e = \sum_{k=0}^{N-1} T h(x(k+1)) u^2(k). \tag{9}$$

A high-quality battery charging control strategy should both pursue short charging time and low charging energy loss. Hence, both the two objectives (8) and (9) should be taken into consideration in the battery charging control. To solve the multi-objective optimization issue, we convert it to a single objective optimization by transforming the energy loss objective into a constraint as

$$J_e \le J_{eM} \tag{10}$$

with J_{eM} denoting the maximum allowed energy loss of the battery, since the charging time objective is with higher importance than the energy loss objective in practice.

According to the objective function in (8) as well as the charging constraints (5) - (7), and (10), the optimal charging control strategy can be formulated as a constrained optimization problem as follows:

$$\begin{aligned} \min_{u(0),\cdots,u(N-1)} \quad & J_t \\ \text{s.t.} \quad & x(k+1) = x(k) + bu(k) \\ & f(x(k+1)) + h(x(k+1))u(k) \leq y_M \\ & x(k+1) \leq x_M, \ \ 0 \leq u(k) \leq u_M \\ & J_e \leq J_{eM}, \ \ x(N) = x_r, \ \ x(0) = x_0. \end{aligned} \tag{11}$$

Although the optimal charging current $u(k)$ $(0 \leq k \leq N-1)$ can be obtained by solving (11), the solution of (11) cannot be directly computed, since the terminal charging time NT is not fixed and the relationship between charging time and charging current cannot be explicitly expressed. Usually, intelligent algorithms are utilized to search its optimal solution, such as genetic algorithm (Zhang et al., 2017), particle swarm optimization algorithm (H. Min et al., 2017), biogeography-based optimization (Liu et al., 2018). But these intelligent algorithms bring a huge computational burden to the charging controller that makes it hard to be implemented in practical charging applications. To reduce the computational cost of solving (11), a two-layer optimization algorithm is proposed here, which narrows the charging time range until the shortest expected charging time is determined in the top layer and uses the bottom-layer optimization method to calculate the corresponding optimal charging current.

3.2 Two-Layer Optimization strategy

The two-layer optimization strategy is presented in-depth as follows. In the bottom layer, for a charging time J_t given by the top layer, the terminal state constraint in the optimization problem (11) is transformed to design the optimal charging current $u(k)$ to drive the terminal state $x(N)$ toward to the desired x_r to the greatest extent. Through such operation, (11) can be transformed into a conventional constrained optimization problem that can be easily and effectively solved by many power techniques such as gradient-based algorithms (Boyd and Vandenberghe, 2004). The top layer is based on a charging time region contraction method, where the time region continues to be narrowed until the minimum expected charging time is obtained that can make difference between the actual and desired states meet the required accuracy in the bottom layer. Thus, the optimal charging current can be obtained that leads to the shortest charging period while guaranteeing the charging constraints in (11).

Bottom layer: charging current optimization: In the bottom layer, we fix the sampling step number N and let the sampling period T change with the charging time J_t given by the top layer, which can make the computational burden at the bottom layer consistent for different charging currents. In other words, the sampling period is calculated as

$$T = \frac{J_t}{N}. \tag{12}$$

Then for a charging time J_t, as stated above, the optimization issue with terminal state constraint in (11) is transformed to minimize the difference between the terminal state $x(N)$ and the desired state x_r as

$$\begin{aligned} \min_{u(0),\cdots,u(N-1)} \quad & (x(N) - x_r)^2 \\ \text{s.t.} \quad & x(k+1) = x(k) + bu(k), \ \ x(0) = x_0 \\ & f(x(k+1)) + h(x(k+1))u(k) \leq y_M \\ & x(k+1) \leq x_M, \ \ 0 \leq u(k) \leq u_M, \ \ J_e \leq J_{eM}. \end{aligned} \tag{13}$$

The battery's state can be written as $x(k) = x(0) + bH_kU$ with $U = [u(0), \cdots, u(N-1)]^T \in \mathbb{R}^N$ and $H_k = [1_k^T, 0_{N-k}^T] \in \mathbb{R}^{1\times N}$, where 1_k and 0_{N-k} denote column vectors with k ones and $N-k$ zeros, respectively. Then, (13) can be rewritten as the following constrained optimization problem:

$$\begin{aligned} \min_{U} \quad & J_1(U) \\ \text{s.t.} \quad & F(U) + G(U)U \leq Y_M \\ & MU \leq X_C, \ \Phi U \leq U_M \\ & TU^T GU \leq J_{eM} \end{aligned} \tag{14}$$

with

$$\begin{aligned} & J_1(U) = b^2U^TH_N^TH_NU + 2(x(0) - x_s)bH_NU + (x(0) - x_r)^2 \\ & F(U) = [f(x(0) + bH_1U), \cdots, f(x(0) + bH_NU)]^T \\ & G(U) = diag\{h(x(0) + bH_1U), \cdots, h(x(0) + bH_NU)\} \\ & Y_M = y_M 1_N, \ \ X_C = (x_M - x(0))1_N, \ \ \Phi = [I_N, -I_N]^T \\ & M = [(bH_1)^T, \cdots (bH_N)^T]^T, \ \ U_M = [u_M 1_N^T, 0_N^T]^T \end{aligned}$$

where I_N denotes an identity matrix with dimensions of $N \times N$ and $diag\{\cdot\}$ represents the diagonal matrix. Note that (14) is a standard nonlinear constrained optimization problem. With employing the barrier method in (Boyd and Vandenberghe, 2004) to solve it, the optimal charging current sequence can be easily obtained.

Top layer: charging time optimization: It is observed that there always exist feasible solutions for (14), since the terminal state constraint in (11) has been converted to a minimization problem in the bottom layer. But when the expected charging time is too short, the battery's terminal SOC can deviate significantly from the desired one. Hence, if the cost function satisfies

$$|x(N) - x_r| > \varepsilon_1 \tag{15}$$

with ε_1 a set tolerance, it means the given charging time from the top layer J_t is less than the minimum required charging time. Otherwise, it denotes that the selection of J_t may be too large. Motivated by this, a charging time region contraction strategy is developed in the top layer, where the charging time region is narrowed from a initial region $[T_{c1_0}, T_{c2_0}]$ until the minimum expected charging time is determined. It is a binary search algorithm. For the k-th iteration step, the middle value of the region in the $k-1$-th iteration $\lambda_k = \frac{T_{c1_{k-1}} + T_{c2_{k-1}}}{2}$ is selected as the charging time for the bottom-layer optimization, where $[T_{c1_{k-1}}, T_{c2_{k-1}}]$ is the charging time region updated after the $k-1$-th iteration. If this charging time can make $|x(N) - x_r| \leq \varepsilon_1$ in the bottom layer, it indicates that the right end point of the charging time region $T_{c2_{k-1}}$ is selected a bit large and it should be replaced by the middle value λ_k, i.e., the charging time region after the k-th iteration is $[T_{c1_{k-1}}, \lambda_k]$. Otherwise, it means that the charging time $T_{c1_{k-1}}$ is not enough for implementing the charging task. Hence, the middle value λ_k should be utilized to replace the left end point of the charging time region $T_{c1_{k-1}}$, and the new charging time region becomes $[\lambda_k, T_{c2_{k-1}}]$. Through these iterations, the charging time region can be reduced at a rate of 50% until it satisfies $|T_{c2_k} - T_{c1_k}| \leq \varepsilon_2$, where ε_2 is the tolerance. Then, the shortest charging time can be determined as $J_t = \frac{T_{c1_k}+T_{c2_k}}{2}$ and the corresponding optimal charging current can obtained by solving (14) with that J_t. The detailed optimization algorithm is summarized as follows:

Algorithm 1:

1) Set the initial charging time region $[T_{c1_0}, T_{c2_0}]$.

2) For the k-th iteration, the intermediate variable is chosen as $\lambda_k = \frac{T_{c1_{k-1}}+T_{c2_{k-1}}}{2}$. Set the charging time as $J_t = \lambda_k$ and use bottom-layer optimization algorithm to calculate (14). If there exists a solution to make $|x(N) - x_r| \leq \varepsilon_1$ in the bottom layer, select $T_{c1_k} = T_{c1_k}$ and $T_{c2_k} = \lambda_k$. Otherwise, set $T_{c1_k} = \lambda_k$ and $T_{c2_k} = T_{c2_k}$.

3) Stop and output the optimal solution of (14) with the charging time $J_t = \frac{T_{c1_k}+T_{c2_k}}{2}$, if $|T_{c2_k} - T_{c1_k}| \leq \varepsilon_2$, where ε_2 is the tolerance. Otherwise, set $k = k+1$ and return to Step 2).

4. SIMULATION RESULTS

In this section, MATLAB/SIMULINK-based simulations are performed to evaluate the effectiveness of the proposed optimal fast charging control method. A lithium-ion battery with a capacity of 2.38 Ah and a nominal voltage of 3.7V is selected, where the mappings from the SOC to its open circuit voltage and internal resistance are shown in Fig. 2 (Ouyang et al., 2018). The upper bounds of the battery's charging current, SOC and terminal voltage are, respectively, selected as 3 C-rate, $x_M = 100\%$, and $y_M = 4.2$ V. The initial and desired SOCs of the battery are $x_0 = 0\%$ and $x_r = 100\%$, respectively. The battery's maximum allowed energy loss is limited to 5% of its recharged energy, which can be approximately calculated as $J_{ew} = 5\% \times 3.7 \times (x_r - x_0) \times 2.38 \times 3600$ J. The initial charging time region is chosen as $[10 \text{ min}, 360 \text{ min}]$. The sampling step number is set as $N = 10$. The tolerances ε_1 and ε_2 are selected as 0.5% and 2 min, respectively.

The results in terms of SOC, charging current, terminal voltage, and energy loss under the proposed optimal fast charging strategy are illustrated in Fig. 3 (a)-(d), respectively. They show that the battery's SOC can be charged from 0% to 99.59% while satisfying the charging constraints within a charging period of 34 min. Note that the actual energy loss (1596 J) is a little larger than the maximum allowed one (1585 J), which is caused by the battery discrete model bias since the simulation is based on the continuous-time model. To demonstrate the superior performance of the designed fast optimal charging control method, the charging results of the traditional CC-CV are provided as comparisons, where the constant charging current is set as 1 C-rate, 2 C-rate, and 3 C-rate, respectively. The corresponding charging results are also shown in Fig. 3 and Table 1. It shows that the objectives of charging time and energy loss are conflicting, where a shorter charging time leads to more energy loss in the charging process. But the proposed fast charging control algorithm can minimize the battery's charging time while constraining the energy loss under the pre-set limit, which hence enables a good balance between these two objectives. It demonstrates the effectiveness and advantages of proposed charging control strategy. Note that users can adjust the suitable energy loss limit in the proposed charging control algorithm to get more appropriate charging current according to their actual demand in practical applications.

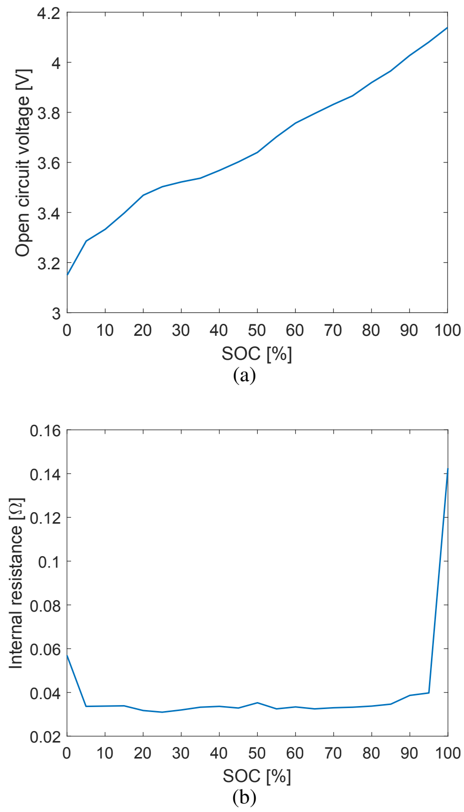

Fig. 2. Relationship between the battery's (a) SOC and open circuit voltage, (b) SOC and internal resistance.

5. CONCLUSIONS

Charging is a crucial process for lithium-ion batteries to replenish and store energy, which calls for a fast charging strategy that minimizes the charging time while guaranteeing the battery's safety in the charging procedure. In this paper, a constrained optimization based charging control strategy is formulated by considering the objectives of charging time, energy loss, and safety-related charging constraints. Then, a two-layer optimization strategy is proposed to solve it to get the optimal charging current that leads to the shortest charging period while guaranteeing the charging constraints with relatively low computational complexity. Simulations have been carried out to validate the proposed optimal fast charging control method, showing performance superior to the conventional CCCV method.

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Table 1. Charging results comparison

	Charging time/min	Energy loss/J
Proposed method	34	1596
CC-CV with 1 C-rate	63.7	700
CC-CV with 2 C-rate	35.6	1346
CC-CV with 3 C-rate	27	1945

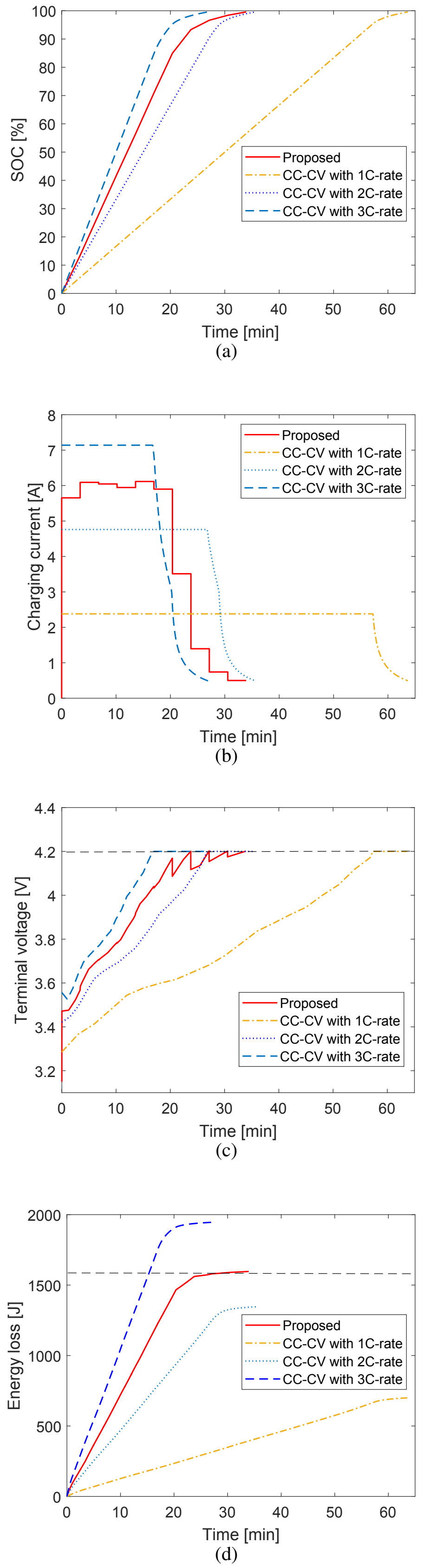

Fig. 3. Battery's responses of (a) SOC, (b) charging current, (c) terminal voltage, (d) energy loss.

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