Road Constrained Labeled Multi Bernoulli Filter based on PDF Truncation for Multi-Target Tracking

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Abstract: In this paper, road constrained filtering is applied to labeled multi Bernoulli (LMB) filter using PDF truncation in multi-road environment. In target tracking systems with road map information, road constraints can effectively improve the estimation performance. To apply multiple road constraints information to the tracking filter, all constraints should not be applied simultaneously and only one should be selected for each estimated trajectory. Then, probability density function (PDF) truncation is conducted which is a constrained filtering technique for inequality constraints. To verify the constrained filtering technique to LMB filter, simulations for multi-target tracking in cluttered environments are carried out. The simulation result shows that the proposed method bounded estimated trajectories on the road effectively and reduced OSPA error.

Keywords: Multi-target Tracking, Road Constrained Filtering, PDF Truncation

1. INTRODUCTION

In a multi-target tracking (MTT) problem, it is essential to estimate states of the multiple targets, which of the number is time-varying. To solve this problem, finite set statistics (FISST) is proposed which is based on random finite set (RFS) (Mahler (2003)). Based on FISST, probability hypothesis density (PHD) filter which propagates first-order moment of the filter is proposed with its variations (Mahler (2003). To improve cardinality estimation performance of the PHD filter, cardinalized PHD (CPHD) filter was introduced that jointly estimates intensity and cardinality distribution (Mahler (2007)). However, PHD and CPHD filters are suboptimal multi-target Bayes-filters since they only consider the first-order moment. The first optimal multi-target Bayesian filter is proposed using labeled multi-Bernoulli RFS which is called δ-generalized labeled multi-Bernoulli filter (δ-GLMB) (Vo et al. (2013)). To reduce the computation complexity of the δ-GLMB, labeled multi-Bernoulli (LMB) and Gibbs sampling based LMB filter are proposed with some approximations (Vo et al. (2017)). These RFS based filters are widely used with applications at radar surveillance systems, autonomous vehicles, and computer vision (Papi (2015), Gostar et al. (2017), He et al. (2017)).

Most of the multi-target tracking systems use range, bearing measurements as information. Not only this information, but physical constraints on each system can be used as additional information. In most physical systems, states are bounded by their physical limitations. These constraints can improve the estimation performance of the filter. Various methods have been developed to deal with constraints, including state estimate projection, moving horizon estimation (MHE), and probability density function (PDF) truncation (Simon (2010), Nesrine et al. (2018)). State estimate projection transforms the unconstrained estimate to the constrained surface. MHE is an optimization-based approach that has large computational load. PDF truncation method truncates the constrained edge of the estimated state which is assumed Gaussian distribution, and then estimate becomes equal to the mean of the truncated PDF. In single-target tracking application, above constrained filtering techniques are applied and showed improved estimation performance (Ondřej et al. (2012), Lai et al. (2017), Hu et al. (2017)).

Constrained filtering is also studied in RFS filter research area. Many studies are related to ground target tracking applications with roadmap information, which provides road constraints (Wong et al. (2012), Yang et al. (2016), Lian et al. (2018)). Yang et al. (2015) decomposed unconstrained state to mutually uncorrelated terms and applied linear equality constraint simulation, but it was verified in a single-target scenario. Also, equality constraint is too restrictive to use in real world. Zheng and Gao (2018) proposed two Gaussian Mixture PHD filter based constrained filter. One is DPN (directional process noise) GM-PHD which adjust process noise according to road direction. The other is SC (state constraint) GM-PHD which project unconstrained estimate to the road. However, it uses equality constraint that does not consider road width, and simulation was done in single target scenario. Papi (2015) proposed constrained GLMB by generalized likelihood function which is derived from the available set of state constraints, but it needs a more detailed explanation of how the multiple constraints are applied appropriately.

In this paper, we propose a road-constrained LMB filter using PDF truncation in multi-road environment. As explained above, existing researches have been conducted on simple situations where there is no road width or one-way road. In multiple road constraints, nearest road constraint to each trajectory is selected every time step and applies PDF truncation to move the trajectories inside the road width. Simulation results show that the estimation performance can be improved compared to conventional LMB filter without computational burden.

2. RFS Based Multi-target Tracking Filter

In this section, the labeled RFS and LMB filter are explained briefly (Vo et al. (2017)).

2.1 Labeled RFS

A labeled RFS is an augmented RFS of state $x \in \mathbb{X}$ and unique labels $l \in \mathbb{L} = \{\alpha_i : i \in \mathbb{N}\}$. The set of labels $\mathbf{X}$ is given by $\mathcal{L}(\mathbf{X}) = \{\mathcal{L}(\mathbf{X}), x \in \mathbf{X}\}$ where $\mathcal{L} : \mathbb{X} \times \mathbb{L} \to \mathbb{L}$ is the projection defined by $\mathcal{L}((x,l)) = l$. The distinct label indicator is defined as (1).

$$\Delta(\mathbf{X}) = \delta_{|\mathbf{X}|}\left[\left|\mathcal{L}(\mathbf{X})\right|\right] \tag{1}$$

$$\delta_Y[X] = \begin{cases} 1, & if\ X = Y \\ 0, & otherwise \end{cases} \tag{2}$$

The integral of a function $f : \mathbb{X} \times \mathbb{L}$ is given by

$$\int f(\mathbf{x})d\mathbf{x} = \sum_{l \in \mathbb{L}} \int f(x,l)dx \tag{3}$$

2.2 Labeled Multi-Bernoulli Filter

A GLMB is a labeled RFS filter with a probability distribution

$$\pi(\mathbf{X}) = \Delta(\mathbf{X}) \sum_{\xi \in \Xi} \omega^{(\xi)}(\mathcal{L}(\mathbf{X}))\left[p^{(\xi)}\right]^{\mathbf{X}} \tag{4}$$

where Ξ is a discrete space, each $p^{(\xi)}(\cdot,l)$ is a probability density, $\omega^{(\xi)}$ is non-negative weight with $\sum_{(I,\xi) \in \mathcal{F}(\mathbb{L}) \times \Xi} \omega^{(\xi)}(I) = 1$.

An LMB is a special case of GLMB RFS (4) with only one element in space Ξ .

$$\begin{aligned} \pi(\mathbf{X}) &= \Delta(\mathbf{X})\omega(\mathcal{L}(\mathbf{X}))p^{\mathbf{X}} \\ &= \Delta(\mathbf{X})\left[1-r\right]^{\mathbb{M}}\left[\mathbf{1}_{\mathbb{M}}\frac{r}{1-r}p\right]^{\mathbf{X}} \end{aligned} \tag{5}$$

where $\mathbb{M} \subset \mathbb{L}$.

Clutter is modeled as a Poisson RFS in (6), then multi-object likelihood is (7).

$$\pi_K(K) = e^{-\langle \kappa,1 \rangle}\kappa^K \tag{6}$$

$$g(Z \mid \mathbf{X}) = \sum_{W \subseteq Z} \pi_D(W \mid \mathbf{X})\pi_K(Z - W) \tag{7}$$

3. Constrained Filtering

Constrained filtering is applied to estimate of the LMB filter at every time step. Since constrained filtering result is not fed back into the LMB filter, this step is simply added after the LMB filter.

To apply road constraint to the LMB filter, the estimate projection method and the PDF truncation method are explained below. Also, selecting appropriate road constraints from multiple road constraints is described.

3.1 Estimate Projection

Suppose discrete linear time-invariant system (8) and linear equality constraints (9).

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{F}x_k + \mathbf{w}_k \\ \mathbf{y}_{k+1} = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k \end{cases} \tag{8}$$

$$\mathbf{D}\mathbf{x}_k = \mathbf{d} \tag{9}$$

where k is the time step and $\mathbf{w}_k, \mathbf{v}_k$ are the zero mean Gaussian noises.

Then unconstrained estimate of the Kalman filter $\hat{\mathbf{x}}_k^+$ can be projected to constraint surface (9) by solving equation (10).

$$\tilde{\mathbf{x}}_k^+ = \arg\min_{\mathbf{x}} (\mathbf{x} - \hat{\mathbf{x}}_k^+)^T (\mathbf{P}_k^+)^{-1} (\mathbf{x} - \hat{\mathbf{x}}_k^+) \tag{10}$$

where $\mathbf{P}_k^+$ is the covariance of the Kalman filter estimate $\mathbf{x}_k^+$.

The constrained estimate, which is a solution of the equation (10) is (11).

$$\tilde{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^+ - \mathbf{P}_k^+ \mathbf{D}^T (\mathbf{D}\mathbf{P}_k^+ \mathbf{D}^T)^{-1} (\mathbf{D}\hat{\mathbf{x}}_k^+ - \mathbf{d}) \tag{11}$$

3.2 PDF Truncation

Suppose discrete linear time-invariant system (8) and linear inequality constraints (12).

$$a_{k,i} \le \boldsymbol{\phi}_{k,i}^T \mathbf{x}_{k,i} \le b_{k,i},\ i = 1,\dots,s \tag{12}$$

where k is the time step, i is the index of the multiple constraints.

The objective of the PDF truncation is transforming Gaussian PDF to standard normal distribution and truncate at the constraint edges. The transformation is written in (13),

$$z_{k,i} = \mathbf{S}_i \mathbf{W}_i^{-1/2} \mathbf{T}_i^T (\mathbf{x}_{k,i} - \tilde{\mathbf{x}}_{k,i}) \sim N(\mathbf{0}, e_1 e_1^T) \tag{13}$$

$$e_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T \tag{14}$$

where $\mathbf{T}_i$ is obtained by Jordan canonical decomposition as (15) and $\mathbf{S}_i$ is obtained by Gram-Schmidt orthogonalization as (16).

$$\tilde{\mathbf{P}}_{k,i} = \mathbf{T}_i \mathbf{W}_i \mathbf{T}_i^T \tag{15}$$

$$\mathbf{S}_i = \begin{bmatrix} \boldsymbol{\rho}_{1,i} \\ \vdots \\ \boldsymbol{\rho}_{n,i} \end{bmatrix} \tag{16}$$

$$\boldsymbol{\rho}_{1,i} = \frac{\boldsymbol{\phi}_{k,i}^T \mathbf{T} \mathbf{W}^{1/2}}{\left(\boldsymbol{\phi}_{k,i}^T \tilde{\mathbf{P}} \boldsymbol{\phi}_{k,i}\right)^{1/2}},\ \boldsymbol{\rho}_{k,i} = e_k - p\sum_{j=1}^{k-1}\left(e_k^T \boldsymbol{\rho}_{j,i}\right)\boldsymbol{\rho}_{j,i} \tag{17}$$

After transformation (13), inequality constraint become (18)

$$c_{k,i} \le \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} z_{k,i} \le d_{k,i} \tag{18}$$

$$c_{k,i} = \frac{a_{k,i} - \phi_{k,i}^T \tilde{\mathbf{x}}_{k,i}}{\left(\phi_{k,i}^T \tilde{\mathbf{P}}_{k,i} \phi_{k,i}\right)^{1/2}},\ d_{k,i} = \frac{b_{k,i} - \phi_{k,i}^T \tilde{\mathbf{x}}_{k,i}}{\left(\phi_{k,i}^T \tilde{\mathbf{P}}_{k,i} \phi_{k,i}\right)^{1/2}} \tag{19}$$

By truncating edge of the Gaussian PDF as (20) and normalize as (22), the result of mean and variance are (23) and (24).

$$\int_{c_{k,i}}^{d_{k,i}} \frac{1}{\sqrt{2\pi}} e^{-\zeta^2/2} d\zeta = \frac{1}{2}\left[erf\left(\frac{d_{k,i}}{\sqrt{2}}\right) - erf\left(\frac{c_{k,i}}{\sqrt{2}}\right)\right] \tag{20}$$

$$erf(t) = \frac{2}{\sqrt{\pi}} \int_0^t \exp(-\gamma^2) d\gamma \tag{21}$$

$$pdf(\zeta) = \begin{cases} \alpha e^{-(\zeta^2/2)} & \zeta \in [c_k, d_k] \\ 0 & otherwise \end{cases} \tag{22}$$

$$\tilde{x}_{k,i+1} = \mathbf{T}_i \mathbf{W}_i^{1/2} \mathbf{S}_i^T \tilde{z}_{k,i+1} + \tilde{x}_{k,i} \tag{23}$$

$$\tilde{\mathbf{P}}_{k,i+1} = \mathbf{T}_i \mathbf{W}_i^{1/2} \mathbf{S}_i^T \operatorname{cov}(\tilde{z}_{k,i+1}) \mathbf{S}_i \mathbf{W}_i^{1/2} \mathbf{T}_i^T \tag{24}$$

3.3 Selecting a constraint from multiple road constraints

Since there are many boundaries on the road, constraints must be applied properly. Figure 1 is an example of multiple road constraints. Blue line is the boundaries of the road, and shaded areas are road constrained areas. Green shaded area is between road 1 and 2, and yellow shaded area is between road 3 and 4. If constraints 1,2,3,4 are applied simultaneously, there are no satisfied area. Figure 2 is another road constraint example. The red triangle is the unconstrained estimate obtained by the tracking filter. If all constraints are applied simultaneously, red triangle will be moved to blue triangle which is far from unconstrained estimate. Otherwise, if only road 1 and 2 are applied, the red triangle will be moved to purple one. From these examples, it is confirmed that select only near road from the unconstrained estimate is appropriate.

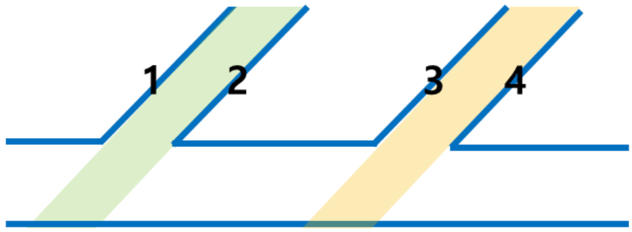

Fig. 1. Multiple road constraints example 1

Fig. 2. Multiple road constraints example 2

To select the nearest road, vertical line distances from the unconstrained estimate to roads should be calculated. In addition, if there is no intersection point between vertical line and the road, the distance between unconstrained estimate and end-point of the road, instead.

4. Performance Evaluation

To analyze the performance of the LMB filter with PDF truncation, it is compared with the estimate projection based LMB filter and conventional LMB filter in simulations. The simulation environment and results are described in this section.

4.1 Simulation Environment

In the simulation, two-dimensional multi-target tracking examples are considered. In Fig. 3, three targets are moving from circle to triangle through black lines at constant velocity 10 m/s. Blue lines are boundaries of the road, and road width of the multiple roads varies from 10 m to 20 m. Total simulation time is 100 s, and Gaussian mixture based LMB filter is used to track multiple targets. The system model is constant velocity model and uses one sensor that provides x, y position with 10 m white Gaussian noise. The probability of detection is 0.98, and probability of survival is 0.99, and probability of gating is 0.99. The Poisson average rate of uniform clutter per scan is 30. All simulations are done with 100 Monte Carlo simulations.

Tracking performance is measured by optimal sub-pattern assignment (OSPA) distance (Schuhmacher et al. (2008)). The parameters for OSPA distance is set as $p = 1$ and $c = 100$.

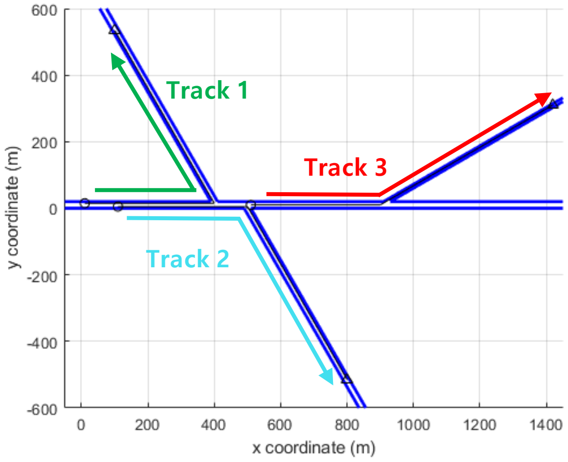

Fig. 3. Road constrained scenario

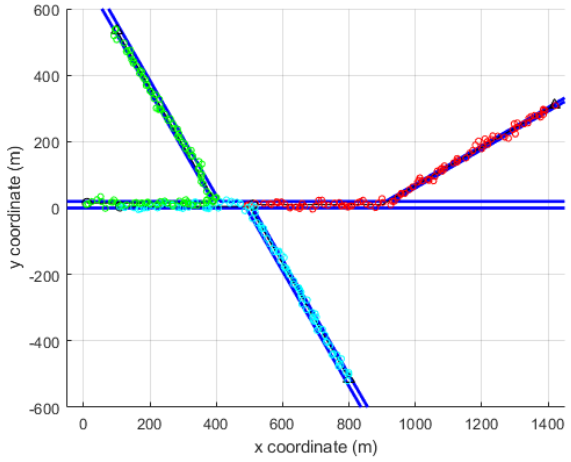

Fig. 4. The estimated trajectory of the LMB filter

4.2 Simulation Results

To verify the effect of the LMB filter with PDF truncation, tracking performance is compared. Figure 4 is the estimated trajectory of the LMB filter and Fig. 5 is the magnified figure. Each color means separate trajectories. Since the LMB filter does not use road constraints, estimated trajectories sometimes off the road. Meanwhile, Fig. 6 is the estimated trajectory of the LMB filter with estimate projection and Fig. 7 is the PDF truncation based filter. As shown in the figures, multiple road constraints caged estimated trajectories on the roads or inside the roads effectively. To compare the results in performance index, OSPA error of the 100 Monte Carlo simulation is plotted in Fig. 8. As expected, OSPA error of the constrained LMB filter is lower than LMB filter. When comparing PDF truncation and estimate projection, PDF truncation is more effective.

Figure 9 shows an example of the PDF truncation of the estimated target distribution. Since we select one road constraint from the multiple constraints at each time step, the equality constraint is one-sided equality. The normal distribution is truncated by road constraint which is shown as dotted vertical line, then mean and variance are recalculated by truncated PDF. This procedure results in reducing OSPA error of the filter.

Also, computation time is compared with Intel Core i7-9700K 3.60 GHz CPU processor and 16 GB RAM. Computational time per 1 simulation of the LMB filter is 4.99 s and 5.00 s for the constrained LMB filter. There is no difference in computation time since computational cost of calculating distance, and PDF truncation procedure are small.

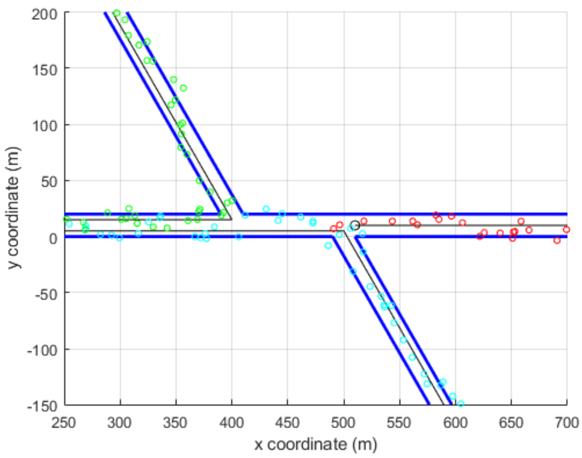

Fig. 5. The magnified figure of Fig. 4

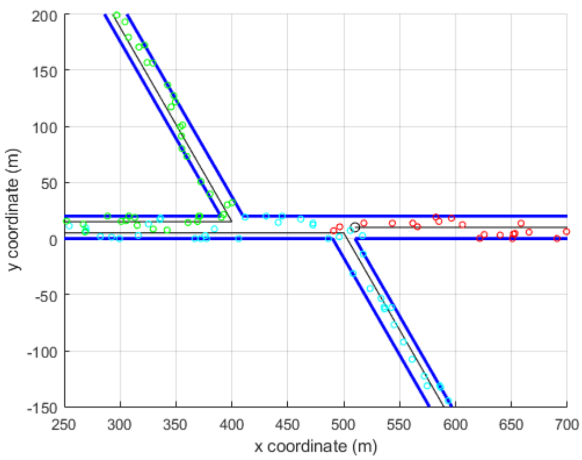

Fig. 6. The magnified trajectory of the estimate projection based filter

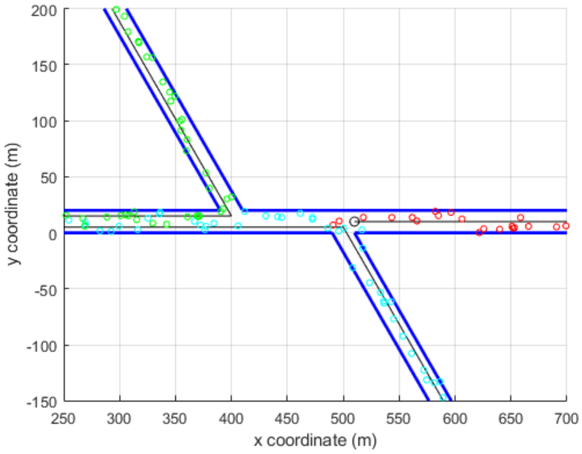

Fig. 7. The magnified trajectory of the truncation based filter

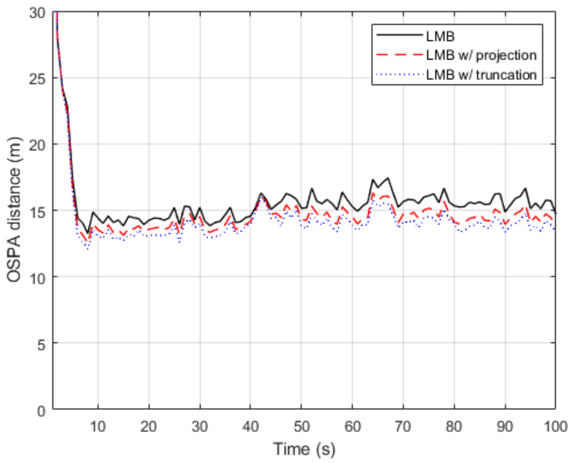

Fig. 8. OSPA distance of the LMB filter and the constrained LMB filter

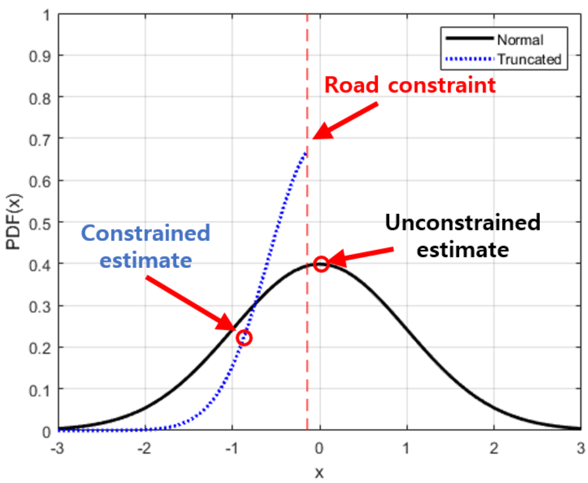

Fig. 9. PDF truncation of the estimated target distribution

Table 1. OSPA distance and computation time for the filtering methods

Filter	OSPA distance [m]	Time [s] (per 1 simulation)
LMB	16.0470	4.99
LMB w/ projection	15.2021	5.00
LMB w/ truncation	14.7302	5.00

5. CONCLUSIONS

In this paper, road constrained filtering was applied to the Gaussian mixture based LMB filter. By selecting nearest road from the estimated trajectories to the multiple road boundaries, estimated trajectories can be bounded in the road properly. In simulation results, the performance of the constrained LMB filter was verified in multiple road boundaries. Computation time was similar since computational cost of the PDF truncation is small.

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