Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Path tracking and coordination control of multi-agent systems: a robust tube-based MPC scheme

Luigi D'Alfonso * **Giuseppe Fedele** ** **Giuseppe Franzè** **

* *GiPStech s.r.l., Rende (CS), Italy, (e-mail:l.dalfonso@gipstech.com)*
** *DIMES, University of Calabria, Italy (e-mail: {giuseppe.fedele,giuseppe.franze}@unical.it)*

Abstract: This paper addresses the reference tracking problem subject to formation constraints for a group of unmanned vehicles. A scheme based on receding horizon control ideas has been developed, whose the main feature consists in avoiding the need to explicitly impose non-convex constraints in the underlying optimization problem. The latter has been achieved by exploiting the properties provided by a novel description of the kinematic evolution when the agents are organized as a swarm. Numerical simulations on a team of five agents described by double integrator models are presented to show the effectiveness of the proposed control architecture.

Keywords: Multi-agent systems, Reference tracking, Swarm models, Model Predictive Control

1. INTRODUCTION

Multi-agent systems have attracted an increasing interest due to the natural occurrence of flocking and formation. In this respect, several contributions can be found in different areas: spacecraft formations, unmanned aerial vehicles, mobile robots, distributed sensor networks to cite a few, see e.g. Egerstedt and Hu (2001), Olfati-Saber et al. (2007). In this context, one of the most interesting and well-known problems concerns with the capability of the multi-agent system to track a given trajectory. The rationale behind this can be found in many examples. For the sake of simplicity, consider the following operating scenario arising in the unmanned vehicle field.
Assume that an operator (autonomous vehicle or human being) needs a support during search and rescue operations in an impervious and difficult to reach region. To facilitate this task, the operator periodically transmits samples of a safe trajectory to the group of vehicles sent by the central operating unit. Then, at each time instant the emergency vehicles receive the proper sampled information to be tracked as best as possible in order to safely accomplish the mission.

Although several approaches have been proposed in literature as testified by the recent survey Oh et al. (2015), a key computational aspect is still the subject of further investigations: *keeping the vehicle formation during the on-line operations by ensuring collision avoidance capabilities amongst the involved agents.*

In fact as it is well-known, these requirements are usually addressed by imposing the satisfaction of non-convex constraints that, in order to be computationally tractable, must be convexified via geometrical arguments such as *ad-hoc* inner approximation algorithms, see Dinh et al. (2012) and references therein. As easily perceivable, this leads to control performance degradations that could become significant within e.g. the unmanned vehicle field of applications (missions in hazardous environments, search and rescue operations, and so on).

By pointing out the attention to the distributed tracking problem, it has been extensively scattered in the literature by means of different methods, such as artificial potential fields Tanner et al. (2007), sliding-mode control Mirkin et al. (2012), adaptive control Peng et al. (2013), impulsive control Han et al. (2016) and output-feedback control Zhang et al. (2011). Nonetheless, a distributed model predictive control (DMPC) approach appears to be more appropriate Christofides et al. (2013) to deal with hard constraints, neighbour interactions and time-varying state references or multi-agent system topologies Cheng et al. (2014).

Starting from these premises and by referring to the multi-agent system as a swarm whose communication graph is undirected, connected and time-varying, the constrained tracking problem of interest consists in achieving the asymptotic convergence of the swarm centroid to a given reference signal.
To this end, an *ad-hoc* kinematic model of the swarm agents is derived whose main properties can be summarized as follows. Under the assumption that the current kinematic conditions are distant more than twice a pre-assigned positive quantity μ, one has that: **1)** at each time instant, agent kinematic trajectories are confined into non-intersecting hyper-balls centered at the corresponding current positions having the same radius μ (no collisions amongst the swarm agents may occur at the same time instant); **2)** given the hyper-ball of an agent at a certain time instant, say $\bar{t}$, it is guaranteed that starting from $\bar{t}$ a finite number of future reference trajectory samples remain confined within this hyper-ball.
Then, such properties allow to develop a distributed re-

ceding horizon control strategy where the local MPC controllers are built up according to:

- collision avoidance requirements addressed by imposing that the sequences of state predictions are jailed within those hyper-balls complying with the property **1)**;
- control horizon lengths computed by following the prescriptions of property **2)**.

One of the main merits of the resulting distributed algorithm concerns with the feasibility retention: if the current agent conditions do not satisfy the prerequisite on the minimum distance (at least greater than 2μ), then the controller considers the last admissible reference samples.

In conclusion, the overall controller architecture is based on two components: a path-planning unit whose reference state trajectories are provided by the kinematic swarm agent models; a distributed MPC algorithm whose single unit is adequately customized to the proposed framework starting from the results of Chisci et al. (2001).

The paper is organized as follows. Section 2 is devoted to state the proposed tracking problem. Section 3 presents a novel description of the swarm kinematic evolution and a brief analysis on its main properties. Then, the distributed model predictive control architecture is developed in Section 4, while Section 5 provides illustrative simulations. Finally, some remarks end the paper.

NOTATION

Let $\mathcal{G} = (\underline{L}, \mathcal{E})$ be a graph with $\underline{L} := \{1, \ldots, L\}$ nodes and $\mathcal{E}$ edges, i.e. $\mathcal{E} := \{(i,j) : i, j \in \underline{L}, i \neq j\}$. The graph $\mathcal{G}$ is connected if there exists a path from any node $i \in \underline{L}$ to any other node $j \in \underline{L} \setminus \{i\}$.

The hyper-ball of center ξ and radius μ is indicated as $\mathcal{B}(\xi, \mu)$.
With 0_n we denote the zero entries vector of $\mathbb{R}^n$, while I_n the $n \times n$ identity matrix.
Given a set $S \subseteq X \times Y \subseteq \mathbb{R}^n \times \mathbb{R}^m$, the projection of S onto X is defined as
$\text{Proj}_X(S) := \{x \in X \,|\, \exists y \in Y \text{ s.t. } (x, y) \in S\}$.

Definition 1. Given the sets $\mathcal{A}, \mathcal{E} \subset \mathbb{R}^n$, $\mathcal{A} \sim \mathcal{E} := \{a \in \mathcal{A} : a + e \in \mathcal{A}, \forall e \in \mathcal{E}\}$ the *Pontryagin-Minkowski Set Difference.* □

2. PROBLEM FORMULATION

Consider a group of L agents described by the following discrete-time state model:

$$x^i(t+1) = A^i x^i(t) + B^i u^i(t) + B^i_d d^i(t), \;\; i = 1, \ldots, L, \quad (1)$$

where $t \in \mathbb{Z}_+ := \{0, 1, \ldots, \}$, $x^i(t) = [{p^i}^T(t) \; {\bar{x}^i}^T(t)]^T \in \mathbb{R}^{n_x}$ denotes the state of the $i-th$ agent, with $p^i(t) \in \mathbb{R}^{n_p}$ the environment position vector, $u^i(t) \in \mathbb{R}^{n_u}$ the input vector and $d^i(t) \in \mathcal{D}^i$ is an exogenous bounded disturbance with $\mathcal{D}^i \subset \mathbb{R}^{n_d}$ a compact subset. Moreover, for each $i-th$ agent the following constraints are prescribed:

$$u^i(t) \in \mathcal{U}^i, \;\; \forall t \geq 0, \quad (2)$$

$$x^i(t) \in \mathcal{X}^i, \;\; \forall t \geq 0, \quad (3)$$

with $\mathcal{U}^i \subset \mathbb{R}^{n_u}$ and $\mathcal{X}^i \subset \mathbb{R}^{n_x}$ compact subsets.

Hereafter, it is assumed that the L agents (1) are organized as a swarm $SW := \{\Sigma^i\}_{i=1}^L$, whose the peculiarity is to track as best as possible a reference trajectory under formation requirements, see e.g. Lozano-Perez (2012).

Throughout the paper, the following hypotheses and definitions are exploited:

- *Reference trajectory* - It is assumed that an external agent, say Σ_{sup}, is in charge of correctly providing to each agent Σ^i, $i = 1, \ldots, L$, and at each time instant t, the same reference trajectory;
- *Vision module* - Agents are equipped with a vision module capable to detect neighbours within a prespecified radius $R_v \in \mathbb{R}^+$;
- *Communication topology* - The team of L agents are indexed by the elements of the set $\underline{L}$. The communication network is represented by a time-varying undirected and connected graph $\mathcal{G}_t = (\underline{L}, \mathcal{E}_t)$;
- *Neighbours* - The time-varying set of neighbours of each agent $\Sigma^i, i = 1, \ldots, L$, is:
$$\begin{aligned} \mathcal{N}^i_t := \{q \in \{1, \ldots, i-1, i+1, \ldots, L\} : \\ p^q(t) \in \mathcal{B}(p^i(t), R_v)\} \end{aligned} \quad (4)$$
- *Bidirectional property* - The edge $(i,j) \in \mathcal{N}^i_t$ iff $(j,i) \in \mathcal{N}^j_t$;
- *Swarm centroid* - At each time instant t
$$\zeta(t) \triangleq \frac{1}{L} \sum_{i=1}^{L} p^i(t) \quad (5)$$

Then, the problem to solve can be stated as follows.

Reference Tracking and Coordination (RTC) - Given the reference trajectory $r(t)$, $\forall t \geq 0$, determine a distributed state feedback policy

$$u^i(t) = g(x^i(t), \{x^k(t)\}, r(t)), \; k \in \mathcal{N}^i_t, \; i = 1, \ldots, L, \quad (6)$$

compatible with (2)-(3) and collision avoidance requirements such that, starting from an admissible initial condition $x(0) = [{x^1}^T(0), {x^2}^T(0), \ldots, {x^L}^T(0)]^T$ the swarm centroid $\zeta(t)$ asymptotically converges to $r(t)$. □

3. THE KINEMATIC MODEL

Consider a swarm SW, topologically characterized by the time-varying connected graph $\mathcal{G}_t = (\underline{L}, \mathcal{E}_t)$ and moving in a n_p-dimensional Euclidean space with positions $z^i(t)$, $i = 1, \ldots, L$. Agent evolutions are described by single-integrator models

$$\dot{z}^i(t) = h^i(t), \quad i = 1, \ldots, L. \quad (7)$$

Given a reference signal $r(t) \in \mathbb{R}^{n_p}$, the r.h.s. of (7) is selected as the combination of an attractive term to $r(t)$ and a hard limiting repulsion function:

$$\begin{aligned} h^i(t) = & -\alpha(z^i(t) - r(t)) + \dot{r}(t) \\ & - M \sum_{j \in \mathcal{N}^i_t} \frac{z^i(t) - z^j(t)}{\left(||z^i(t) - z^j(t)||^2 - 4\mu^2\right)^2}, \quad i = 1, \ldots, L, \end{aligned} \quad (8)$$

with $M \in \mathbb{R}^{n_p \times n_p}$ any full rank matrix and the scalar $\alpha > 0$ adequately chosen to improve as much as possible the capability to track $r(t)$. Notice that the bidirectional property of SW ensures that

$$\sum_{i=1}^{L}\sum_{j\in\mathcal{N}_t^i}\frac{z^i(t)-z^j(t)}{(||z^i(t)-z^j(t)||^2-4\mu^2)^2}=0_{n_p},\quad \forall t \tag{9}$$

and, in turn, the swarm centroid exponentially converges to $r(t)$ with a rate 2α as it results from

$$\dot{\zeta}(t)-\dot{r}(t)=-\alpha(c(t)-r(t)) \tag{10}$$

Then, the following statements straightforwardly come out.

Statement 1 - *If* $\|z^i(0)-z^j(0)\|>2\mu,\ \forall(i,j),\ j\neq i$, *then* $\|z^i(t)-z^j(t)\|>2\mu$ *holds true* $\forall t$ *and* $\forall(i,j),\ j\neq i$.

It is important to underline that the repulsion function (8) could lead to node disconnections within the graph $\mathcal{G}_t$. In fact at a certain time instant $\bar{t}$, if an agent Σ^i is such that

$$\mathcal{B}(p^i(\bar{t}),R_v)\cap\mathcal{B}(p^j(\bar{t}),R_v)=\emptyset,\ \ \forall j\in\underline{L}\setminus\{i\}, \tag{11}$$

one has $\mathcal{G}_{\bar{t}}=(\underline{L}\setminus\{i\},\mathcal{E}_{\bar{t}})$. In this case, the i-th agent becomes a *singleton*, i.e. $\mathcal{N}_t^i=\emptyset$, and the model (7)-(8) reduces to

$$\dot{z}^i(t)=-\alpha(z^i(t)-r(t))+\dot{r}(t) \tag{12}$$

Nonetheless, notice that (12) ensures that Σ^i will converge to $r(t)$ and, as a consequence, it will again interact with the connected graph as soon as its distance from $\mathcal{G}_t$ is less than μ.

Statement 2 - *Let* T_s *be the sampling time. There exists an integer* $k_M^i\geq 0$ *such that*

$$k_M^i=\max_k\left\{||p^i(t)-z^i(t+k)||\leq\mu\right\},\ i=1,\ldots,L. \tag{13}$$

where $k\triangleq q*T_s$.

The latter has the following meaning: the sequence of kinematic samples $\{z^i(t+k)\}_{k=0}^{k_M^i}$ of each agent Σ^i strictly lies within the corresponding hyper-ball $\mathcal{B}(p^i(t),\mu)$. The rationale behind (13) can be illustrated by means of the sketch in Fig. 1. There, two agents Σ^i and Σ^j move within a planar environment by tracking their kinematic samples $\{z^i(t+k)\}_{k=0}^{1}\subset\mathcal{B}(p^i(t),\mu)$ and $\{z^j(t+k)\}_{k=0}^{2}\subset\mathcal{B}(p^j(t),\mu)$, respectively. The number of usable reference signal samples could be different for each involved agent. According to this, it is guaranteed that no collisions amongst neighbours can occur.

4. A DISTRIBUTED MPC ARCHITECTURE

In this section, the swarm kinematics properties are exploited in order to derive a computable MPC scheme capable to comply with the **RTC** problem.

4.1 Basic MPC unit

By recalling that:

- the constraint sets $\mathcal{X}^i$ and $\mathcal{U}^i$ in (2)-(3) are compact, convex and containing the origin in their interiors;
- the pair (A^i,B^i) is stabilizable;
- there exists a stabilizing state feedback law $u^i(t)=F^ix^i(t)$ for the unconstrained and disturbance-free model (1) such that the closed-loop matrix $\Phi^i:=A^i+B^iF^i$ is Schur,

the approach of Chisci et al. (2001) will be considered to deal with disturbance effects.

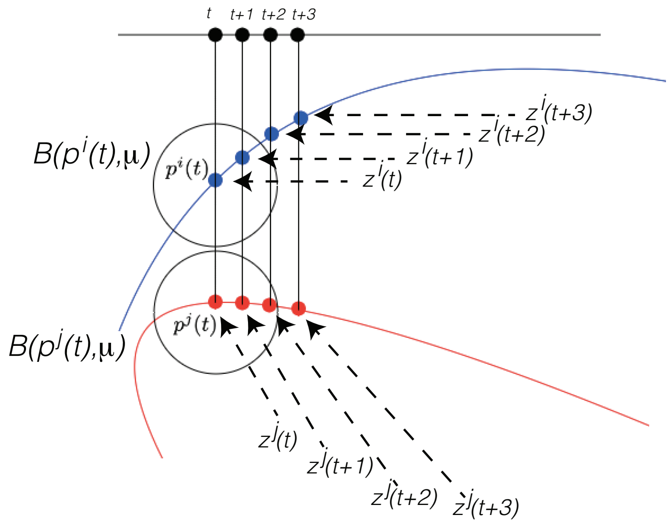

Fig. 1. Swarm agents: the kinematic evolution

Specifically, the family of virtual commands $\mathbf{u^i}^{MPC}(t)$ is parametrized as follows:

$$u^{i^{MPC}}(t+k|t)=F^ix^i(t+k|t)+c^i(t+k|t),\ \forall k\geq 0, \tag{14}$$

with $c^i(t+k|t)$ a perturbation with respect to the nominal feedback $F^ix^i(t+k|t)$. Let $\mathbf{c^i}(t)\triangleq\{c^{i^T}(t|t),\ldots,c^{i^T}(t+N^i-1|t)\}$, then by considering the following quadratic cost:

$$J_{N^i}(t):=\sum_{k=0}^{\infty}c^{i^T}(t+k|t)\Psi c^i(t+k|t),\ \ \Psi^T=\Psi>0, \tag{15}$$

at each time instant t a robust MPC solution complying with persistent disturbances is:

$$\mathbf{c^i}^*(t+k|t)\triangleq\arg\min_{\mathbf{c^i}(t)}J_{N^i}(t) \tag{16}$$

subject to

$$F^ix^i(t+k|t)+c^i(t+k|t)\in\mathcal{U}_k^i,\ \ k=0,1,\ldots,N^i-1, \tag{17}$$

$$x^i(t+k|t)\in\mathcal{X}_k^i,\ \ k=0,1,\ldots,N^i-1, \tag{18}$$

$$x^i(t+N^i|t)\in\Xi_0^i\sim\sum_{q=0}^{N^i-1}\left(\Phi^i\right)^qB_d^i\mathcal{D}^i, \tag{19}$$

$$c^i(t+k|t)=0,\ \ k\geq N^i, \tag{20}$$

where

$$\mathcal{X}_k^i\triangleq\mathcal{X}\sim\sum_{q=0}^{k-1}\left(\Phi^i\right)^qB_d^i\mathcal{D}^i \tag{21}$$

$$\mathcal{U}_k^i\triangleq\mathcal{U}^i\sim\sum_{q=0}^{k-1}F\left(\Phi^i\right)^qB_d^i\mathcal{D}^i \tag{22}$$

and Ξ_0^i is the *maximal output admissible set* (the largest d-invariant set) Gilbert et al. (1995):

$$\Xi_0^i\triangleq\{x:\Phi^ix\in\mathcal{X}_k^i,\ F\Phi^ix\in\mathcal{U}_k^i,\forall k=0,1,\ldots,k_0^i\} \tag{23}$$

with the positive integer k_0^i computed by using the procedure developed in Gilbert and Tan (1991).

4.2 The distributed algorithm

The following two considerations open the doors to a distributed formulation for solving the **RTC** problem:

(1) the set-membership property of **Statement 1** ensures that along the kinematics state trajectory tube no collisions occur when the agents jointly move;
(2) **Statement 2** proves that, given the hyper-ball $\mathcal{B}(p^i(t),\mu)$, there exists an integer k_M^i such that the future kinematic samples $z^i(t+k) \in \mathcal{B}(p^i(t),\mu)$, $\forall k = 1,\ldots,k_M^i$.

In order to simplify the next developments, these assumptions are made:

Assumption 1 - All the agents Σ^i receive the sequence of reference signal samples at the same time instant. □

Assumption 2 - At each time instant t, data are instantaneously shared amongst the agents of the graph $\mathcal{G}_t$. □

Moreover, the vision module is subject to the following constraint $R_v \geq 2\mu$, so that the **Statement 1** prescriptions can be evaluated during the on-line operations.

The control architecture for each agent consists of two ingredients: a **Path Planner** algorithm and a **MPC controller** each tuned with respect to the agent Σ^i, $i = 1,\ldots,L$. Then, the idea can be summarized as follows:

(1) at each time instant t, the connected graph $\mathcal{G}_t$ is updated by using the set-membership condition (11);
(2) according to **Statement 1**, each agent Σ^i evaluates $\|z^i(t) - z^j(t)\|, \forall j \in \mathcal{N}_t^i$ and instantaneously shares the result with its neighbours;
(3) at each time instant t, all the agents Σ^i, $i = 1,\ldots,L$, jointly receive the reference signal sample $r(t)$ from Σ_{sup};
(4) the new sample $r(t)$ will be exploited to generate the kinematic state trajectories $z^i(\cdot)$ by means of (7)-(8) if and only if $\|z^i(t) - z^j(t)\| > 2\mu$, $\forall (i,j), j \neq i$; otherwise the sample $r(t)$ is discarded and the old data $r(t-1)$ is kept without updating $z^i(\cdot)$;
(5) the **MPC controller** computes the admissible control actions $u^i(t)$ in a distributed receding horizon fashion.

Such an abstract procedure can be formally recast into a computable algorithm. To this end, the first question relies on which is the control horizon length of the input sequence parametrization (14) pertaining to the local MPC controller. Notice that at each time instant the **Statement 2** provides the number of admissible kinematics samples $z^i(\cdot)$; then the control horizon length for the MPC optimization can be chosen exactly equal to k_M^i, because it is ensured that the related kinematic evolution remains confined within $\mathcal{B}(p^i(t),\mu)$. As a *quid pro quo*, this could give rise to the following consequences:

- different values: $N^i \neq N^j, \forall i \neq j$;
- time dependency: $N^i(t), \forall i$.

As a matter of fact, the time-varying nature of the control horizon length could make infeasible the resulting MPC strategy. To formally overcome such an hitch and according to the robust Bellman optimality principle Mayne (2001), the following inequalities has to be satisfied:

$$N^i(0) \geq N^i(1) \geq \cdots \geq N^i(t), \forall t \geq 0, i = 1,\ldots,L. \quad (24)$$

A second aspect to address concerns with the computations of k_M^i, $i = 1,\ldots,L$. This can be straightforwardly done by using the above condition (24). Accordingly, a set of buffer units of length $N^i(0)$ is used and the new reference sample $r(t)$ is there stored by discarding the oldest data (a *first-input-first-output* strategy). Then, the integers k_M^i, $i = 1,\ldots,L$, are easily computed by evaluating the set-membership argument of (13).

Finally, the optimization of Section 4.1 must be customized according to the above developments that prescribe:

(1) the $k-$steps ahead state predictions must be subsets of the hyper-ball $\mathcal{B}(p^i(t),\mu)$, $k = 0,\ldots,N^i(t), \forall t \geq 0$;
(2) a time-varying framework in the involved variables comes out.

As a consequence, the optimization (15)-(20) is re-written as follows:

DMPC-$\mathcal{P}^i(t)$:

$$\mathbf{c^i}^*(t+k|t) \triangleq \arg\min_{\mathbf{c}^i(t)} J_{N^i(t)}(t) \quad (25)$$

subject to

$$F^i(t)x^i(t+k|t) + c^i(t+k|t) \in \mathcal{U}_k^i, k = 0,1,\ldots,N^i(t)-1 \quad (26)$$

$$p^i(t+k|t) \in Pro_{p^i}\left(\mathcal{X}_k^i\right) \subset \mathcal{B}(p^i(t),\mu), k = 0,1,\ldots,N^i(t)-1 \quad (27)$$

$$\bar{x}^i(t+k|t) \in Pro_{\bar{x}^i}\left(\mathcal{X}_k^i\right), \ k = 0,1,\ldots,N^i(t)-1 \quad (28)$$

$$p^i(t+N^i(t)|t) \in Pro_{p^i}\left(\Xi_0^i(t) \sim \sum_{q=0}^{N^i(t)-1} \left(\Phi^i\right)^q B_d^i \mathcal{D}^i\right) \subset \mathcal{B}(p^i(t),\mu) \quad (29)$$

$$\bar{x}^i(t+N^i(t)|t) \in Pro_{\bar{x}^i}\left(\Xi_0^i(t) \sim \sum_{i=0}^{N^i(t)-1} \left(\Phi^i\right)^q B_d^i \mathcal{D}^i\right) \quad (30)$$

$$c^i(t+k|t) = 0, \ k \geq N^i(t) \quad (31)$$

For the sake of simplicity, assume that the following information are available: the initial control horizon lengths $N^i(0)$, $i = 1,\ldots,L$; initial pairs $(\Xi_0^i(0), F^i(0))$; the connected graph $\mathcal{G}_0$. Moreover, it is hypothesized without loss of generality that the **Buffer-**Σ^i are initialized according to $N^i(0), i = 1,\ldots,L$.

Then, the above developments translate into the following algorithm.

Swarm-DMPC-Algorithm - Agent $i-th$

Input: $\mathcal{G}_t$, R_v, $x^i(t|t)$, $\{x^j(t|t)\}_{\forall j \in \mathcal{G}_t}$;
Output: $u^{i^*}(t|t)$;
1: **Update** $\mathcal{G}_t$;
2: **Receive** $r(t)$ from Σ_{sup};
3: **Update** the unit **Buffer-**Σ^i
4: **if** $i \in \mathcal{G}_t$ **then** **CONNECTED AGENT**

5: **Evaluate** the euclidean distances
$$d_{ij} := \|p^i(t) - p^j(t)\|, \forall (i,j) \in \mathcal{G}_t;$$
6: **if** $d_{ij} > 2\mu, \forall (i,j) \in \mathcal{G}_t$, **then Activate**==true;
7: **end if**
8: **if Activate**==false **then Store** $r(t)$
9: **Solve** the optimization **DMPC**-$\mathcal{P}^i(t)$, by considering $(\Xi_0^i(t-1), F^i(t-1))$;
10: **Apply** $u^{i^*}(t|t) = F^i(t-1)x^i(t|t) + c^{i^*}(t|t)$;
11: **Goto** Step 26
12: **end if**
13: **Determine** $N^i(t)$ via (13) under the satisfaction of (24);
14: **Generate** $z^i(t+k), k = 0, 1, \ldots, N^i(t)$, by means of (7)-(8);
15: **Update** the pair $(\Xi_0^i(t), F^i(t))$ with respect to the current state condition $x^i(t)$;
16: **Solve** the optimization **DMPC**-$\mathcal{P}^i(t)$;
17: **Apply** $u^{i^*}(t|t) = F^i(t)x^i(t|t) + c^{i^*}(t|t)$;
18: **Send** $x^i(t+1)$ to all the connected agents in $\mathcal{G}_t$;
19: **else** **NOT CONNECTED AGENT**
20: **Determine** $N^i(t)$ via (13) and subject to (24);
21: **Generate** $z^i(t+k), k = 0, 1, \ldots, N^i(t)$, by means of (7)-(8);
22: **Update** the pair $(\Xi_0^i(t), F^i(t))$ with respect to the current state condition $x^i(t)$;
23: **Solve** the optimization **DMPC**-$\mathcal{P}^i(t)$;
24: **Apply** $u^{i^*}(t|t) = F^i(t)x^i(t|t) + c^{i^*}(t|t)$;
25: **end if**
26: $t \leftarrow t+1$ and goto **Step 1**.

Feasibility and closed-loop stability of the **Swarm-DMPC** Algorithm are stated in the next proposition.

Proposition 1. Let $x(0)$ be given. Then, the **Swarm-DMPC** *Algorithm always satisfies the prescribed constraints, the swarm centroid $\zeta(t)$ asymptotically tracks the reference signal $r(t)$ and ensures that the closed-loop state trajectories are asymptotically stable.*

Proof - The recursive feasibility property of the **Swarm-DMPC** Algorithm is proved by investigating the admissibility at each time instant t of the optimization problem **DMPC**-$\mathcal{P}^i(t)$.

By referring to the connected agents, two scenarios are considered. Under a *normal operating phase*, i.e. the prescriptions of **Statement 1** are always satisfied, the feasibility arises from the fact that if an optimal solution there exists at t, then at the next time instant the d-invariance property of $\Xi_0^i(t)$ ensures that the state trajectory is at least confined within $\Xi_0^i(t)$ and, by construction, the transition to the new hyper-ball $\mathcal{B}(p^i(t+1), \mu)$ is guaranteed. Conversely if there exists at least a pair $(i,j), i, j \in \mathcal{G}_t$ such that $d_{ij} < 2\mu$, an *anomalous scenario* results. In this case, even if the received reference sample $r(t)$ cannot be exploited for updating the hyper-ball $\mathcal{B}(p^i(t), \mu)$, the pair $(\Xi_0^i(t-1), F^i(t-1))$ is still admissible at the current time instant. Then, the optimization **DMPC**-$\mathcal{P}^i(t)$ has always a solution because the state trajectory could remain indefinitely jailed in $\Xi_0^i(t-1)$.

Notice that at each time instant the not-connected agents operate in a completely decentralized fashion and, therefore, feasibility retention can be simply proved.

The closed-loop asymptotic stability also comes out by retracing the same reasoning and it is omitted for the sake of brevity. □

5. SIMULATIONS

In this section, the effectiveness of the proposed strategy is evaluated by considering a team of five autonomous vehicles described by double integrator models and discretized via a forward Euler method with a sampling time $T_s = 0.8\,[s]$. The agent state is $x^i = [p_x^i, p_y^i, v_x^i, v_y^i]^T \in \mathbb{R}^4$ and the following point-wise input constraints are prescribed: $|u_x^i(t)| \leq 3[m/s^2],\ |u_y^i(t)| \leq 3[m/s^2], i = 1, \ldots, L,\ \forall t \geq 0.$

The **RTC** problem has been faced by referring to the signal $r(t) = [t, 5\sin(0.1t)]^T$. By assuming the initial topology of Fig. 2 and under the initial conditions $p^1(0) = -0.0300, 0.0306]^T$, $p^2(0) = [2.5306, -4.0604]^T$, $p^3(0) = [0.5656, -9.5658]^T$, $p^4(0) = [-4.4304, -9.1734]^T$, $p^5(0) = [-3.4636, -14.3047]^T$, $v^i(0) = [0, 0]^T$, $i = 1, ..., 5$, the agent kinematics is derived by using the following knobs: $\alpha = 1$, $M = -50\, I_2$, $\mu = 2[m]$ and $R_v = 6[m]$.

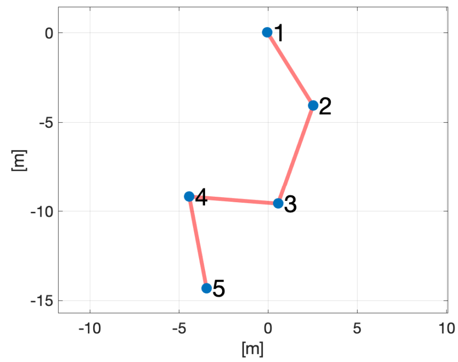

Fig. 2. Initial swarm topology

All the relevant numerical results have been collected in Figs. 3-5. First, notice that the prescriptions of the **RTC** problem have been satisfied: the swarm centroid asymptotically tracks the reference $r(t)$, Figs. 3-4; the saturation constraints are fulfilled, Fig. 5.

By analysing the snapshots of Fig. 3, it is interesting to underline that, as expected, the swarm formation is kept at each time instant, while the control horizon lengths of each local MPC optimizations change according to the feasibility condition (24): starting from $N^i(0) = 3, i = 1, \ldots, 5$ until $N^i(t) = 1, i = 1, \ldots, 5, \forall t \geq 39.24\,[s]$. Finally, in Fig. 3 it is also shown the time-varying nature of the swarm topology, where the connections amongst the five agents are explicitly reported.

6. CONCLUSIONS

In this paper, a distributed model predictive control strategy has been proposed for addressing tracking problems for a class of multi-agent systems. The main effort was devoted to mitigate as much as possible the occurrence of non-convex constraints when formation requirements are of interest. This has been formally achieved by harmonizing into a unique framework the properties deriving from

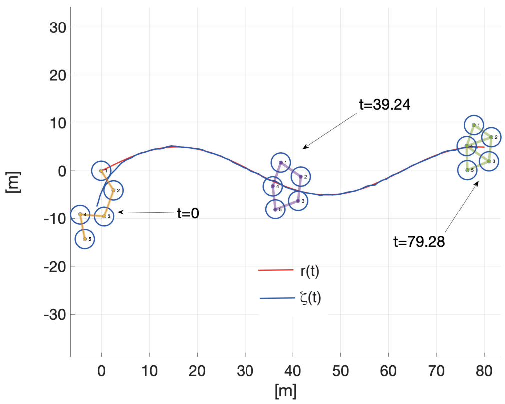

Fig. 3. Swarm centroid vs reference signal

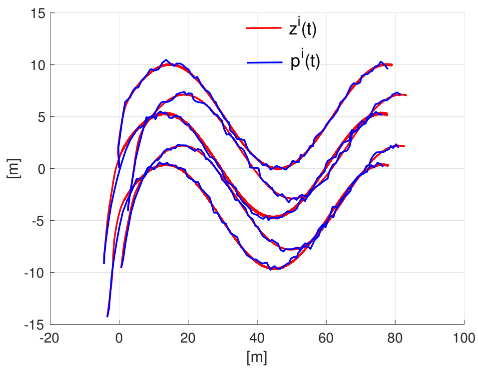

Fig. 4. Kinematic and dynamical behaviours

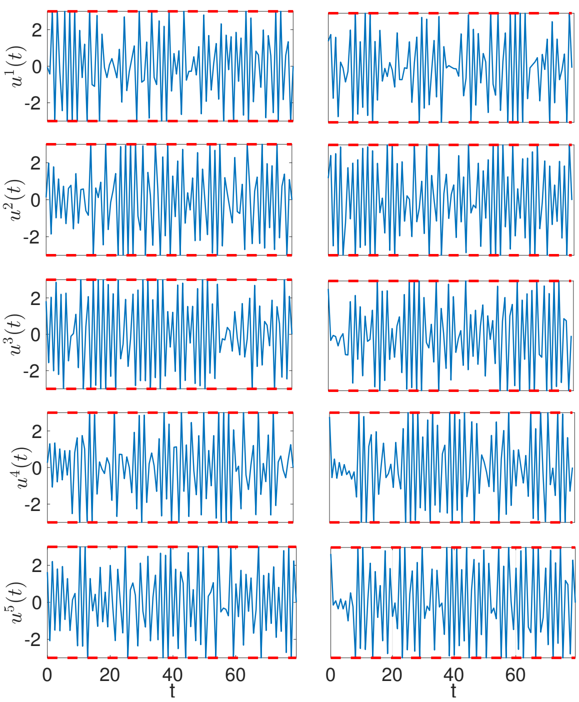

Fig. 5. Command input evolutions. The dashed lines refer to the prescribed constraints

a novel description of the kinematic evolution with the capabilities of the robust MPC philosophy. Finally, some simulations have been carried out with the aim to show effectiveness and benefits of the proposed approach.

REFERENCES

Cheng, Z., Zhang, H.T., Fan, M.C., and Chen, G. (2014). Distributed consensus of multi-agent systems with input constraints: A model predictive control approach. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 62(3), 825–834.

Chisci, L., Rossiter, J.A., and Zappa, G. (2001). Systems with persistent disturbances: predictive control with restricted constraints. *Automatica*, 37(7), 1019–1028.

Christofides, P.D., Scattolini, R., de la Pena, D.M., and Liu, J. (2013). Distributed model predictive control: A tutorial review and future research directions. *Computers & Chemical Engineering*, 51, 21–41.

Dinh, Q.T., Michiels, W., Gros, S., and Diehl, M. (2012). An inner convex approximation algorithm for bmi optimization and applications in control. In *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)*, 3576–3581. IEEE.

Egerstedt, M. and Hu, X. (2001). Formation constrained multi-agent control. *IEEE transactions on robotics and automation*, 17(6), 947–951.

Gilbert, E.G., Kolmanovsky, I., and Tan, K.T. (1995). Discrete-time reference governors and the nonlinear control of systems with state and control constraints. *International Journal of robust and nonlinear control*, 5(5), 487–504.

Gilbert, E.G. and Tan, K.T. (1991). Linear systems with state and control constraints: The theory and application of maximal output admissible sets. *IEEE Transactions on Automatic control*, 36(9), 1008–1020.

Han, G.S., Guan, Z.H., Li, J., He, D.X., and Zheng, D.F. (2016). Multi-tracking of second-order multi-agent systems using impulsive control. *Nonlinear Dynamics*, 84(3), 1771–1781.

Lozano-Perez, T. (2012). *Autonomous robot vehicles*. Springer Science & Business Media.

Mayne, D.Q. (2001). Control of constrained dynamic systems. *European Journal of Control*, 7(2-3), 87–99.

Mirkin, B., Gutman, P.O., and Shtessel, Y. (2012). Asymptotic sliding mode control approach to adaptive distributed tracking problem for multi-agent nonlinear delayed systems. *International Journal of Control*, 85(11), 1671–1682.

Oh, K.K., Park, M.C., and Ahn, H.S. (2015). A survey of multi-agent formation control. *Automatica*, 53, 424–440.

Olfati-Saber, R., Fax, J.A., and Murray, R.M. (2007). Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, 95(1), 215–233.

Peng, Z., Wang, D., Zhang, H., Sun, G., and Wang, H. (2013). Distributed model reference adaptive control for cooperative tracking of uncertain dynamical multi-agent systems. *IET Control Theory & Applications*, 7(8), 1079–1087.

Tanner, H.G., Jadbabaie, A., and Pappas, G.J. (2007). Flocking in fixed and switching networks. *IEEE Transactions on Automatic control*, 52(5), 863–868.

Zhang, H., Lewis, F.L., and Das, A. (2011). Optimal design for synchronization of cooperative systems: state feedback, observer and output feedback. *IEEE Transactions on Automatic Control*, 56(8), 1948–1952.