Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Leader-Following Consensus Control of Nabla Discrete Fractional Order Multi-Agent Systems ⋆

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Abstract: This paper studies a consensus problem for discrete-time linear nabla fractional order multi-agent systems with Riemann-Liouville difference operator. With the help of the discrete fractional Lyapunov direct method, a state feedback stabilization problem of a discrete-time linear nabla fractional order system is firstly analyzed. Then a distributed consensus control law is proposed for a discrete-time linear nabla fractional order multi-agent system. Some sufficient conditions are presented to guarantee that the leader-following consensus can be achieved by the proposed algorithm. The control gain is determined according to an algebraic Riccati inequality. Finally, simulation results are presented to demonstrate the effectiveness of theoretical analysis.

Keywords: Discrete nabla fractional order system, multi-agent system, leader-following consensus, discrete fractional Lyapunov direct method, Algebraic Riccati inequality.

1. INTRODUCTION

Coordination control of multi-agent systems (MASs) has attracted much more attention in many research fields due to its wide applications such as wireless sensor networks, unmanned air vehicle formations and smart grids (see Hu and Hu (2008), Zheng et al. (2019), Dong et al. (2016) and Ansari et al. (2016)). As one of the key issues of coordination control problems, consensus control focuses on developing appropriate protocols to achieve an agreement for all agents by using the information of neighbor agents. So far, there have been a large number of research results in the literature such as leaderless consensus, the leader-following consensus and robust consensus (see Hu et al. (2013), Yu and Xia (2017), Girejko and Malinowska (2019) and Li et al. (2018)).

Up to date, most of the existing works related to consensus control of MASs focused on the integer-order dynamics. However, some physical phenomena with memory properties and practical engineering systems can not be well described by integer-order dynamics, such as microorganisms moving in macromolecule fluids, unmanned aerial vehicles flying in snow (see Ren et al. (2019) and Chen et al. (2018)). Motivated by these observations, consensus control of fractional order multi-agent systems(FOMASs) has been paid much attention by some researchers. For example, a consensus problem of FOMASs was addressed firstly in Cao et al. (2010), which showed that the convergence speed of the fractional order consensus algorithms can be improved by a varying order strategy. A consensus problem of FOMASs with input delays was studied in Shen and Cao (2012) by applying the Laplace transform and Nyquist stability theorem. A fractional Lyapunov direct method and the adaptive control law were used to study the nonlinear fractional order leaderless and leader-following consensus control algorithms in Gong (2016). An adaptive distributed control strategy was proposed in Mo et al. (2019) to solve a leaderless consensus problem of FOMASs with unknown nonlinearities and external disturbance by applying neural networks.

All previous work about consensus control of FOMASs just considered continuous cases, discrete fractional order calculus may play a more important role than the continuous counterparts in real applications. Very recently, there has been an increasing interest to study consensus control of discrete FOMASs. In Liu et al. (2015), the authors analyzed the convergence problem of a discrete FOMAS with a leader based on the Grunwald-Letnikov's definition of fractional order operators. By using the finite difference method, Shahamatkhah and Tabatabaei (2018) studied the leader-following consensus of discrete FOMASs, where the dynamics of the agents and the leader are considered as

⋆ This work is supported partially by National Nature Science Foundation of China under Grants 61473061, 71503206, 61104104, the Sichuan Science and Technology Program under Grant 2020YF-SY0012, the Fundamental Research Funds for the Central Universities Grant JBK2002021, and the Program for New Century Excellent Talents in University under Grant NCET-13-0091.

first-order or second-order fractional order systems. In order to reduce numerical errors in discrete FOMASs, some researchers turned to use fractional sum and difference with Riemann-Liouville and Caputo definitions to study discrete fractional systems. For example, in Chen (2011), the authors discussed the stability of nonlinear fractional difference equations with Caputo delta fractional difference operators via fixed point theory.

However, according to Cermak and Nechvatal (2010), the delta fractional difference of a given function is often defined at points different from those of the function's original domain which may cause some unpleasant phenomenons. For this reason, Cermak et al. (2013) analyzed the stability region of nabla linear fractional order system with Riemann-Liouville difference operator. Wu et al. (2017) provided asymptotic stability conditions for nonlinear nabla discrete fractional order system with Riemann-Liouville definition by applying the Lyapunov second direct method. Wei et al. (2019) presented some sufficient conditions on the stability of discrete fractional order systems by computing the fractional difference of Lyapunov functions based on Riemann-Liouville, Caputo, Grunwald-Letnikov definitions. However, there is still few results on consensus control of discrete FOMASs with nabla fractional difference operators.

In this paper, a consensus problem of a linear nabla discrete FOMAS with Riemann-Liouville difference operator is addressed. The contributions of this paper are summarized as follows: First, based on the discrete fractional Lyapunov direct method, the stabilizaiton problem of nabla linear fractional order system is considered. Second, a leader-following consensus control is proposed for a FOMAS and the consensus analysis is provided using Riccati inequality as well. Third, numerical results are given to validate the proposed controls of the nabla linear fractional order systems.

The remainder of this paper is organized as follows. In Section 2, some preliminaries of graph theory and the nabla fractional difference are revisited. In Section 3, several sufficient conditions are presented to guarantee the stability of the closed-loop discrete fractional order multi-agent system. In Section 4, numerical examples are provided to demonstrate the proposed method. Finally, we give some conclusions in Section 5.

2. PRELIMINARIES

2.1 Graph theory

Let $G = (V, \mathcal{E})$ be a weighted undirected graph with the set of nodes $V = \{v_1, \dots, v_N\}$ and the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The set of neighbors of node v_i is denoted by $\mathcal{N}_i = \{v_j \in V : (v_i, v_j) \in \mathcal{E}\}$. A weighted adjacency matrix $\mathcal{A} = (a_{ij})_{N\times N}$ with nonnegative entries is defined as $a_{ij} > 0$ if $e_{ij} \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. A diagonal matrix $\mathcal{D} = diag\{d_1, d_2, \dots, d_N\}$ is the degree matrix whose diagonal elements are defined by $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$.The Laplacian matrix of the graph is denoted by $\mathcal{L} = (l_{ij})_{N\times N}$ as $\mathcal{L} = \mathcal{D} - \mathcal{A}$. We use another graph $\overline{G}$ to describe the interaction network topology of the leader-following multi-agent system, which contains N follower agents labeled by $v_i \in V$ and one leader labeled by v_0. The leader adjacency matrix is given by $\overline{B} = \{b_1, \dots, b_N\}$, where b_i is a positive number if the follower agent i can receive the information from the leader, and 0 otherwise.

Lemma 1. (Hu and Hong (2007)) If the undirected graph $\overline{G}$ is connected, then the matrix $H = \mathcal{L} + \overline{B}$ is a symmetric positive definite matrix.

2.2 Discrete fractional calculus

Definition 1. [**Nabla difference**] (Goodrich and Peterson (2015)) Given a function $f : \mathbb{N}_{a+1-m} \to \mathbb{R}$, its m-th nabla difference is defined by

$$ {}_a\nabla_n^m f(n) = \sum_{j=0}^{n} (-1)^j \binom{m}{j} f(n-j) \tag{1}$$

where $m \in \mathbb{Z}_+, n \in \mathbb{N}_{a+1} = \{a+1, a+2, \dots\}, a \in \mathbb{R}$, $\binom{p}{q} \triangleq \frac{\Gamma(p+1)}{\Gamma(q+1)\Gamma(p-q+1)}$ and $\Gamma(\cdot)$ is the Gamma function.

Definition 2. [**Nabla fractional sum**](Goodrich and Peterson (2015)) Given a function $f : \mathbb{N}_{a+1} \to \mathbb{R}$, its α-th nabla fractional sum is defined by

$$ {}_a\nabla_n^{-\alpha} f(n) = \sum_{j=0}^{n-a-1} (-1)^j \binom{-\alpha}{j} f(n-j) \tag{2}$$

where $\alpha \in \mathbb{R}_+, n \in \mathbb{N}_{a+1}$ and $a \in \mathbb{R}$.

Definition 3. [**Nabla Riemann-Liouville fractional difference**](Goodrich and Peterson (2015)) Given a function $f : \mathbb{N}_{a+1-m} \to \mathbb{R}$, the α-th nabla Riemann-Liouville fractional difference is defined by

$$ {}_a^R\nabla_n^\alpha f(n) = \nabla_a^m \nabla_n^{\alpha-m} f(n) \tag{3}$$

where $\alpha \in (m-1, m), m \in \mathbb{Z}_+, n \in \mathbb{N}_{a+1}, a \in \mathbb{R}$ and $\nabla f(n) = f(n) - f(n-1)$ represents the conventional nabla operator.

Lemma 2. (Wu et al. (2017)) Consider the following R-L nabla fractional dynamical system ${}_a^R\nabla_n^\alpha x(n) = f(n, x(n)), x(a+1) = C$ with $0 < \alpha < 1, n \in \mathbb{N}_{a+1}$ and let $x = 0$ be an equilibrium point of the system.If there exists a positive definite function $V(n, x(n))$ such that

$$ \gamma_1(\|x(n)\|) \leq V(n, x(n)) \leq \gamma_2(\|x(n)\|), n \in \mathbb{N}_{a+1} \tag{4}$$

and

$$ {}_a^R\nabla_n^\alpha V(n, x(n)) \leq -\gamma_3(\|x(n)\|), \tag{5}$$

where γ_1, γ_2 and γ_3 are discrete class-$\mathcal{K}$ functions, then the equilibrium point $x = 0$ is asymptotically stable.

Remark 1. For a positive definite function $V = x^T P x$, the following inequalities are always hold:

$$ \lambda_{min}(P)\,\|x\|^2 \leq x^T P x \leq \lambda_{max}(P)\|x\|^2 \tag{6}$$

where $\lambda_{min}(P)$ and $\lambda_{max}(P)$ denotes the minimum and maximum eigenvalue of matrix P respectively.

Lemma 3. (Wei et al. (2019)) For any discrete time $n \in \mathbb{N}_{a+1}$ and $0 < \alpha < 1$,the following inequality involving the R-L difference holds:

$$ {}_a^R\nabla_n^\alpha x^T(n) P x(n) \leq 2x^T(n) P {}_a^R\nabla_n^\alpha x(n) \tag{7}$$

where $x(n) \in \mathbb{R}^k, k \in \mathbb{Z}_+, P \in \mathbb{R}^{k\times k}$ is a positive definite matrix.

Lemma 4. The linear nabla discrete fractional order system

$$ {}_a^R\nabla_n^\alpha x(n) = Ax(n) \tag{8}$$

is asymptotically stable if there exists a positive definite matrix $P \in \mathbb{R}^{k\times k}$ such that

$$A^T P + PA < 0 \tag{9}$$

Proof. Let $V(n) = x^T(n)Px(n)$ be a Lyapunov function for system (8).Computing the fractional difference with respect to system (8), while using inequality (7), then we obtain:

$$\begin{aligned} {}_a\nabla_k^\alpha v(n) &\le 2x^T(n)P_a\nabla_k^\alpha x(n) \\ &\le ({}_a\nabla_k^\alpha x(n))Px(n) + x^T P_a\nabla_k^\alpha x(n) \\ &= (Ax(n))^T Px(n) + x^T P(Ax(n)) \\ &= x^T(n)(PA + A^T P)x(n) < 0. \end{aligned} \tag{10}$$

According to Lemma 2,the equilibrium point of system (8) is asymptotically stable.

3. MAIN RESULTS

3.1 Stabilization of discrete linear nabla fractional order systems

Consider a discrete linear nabla fractional order system as follows:

$${}_a^R\nabla_n^\alpha x(n) = Ax(n) + Bu(n) \tag{11}$$

where $0 < \alpha < 1$, ∇^α is the R-L difference, $x(n) \in \mathbb{R}^k$ and $u(n) \in \mathbb{R}^m$ are the state and control input respectively, $A \in \mathbb{R}^{k\times k}$ and $B \in \mathbb{R}^{k\times m}$ are constant matrices.

A state-feedback controller is designed for the nabla fractional order system (11) as follows:

$$u(n) = Kx(n) \tag{12}$$

where K is the feedback gain to be determined later.

Now we give a main result to show under what condition the closed-loop system of system (11) under the controller (12) is asymptotically stable.

Theorem 1. Suppose that (A, B) is stabilizable. The linear nabla fractional system (11) is asymptotically stable under the state feedback control law (12) with the control gain given by $K = B^T P$, where P is the solution of the following inequality:

$$PA + A^T P + 2PBB^T P + \theta I_k < 0, \tag{13}$$

and θ is a positive number.

Proof.

Consider the Lyapunov function candidate:

$$V(n) = x^T(n)Px(n). \tag{14}$$

Applying Lemma 3, the fractional difference of Lyapunov function (14) along the trajectories of system (11) is given as follows:

$$\begin{aligned} {}_a\nabla_k^\alpha V(n) &\le 2x^T(n)P_a\nabla_k^\alpha x(n) \\ &= 2x^T(n)P(Ax(n) + BB^T Px(n)) \\ &= x^T(n)(PA + A^T P + 2PBB^T P)x(n) \\ &\le -\theta x^T(n)x(n) \\ &= -\theta \|x(n)\|^2. \end{aligned} \tag{15}$$

From Lemma 2, the equilibrium point of system (11) is asymptotically stable.

3.2 Consensus analysis of multi-agent systems

Consider a nabla fractional multi-agent system consisting of N agents and a leader. The dynamics of each agent is given by

$${}_a\nabla_k^\alpha x_i(n) = Ax_i(n) + Bu_i(n), i \in \{1, 2, \dots, N\} \tag{16}$$

where $0 < \alpha < 1$, ∇^α is the R-L difference, $x_i(n) \in \mathbb{R}^k$ and $u_i(n) \in \mathbb{R}^m$ are the state and control input of the i-th agent respectively, $A \in \mathbb{R}^{k\times k}$ and $B \in \mathbb{R}^{k\times m}$ are constant matrices. The leader dynamics is described as

$${}_a\nabla_k^\alpha x_0(n) = Ax_0(n), \tag{17}$$

where $0 < \alpha < 1$, $x_0(n) \in \mathbb{R}^k$.

Definition 4. The multi-agent system (16)-(17) achieves leader-following consensus if:

$$\lim_{n\to\infty} \|x_i(n) - x_0(n)\| = 0. \tag{18}$$

In order to achieve the leader-following consensus, we need to propose a distributed control law for each agent. By using the relative state information in the neighborhood, a distributed controllers $u_i(n)$ is designed for agent i as follows:

$$u_i(n) = K\big(\sum_{j\in\mathcal{N}_i} a_{ij}(x_j(n) - x_i(n)) + b_i(x_0(n) - x_i(n))\big) \tag{19}$$

where K is the control gain matrix to be designed later.

A main result is given to show that the closed-loop multi-agent system can be stabilized by the control law (19).

Theorem 2. Suppose that (A, B) is stabilizable and the interaction network $\overline{G}$ is connected. The leader-following consensus can be achieved for the linear nabla fractional multi-agent system (16)-(17) under the feedback control law (19) with the gain matrix given by $K = B^T P$, where P is the unique positive definite solution of the following Riccati inequality:

$$PA + A^T P - 2\beta PBB^T P + \beta I_k < 0 \tag{20}$$

and $\beta = \rho(H)$, $\rho(H)$ is the smallest nonzero eigenvalue of the matrix H.

Proof. Because (A, B) is stabilizable,then there exists a matrix $P > 0$ such that the Riccati inequakity holds(Ren et al. (2019)).

Let $e_i(n) = x_i(n) - x_0(n), i = 1, 2, \dots, N$, from (16)-(19), we have

$${}_a\nabla_k^\alpha e_i(n) = Ae_i(n) + \tag{21}$$

$$BK\big(\sum_{j\in\mathcal{N}_i} a_{ij}(e_j(n) - e_i(n)) - d_i e_i(n)\big). \tag{22}$$

Denote $e(n) = (e_1^T(n), \dots, e_N^T(n))^{\mathrm{T}}$. By using the Kronecker product and Lemma 1, (21) can be rewritten in the following compact form

$${}_a\nabla_k^\alpha e(n) = (I_N \otimes A - H \otimes BK)e(n). \tag{23}$$

We construct the following Lyapunov function candidate:

$$V(n) = \frac{1}{2}e^T(n)(I_N \otimes P)e(n), \tag{24}$$

where $P > 0$ is a solution of Riccati inequality (20).

Denote $\hat{A} = \frac{1}{2}(A^T P + PA)$ and $\hat{B} = PBB^T P$.With the help of Lemma 3, the α-th order fractional difference of $V(n)$ along the trajectory of system (23) is given by

$$
\begin{aligned}
{}_a\nabla_k^\alpha V(n) &\leq e^T(n)(I_N \otimes P)\nabla^\alpha e(n) \\
&\leq e^T(n)(I_N \otimes P)(I_N \otimes A - H \otimes BK)e(n) \\
&\leq e^T(n)((I_N \otimes \hat{A}) - (H \otimes PBB^T P))e(n) \\
&\leq e^T(n)(I_N \otimes \hat{A} - H \otimes \hat{B})e(n), \qquad (25)
\end{aligned}
$$

Since $H = \mathcal{L} + \overline{B}$ is a symmetric positive definite matrix, there exists an orthogonal matrix $T \in \mathbb{R}^{N\times N}$ such that

$$THT^T = \Lambda = diag(\lambda_1, \ldots, \lambda_N)$$

where $\lambda_1, \ldots, \lambda_N$ are the eigenvalues of matrix H. Let $\tilde{e}(n) = (T \otimes I_n)e(n)$, then the system (25) is transformed to

$$
\begin{aligned}
{}_a\nabla_k^\alpha V(n) &= \tilde{e}^T(n)(I_N \otimes \hat{A} - \Lambda \otimes \hat{B})\tilde{e}(n) \\
&= \sum_{i=1}^{N} \tilde{e}_i^T(n)(\hat{A} - \lambda_i PBB^T P)\tilde{e}_i(n) \\
&\leq \sum_{i=1}^{N} \tilde{e}_i^T(n)(\hat{A} - \beta PBB^T P)\tilde{e}_i(n) \\
&\leq -\frac{\beta}{2}\sum_{i=1}^{N} \tilde{e_i}^T(n)\tilde{e}(n)_i \\
&= -\frac{\beta}{2}e^T(n)e(n) \qquad (26)
\end{aligned}
$$

It is noticed that $V(n) \leq \frac{1}{2}\lambda_N(P)e^T(n)e(n)$, we have

$$ {}_a\nabla_k^\alpha V(n) \leq -\frac{\beta}{\lambda_{max}(P)}V(n) \qquad (27)$$

where $\lambda_{max}(P)$ is the largest eigenvalue of the matrix P. From Lemma 2, the system (23) is asymptotically stable about its equilibrium point, which implies that the leader-following consensus is achieved.

Remark 2. Even though we assume that the interaction network $\overline{G}$ is undirected in this paper, the Theorem 2 can also be extended to the case with balanced directed networks.

4. NUMERICAL SIMULATION

In this section,we give two examples to demonstrate the efficiency of the proposed method.

Example 1. Consider a linear nabla discrete fractional order system described by (11), where $0 < \alpha < 1$ and

$$A = \begin{bmatrix} -2 & -2 & -1 \\ -3 & -1 & -1 \\ 1 & 0 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ -1 & -1 \end{bmatrix},$$

It is not difficult to see that (A, B) is stabilizable(Wei et al. (2018)).Let $a = 0, \alpha = 0.98, x_1(1) = 2, x_2(1) = -1, x_3(1) = 0.5$. If we select $u(t) = 0$, Fig. 1 shows that the system is unstable.

By solving the inequality (13) by the YALMIP toolbox in MATLAB with $\theta = 0.2679$, we have

$$P = \begin{bmatrix} 0.1533 & -0.0821 & -0.0062 \\ -0.0821 & 0.1591 & -0.0106 \\ -0.0062 & -0.0106 & 0.0713 \end{bmatrix},$$

and further, the gain matrix is given by

$$K = \begin{bmatrix} -0.1471 & 0.0927 & -0.0651 \\ 0.0883 & -0.1486 & -0.0607 \end{bmatrix}.$$

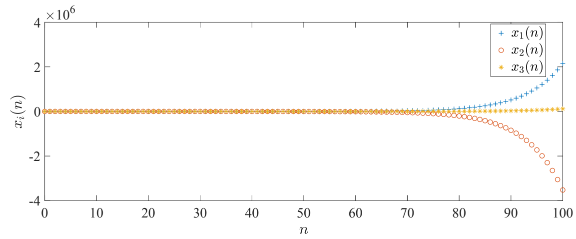


Fig. 1. State evolution of the nabla fractional system

Using the control law (12), the state trajectory of the closed-loop system (11) is shown in Fig. 2, which shows that the equilibrium is asymptotically stable.

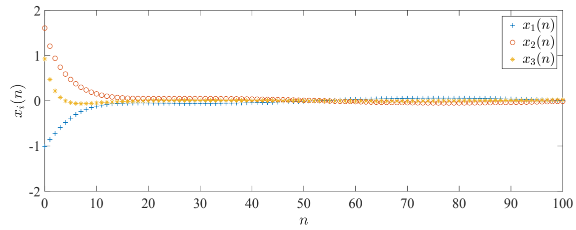


Fig. 2. State evolution of the system in (11) with $u(n) = Kx(n)$

Example 2. Consider a discrete nabla fractional order multi-agent system consisting of a leader and four followers. The interaction network topology is illustrated as in Fig. 3. Then the Laplacian matrix $\mathcal{L}$ and the leader

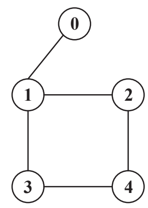


Fig. 3. The undireted topology of system (16)-(17)

adjacency matrix $\overline{B}$ are given as follows

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The agent dynamics are described by (16)-(17) with $0 < \alpha < 1$ and

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0.7 \\ 0.5 & 1 \end{bmatrix}.$$

It is easy to verify that (A, B) is stabilizable Wei et al. (2018) and according to a straightforward calculation shows that the smallest nonzero eigenvalue of $H = \mathcal{L} + \overline{B}$ is $\beta = 0.1864$.Then,by Solving the Riccati inequality (20) with YALMLP toolbox in MATLAB leads to

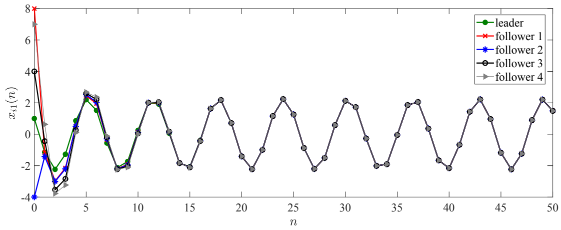
$$P = \begin{bmatrix} 0.1078 & -0.0244 \\ -0.0244 & 0.1399 \end{bmatrix},$$

and further,

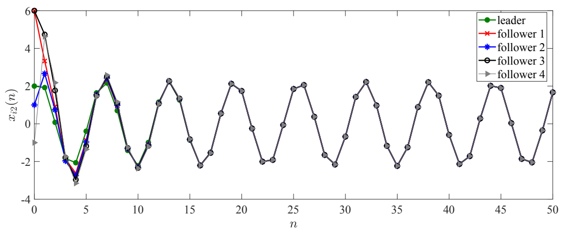
$$K = \begin{bmatrix} 0.0956 & 0.0456 \\ 0.0511 & 0.1228 \end{bmatrix}.$$

We take $a = 0, \alpha = 0.8$ and choose the initial condition as $x_0(1) = [1, 2]^T, x_1(1) = [8, 6]^T, x_2(1) = [-4, 1]^T, x_3(1) = [4, 6]^T, x_4(1) = [7, -1]^T$.

Under the proposed control law (19), the state trajectories, $x_i(n)$, are shown in Fig. 4 and the state error trajectories, $e_i(n) = x_i(n) - x_0(n)$ $(i = 1, 2, 3, 4)$, are shown in Fig. 5, respectively. The two simulation results show that the four agents can follow the leader.

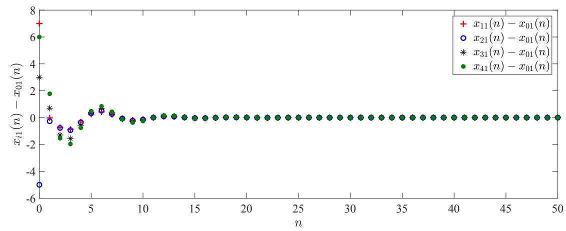


(a) $x_{i1}(n)$

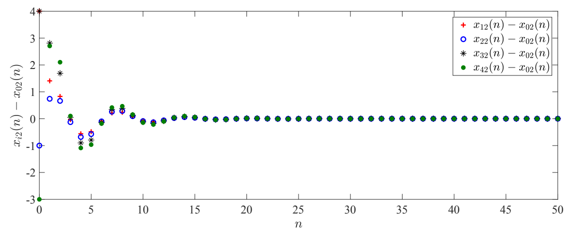


(b) $x_{i2}(n)$

Fig. 4. The state trajectories of the leader and the followers



(a)



(b)

Fig. 5. The error trajectories between the leader and the followers

5. CONCLUSION

In this paper, the consensus problem of a discrete linear nabla fractional order multi-agent system with Riemann-Liouville difference operator has been investigated. Firstly, the stabilization problem of discrete linear nabla fractional order system has been analyzed by using the discrete fractional Lyapunov direct method. Secondly, distributed controllers have been proposed for agents to realize the leader-following consensus for a discrete linear nabla fractional order multi-agent system through Riccati inequality. Finally, numerical simulation results have been presented to demonstrate the theoretical results. In the future, we will further consider the consensus control of fractional order multi-agent systems in some more practical cases with time-varying network topology and privacy preservation.

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