Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Wave energy control: status and perspectives 2020 ⋆

John V. Ringwood *

* *Centre for Ocean Energy Research, Maynooth University, Ireland (e-mail: john.ringwood@mu.ie)*

Abstract: Wave energy has a significant part to play in providing a carbon-free solution to the world's increasing appetite for energy. In many countries, there is sufficient wave energy to cater for the entire national demand, and wave energy also has some attractive features in being relatively uncorrelated with wind, solar and tidal energy, easing the renewable energy dispatch problem. However, wave energy has not yet reached commercial viability, despite the first device designs being proposed in 1898. Control technology can play a major part in the drive for economic viability of wave energy and this paper charts the progress made since the first wave energy control systems were suggested in the 1970s, and examines current outstanding challenges for the control community.

Keywords: Wave energy, modelling, control, estimation, forecasting, sensitivity, robust control, nonlinearity

1. INTRODUCTION

The first patented wave energy converter appeared in 1898 (McCormick, 2013), yet wave energy has still to make a commercial impact. A cursory comparison with wind energy (Ringwood and Simani, 2015), which has achieved considerable commercial success, both onshore and offshore, is revealing:

(1) In wind energy, the energy flux is unidirectional, but bi-directional in wave energy.
(2) In wind energy, there is a single primary resource variable, wind speed, while waves are described by both amplitude and period.
(3) Energy can be relatively easily spilled by pitch/yaw control in wind turbines, while it is difficult to spill energy for most wave energy converters (WECs), and therefore retain power below the rated value for the power take-off system. This difficulty leads to typically poor capacity factor values for wave devices.
(4) The energy density is considerably greater in the sea than in the air, leading to greater survivability demands, with associated large capital costs.

Item (3) above is very much a function of the device design, but difficult to achieve with WECs. Some WECs can be partially or completely submerged (Rafiee and Fiévez, 2015), while others can significantly alter their hydrodynamic gain through the use of movable flaps (Papillon et al., 2019), and air valve throttling is also possible for oscillating waver column (OWC) devices (Henriques et al., 2016). However, many WECs must enter a survival mode, where no power is produced, beyond a certain level of wave excitation. Of course, the addition of such adaptive device features enriches the control problem by providing the control engineer with something similar to the pitch/torque control for wind turbines. An example, where a WEC was controlled to have a power curve similar to that of a pitch-controlled wind turbine, was shown by Papillon et al. (2019).

In general, however, the control problem for WECs consists of manipulation of the load force/torque on the power take-off (PTO) system, with the following objectives:

(a) Maximise captured power
(b) Ensure that physical device constraints (e.g. force, displacement) are not violated

Item (a) above is, in fact, suboptimal in the sense that maximising captured power alone may not maximise the economic return. Since the wave energy itself is free, maximising conversion 'efficiency' is not the aim, but rather to minimise the cost of converted energy. However, most control engineers focus on (a) due to the difficulty of articulating the effect of control actions on operational costs, though some studies, e.g. Nielsen et al. (2017), have taken a step in this direction. The manipulation of device torque/force to maximise captured power is also characteristic of a wind turbine in Region 2 (between cut-in speed and rated wind speed) (Ringwood and Simani, 2015).

Regarding (b) above, this represents a fairly standard constraint in many control application fields. However, it should be noted that, in the WEC case, once the wave excitation exceeds a certain level, there may be no control solution (for the PTO force) that simultaneously satisfies force and displacement constraints. This is demonstrated geometrically by Bacelli and Ringwood (2013) (see Fig.6) and may be intuitively understood by the fact that, given sufficient force, the displacement can be constrained, while the displacement must be extended if the constraining force available is small. If no viable control solution is

⋆ This paper is based upon work supported by Science Foundation Ireland under Grant no. 13/IA/1886 and Grant No. 12/RC/2302 for the Marine Renewable Ireland (MaREI) centre.

available, the PTO must be locked, or the device must enter some form of survival mode, if structural damage is to be avoided.

Finally, the achievement of (a) above needs to be considered across the entire drive train, as shown in Fig.1, taking into account the various changes in energy form and any non-ideal efficiencies in components. Furthermore, the system may contain significant nonlinearity in either the hydrodynamic model, or the PTO sections, and we note that both the incident sea state (and instantaneous free surface elevation) are largely unknown, along with the condition of any grid connection, if the system is producing electrical power. One alternative use for wave energy is the production of potable water, possibly through reverse osmosis, since pressurised fluid is naturally produced in many WEC PTO systems (Bacelli et al., 2009).

2. MATHEMATICAL MODELS

A fundamental component of any model-based control system is an accurate, validated, mathematical model. This model, or a simplified version of it, provides a foundation upon which to develop the energy-maximising control algorithm, and also provides a simulation platform upon which to evaluate the performance of the controlled system. However, care should be taken to use the highest available model fidelity for simulation, since the use of the exact same model for control design and performance evaluation is likely to lead to very predictable, but potentially misleading, results. In general, wave energy control systems do not enjoy the same intuitive appeal as tuning a setpoint-following PID feedback loop by hand, though some model-free approaches have emerged, both at the sea state level (Anderlini et al., 2016) and wave-by-wave level (Davidson et al., 2018). Ultimately, for most WECs, the performance achievable by a model-based WEC controller is heavily dependent on the fidelity, and computational performance, of the model upon which the controller is based. Therefore, considerable attention is given, in this section, to the development of mathematical WEC models.

2.1 Hydrodynamic models

A wide variety of hydrodynamic modelling approaches are available to the wave energy researcher (Penalba et al., 2017a; Davidson and Costello, 2020). At the top end of the fidelity spectrum are hydrodynamic models based on computational fluid dynamics (CFD) (Windt et al., 2018) and smoothed particle hydrodynamics (SPH) (Omidvar et al., 2012). These are computationally intensive models, implementing the Navier-Stokes equations, where the full fluid domain is discretised and, despite the application of parallel processing, can typically take around 10^3 s to compute 1 s of simulated time. In general, the application of SPH (which caters well for separated multi-phase flow) for wave energy applications is mainly focussed on extreme sea states and device survival, and is generally even more computationally demanding than CFD, though recent implementation on graphics accelerator cards can give computational performance rivaling that of CFD (Gotoh and Khayyer, 2018).

A more computationally economical hydrodynamic modelling solution is to focus on the wave/device interaction by modelling hydrodynamic effects only on the WEC hull surface, resulting in boundary element methods (BEMs), based on potential flow (Papillon et al., 2020). In general, in the consideration of devices of arbitrary shape and multiple degrees of freedom, such methods are inherently numerical, though some analytic or semi-analytic solutions for particular shapes have been derived (Havelock, 1954; Yeung, 1981).

One of the drawbacks is that there are the implicit assumptions of irrotational and inviscid flow, obviating the possibility of inclusion of viscous forces, which generally increase with relative device/fluid velocity. Typically, a Morison-like drag term:

$$F_v = \frac{1}{2}\rho C_d A v|v| \tag{1}$$

is added to potential flow models to cater for viscous drag, with ρ being the fluid density, A a reference (e.g. cross-sectional) area, and v the device relative velocity. However, the determination of C_d, the drag coefficient, is far from straightforward (Giorgi and Ringwood, 2017b).

Nevertheless, BEMs have proven very popular in wave energy applications, for a range of functions, including simulation, power production assessment and control design. BEMs offer a range of levels of modelling complexity , from linear hydrodynamics to ‘fully nonlinear’ approaches (Papillon et al., 2020). The assumptions of an incompressible and inviscid fluid lead to a set of equations for potential flow theory, which, for a velocity potential ϕ and a free-surface elevation η, lead to the Laplace equation

$$\nabla^2\phi = 0, \tag{2}$$

along with a variety of boundary conditions relating to the free surface, the seabed, the boundary of the device itself, and a far-field radiation condition. Additionally, applying the assumption of irrotational flow, Euler’s equation leads to Bernoulli’s equation, giving access, knowing the velocity potential, to the pressure

$$p = -\rho g z - \rho\frac{\partial\phi}{\partial t} - \frac{1}{2}\rho|\boldsymbol{\nabla}\phi|^2, \tag{3}$$

with z the vertical displacement. Eq. (3), via integration of the pressure, gives the resulting hydrodynamic forces $\boldsymbol{F}_{hydro}$ acting on a surface (S) of normal n

$$\boldsymbol{F}_{hydro} = -\iint_S pn ds. \tag{4}$$

Applying Newton’s second law, the equation of motion for a floating body can be expressed as

$$M\ddot{\boldsymbol{X}} = \boldsymbol{F}_{hydro} - m\boldsymbol{g} + \boldsymbol{F}_{exter}, \tag{5}$$

with m the mass of the body, M its mass matrix, g the gravity, $\boldsymbol{F}_{exter}$ represents additional external forces (due to mooring, PTO, etc.). Assuming a decomposition of the total potential into a linear superposition of the incident potential ϕ_0, a diffracted potential ϕ_D, and a radiated potential ϕ_R:

$$\phi = \phi_0 + \phi_D + \phi_R, \tag{6}$$

leads to the venerated Cummins equation:

$$\begin{aligned}(M + m_\infty)\dot{v}(t) + \int_0^{+\infty} h_r(\tau)v(t-\tau)d\tau + K_b x(t) = \\ \int_{-\infty}^{t} h_{ex}(\tau)\eta(t-\tau)d\tau.\end{aligned} \tag{7}$$

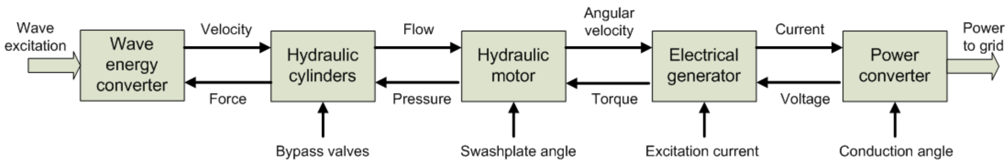

Fig. 1. Complete drive train for electricity production

where m_∞ is the added mass at infinite frequency, K_b is the restoring force stiffness, η is the free surface elevation, and $v(t) = \dot{x}(t)$. Typically, the non-parametric hydrodynamic quantities $h_{ex}(t)$ and $h_r(t)$, representing the excitation force and radiation damping dynamics respectively, are calculated numerically using linear boundary-element potential methods such as WAMIT (Lee, 1995) or NEMOH (Babarit and Delhommeau, 2015) (a comparative study was carried out by Penalba et al. (2017b)) which perform the calculations in the frequency domain, or ACHIL3D (Clément, 1999), where time-domain calculations are used.

A variety of nonlinear extensions to potential flow models have been documented by Papillon et al. (2020). These can be broadly classified as body-exact, weak-scatterer, or fully nonlinear. In the body-exact representation, the instantaneous wetted surface is computed, allowing nonlinear Froude-Krylov (NLFK) forces to be represented, while the representation of radiation forces can be linear (Gilloteaux, 2007) or nonlinear (Letournel et al., 2018). A computationally attractive solution, using an analytic solution route, is provided by Giorgi and Ringwood (2017a), but is restricted to axisymmetric devices. One advantage of these body-exact models is their ability to capture parametric resonance (Giorgi and Ringwood, 2018), which is a known phenomenon in spar-type heaving WECs.

The next level of nonlinear complexity, the weak scatterer approach (Kim et al., 2011), in addition to using the exact wetted surface, avoids the use of purely linear free surface boundary conditions. Instead, the free-surface boundary condition is linearised around the instantaneous incident wave elevation. Finally, fully nonlinear methods make no assumptions about the body motion and wave steepness, so large amplitude motion and extreme sea-states can be considered. The development of fully nonlinear models is based on the work of Longuet-Higgins and Cokelet (1976), who introduced the so-called Mixed Eulerian Lagrange (MEL) method.

Hydrodynamic models which have some nonlinear representational capability, but also a frequency domain flavour, include those of Folley and Whittaker (2010) and Mérigaud and Ringwood (2017a); their respective 'spectral' and 'harmonic balance' models are not unrelated. These models, being essentially frequency domain based, are computationally efficient, and provide a platform for simulation, power production assessment, and possibly model-based control design (for more, see Section 2.3).

One alternative to using physics-based hydrodynamic models is to determine parametric models from measured data. This can be done from both experimental data (Giorgi et al., 2019), or data generated from hi-fidelity numerical (e.g. CFD) models (Giorgi et al., 2016). Both approaches have their advantages and drawbacks; data from numerical models are only as good as the model used to generate them, while physical wave tanks have their own problems, including reflections and measurement errors. Both linear and nonlinear parametric models can be determined from data, with the data-based linear models perhaps more representative of the real operating regime (Davidson et al., 2015) than physical models based on small variations around the equilibrium point (see Section 2.4 for more on linearisation).

2.2 Power take-off and wave-to-wire models

Given the wide diversity in PTO and PTO component types, no attempt is made here to cover the plethora of PTO models. One might argue that PTOs are more straightforward to model, since they can be decomposed into component parts, and WEC PTOs usually contain components found in other application areas, which have well-established mathematical models. Nevertheless, it is essential that PTO aspects which are important to the control performance are modelled and that a fidelity balance is achieved between the hydrodynamic and PTO models, in a wave-to-wire perspective.

In general, a crucial aspect in PTO modelling is to consider non-ideal PTO efficiency. This has been studied in some depth by Hansen (2013) and one of the consequences of ignoring non-ideal efficiency at the control design stage is that average negative power production (i.e. power consumption) can result (Bacelli et al., 2015). It must also be considered that PTO efficiency ratings may differ in the forward and reverse (reactive power) directions (Hansen, 2013).

A major challenge in modelling PTO dynamics is to decide on what frequency range to consider. Compared to the hydrodynamic and mechanical dynamics, with typical time constants in the region of 0.1-15 s, some PTO dynamics corresponding to the hydraulic, electrical and electronic systems can be considerably faster, encouraging the modeller to ignore the high-frequency effects. However, ignoring fluid compressibility in hydraulic circuits can lead to the unwelcome presence of water-hammer effects, while the electrical dynamics of generators and their electronic control (power converter) circuitry are orders of magnitude faster than the mechanical/hydrodynamic WEC components. Some studies exist which have looked at the potential effects of model simplification at the control design stage, including that by Penalba and Ringwood (2018).

A number of studies have developed complete wave-to-wire models, including those by Bailey et al. (2016), Forehand et al. (2015) and Josset et al. (2007), for a range of devices. A review of wave-to-wire models is provided in (Penalba and Ringwood, 2016). One issue, characteristic of the wave energy area, and no doubt related to the wide disparity of domain knowledge needed to develop wave-to-wire WEC models, is that many studies overemphasise the fidelity of the electrical subsystem, to the detriment of the hydrodynamics, or vice versa. This imbalance no doubt reflects the principal expertise of those performing the study. Some efforts have been made to extend both hydrodynamic and PTO aspects to high fidelity, though the computational cost is significant Penalba and Ringwood (2019); Penalba et al. (2018).

2.3 Models for model-based control design

The suitability of various WEC models for control design inevitably depends, to a large extent, on the control design method. This is addressed in some detail in Section 3 but, for the moment, we can observe two broad categories of controller: (a) those based on an analytical design philosophy, and (b) those employing on-line numerical optimisation. The WEC models for (a) should ideally have a relatively simple parametric structure, while those for (b) must be computationally efficient. Clearly, CFD-based WEC hydrodynamic models cannot satisfy either of these requirements (though CFD models provide hi-fidelity evaluation capability). In their raw form, BEM-based hydrodynamic models can also be difficult to handle, since even the linear (7) represents an integro-differential equation and the quantities $h_r(t) \leftrightarrow H_r(\omega)$ and $h_{ex}(t) \leftrightarrow H_{ex}(\omega)$ are given in nonparametric form. For nonlinear BEM codes, the complexity naturally increases.

A common simplification is to replace the convolution integrals in (7) with finite-order parametric models. Approximations can be determined in either the time or frequency domain, depending on the manner in which $h_r(t) \leftrightarrow H_r(\omega)$ is determined, and the intended (time/frequency domain) use of the finite-order approximation. For example, WAMIT (Lee, 1995) and NEMOH (Babarit and Delhommeau, 2015) use frequency-domain analysis to determine $H_r(\omega)$ directly and approximations based on such data are usually based on frequency-domain error criteria. In such a case, state-space forms (Perez and Fossen, 2007) or transfer function forms (McCabe et al., 2005) may be determined using frequency-domain identification (Levy, 1959). Alternatively, if $h_r(t)$ is directly produced, for example from the time-domain code ACHIL3D (Clément, 1999), time-domain impulse-response fitting can be employed, typically using the method by Prony (1795).

A recent alternative to the above approaches is presented in (Faedo et al., 2018a). Some advantages of the moment-matching method are that the user can choose to match the frequency response *exactly* at a specific number of frequencies, the approximation is guaranteed to be passive, there is a significant complexity reduction for the case of multi-body/arrays (Y. Peña-Sanchez and Ringwood, 2019), and the loss function decreases monotonically.

For the case of nonlinear WEC models, the situation is less clear, since bespoke control solutions are necessary, though a reasonable variety of nonlinear models can be catered for within the class of MPC-like WEC controllers (Faedo et al., 2017) which employ numerical optimisation, and don't have a strong dependence on the WEC model structure. However, in such cases, the model must be computationally compact, and certain nonlinear model structures may lead to non-convex optimisation problems, with resulting uncertainty in achieving an optimal control solution.

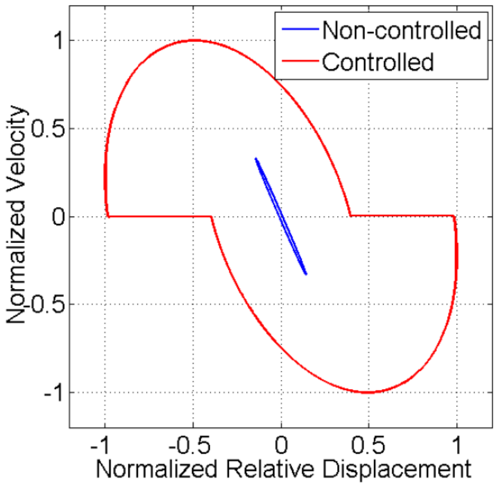

Fig. 2. Operational space covered by an uncontrolled and (latching) controlled spherical heaving buoy

2.4 Validity of linearised models

While linear WEC models are attractive for their relative simplicity, intuitive appeal and computational efficacy, considerable care needs to be taken in ensuring that linearising assumptions are valid. For traditional setpoint-tracking control systems, the control system ensures that the system operation is around the setpoint and actively tries to reduce the variance around the setpoint. In contrast, energy-maximising WEC controllers tend to exaggerate device motion, defying any assumptions of small motion around an equilibrium point. Fig.2 demonstrates the significant increase in operational envelope for a spherical heaving WEC under latching control, in regular waves. In particular, care must be taken if validating a model under uncontrolled conditions and then attempting to utilise such a model as a platform for control design. Fig.3, taken from (Giorgi et al., 2017), shows the relative fidelity (compared to a CFD 'gold standard') of a range of linear and nonlinear WEC hydrodynamic models, under both uncontrolled and controlled conditions. Perhaps unsurprisingly, all models validate well under uncontrolled conditions (blue circles), but the increase in potential NLFK forces (from increased displacement excursions) and viscous drag (due to increased relative device/fluid velocity) cause a significant fall off in fidelity, for most models, under controlled conditions (red crosses).

3. WEC CONTROL

In this paper, no attempt will be made to perform a comprehensive review of all WEC control strategies; the interested reader is referred to (Ringwood et al., 2014) and (Faedo et al., 2017) for such details. Rather, this section attempts to chart the major milestones in the development of WEC control technology and to focus on some recent developments in the area.

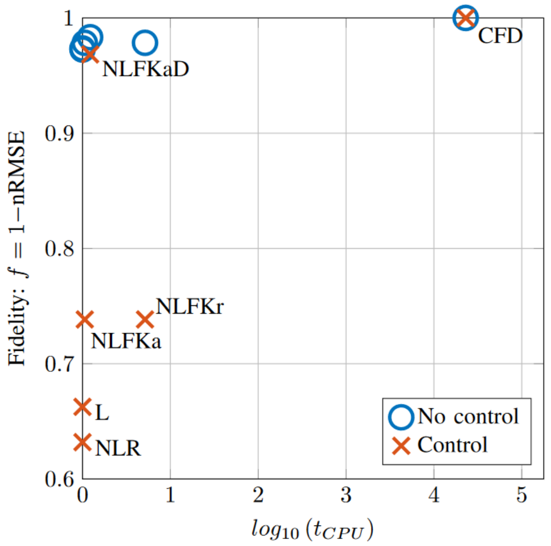

Fig. 3. Model validation under controlled and uncontrolled conditions. For explanation of the model identifiers, see (Giorgi et al., 2017)

3.1 The Classical era

The *Classical* era in the development of wave energy control systems began with the pioneering work of Budal and Falnes (1975), and collected in the widely read reference of Falnes (2002). The work was characterised by linear WEC models, monochromatic analysis and control methods based on complex-conjugate impedance matching, and latching. It set the stage for further developments in latching (Babarit and Clément, 2006) and freewheeling/declutching (Wright et al., 2003; Babarit et al., 2009).

The essence of these methods originates from the consideration of the force-to-velocity model of a WEC, which is obtained from (7) in the frequency domain (Falnes, 2002) as:

$$\frac{V(\omega)}{F_{ex}(\omega) + F_u(\omega)} = \frac{1}{Z_i(\omega)}, \tag{8}$$

where $Z_i(\omega)$ is termed the intrinsic impedance of the system:

$$Z_i(\omega) = B_r(\omega) + \jmath\omega \left[M + M_a(\omega) - \frac{K_b}{\omega^2} \right], \tag{9}$$

and $B_r(\omega)$ is the radiation resistance and M_a is added mass. The model in (8) allows the derivation of conditions for optimal energy absorption in the frequency domain (Falnes, 2002) as:

$$Z_{PTO}(\omega) = Z_i^*(\omega) \tag{10}$$

where $(\ )^*$ denotes the complex conjugate and $F_{PTO} = Z_{PTO}V(\omega)$. The result in (10) has a number of important implications:

- It is frequency dependent, implying that there is a different optimal impedance for each frequency - how to specify a single frequency for irregular seas containing a mixture of frequencies?
- Since $h_r(t)$ is causal, $h_c(t) = \mathcal{F}^{-1}(Z_{PTO}(\omega))$ is anti-causal, requiring future knowledge of the excitation force.
- Since force and velocity can have opposite signs, the PTO may *supply* power for some parts of the sinusoidal cycle. This is akin to reactive power in power systems. Such a phenomenon places particular demands on PTO systems, not only in terms of the need to facilitate bi-directional power flow, but also that the peak reactive power can be significantly greater than active power peak (Shek et al., 2007). The optimal *passive* PTO is provided by $R_{PTO} = |Z_i(\omega)|$, which avoids the need for the PTO to supply power, but results in suboptimal control.
- The optimal control in (10) takes no account of physical constraints in the WEC/PTO, where there are likely to be limitations on displacement or relative displacement, PTO force and there may be external constraints imposed by electrical grid regulations.

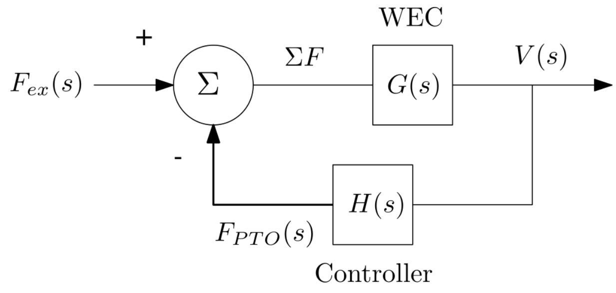

Fig. 4. ACC controller structure, where $G(s)$ represents $1/Z_i$ and $H(s)$ represents $Z_{PTO} = Z_i^*$

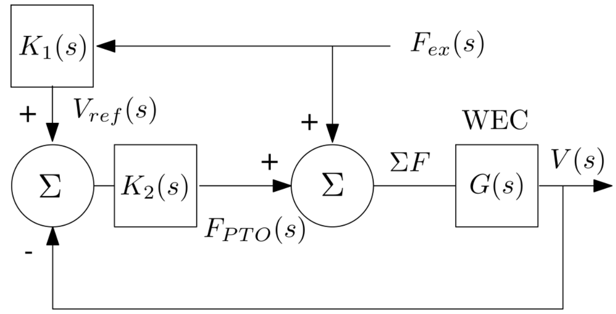

Fig. 5. AVT controller structure, which calculates the optimal velocity profile, prior to the use of a tracking control loop to achieve that velocity profile

The condition in (10) can alternatively be expressed in terms of an optimal velocity profile as:

$$V^{ref}(\omega) = F_{ex}(\omega)/(2B_r(\omega)) \tag{11}$$

where $B_r = 1/2(Z_i + Z_i^*)$ is the real part of Z_i. The condition in (11) is a condition on the amplitude of $V^{opt}(\omega)$, with the restriction that $v^{opt}(t)$ be in phase with $f_{ex}(t)$. This phase condition, considered separately, forms the basis for latching and declutching control. While latching and declutching are suboptimal in only satisfying the phase matching condition, they are passive, with no requirement for reactive power. From (10) and (11), two corresponding control structures, denoted as approximate complex conjugate (ACC) and approximate velocity tracking (AVT), can be identified, shown in Figs. 4 and 5, respectively. In particular, the ACC control structure is appealing, since no knowledge of f_{ex} is required. Furthermore, Hansen (2013) demonstrates that by parameterising Z_{PTO} as

$$Z_{PTO}(s) = M_c s^2 + B_c s + K_c \tag{12}$$

there is some redundancy in the choice of M_c, B_c and K_c, with only one of M_c or K_c required to achieve the complex conjugate. However, the three options of $[M_c, B_c]$, $[B_c, K_c]$, and $[M_c, B_c, K_c]$ have different characteristics in how F_{PTO} is manipulated. Ultimately, however, the AVT structure, though requiring knowledge of F_{ex}, provides more flexibility for panchromatic operation (see Section 3.2).

One of the critical limitations of basic complex conjugate (CC) control is the focus on a single frequency at which optimum power transfer is achieved. For the more realistic panchromatic case, users are forced to pick a single characteristic frequency, such as the device resonant period, or the sea spectrum energy period (T_E), though attempts have been made to extend the basic CC control to multiple frequencies (Song et al., 2016).

3.2 The Numerical era

This era saw the application of numerical optimisation techniques, typically employed in traditional control, such as model predictive control (MPC), to wave energy systems, led by Hals et al. (2011), who combined energy maximisation and forecasting, Bacelli et al. (2011), who introduced a parameterisation of the system variables using harmonic basis functions, Cretel et al. (2011), who used a first-order hold in the system model, and Li et al. (2012), who used dynamic programming to solve the WEC MPC problem. Each method uses constrained optimisation to solve for the optimal control, therefore recognising physical constraints on displacement and control force. Also, since the control problem is formulated in the time domain, panchromatic capability was easily included. All of these control solutions are based on the AVT control structure of Fig.5, so require a forecast of the free surface or the excitation force, which may diminish the control effectiveness. Some consideration to the impact of non-exact forecasts is given by Fusco and Ringwood (2011a).

Around the same time, Scruggs (2011) developed a causal solution to the WEC control problem, using a variant of linear quadratic Gaussian (LQG) control. In the original formulation of (Scruggs, 2011), constraints were not considered, but were introduced, in a statistical framework, in a subsequent study (Scruggs, 2017).

One other potential difficulty with the optimal numerical methods is that, unlike the traditional quadratic LQG problems of tracking control, the energy maximisation problem, involving the product of velocity and force, is not guaranteed to be convex. In some cases, an extra (quadratic) term is added to the performance function, which has the added bonus of providing a soft constraint on control energy, but may bias the optimal answer. A further complication, to achieving a globally optimum solution, or indeed any viable control solution, can relate to constraints. In Paparella and Ringwood (2016), an extra constraint, to limit the flow of reactive power, is applied, but with significant adverse effects on the real-time computational capability, while the geometric formulation of Bacelli and Ringwood (2013) shows, in diagrammatic form, the interrelationship between the energy performance surface, displacement and force constraints, and the wave excitation force, see Fig.6.

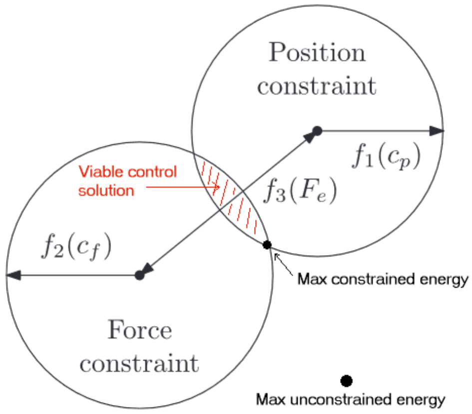

Fig. 6. Geometric representation of constraints on parametric space of PTO force. Note that the control solution space vanishes as the excitation force, F_e, increases. Adapted from (Bacelli and Ringwood, 2013)

This geometric representation allows the PTO constraints to be optimised in the knowledge of the optimal control solution, suggesting the possibility of co-design of the device and control. Indeed, given the family of numerically optimised controllers under current consideration, it makes perfect sense to also consider other aspects of the WEC within an optimisation framework, especially when they impact the control problem, and vice-versa. Besides the study of Bacelli and Ringwood (2013), concerning PTO constraints, Garcia-Rosa and Ringwood (2015) considered the interplay between the control problem and that of WEC geometry optimisation, while the interaction between optimal WEC array layout and WEC control was considered in Garcia-Rosa et al. (2015). The co-design of the WEC control, along with the parameters of a permanent magnet linear generator, was considered by O'Sullivan and Lightbody (2017).

Finally, numerical control solutions have been extended to arrays of WECs, demonstrating a significant performance improvement with global array control (using a fully interactive system model), as demonstrated in (Bacelli et al., 2013; Li and Belmont, 2014). A more complete review of numerical WEC control methods is given in (Faedo et al., 2017).

Parameterisations Following the initial work of Bacelli et al. (2011), a range of parameterisation options for various system variables have been considered. This is motivated by the fact that signals in WEC systems are generally smooth, and relatively poorly parameterised by the default zero-order hold (ZOH) rectangular basis functions, see Fig.7, though the first-order hold of Cretel et al. (2011) does provide a significant improvement, albeit with the introduction of a 1-step delay. Indeed, a comparative study by Genest and Ringwood (2016a) suggests that a pseudospectral parameterisation reduces the control computation by a factor of 3, while providing at least the same level of control performance.

A variety of parameterisation options have been considered, considering various basis functions and system variables, including the differential flatness approach

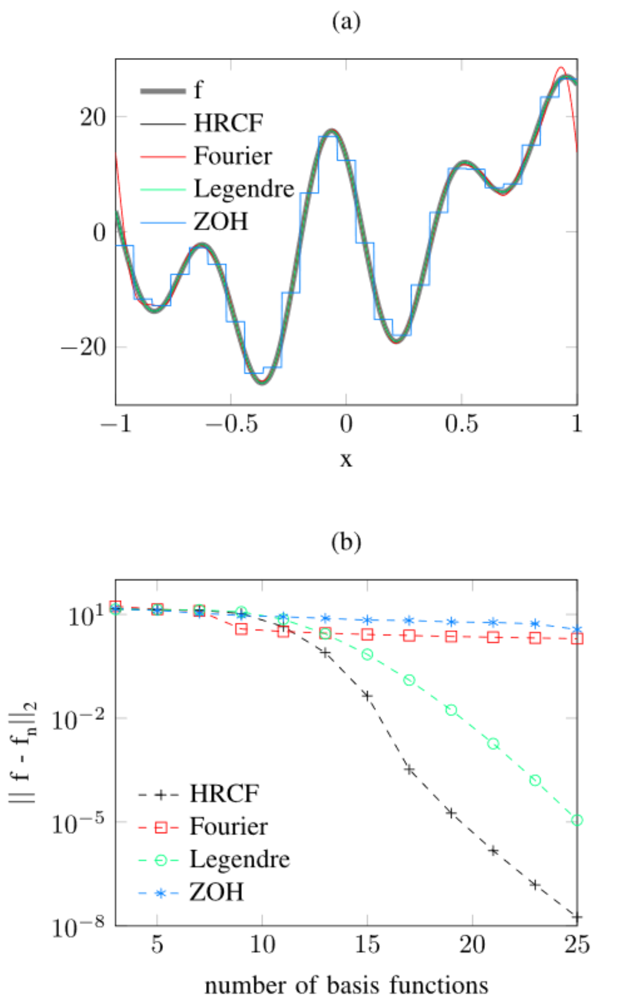

Fig. 7. Basis function approximations (a) Form, and (b) Relative approximation error. Taken from (Genest and Ringwood, 2016b)

of Li (2017) (employing Laguerre basis functions), the 'shape-based' approach of Abdelkhalik et al. (2016), and Mérigaud and Ringwood (2017a). One advantage that traditional (ZOH) MPC has is that it naturally lends itself to receding-horizon operation, while spectral/pseudospectral approaches employing periodic basis functions are problematic over a short duration prediction window. A number of solutions to this are available, including the modified (half-range Chebychev-Fourier) aperiodic basis functions of Genest and Ringwood (2016b), or the use of windowing functions (Auger et al., 2018). The use of moment-matching (Faedo et al., 2018b) also shows considerable promise in the computation/performance tradeoff, compared to standard MPC, and can be shown to encompass pseudospectral techniques as a special case. One particular advantage of the moment-matching framework is the ability to generalise, reasonably smoothly, to nonlinear systems.

Nonlinear models A variety of WEC controllers have been adapted to deal with various aspects of nonlinearity. Most of these controllers come from the MPC/pseudo-spectral domains, focussing on specific nonlinearities including mooring force (Richter et al., 2012), viscous drag (Bacelli et al., 2015), or nonlinear restoring force (Li, 2015). Mérigaud and Ringwood (2017b) highlight the importance of correctly articulating velocity-dependent nonlinearity (e.g viscous drag), and the role that various constraints (displacement, force, passivity) play in exaggerating, or attenuating, various nonlinear effects.

Also, some approaches to control of WEC systems, while recognising non-ideal PTO efficiency, has been considered (Bacelli et al., 2015; Genest et al., 2014; Tona et al., 2015). One of the strong messages here is that, if non-ideal PTO efficiency is ignored, the resulting incorrect control may, in fact, produce negative net energy.

Despite these ventures into the nonlinear world, there is still no option to address the more general nonlinear model structures of Sections 2.1 and 2.2 within WEC controller structures. The difficulty centres around bridging the gap between the nonlinear model parameterisations (CFD models being the ultimate example) and the nonlinear parameterisation that can be tolerated within nonlinear WEC control structures. One attempt to build such a bridge may come in the form of nonlinear model reduction methods, where some application to the wave energy space has recently taken place (Faedo et al., 2020).

One reasonably straightforward method to bridge the divide between nonlinear (including CFD) WEC models is to use the nonlinear model to generate data, from which a particular nonlinear parameterisation (suitable for control) can be fitted (Giorgi et al., 2019). This concept can, of course, be extended to recorded tank data (Giorgi et al., 2019). However, as previously mentioned, care must be taken in the design of experiment used to elicit the data (Davidson et al., 2016).

3.3 The new generation

This is a somewhat diverse grouping, but hopefully reflects the main current and new directions in wave energy control. One significant emerging area is robust WEC control, specifically targetting uncertainty in the model, for model-based control design. However, this control area is dealt with separately in Section 5.

Apart from the incremental improvements in MPC and MPC-like control strategies, several important new directions are being explored. An interesting aspect is that, in many cases, rather than adding complexity, there is a drive to *reduce* complexity. This complexity reduction is associated with ways of trying to avoid some of the particular problems that make wave energy control so challenging. These include:

(a) Avoiding the need for excitation force estimation
(b) Avoiding the need for excitation force forecasting
(c) Avoiding the need for numerical optimisation, which has the consequent difficulties of convexity and convergence.

One particular advantage of the feedback configuration of the ACC controller (see Fig.4) is that no measurement, either current or future, of F_{ex} is required. However, in its original form, the ACC controller can only cater for monochromatic waves. With some small compromise on full optimality (achieved only for monochromatic waves), the controller of Nielsen et al. (2013) deals with stochastic wave climates, while also naturally providing a causal control solution. One small issue is that the control problem is solved over an infinite time horizon, which might present a challenge for real-time implementation.

The last decade has also seen the application of machine learning techniques to wave energy modelling and control. However, the first application of neural networks to wave energy control was documented by Beirao et al. (2007), who used an internal model control (IMC) structure, implementing phase and amplitude control, to incorporate a neural network model of the Archimedes Wave Swing. More recent applications include the use of a neural network model by Anderlini et al. (2017), though the use of a simple (single frequency) CC controller seems to be a little at odds with the model complexity.

The causal approach of Scruggs et al. (2013) avoids the need for forecasting, and has been progressively refined to include nonlinear dynamics and PTO losses (Nie et al., 2016), and finite stroke constraints (Scruggs, 2017). The causal WEC controller of Shi et al. (2019) is also noteworthy in the innovative use of Bayesian learning.

Finally, a recent control design by Garcia-Violini et al. (2020) attempts to initially articulate the panchromatic CC problem in nonparametric form, and then to use the FOAMM toolbox (Faedo et al., 2018a) to determine a parametric LTI model. With such a configuration, the implementation of constraints is suboptimal, but the controller returns performance comparable with MPC and better than the 'simple and effective' (SE) controller of Fusco and Ringwood (2012), which also targets simplicity by using an instantaneous frequency tracker to implement constrained CC control, as shown in Fig.8. A somewhat similar approach, with a more traditional controller identification, without considering constraints, is shown in Bacelli et al. (2020).

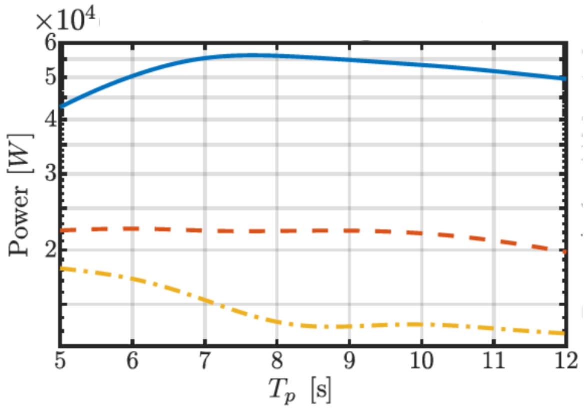

Fig. 8. Performance of a LiTe-Con controller (dashed red), compared to a high performance moment-based controller (solid blue) and an SE controller (orange dash-dot). Taken from (Garcia-Violini et al., 2020)

4. ESTIMATION AND FORECASTING

In this section, pertinent and recent developments in excitation force (and/or wave) estimation and forecasting are profiled. In general, since the excitation force is essentially a low-pass filtered version of the incident free surface elevation (FSE), it is an easier quantity both to estimate and forecast. However, dealing first with *wave* forecasting, there are some key points:

- If an *ideal* low-pass (LP) filtered version of the FSE is available, then the forecasting fidelity can be extended (Fusco and Ringwood, 2010). With a realistic LP filter phase delay, the forecasting challenge is effectively extended, somewhat nullifying the value of the filtering.
- It's difficult to justify the use of nonlinear wave forecasting models (Fusco and Ringwood, 2010), particularly for WEC devices operating in the power production region.
- Up-wave measurements have the potential to improve wave forecasts (Mérigaud et al., 2018; Blenkinsopp et al., 2012), although this is not always the case (Paparella et al., 2014). The capital cost and maintenance of data buoys or measurement systems must be considered against their potential benefit over time-series only techniques.
- It's difficult to beat a simple autoregressive (AR) model for wave forecasting (Peña-Sanchez et al., 2020a)! A recent data-based method, employing Gaussian process (GP) models (Shi et al., 2018), has shown promise and has the added advantage of producing reliable error bounds.

In relation to excitation force forecasting, there is an additional device dependence, which determines the severity of the non-causal control problem (Fusco and Ringwood, 2011b). For example, if the radiation impulse response is of long duration, the control problem is significantly non-causal, but this is alleviated somewhat by the fact that slow (large) devices are designed to operate in longer waves, easing the forecasting problem.

Regarding the estimation of excitation force, this represents a somewhat unusual estimation problem, in that the unknown quantity is an input, rather than a state. The survey of Peña-Sanchez et al. (2020b) examines the relative performance of a set of excitation force estimators, under a range of measurement noise conditions. The use of a CFD-based numerical wave tank for simulation avoids tuning the simulation to any particular input signal model in the estimators.

Finally, the arrangement of WECs in an array presents a more complex estimation/forecasting problem, due to the added presence of radiated waves, but also allows the motion of each WEC to be used as a measurement point, providing more information for estimation and forecasting of excitation force on each individual WEC. The study of Peña-Sanchez et al. (2018) (using a Kalman filter estimator and AR forecaster) demonstrates that, broadly speaking, the added complexity of the wave field is compensated by the extra information available. The array estimator of Zou and Abdelkhalik (2018) also uses information from the complete WEC array to estimate the F_{ex} on individual devices, but prioritises the motion information from the device on which the estimation is currently focussed.

5. SENSITIVITY AND ROBUSTNESS

One aspect of WEC control that is receiving increasing attention is the area of control sensitivity to modelling errors. Recently (Ringwood et al., 2020), the two main WEC control structures, ACC and AVT, shown in Figs.4 and 5 respectively, were examined in terms of their comparative sensitivity and robustness properties. Some of the analysis gives cause for alarm, for example the system sensitivity function for the ACC controller, copied here in Fig.9. From

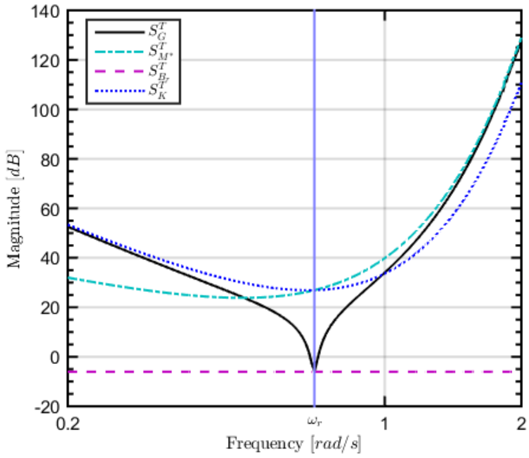

Fig. 9. Sensitivity functions for an ACC controller for a cylindrical point absorber. The vertical blue line represents the resonant frequency $\omega_r = \sqrt{K/(M + m_\infty)}$. Individual sensitivity to mass (M), damping (B), and spring term (K) are shown, as well as overall sensitivity, S_G^T. Taken from (Ringwood et al., 2020)

.

Fig.9, it can be seen that quite alarming levels of closed-loop sensitivity to modelling errors are achieved away from the device resonance. At the device resonant frequency, sensitivity is quite good ($= 1/2$), but the controller is, in fact, inactive at this frequency. Despite this high closed-loop sensitivity, there may be mitigating issues:

- The sensitivity of power production is usually lower (Ringwood et al., 2020).
- If the WEC has a broad bandwidth (e.g. a flap), the requirement for, and sensitivity of, the control system may be lower, since the controller is required to do less work (Folley et al., 2015).

Nevertheless, there are reasons for concern and the use of (particularly linear) models, validated under uncontrolled conditions, for model-based control design, is likely to lead to an unacceptable mismatch between system and controller. One solution is to *adapt* the controller, keeping it sensitive to WEC model variations (Davidson et al., 2018; Zhan et al., 2018), though robust control approaches are also now also emerging (Garcia-Violini and Ringwood, 2019). Applying robust control to the velocity tracking loop of an AVT control can draw on tradition robust control techniques e.g. (Wahyudie et al., 2015).

6. WHERE TO NOW ?

Considerable progress has been made in WEC control over the past 3 decades. However, challenges still remain in accurately addressing WEC system (hydrodynamic and PTO) nonlinearity in its true form. There is significant challenge in the accurate representation of WEC hydrodynamics, in a form suitable for model-based control, given the degree to which WEC controllers themselves increase the operational space, and consequently the control challenge, of WECs. Finally, while the numerical AVT optimisation methods give a concise facility to implement hard physical constraints, they must include estimators and forecasters to deal with the inherent non-causal nature of the control problem. Feedback controller configurations are attractive, in eliminating the need for an estimator/forecaster, but struggle with hard constraints and pan-chromatic operation, while the sub-optimality of causal controllers must be balanced with the inevitable forecasting errors of their non-causal counterparts. A rich control problem indeed!

REFERENCES

Abdelkhalik, O., Robinett, R., Zou, S., Bacelli, G., Coe, R., Bull, D., Wilson, D., and Korde, U. (2016). On the control design of wave energy converters with wave prediction. *J. of Ocean Eng. and Marine Energy*, 2(4), 473–483.

Anderlini, E., Forehand, D., Bannon, E., and Abusara, M. (2017). Reactive control of a wave energy converter using artificial neural networks. *International journal of marine energy*, 19, 207–220.

Anderlini, E., Forehand, D.I., Stansell, P., Xiao, Q., and Abusara, M. (2016). Control of a point absorber using reinforcement learning. *IEEE Transactions on Sustainable Energy*, 7(4), 1681–1690.

Auger, C., Merigaud, A.P.L., and Ringwood, J.V. (2018). Receding-horizon pseudo-spectral control of wave energy converters using periodic basis functions. *IEEE Transactions on Sustainable Energy*.

Babarit, A. and Clément, A.H. (2006). Optimal latching control of a wave energy device in regular and irregular waves. *Applied Ocean Research*, 28(2), 77–91.

Babarit, A. and Delhommeau, G. (2015). Theoretical and numerical aspects of the open source BEM solver NEMOH. In *11th European Wave and Tidal Energy Conference, Nantes*.

Babarit, A., Guglielmi, M., and Clement, A.H. (2009). Declutching control of a wave energy converter. *Ocean Engineering*, 36(12-13), 1015–1024.

Bacelli, G., Balitsky, P., and Ringwood, J.V. (2013). Coordinated control of arrays of wave energy devices—benefits over independent control. *IEEE Transactions on Sustainable Energy*, 4(4), 1091–1099.

Bacelli, G., Genest, R., and Ringwood, J.V. (2015). Nonlinear control of flap-type wave energy converter with a non-ideal power take-off system. *Annual Reviews in Control*, 40, 116–126.

Bacelli, G., Gilloteaux, J.C., and Ringwood, J. (2009). A predictive controller for a heaving buoy producing potable water. In *2009 European Control Conference (ECC)*, 3755–3760. IEEE.

Bacelli, G., Nevarez, V., Coe, R.G., and Wilson, D. (2020). Feedback resonating control for a wave energy converter. *IEEE Trans. on Industry Apps.*, 20, 1862–1868.

Bacelli, G. and Ringwood, J.V. (2013). A geometric tool for the analysis of position and force constraints in wave energy converters. *Ocean Engineering*, 65, 10–18.

Bacelli, G., Ringwood, J.V., and Gilloteaux, J.C. (2011). A control system for a self-reacting point absorber wave energy converter subject to constraints. *IFAC Proceedings Volumes*, 44(1), 11387–11392.

Bailey, H., Robertson, B.R., and Buckham, B.J. (2016). Wave-to-wire simulation of a floating oscillating water column wave energy converter. *Ocean Eng.*, 125, 248–260.

Beirao, P., Mendes, M., Valério, D., and da Costa, J.S. (2007). Control of the archimedes wave swing using neural networks. In *7th European Wave and Tidal Energy Conference, Porto.*

Blenkinsopp, C.E., Turner, I.L., Allis, M.J., Peirson, W.L., and Garden, L.E. (2012). Application of lidar technology for measurement of time-varying free-surface profiles in a laboratory wave flume. *Coastal Eng.*, 68, 1–5.

Budal, K. and Falnes, J. (1975). Power generation from ocean waves using a resonant oscillating system. *Marine Science Communication*, 1, 269–288.

Clément, A. (1999). Using differential properties of the green function in seakeeping computational codes. In *Proc. European Control Conf., Karlsruhe, Germany*, 3755–3760.

Cretel, J.A., Lightbody, G., Thomas, G.P., and Lewis, A.W. (2011). Maximisation of energy capture by a wave-energy point absorber using model predictive control. *IFAC Proceedings Volumes*, 44(1), 3714–3721.

Davidson, J. and Costello, R. (2020). Efficient nonlinear hydrodynamic models for wave energy converter design—a scoping study. *J. Marine Science and Engineering*, 8(1), 35.

Davidson, J., Genest, R., and Ringwood, J.V. (2018). Adaptive control of a wave energy converter. *IEEE Trans. on Sustainable Energy*, 9(4), 1588–1595.

Davidson, J., Giorgi, S., and Ringwood, J.V. (2015). Linear parametric hydrodynamic models for ocean wave energy converters identified from numerical wave tank experiments. *Ocean Engineering*, 103, 31–39.

Davidson, J., Giorgi, S., and Ringwood, J.V. (2016). Identification of wave energy device models from numerical wave tank data—part 1: Numerical wave tank identification tests. *IEEE Trans. on Sustainable Energy*, 7(3), 1012–1019.

Faedo, N., Dores, F., and Ringwood, J.V. (2020). Nonlinear model reduction for wave energy systems: A moment-based approach. *IEEE Trans. on Control Systems Technology, in press.*

Faedo, N., Olaya, S., and Ringwood, J.V. (2017). Optimal control, mpc and mpc-like algorithms for wave energy systems: An overview. *IFAC Journal of Systems and Control*, 1, 37–56.

Faedo, N., Peña-Sanchez, Y., and Ringwood, J.V. (2018a). Finite-order hydrodynamic model determination for wave energy applications using moment-matching. *Ocean Engineering*, 163, 251–263.

Faedo, N., Scarciotti, G., Astolfi, A., and Ringwood, J.V. (2018b). Energy-maximising control of wave energy converters using a moment-domain representation. *Control Engineering Practice*, 81, 85–96.

Falnes, J. (2002). *Ocean waves and oscillating systems: linear interactions including wave-energy extraction.* Cambridge University Press.

Folley, M. and Whittaker, T. (2010). Spectral modelling of wave energy converters. *Coastal Eng.*, 57(10), 892–897.

Folley, M., Henry, A., and Whittaker, T. (2015). Contrasting the hydrodynamics of heaving and surging wave energy converters. In *Proceedings of the 11th European Wave and Tidal Energy Conference, Nantes.*

Forehand, D.I., Kiprakis, A.E., Nambiar, A.J., and Wallace, A.R. (2015). A fully coupled wave-to-wire model of an array of wave energy converters. *IEEE Trans. on Sustainable Energy*, 7(1), 118–128.

Fusco, F. and Ringwood, J.V. (2010). Short-term wave forecasting for real-time control of wave energy converters. *IEEE Trans. on Sustainable Energy*, 1(2), 99–106.

Fusco, F. and Ringwood, J.V. (2011a). A model for the sensitivity of non-causal control of wave energy converters to wave excitation force prediction errors. In *Proc. of the 9th European Wave and Tidal Energy Conf. (EWTEC), Southampton.*

Fusco, F. and Ringwood, J.V. (2011b). A study of the prediction requirements in real-time control of wave energy converters. *IEEE Trans. on Sustainable Energy*, 3(1), 176–184.

Fusco, F. and Ringwood, J.V. (2012). A simple and effective real-time controller for wave energy converters. *IEEE Trans. on sustainable energy*, 4(1), 21–30.

Garcia-Rosa, P.B., Bacelli, G., and Ringwood, J.V. (2015). Control-informed optimal array layout for wave farms. *IEEE Trans. on Sustainable Energy*, 6(2), 575–582.

Garcia-Rosa, P.B. and Ringwood, J.V. (2015). On the sensitivity of optimal wave energy device geometry to the energy maximizing control system. *IEEE Trans. on Sustainable Energy*, 7(1), 419–426.

Garcia-Violini, D., Pena-Sanchez, Y., Faedo, N., and Ringwood, J. (2020). An energy-maximising linear time-invariant controller (lite-con) for wave energy devices. *IEEE Trans. on Sustainable Energy, in press.*

Garcia-Violini, D. and Ringwood, J.V. (2019). Energy maximising robust control for spectral and pseudospectral methods with application to wave energy systems. *Int. Journal of Control*, (just-accepted), 1–27.

Genest, R., Bonnefoy, F., Clément, A.H., and Babarit, A. (2014). Effect of non-ideal power take-off on the energy absorption of a reactively controlled one degree of freedom wave energy converter. *Applied Ocean Research*, 48, 236–243.

Genest, R. and Ringwood, J.V. (2016a). A critical comparison of model-predictive and pseudospectral control for wave energy devices. *J. Ocean Engineering and Marine Energy*, 2(4), 485–499.

Genest, R. and Ringwood, J.V. (2016b). Receding horizon pseudospectral control for energy maximization with application to wave energy devices. *IEEE Trans. on Control Systems Technology*, 25(1), 29–38.

Gilloteaux, J.C. (2007). *Mouvements de grande amplitude d'un corps flottant en fluide parfait. Application à la récupération de l'énergie des vagues.* Ph.D. thesis, Ecole Centrale Nantes.

Giorgi, G., Penalba, M., and Ringwood, J.V. (2017). Nonlinear hydrodynamic models for heaving buoy wave energy converters. In *Proc. Asian Wave and Tidal Energy Conf. (AWTEC), Singapore*, 144–153.

Giorgi, G. and Ringwood, J.V. (2017a). Computationally efficient nonlinear froude–krylov force calculations for heaving axisymmetric wave energy point absorbers. *J. Ocean Eng. and Marine Energy*, 3(1), 21–33.

Giorgi, G. and Ringwood, J.V. (2017b). Consistency of viscous drag identification tests for wave energy applications. In *Proc. 12th European Wave and Tidal Energy Conf. (EWTEC), Cork, Ireland*, volume 27.

Giorgi, G. and Ringwood, J.V. (2018). Articulating parametric nonlinearities in computationally efficient hydrodynamic models. *IFAC Proceedings*, 51(29), 56–61.

Giorgi, S., Davidson, J., Jakobsen, M., Kramer, M., and Ringwood, J.V. (2019). Identification of dynamic models for a wave energy converter from experimental data. *Ocean Engineering*, 183, 426–436.

Giorgi, S., Davidson, J., and Ringwood, J.V. (2016). Identification of wave energy device models from numerical wave tank data—part 2: Data-based model determination. *IEEE Trans. on Sus. Energy*, 7(3), 1020–1027.

Gotoh, H. and Khayyer, A. (2018). On the state-of-the-art of particle methods for coastal and ocean engineering. *Coastal Engineering J.*, 60(1), 79–103.

Hals, J., Falnes, J., and Moan, T. (2011). Constrained optimal control of a heaving buoy wave-energy converter. *J. Offshore Mechanics and Arctic Eng.*, 133(1), 011401.

Hansen, R.H. (2013). *Design and control of the powertake-off system for a wave energy converter with multiple absorbers*. Ph.D. thesis, Aalborg University,The Faculty of Engineering and Science.

Havelock, T.H. (1954). The forces on a submerged body moving under waves. *Quarterly Trans. of the Inst. of Naval Architects*, 96(1), 77–88.

Henriques, J., Gato, L., Lemos, J., Gomes, R., and Falcão, A. (2016). Peak-power control of a grid-integrated oscillating water column wave energy converter. *Energy*, 109, 378–390.

Josset, C., Babarit, A., and Clément, A. (2007). A wave-to-wire model of the searev wave energy converter. *Proc. of the Inst. of Mech. Engineers, Part M: J. of Engineering for the Maritime Environment*, 221(2), 81–93.

Kim, Y., Kim, K.H., Kim, J.H., Kim, T., Seo, M.G., and Kim, Y. (2011). Time-domain analysis of nonlinear motion responses and structural loads on ships and offshore structures: development of wish programs. *Int. J. of Naval Architecture and Ocean Eng.*, 3(1), 37–52.

Lee, C.H. (1995). *WAMIT theory manual*. MIT, Department of Ocean Engineering.

Letournel, L., Chauvigné, C., Gelly, B., Babarit, A., Ducrozet, G., and Ferrant, P. (2018). Weakly nonlinear modeling of submerged wave energy converters. *Applied Ocean Research*, 75, 201–222.

Levy, E. (1959). Complex curve fitting. *IRE Trans. on Automatic Control*, AC-4, 37–43.

Li, G. (2015). Predictive control of a wave energy converter with wave prediction using differential flatness. In *2015 54th IEEE Conf. on Decision and Control (CDC)*, 3230–3235. IEEE.

Li, G. (2017). Nonlinear model predictive control of a wave energy converter based on differential flatness parameterisation. *Int. J. of Control*, 90(1), 68–77.

Li, G. and Belmont, M.R. (2014). Model predictive control of sea wave energy converters–part ii: The case of an array of devices. *Renewable Energy*, 68, 540–549.

Li, G., Weiss, G., Mueller, M., Townley, S., and Belmont, M.R. (2012). Wave energy converter control by wave prediction and dynamic programming. *Renewable Energy*, 48, 392–403.

Longuet-Higgins, M.S. and Cokelet, E. (1976). The deformation of steep surface waves on water-i. a numerical method of computation. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, 350(1660), 1–26.

McCabe, A.P., Bradshaw, A., and Widden, M.B. (2005). A time-domain model of a floating body using transforms. In *6th Euro. Wave and Tidal Energy Conf.*

McCormick, M.E. (2013). *Ocean wave energy conversion*. Courier Corporation.

Mérigaud, A., Herterich, J., Flanagan, J., Ringwood, J., and Dias, F. (2018). Incorporating wave spectrum information in real-time free-surface elevation forecasting: Real-sea experiments. *IFAC Proc.*, 51(29), 232–237.

Mérigaud, A. and Ringwood, J.V. (2017a). A nonlinear frequency-domain approach for numerical simulation of wave energy converters. *IEEE Trans. on Sustainable Energy*, 9(1), 86–94.

Mérigaud, A. and Ringwood, J.V. (2017b). Optimal trajectories, nonlinear models and constraints in wave energy device control. *IFAC Proc.*, 50(1), 15645–15650.

Nie, R., Scruggs, J., Chertok, A., Clabby, D., Previsic, M., and Karthikeyan, A. (2016). Optimal causal control of wave energy converters in stochastic waves–accommodating nonlinear dynamic and loss models. *Int. J. of Marine Energy*, 15, 41–55.

Nielsen, K.M., Pedersen, T.S., Andersen, P., and Ambühl, S. (2017). Optimizing control of wave energy converter with losses and fatigue in power take off. *IFAC-PapersOnLine*, 50(1), 14680–14685.

Nielsen, S.R., Zhou, Q., Kramer, M.M., Basu, B., and Zhang, Z. (2013). Optimal control of nonlinear wave energy point converters. *Ocean Eng.*, 72, 176–187.

Omidvar, P., Stansby, P.K., and Rogers, B.D. (2012). Wave body interaction in 2d using smoothed particle hydrodynamics (sph) with variable particle mass. *Int. J. for Numerical Methods in Fluids*, 68(6), 686–705.

O'Sullivan, A.C. and Lightbody, G. (2017). Co-design of a wave energy converter using constrained predictive control. *Renewable energy*, 102, 142–156.

Paparella, F., Monk, K., Winands, V., Lopes, M., Conley, D., and Ringwood, J.V. (2014). Up-wave and autoregressive methods for short-term wave forecasting for an oscillating water column. *IEEE Trans. on Sustainable Energy*, 6(1), 171–178.

Paparella, F. and Ringwood, J.V. (2016). Optimal control of a three-body hinge-barge wave energy device using pseudospectral methods. *IEEE Trans. on Sustainable Energy*, 8(1), 200–207.

Papillon, L., Costello, R., and Ringwood, J.V. (2020). Boundary element methods in potential flow theory: A comprehensive review with a focus on wave energy applications. *J. of Ocean Eng. and Marine Energy, under review.*

Papillon, L., Wang, L., Tom, N., Weber, J., and Ringwood, J. (2019). Parametric modelling of a reconfigurable wave energy device. *Ocean Engineering*, 186, 106105.

Peña-Sanchez, Y., Garcia-Abril, M., Paparella, F., and Ringwood, J.V. (2018). Estimation and forecasting of excitation force for arrays of wave energy devices. *IEEE Trans. on Sustainable Energy*, 9(4), 1672–1680.

Peña-Sanchez, Y., Mérigaud, A., and Ringwood, J.V. (2020a). Short-term forecasting of sea surface elevation for wave energy applications: The autoregressive model revisited. *IEEE J. of Oceanic Engineering, in press.*

Peña-Sanchez, Y., Windt, C., Davidson, J., and Ringwood, J.V. (2020b). A critical comparison of excitation force estimators for wave energy devices. *IEEE Trans. on Control Systems Technology, in press.*

Penalba, M. and Ringwood, J. (2018). A reduced wave-to-wire model for controller design and power assessment of wave energy converters. In *Proc. 3rd International Conf. on Renewable Energies Offshore (RENEW 2018), Lisbon, Portugal*, 379.

Penalba, M., Davidson, J., Windt, C., and Ringwood, J.V. (2018). A high-fidelity wave-to-wire simulation platform for wave energy converters: Coupled numerical wave tank and power take-off models. *Applied Energy*, 226, 655–669.

Penalba, M., Giorgi, G., and Ringwood, J.V. (2017a). Mathematical modelling of wave energy converters: a review of nonlinear approaches. *Renewable and Sustainable Energy Reviews*, 78, 1188–1207.

Penalba, M., Kelly, T., and Ringwood, J.V. (2017b). Using NEMOH for modelling wave energy converters: A comparative study with WAMIT. In *Proc. 12th European Wave and Tidal Energy Conference (EWTEC2017), Cork, Ireland*, volume 27, 631–1.

Penalba, M. and Ringwood, J.V. (2016). A review of wave-to-wire models for wave energy converters. *Energies*, 9(7), 506.

Penalba, M. and Ringwood, J.V. (2019). A high-fidelity wave-to-wire model for wave energy converters. *Renewable energy*, 134, 367–378.

Perez, T. and Fossen, T. (2007). Time-domain models of marine surface vessels for simulation and control design based on seakeeping computations. In *Proc. 7th IFAC Conf. on Manoeuvring and Control of Marine Craft.*

Prony, R. (1795). Essai experimental et analytique: Sur les lois de la dilatabilite des fluides elastiques et sur celles de la force expensiver de la vapeur del'eau et de la vapeur de l'alkool, a differentes temperatures. *Paris J. l'Ecola Polytechnique*, 1, 24–76.

Rafiee, A. and Fiévez, J. (2015). Numerical prediction of extreme loads on the ceto wave energy converter. In *Proc. 11th European Wave and Tidal Energy Conf. (EWTEC). Nantes, France.*

Richter, M., Magana, M.E., Sawodny, O., and Brekken, T.K. (2012). Nonlinear model predictive control of a point absorber wave energy converter. *IEEE Trans. on Sustainable Energy*, 4(1), 118–126.

Ringwood, J.V., Bacelli, G., and Fusco, F. (2014). Energy-maximizing control of wave-energy converters: The development of control system technology to optimize their operation. *IEEE Control Systems*, 34(5), 30–55.

Ringwood, J.V., Mérigaud, A., Faedo, N., and Fusco, F. (2020). An analytical and numerical sensitivity and robustness analysis of wave energy control systems. *IEEE Trans. on Control Systems Technology, in press.*

Ringwood, J.V. and Simani, S. (2015). Overview of modelling and control strategies for wind turbines and wave energy devices: Comparisons and contrasts. *Annual Reviews in Control*, 40, 27–49.

Scruggs, J. (2011). Multi-objective optimal causal control of an ocean wave energy converter in random waves. In *OCEANS'11 MTS/IEEE KONA*, 1–6. IEEE.

Scruggs, J. (2017). Causal control design for wave energy converters with finite stroke. *IFAC Proceedings*, 50(1), 15678–15685.

Scruggs, J., Lattanzio, S., Taflanidis, A., and Cassidy, I. (2013). Optimal causal control of a wave energy converter in a random sea. *App. Ocean Res.*, 42, 1–15.

Shek, J., Macpherson, D., Mueller, M., and Xiang, J. (2007). Reaction force control of a linear electrical generator for direct drive wave energy conversion. *Renewable Power Generation, IET*, 1(1), 17–24.

Shi, S., Patton, R., Abdelrahman, M., and Liu, Y. (2019). Learning a predictionless resonating controller for wave energy converters. In *ASME 38th Intl. Conf. on Ocean, Offshore and Arctic Eng., Glasgow*, 95619.

Shi, S., Patton, R.J., and Liu, Y. (2018). Short-term wave forecasting using Gaussian process for optimal control of wave energy converters. *IFAC-PapersOnLine*, 51(29), 44–49.

Song, J., Abdelkhalik, O., Robinett, R., Bacelli, G., Wilson, D., and Korde, U. (2016). Multi-resonant feedback control of heave wave energy converters. *Ocean Engineering*, 127, 269–278.

Tona, P., Nguyen, H., Sabiron, G., and Creff, Y. (2015). An efficiency-aware model predictive control strategy for a heaving buoy wave energy converter. In *Proc. 11th European Wave and Tidal Energy Conf. (EWTEC). Nantes, France.*

Wahyudie, A., Jama, M., Saeed, O., Noura, H., Assi, A., and Harib, K. (2015). Robust and low computational cost controller for improving captured power in heaving wave energy converters. *Renewable Energy*, 82, 114–124.

Windt, C., Davidson, J., and Ringwood, J.V. (2018). High-fidelity numerical modelling of ocean wave energy systems: A review of computational fluid dynamics-based numerical wave tanks. *Renewable and Sustainable Energy Reviews*, 93, 610–630.

Wright, A., Beattie, W., Thompson, A., Mavrakos, S., Lemonis, G., Nielsen, K., Holmes, B., and Stasinopoulos, A. (2003). Performance considerations in a power take-off unit based on a non-linear load. In *Proc. 5th European Wave Energy Conf.* Cork.

Y. Peña-Sanchez, N.F. and Ringwood, J.V. (2019). Moment-based parametric identification of arrays of wave energy converters. In *—Proc. American Control Conf. (ACC), Philadelphia, PA*, 4785—-4790.

Yeung, R.W. (1981). Added mass and damping of a vertical cylinder in finite-depth waters. *Applied Ocean Research*, 3(3), 119–133.

Zhan, S., Wang, B., Na, J., and Li, G. (2018). Adaptive optimal control of wave energy converters. *IFAC Proceedings*, 51(29), 38–43.

Zou, S. and Abdelkhalik, O. (2018). Consensus estimation in arrays of wave energy converters. *IEEE Trans. on Sustainable Energy*, 10(2), 943–951.