Distributed global actuator fault-detection scheme for a class of linear multi-agent systems with disturbances

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Abstract: This paper proposes a distributed methodology for the detection of actuator faults in a class of linear multi-agent systems in the presence of disturbances. A cascade of fixed-time observers is introduced to give an exact estimate of the global system state for each agent whereby the convergence time is estimated regardless of the initial conditions. Distributed observers are then designed using Linear Matrix Inequalities (LMI), by employing the $\mathcal{H}_-$ and $\mathcal{H}_\infty$ norms. The proposed method ensures global actuator fault detection, where each agent is capable of detecting not only its faults but also those that occur at any other part of the system. Numerical simulation results are carried out to show the effectiveness of the proposed approach.

Keywords: State reconstruction, Multi-agent system, Fixed-time observer, Fault Detection.

1 Introduction

In recent years, the issue of global fault detection in multi-agent systems (MAS) have become the subject of intensive and widespread research due to their potential use. They are present in various applications ranging from the flocking of mobile vehicles, formation control in spacecraft flights to unmanned aerial vehicles, to mention a few (See Cao et al. [2013], Gómez-Gutiérrez et al. [2015]). On the other hand, many approaches have been proposed for the design of fault detection and isolation (FDI) filters that can identify the presence of a fault in a dynamical system before it leads to a potentially catastrophic failure, e.g., Zhang and Jiang [2008].

Most of the available literature on model-based global actuator FD (i.e., faults at any part of the system) for multi-agent systems focuses, to a large extent, on centralized approaches whereby the fault detection module has access to all available measurements (see Chen and Patton [2012], Ding et al. [2008], Menon and Edwards [2014]), which is often impossible in real-life applications. Some works also focus on distributed filters, which depend on relative information (See Stanković et al. [2010], Nguyen et al. [2017]). In these works, on the one hand, only the faults of the agent and its neighbors can be detected by an agent, on the other hand, inputs are assumed to be exchangeable, which is not always feasible, namely in sensible cooperative operations. In the above-mentioned cooperative control tasks, it is crucial to consider both the dynamics of the agent involved and the dynamics of the formation to ensure that the overall group goals are met by adapting control for recovery.

It is well known that there are issues that can affect MAS performance. Some of them are faults occurring at unknown times and the fact that the FDI information is not always trusted locally due to possible communication malicious attacks (e.g., [Teixeira et al. 2010]). Those problems may compromise the FD and, eventually, the reconfiguration scheme. Therefore, agents need to overcome these mentioned issues for guaranteeing the security of the whole MAS, particularly those that accomplish sensitive operations such as acting as monitoring nodes to the dynamics of the MAS and must synchronously achieve fault detection and identification.

This paper proposes a distributed methodology for the detection of actuator faults in a class of linear MAS with unknown disturbances, aiming to consider those mentioned problems of fault detection and identification for a class of systems. The approach proposed is based on the results of Zuo et al. [2018] whereby distributed observers for high-order integrator multi-agent systems are designed to estimate the leader state. More specifically, we extend the work to the problem of distributed state estimation for all agents with disturbances. A global model based on these estimates is then used for each agent to detect actuator faults for each agent in a distributed manner. The FDI problem investigated here is built upon observer-based residual generators using sensitivity techniques. Our approach relies on a mixed $\mathcal{H}_-/\mathcal{H}_\infty$ indexes Ding et al. [2008]. This method combines, on the one hand, the $\mathcal{H}_\infty$ constraint that describes the robustness of residuals to disturbances, and the minimum singular value $\mathcal{H}_-$ index, on the other, that maximizes the minimum effect of faults on the residuals. The combination of the two achieves the main objective of robust FD.

The main features of the proposed scheme are highlighted in the following. The formulation of the distributed actuator FDI problem for a class of linear multi-agent systems with disturbances is performed through the use of a cascade of fixed-time sliding mode observers, where each agent having access to their state, can complete the global state by concatenating the estimated states obtained from the observer. An LMI-based approach is applied to design distributed global residual signals at each agent, based on mixed $\mathcal{H}_-/\mathcal{H}_\infty$ norms. The above-combined approaches allow treating the actuator fault detection problem, albeit keeping the distributed design, as information obtained by each interacting agent only comes from its neighbours.

Notation : For any non-negative real number α the function $x \rightarrow \lceil x \rfloor^\alpha$ is defined as $\lceil x \rfloor^\alpha = |x|^\alpha \ \mathrm{sign}(x)$ for any $x \in \mathbb{R}$. We define $\lceil x \rfloor^\alpha = [\mathrm{sign}(x_1)\lceil x_1 \rfloor^\alpha, \mathrm{sign}(x_1)\lceil x_2 \rfloor^\alpha, ..., \mathrm{sign}(x_N)\lceil x_N \rfloor^\alpha]^T$, where $x = [x_1, x_2, ..., x_N]^T \in \mathbb{R}^N$. $\lambda_{min}([\cdot])$ represents the smallest eigenvalue of square matrix $[\cdot]$, and $\lambda_{max}([\cdot])$ the largest one. $||(\cdot)||_n$ denotes the $n-$norm of vector $(\cdot)$. Throughout this paper, for a real matrix $P \in \mathbb{R}^{n\times n}$, $P > 0$ means that P is symmetric and positive-definite. For an arbitrarily real matrix X and two real symmetric matrices Y and Z, the matrix: $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$ is a real symmetric matrix, where $*$ implies symmetry.

2 Problem Statement

Consider N agents interacting to achieve a common objective, indexed $k = 1, 2, \dots, N$. The dynamics of the k^{th} agent are given by:

$$\begin{cases} \dot{x}_k(t) = Ax_k(t) + B_u(u_k(x) + f_k(t)) + B_e\xi_k(t) \\ y_k(t) = Cx_k(t) \end{cases} \tag{1}$$

where $x_k = [x_{k,1}, x_{k,2}, ..., x_{k,n}]^T \in \mathbb{R}^n$ and $u_k(x) \in \mathbb{R}$ are the state vector and the control input respectively of the k^{th} agent. Note that the control and FDI problems are independent and $u_k(x) = P_k(x_k, \bar{x}_j)$, where P_k is the control protocol of agent k and is a function of its state along with information received by their neighbours, which might be their state, their estimates of the global state or leader's state, depending on the control problem at hand. $\xi_k \in \mathbb{R}$ represents the disturbances and is an unknown bounded scalar. $f_k \in \mathbb{R}$ represents the fault signal where $f_k = 0$ is equivalent to a fault-free system and $f_k \neq 0$ indicates the occurrence of an actuator fault in agent k. Here, it is assumed that agent k can measure all of its state. The state matrices A and B_u are expressed as:

$$A = \begin{bmatrix} 0 & a_{1,2} & a_{1,3} & \dots & a_{1,n} \\ 0 & 0 & a_{2,3} & \dots & a_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & a_{n-1,n} \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \in \mathbb{R}^{n\times n}, \quad B_u = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathbb{R}^n$$

$y_k(t) \in \mathbb{R}^n$ is the measured output of agent i, with $C = I \in \mathbb{R}^{n\times n}$. Each agent receives its neighbours' estimates of other agents as well as its own measurements.

Assumption 1. It is assumed that $B_e \in \mathbb{R}^n$ is a known matrix with $rank(B_e, B_u) \neq 0$ and $[\zeta_{k,1}, \zeta_{k,2}, ..., \zeta_{k,n}]^T = B_e\xi_k$.

Let us denote $\mathcal{G} = (\varrho, \epsilon)$ the representation of the topology composed of N agents, $\varrho = \{1, ..., N\}$ is the node set consisting of N nodes, and $\epsilon \subseteq \varrho \times \varrho$ is the fixed edge set. $\mathbb{J}_i \subset \{1, ..., N\}\backslash\{i\}$ is the non-empty subset of agents that agent i can sense and interact with. The adjacency matrix $\Upsilon = [\upsilon_{ij}] \in \mathbb{R}^{N\times N}$ is defined by $\upsilon_{ij} > 0$ when the i^{th} agent can receive information from the j^{th} agent and $\upsilon_{ij} = 0$ otherwise. Let D be the degree diagonal matrix, with $d_i = \sum_{j=1}^N \upsilon_{ij}$. The Laplacian matrix $\mathcal{L}$ is defined as:

$$\mathcal{L} = D - \Upsilon \quad \in \mathbb{R}^{N\times N} \tag{2}$$

Let $\mathcal{L}_k \in \mathbb{R}^{(N-1)\times(N-1)}$ be the Laplacian matrix defined without agent k, and $\mathcal{L}^k = diag(\ell_1^k, \dots, \ell_{k-1}^k, \ell_{k+1}^k, \dots, \ell_N^k)$ $\in \mathbb{R}^{(N-1)\times(N-1)}$ be the associated diagonal matrix defining the interconnections between agent k and the remaining agents, $\ell_i^k > 0$ if the information of agent k is accessible by the i^{th} agent, otherwise $\ell_i^k = 0$.

Assumption 2. In this paper, graph $\mathcal{G}$ is considered to be undirected and connected (Fax and Murray [2004]), and self-connections are not allowed (i.e., the diagonal of matrix Υ is null).

The purpose of this paper is that each agent is able to detect an actuator fault in the entire network of agents with disturbances solely by using information exchanged between neighbouring agents.

3 Distributed global actuator fault-detection scheme

The distributed FD scheme is done through the use of a cascade of fixed-time observers to reconstruct the global state of the overall system.

3.1 State reconstruction

The observer proposed here, for each agent i is able to obtain an estimate of all other agents in the network. Denoting by $\hat{x}_{k,m}^i$, agent i's estimate of the m^{th} state variable of agent k and by $x_{k,m}$ the m^{th} measured state variable of agent k. For all $i = 1, \dots, N$, $k = 1, \dots, N$, $k \neq i$ the proposed distributed fixed-time observer has the following structure:

$$\begin{aligned} \dot{\hat{x}}_{k,m}^i &= a_{m,m+1}\hat{x}_{k,m+1}^i + \dots + a_{m,n}\hat{x}_{k,n}^i \\ &+ \alpha_m^k \mathrm{sign}\Big(\sum_{j=1}^N \upsilon_{ij}(\hat{x}_{k,m}^j - \hat{x}_{k,m}^i) + \ell_i^k(x_{k,m} - \hat{x}_{k,m}^i)\Big) \\ &+ \eta_m^k \Big\lceil \sum_{j=1}^N \upsilon_{ij}(\hat{x}_{k,m}^j - \hat{x}_{k,m}^i) + \ell_i^k(x_{k,m} - \hat{x}_{k,m}^i)\Big\rfloor^2, \\ m &= \{1, 2, ..., n-1\} \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{\hat{x}}_{k,n}^i &= \alpha_n^k \mathrm{sign}\Big(\sum_{j=1}^N \upsilon_{ij}(\hat{x}_{k,n}^j - \hat{x}_{k,n}^i) + \ell_i^k(x_{k,n} - \hat{x}_{k,n}^i)\Big) \\ &+ \eta_n^k \Big\lceil \sum_{j=1}^N \upsilon_{ij}(\hat{x}_{k,n}^j - \hat{x}_{k,n}^i) + \ell_i^k(x_{k,n} - \hat{x}_{k,n}^i)\Big\rfloor^2 \end{aligned}$$

The error is defined by:

$$\varepsilon_{k,m}^i = \hat{x}_{k,m}^i - x_{k,m} \tag{4}$$

Differentiating equation (4) yields the dynamics of the observation error:

$$\dot{\varepsilon}^i_{k,m} = a_{m,m+1}\varepsilon^i_{k,m+1} + \ldots + a_{m,n}\varepsilon^i_{k,n} + \alpha^k_m \text{sign}\Big(\sum_{j=1}^{N} \upsilon_{ij}(\varepsilon^j_{k,m} - \varepsilon^i_{k,m}) - \ell^k_i \varepsilon^i_{k,m}\Big) + \eta^k_m \Big\lceil \sum_{j=1}^{N} \upsilon_{ij}(\varepsilon^j_{k,m} - \varepsilon^i_{k,m}) - \ell^k_i \varepsilon^i_{k,m}\Big\rfloor^2 - \zeta_{k,m}, \quad m = \{1, 2, ..., n-1\} \tag{5}$$

$$\dot{\varepsilon}^i_{k,n} = \alpha^k_n \text{sign}\Big(\sum_{j=1}^{N} \upsilon_{ij}(\varepsilon^j_{k,n} - \varepsilon^i_{k,n}) - \ell^k_i \varepsilon^i_{k,n}\Big) + \eta^k_n \Big\lceil \sum_{j=1}^{N} \upsilon_{ij}(\varepsilon^j_{k,n} - \varepsilon^i_{k,n}) - \ell^k_i \varepsilon^i_{k,n}\Big\rfloor^2 - U_k$$

with $U_k = u_k + \zeta_{k,n} + f_k$.

Assumption 3. It is assumed that inputs u_k of each agent, disturbances $\zeta_{k,m}$ with $m = 1, 2, ..., n$ and faults f_k are bounded by known constants such that $|u_k| \leqslant u_{max}$, $|\zeta_{k,m}| \leqslant \zeta_{max}$ and $|f_k| \leqslant f_{max}$ respectively with $u_{max}, \zeta_{max}, f_{max} \in \mathbb{R}^+$, and $U_{max} = u_{max} + \zeta_{max} + f_{max}$.

Equations (5) can be rewritten in a more compact form:

$$\dot{X}^k_m = a_{m,m+1}X^k_{m+1} + \ldots + a_{m,n}X^k_n - \alpha^k_m \text{sign}((\mathcal{L}_k + \mathcal{L}^k)X^k_m) - \eta^k_m \lceil(\mathcal{L}_k + \mathcal{L}^k)X^k_m\rfloor^2 - \mathbf{1}\xi_{k,m}, \qquad m = \{1, 2, ..., n-1\} \tag{6}$$

$$\dot{X}^k_n = -\alpha^k_n \text{sign}((\mathcal{L}_k + \mathcal{L}^k)X^k_n) - \eta^k_n \lceil(\mathcal{L}_k + \mathcal{L}^k)X^k_n\rfloor^2 - \mathbf{1}U_k$$

Each agent k concatenates the estimation errors in the vector X^k_m i.e. $X^k_m = [\varepsilon^1_{k,m}, \varepsilon^2_{k,m}, ..., \varepsilon^N_{k,m}]^T$. $\mathbf{1}$ denotes a vector containing ones. The fixed-time convergence property of the estimation errors is summarized in the following theorem:

Theorem 1. Considering Assumptions 1-3 are satisfied with $k = 1, 2, ..., N$, the observer gains are expressed as:

$$\eta^k_m = \frac{\sigma^k \sqrt{N}}{(2\lambda_{min}(\mathcal{L}_k + \mathcal{L}^k))^{\frac{3}{2}}} \qquad \forall m = 1, 2, ..., n \tag{7}$$

$$\alpha^k_m = \zeta_{max} + \sigma^k \sqrt{\frac{\lambda_{max}(\mathcal{L}_k + \mathcal{L}^k)}{2\lambda_{min}(\mathcal{L}_k + \mathcal{L}^k)}} \quad \forall m = 1, 2, ..., n-1 \tag{8}$$

$$\alpha^k_n = U_{max} + \sigma^k \sqrt{\frac{\lambda_{max}(\mathcal{L}_k + \mathcal{L}^k)}{2\lambda_{min}(\mathcal{L}_k + \mathcal{L}^k)}} \tag{9}$$

The distributed observer described by Eq. (3) achieves the convergence of the observation errors to zero in a finite time, where $\sigma^k > 0$, and this time is bounded by:

$$T^k_o := \frac{n\pi}{\sigma^k} \tag{10}$$

Proof 1. The proof consists of showing the fixed-time stability of (6) in a recursive manner. For that, consider the following Lyapunov function associated with the n^{th} dynamics of the agents:

$$V^k_n = \frac{1}{2}(X^k_n)^T(\mathcal{L}_k + \mathcal{L}^k)(X^k_n) \tag{11}$$

Differentiating (11) results in:

$$\begin{aligned}\dot{V}^k_n &= (X^k_n)^T(\mathcal{L}_k + \mathcal{L}^k) \\ &\times (-\alpha^k_n \text{sign}((\mathcal{L}_k + \mathcal{L}^k)X^k_n) - \eta^k_n \lceil(\mathcal{L}_k + \mathcal{L}^k)X^k_n\rfloor^2) \\ &- (X^k_n)^T(\mathcal{L}_k + \mathcal{L}^k)\mathbf{1}U_k \\ &\leqslant -(\alpha^k_n - U_{max})||(\mathcal{L}_k + \mathcal{L}^k)X^k_n||_1 \\ &- \eta^k_n N^{-\frac{1}{2}}(2\lambda_{min}(\mathcal{L}_k + \mathcal{L}^k))^{\frac{3}{2}}(V^k_n)^{\frac{3}{2}} \\ &\leqslant -\sigma^k(V^k_n)^{\frac{1}{2}} - \sigma^k(V^k_n)^{\frac{3}{2}}\end{aligned} \tag{12}$$

This guarantees that X^k_n containing the n^{th} state variables is fixed-time stable at the origin with the settling time bounded by $T^k_1 = \frac{\pi}{\sigma^k}$, the dynamics of X^k_{n-1} are written in the form (13) after the convergence of X^k_n.

$$\dot{X}^k_{n-1} = -\alpha^k_{n-1}\text{sign}((\mathcal{L}_k + \mathcal{L}^k)X^k_{n-1}) - \eta^k_{n-1}\lceil(\mathcal{L}_k + \mathcal{L}^k)X^k_{n-1}\rfloor^2 - \zeta_{k,n-1} \tag{13}$$

Similarly, we have :

$$\begin{aligned}\dot{V}^k_{n-1} &\leqslant -(\alpha^k_{n-1} - \zeta_{max})||(\mathcal{L}_k + \mathcal{L}^k)X^k_{n-1}||_1 \\ &- \eta^k_{n-1}N^{-\frac{1}{2}}(2\lambda_{min}(\mathcal{L}_k + \mathcal{L}^k))^{\frac{3}{2}}(V^k_{n-1})^{\frac{3}{2}} \\ &\leqslant -\sigma^k(V^k_{n-1})^{\frac{1}{2}} - \sigma^k(V^k_{n-1})^{\frac{3}{2}}\end{aligned}$$

X^k_{n-1} converges to 0 in a fixed time bounded by $T^k_2 = 2T^k_1$. Recursively, the dynamics of X^k_1 reduce to:

$$\dot{X}^k_1 = -\alpha^k_1 \text{sign}((\mathcal{L}_k + \mathcal{L}^k)X^k_1) - \eta^k_1\lceil(\mathcal{L}_k + \mathcal{L}^k)X^k_1\rfloor^2 - \zeta_{k,1}$$

Following the same reasoning, X^k_1 converges to 0 within a fixed-time horizon bounded by $T^k_o := T^k_n = nT^k_1$. This concludes the proof of Theorem 1.

Theorem 1 guarantees that this observer could recover the agent k's state within a fixed time by the remaining agents if Assumptions 1-3 are satisfied. Therefore, it is safe to use $\hat{x}^i_{k,m}$ in the residual generation and evaluation. One could note that the settling estimation time T^k_o, obtained in Theorem 1 is independent of the initial observation errors for each corresponding agent.

Remark 1. It is worth noting similarly to the reasoning in [Menard et al. 2017] where the effect of measurement noise and disturbances on errors are analysed for fixed-time observers, it can be shown that in the presence of measurement noise, the estimation errors (4) due to the disturbances decrease as α^k_m, α^k_n and η^k_m increase, but at the same time increase due to noise as these parameters increase. Hence, through a reasonable choice of the settling time a good compromise between robustness and a sufficiently fast estimation can be achieved.

3.2 Residual generation and fault detection

In the following, an LMI-based approach is used to design a distributed actuator FD scheme residual generator based on mixed $\mathcal{H}_-/\mathcal{H}_\infty$ norms. The use of this approach rather than the one that involves the reconstruction of the disturbances and faults for this system is supported by the fact that the fault detection scheme is less computationally demanding and the gains are only computed once. Furthermore, its performance is less dependant on the sampling period. The approximation of the equivalent information injections by low pass filters at each step may also introduce some delays that could lead to instability for high order systems. Using the proposed distributed observers, each agent can complete the global state by concatenating the estimated states obtained from the observer. For an agent k, the corresponding global state vector can be expressed as $X_G^k = [(\hat{x}_k^1)^T, (\hat{x}_k^2)^T, \ldots, (\hat{x}_k^N)^T]^T \in \mathbb{R}^{(n \cdot N)}$, where $\hat{x}_k^i$ is the agent i's estimate of the state of agent k. When $i = k$, $\hat{x}_k^i = x_k^i$, since each agent has access to their own measurements. Using (1), the associated global state space representation can be written in the following form:

$$\begin{cases} \dot{X}_G^k(t) = \bar{A}^k X_G^k(t) + \bar{B}_u^k(\hat{U}_G^k(t) + F_G^k(t)) + \bar{B}_e^k E_G^k(t) \\ Y^k(t) = X_G^k(t) \end{cases} \tag{14}$$

where $E_G^k = [\xi_1, \xi_2, ..., \xi_N]^T$ and $F_G^k = [f_1, f_2, ..., f_N]^T$. $\hat{U}_G^k(\hat{x}(t)) = [\hat{u}_1, \hat{u}_2, ..., \hat{u}_N]^T$ is the reconstructed input, where $\hat{u}_k = u_k$ since each agent has access to its own control input. Matrices $\bar{A}^k$, $\bar{B}_u^k$, $\bar{B}_e^k$ and C^k are defined by block diagonal notation $\bar{Z}^k = diag(Z^1, Z^2, \ldots, Z^N)$. Differentiating $Y^k(t)$ in (14) yields to:

$$\dot{Y}^k(t) = \bar{A}^k X_G^k(t) + \bar{B}_u^k(\hat{U}_G^k(t) + F_G^k(t)) + \bar{B}_e^k E_G^k(t) \tag{15}$$

Let us denote by $Y_G^k(t)$ the new output described as:

$$Y_G^k(t) = \dot{Y}^k(t) - \bar{B}_u^k \hat{U}_G^k(t) \tag{16}$$

Equations (14) can be re-written as:

$$\begin{cases} \dot{X}_G^k(t) = \bar{A}^k X_G^k(t) + \bar{B}_u^k(\hat{U}_G^k(t) + F_G^k(t)) + \bar{B}_e^k E_G^k(t) \\ Y_G^k(t) = \bar{C}^k X_G^k(t) + \bar{D}_u^k F_G^k(t) + \bar{D}_e^k E_G^k(t) \end{cases} \tag{17}$$

with $\bar{C}^k = \bar{A}^k$, $\bar{D}_u^k = \bar{B}_u^k$ and $\bar{D}_e^k = \bar{B}_e^k$.

Remark 2. It should be also highlighted that $\dot{Y}^k(t)$ and thus $Y_G^k(t)$, can easily be obtained using a fixed-time non-recursive differentiator.

From system (17) and Remark 2, it is clear that one can apply a mixed $\mathcal{H}_-/\mathcal{H}_\infty$ norm optimization technique. A classical FD scheme is comprised of a residual generator and a residual evaluation process. To design the residual generator, the following fault detection observer is used:

$$\dot{\hat{X}}_G^k(t) = \bar{A}^k \hat{X}_G^k(t) + \bar{B}_u^k \hat{U}_G(t) + H^k(Y_G^k(t) - \hat{Y}_G^k(t)) \tag{18}$$

$$\hat{Y}_G^k(t) = \bar{C}^k \hat{X}_G^k(t) \tag{19}$$

$$r^k(t) = V^k(Y_G^k(t) - \hat{Y}_G^k(t)) \tag{20}$$

where for each agent k, $\hat{X}_G^k \in \mathbb{R}^{(n \cdot N)}$ and $\hat{Y}_G^k \in \mathbb{R}^{(n \cdot N)}$ represent, respectively, the state and output estimation vectors. $r^k \in \mathbb{R}^{(n \cdot N)}$ is the residual signal vector. $H^k \in \mathbb{R}^{(n \cdot N) \times (n \cdot N)}$ and $V^k \in \mathbb{R}^{(n \cdot N) \times (n \cdot N)}$ are the gain matrices representing the observer gain and the residual post filter weight, respectively. Considering the residual error $e^k(t) = X_G^k - \hat{X}_G^k$, one can write:

$$\dot{e}^k(t) = \underline{A}^k e^k(t) + \underline{B}_e^k E_G^k(t) + \underline{B}_u^k F_G^k(t) \tag{21}$$

$$r^k(t) = V^k(\bar{C}^k e^k(t) + \bar{D}_e^k E_G^k(t) + \bar{D}_u^k F_G^k(t)) \tag{22}$$

with $\underline{A}^k = (\bar{A}^k - H^k \bar{C}^k)$, $\underline{B}_e^k = (\bar{B}_e^k - H^k \bar{D}_e^k)$ and $\underline{B}_u^k = (\bar{B}_u^k - H^k \bar{D}_u^k)$. In the frequency domain, (22) can be written as:

$$r^k(s) = T_d^k(s) E_G^k(s) + T_f^k(s) F_G^k(s) \tag{23}$$

with $T_d^k(s) = V^k \bar{C}^k (sI - \underline{A}^k)^{-1} \underline{B}_e^k + V^k \bar{D}_e^k$ and $T_f^k(s) = V^k \bar{C}^k (sI - \underline{A}^k)^{-1} \underline{B}_u^k + V^k \bar{D}_u^k$. In this paper, we use the $\mathcal{H}_\infty$ norm to measure the maximum effect of the disturbances on the residual and the $\mathcal{H}_-$ index to measure the minimum effect of the fault on the residual. For system (17), designing the fault detection observer (18)-(20) is equivalent to find matrices H^k and V^k, using the combined $\mathcal{H}_-/\mathcal{H}_\infty$ strategy to guarantee sensitivity of the residuals to the faults and robustness against perturbation. Therefore, the objective is to minimize the following:

$$\min J^k = \min \frac{||T_d^k(s)||_\infty}{||T_f^k(s)||_-} \tag{24}$$

where J^k represents a trade-off between sensitivity and robustness. In this section, the approach in Wang et al. [2007] is used. For given $\gamma_k > 0$ and $\beta_k > 0$, the error system (20)-(21) is asymptotically stable and the following are satisfied:

$$||T_d^k(s)||_\infty < \gamma_k \tag{25}$$

$$||T_f^k(s)||_- > \beta_k \tag{26}$$

if there exist matrices $P^k > 0$, $Q^k > 0$, $M^k > 0$ and H^k such that the following LMIs hold:

$$\begin{bmatrix} P^k \underline{A}^k + (\underline{A}^k)^T P^k + (\bar{C}^k)^T M^k \bar{C}^k & P^k \underline{B}_e^k + (\bar{C}^k)^T M^k \bar{D}_e^k \\ * & -\gamma_k^2 I + (\bar{D}_e^k)^T M^k \bar{D}_e^k \end{bmatrix} < 0 \tag{27}$$

$$\begin{bmatrix} Q^k \underline{A}^k + (\underline{A}^k)^T Q^k - (\bar{C}^k)^T M^k \bar{C}^k & (\bar{C}^k)^T M^k \bar{D}_u^k - P^k \underline{B}_u^k \\ * & \beta_k^2 I - (\bar{D}_u^k)^T M^k \bar{D}_u^k \end{bmatrix} < 0 \tag{28}$$

with $M^k = (V^k)^T V^k$.

Remark 3. Due to the structure of the proposed scheme, a consensus on the occurrence of a fault is naturally required for the fault to be detected (when all agents agree on its occurrence). This also serves to discern external malicious attacks in the network, while a node exchanges the reconstructed state, the latter contains the original data transferred from its neighbours and that can be a reference for them to confirm the validity of the data/detection process. Furthermore, synchronization of the isolation step is assured, the proposed observer allows the agents to avoid transmission delays and thus trigger recovery decisions in a synchronous manner, allowing for more effective corrective measures to be taken.

3.2.1 Residual evaluation The remaining task for fault detection and isolation (identifying the faulty agent in the fleet) is to evaluate the obtained residuals, when the convergence of the estimation errors is obtained (which depends on the parameters of the cascade of fixed-time observers as well as those of the fixed-time differentiator). The residuals are evaluated after this settling time, by comparing the generated residuals with a threshold defined hereafter. The selected evaluation function $J_i^k(t)$ (see Ding [2008]) is expressed as follows:

$$J_i^k(t) = ||r_i^k(t)|| = \left[(r_i^k)^T (r_i^k) \right]^{\frac{1}{2}} \tag{29}$$

Let us denote by $J_{i_{th}}^k = \sup_{faultfree} ||J_i^k(t)||_\infty$ the threshold. The following fault detection logic is used as a decision logic:

$$J_i^k(t) > J_{i_{th}}^k \Rightarrow \text{Fault detected,} \tag{30}$$

$$J_i^k(t) \leqslant J_{i_{th}}^k \Rightarrow \text{No fault detected.} \tag{31}$$

4 Illustrative Example

In this section, an illustrative numerical example is provided to show the effectiveness of the proposed scheme. Consider a team of 4 agents governed by double integrator dynamics with unknown disturbances ($n = 2$), a special case of system (1), where: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B_e = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$, $B_u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The communication among each agent is given according to the topology in Fig. 1. The associated

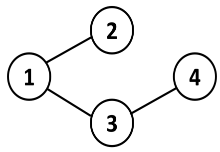

Fig. 1. Communication topology.

interconnection matrices as specified by the given topology are : $\mathcal{L}^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\mathcal{L}^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\mathcal{L}^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\mathcal{L}^4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\mathcal{L}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$, $\mathcal{L}_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$, $\mathcal{L}_3 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\mathcal{L}_4 = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$.

In this example, a consensus algorithm in the presence of a group reference velocity is used for each agent (with the velocity reference set to $v^d = 10m/s$ for each agent).

$$u_i = \dot{v}^d - \kappa(v^d - x_{i,2}) - \sum_{j=1}^{N} a_{ij}[(x_{i,1} - x_{j,1}) - \nu(x_{i,2} - x_{j,2})]$$

where κ and ν are the consensus gains set to 5 and 2.5 respectively. To check the robustness of the proposed scheme, two types of perturbations are considered: band-limited Gaussian white noise of power 0.005 for agents 2 and 4, and high frequency noise for agents 1 and 3 that are modelled as follows: $\xi_1(t) = 3.5\sin(150t)$ and $\xi_3(t) = (2.5\sin(350t))^{0.2}$. In order to show the effectiveness of our approach, multiple types of abrupt faults are considered, a ramp $f_1(t) = t - 5$ for $t \in [5, 15]$, two rectangular faults $f_2(t)$ and $f_4(t)$, and an exponential fault $f_3(t) = -e^{1.3-0.1/(t-10)}$ are added at various instants, and with different amplitudes. They are assumed to occur in agents 1, 2, 3 and 4 respectively as shown in Fig. 2. Using these matrices and by choosing $T_o^1 = T_o^2 = T_o^3 = T_o^4 = 0.5\,s$, from Theorem 1, the resulting σ^1, σ^2, σ^3 and σ^4 allow us to define the gains of the cascade observers expressed in Eq. (7)-(9):

$$\begin{cases} \alpha_1^1 = 23.45, & \alpha_2^1 = 38.45, & \eta_{1,2}^1 = 14.01. \\ \alpha_1^2 = 35.16, & \alpha_2^2 = 50.16, & \eta_{1,2}^2 = 45.30. \\ \alpha_1^3 = 23.45, & \alpha_2^3 = 38.45, & \eta_{1,2}^3 = 14.01. \\ \alpha_1^4 = 35.16, & \alpha_2^4 = 50.16, & \eta_{1,2}^4 = 45.30. \end{cases}$$

Figure 3 shows the evaluation functions $J_i^1(t)$, $J_i^2(t)$, $J_i^3(t)$ and $J_i^4(t)$ of the residual vectors generated by agent 1, 2, 3 and 4 respectively. It is worth noting that these functions make sense only after the observers and differentiators converge. This is achieved after about 1.5 seconds (dashed

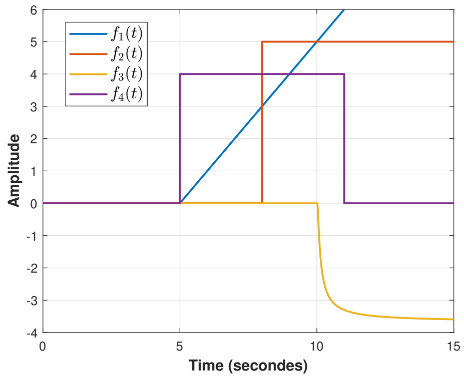

Fig. 2. Fault signals in agent 1, 2, 3 and 4.

blue vertical lines are added as delimiters in order to illustrate this). Each agent generates four evaluation functions, one from its own output and three from the estimates of the state of the three other agents. Each of these functions is capable of detecting the corresponding agent's fault from the point of view of the agent that generates it. Fig. 4(b) for example displays the functions $J_1^2(t)$, $J_2^2(t)$, $J_3^2(t)$ and $J_4^2(t)$ that detect faults occurring in agents 1, 2, 3 and 4 respectively from the point of view of agent 2. It is shown that, even though agent 2 does not directly communicate with neither agent 3 and 4, it uses their state estimates to synchronise fault detection. The functions are robust with respect to disturbances and faults can be easily distinguished. In accordance with this logic the same can be said for the other agents as the same pattern for fault detection is followed. These results show that our approach is useful for the problem of synchronised distributed actuator FD in a network of communicating multi-agent systems, where each agent can detect faults occurring across the entirety of the system and isolate the faulty agent.

5 Conclusion

In this paper, the problem of robust distributed fault detection in a class of linear multi-agent systems with unknown disturbances has been investigated. The problem was solved using a distributed scheme. This scheme was done by using a cascade of fixed-time observers for each agent to give an exact estimate of the global system state whereby the convergence time is estimated regardless of the initial conditions. An LMI-based residual generating observer was then used, by establishing feasibility conditions satisfying mixed $\mathcal{H}_-/\mathcal{H}_\infty$ constraints when convergence conditions of the fixed-time observers are obtained. The efficacy of our proposed scheme has been shown through an illustrative numerical example.

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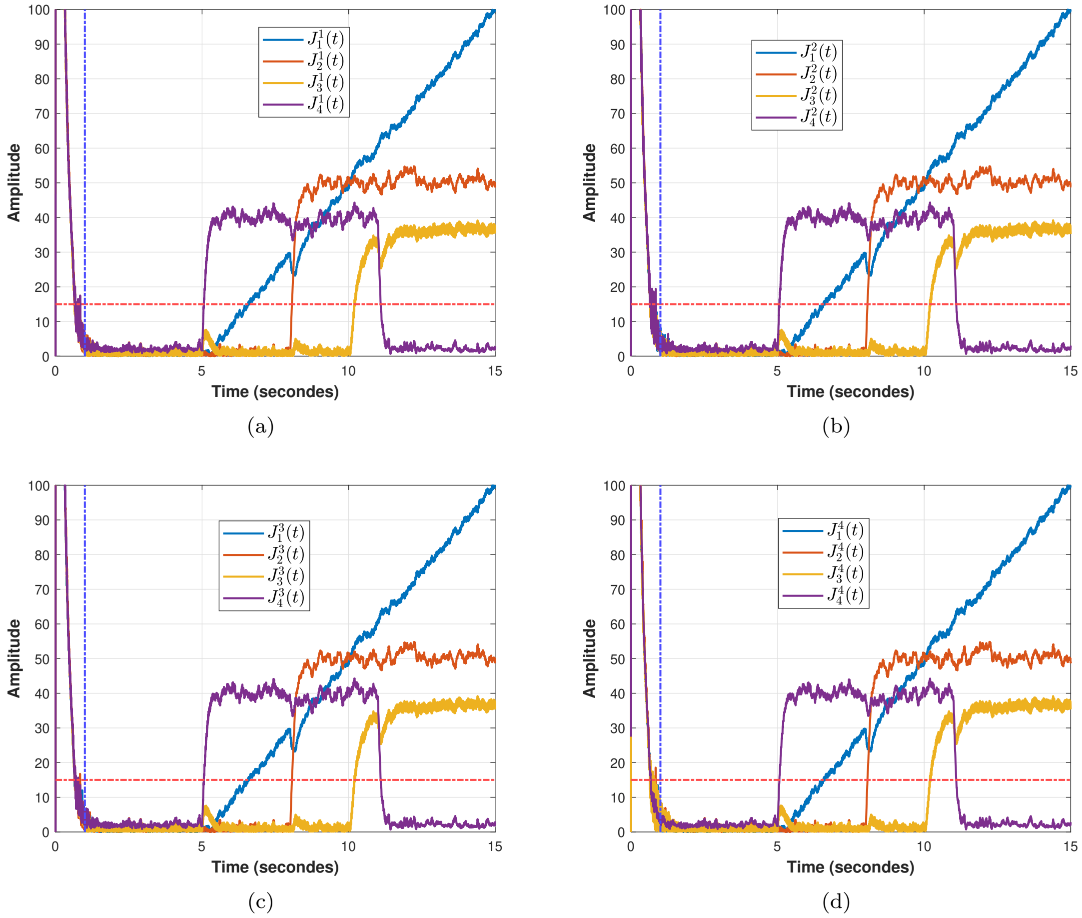

Fig. 3. Residual evaluation functions: (a) $J_i^1(t)$ of agent 1 (b) $J_i^2(t)$ of agent 2 (c) $J_i^3(t)$ of agent 3 (d) $J_i^4(t)$ of agent 4. The dashed blue lines represent the convergence time after which the functions should be considered. The dashed red lines represent the thresholds.

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