Distributed Diffusion Unscented Kalman Filtering Algorithm with Application to Object Tracking ⋆

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Abstract: In this paper, a distributed diffusion unscented Kalman filtering algorithm based on covariance intersection strategy (DDUKF-CI) is proposed for object tracking. By virtue of the pseudo measurement matrix, the standard unscented Kalman filtering (UKF) is transformed to the information form that can be fused by the diffusion strategy. Then, intermediate information from neighbors are fused based on the diffusion framework to attain better estimation performance. Considering the unknown correlations in sensor networks, covariance intersection (CI) strategy is combined with the diffusion algorithm. Moreover, it is proved that the estimation error of the proposed DDUKF-CI is exponentially bounded in mean square using the stochastic stability theory. Finally, the performances of the proposed algorithm and the weighted average consensus unscented Kalman filtering (CUKF) are compared in a target tracking problem with a sensor network.

Keywords: Distributed diffusion nonlinear filtering; UKF; Covariance intersection.

1. INTRODUCTION

In recent years, wireless sensor networks have attracted considerable attention on account of a wide range of engineering applications, such as collaborative surveillance, environment monitoring, target tracking and image processing Zhao and Guibas (2004). Distributed state estimation (DSE) is one of the fundamental problems in wireless sensor networks, which can be performed in either consensus or diffusion setting. However, consensus based methods impose timing constraints in that the consensus step should be executed faster than Kalman filtering operation in the network because of the consensus iteration Talebi and Werner (2018). When the fusion step is required to be operated on the same time scale as the Kalman filtering step, the diffusion strategy is practically more efficient Jia et al. (2017). Diffusion Kalman filtering was proposed in Cattivelli and Sayed (2010). The algorithm utilizes the local Kalman filter in Olfati-Saber (2007) to compute the prior estimate with the information from neighbors, then as opposed to the iterative consensus method, a convex combination of the prior estimates of the neighbors, was applied. Compared with the consensus strategy, the diffusion strategy can carry out real-time fusion with the measurement streaming in and has faster convergence speed and lower mean-square error Tu and Sayed (2012). Based on the CI method, Hu et al. (2012) proposed a diffusion Kalman filtering algorithm that can guarantee the stability of estimates regardless of the locally uniform observability by the measurements of its neighbors. Vahidpour et al. (2019) proposed diffusion algorithms that allow partial exchanging of the entries of the intermediate estimates among neighbors to reach a trade-off between estimation accuracy and communication cost. To avoid sharing raw measurements among neighbors, Wang et al. (2017) derived an algorithm to calculate the local estimate only using individual estimates rather than individual measurements from neighbors. Lorenzo (2014) enabled the distributed diffusion filtering in the Markov system.

This paper proposes a novel distributed diffusion nonlinear filtering algorithm based on CI over a sensor network. The main contributions are threefold: (1) To the best of our knowledge, most of the existing literature about diffusion filtering merely consider the linear system. The diffusion framework proposed in this paper aims at nonlinear systems. (2) The common method of proving stability of the estimation error is to show that the error covariance matrix recursion converges to the solution of a Lyapunov equation, which, however, requires strong assumptions such as the linear time-invariant model. In this paper, we employ the stochastic stability theory to prove the stability without these assumptions. (3) Compared with Hu et al. (2012), we not only consider the nonlinear system but also prove that the estimation error in this work is exponentially bounded in the mean square.

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The rest of the paper is outlined as follows: Section 2 introduces the system model and some necessary preliminaries. The DDUKF-CI algorithm is presented in Section 3. The stability analysis is provided in Section 4. Numerical simulations are presented to show the effectiveness of the proposed algorithm in Section 5. Finally, Section 6 gives the conclusion remarks.

2. PROBLEM STATEMENT AND PRELIMINARIES

2.1 System model

A discrete-time dynamic system with an N-sensor network in which each sensor possesses the capacity of sensing, communication and computation can be described by

$$\begin{cases} \boldsymbol{x}_k = \boldsymbol{f}(\boldsymbol{x}_{k-1}) + \boldsymbol{w}_{k-1} \\ \boldsymbol{z}_k^i = \boldsymbol{h}^i(\boldsymbol{x}_k) + \boldsymbol{v}_k^i \end{cases} \quad i \in \mathcal{N}, \tag{1}$$

where $\boldsymbol{x}_k \in \mathbb{R}^n$ is the state vector at discrete-time instant k, $\boldsymbol{z}_k^i \in \mathbb{R}^m$ is the measurement vector of the ith sensor. The process noise $\boldsymbol{w}_{k-1}$ and ith sensor's measurement noise $\boldsymbol{v}_k^i$ are mutually uncorrelated zero-mean white Gaussian processes with covariance matrices Q_{k-1} and R_k^i, respectively. $\boldsymbol{f}: \mathbb{R}^n \to \mathbb{R}^n$ describes the nonlinear state transition function and $\boldsymbol{h}^i : \mathbb{R}^n \to \mathbb{R}^m$ describes the nonlinear measurement function of the ith sensor, respectively.

2.2 Network communication topology

In the following, the network communication topology is denoted by an undirected graph $(\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ denotes the set of all sensor nodes in the network and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the edge set. $N = |\mathcal{N}|$. Node j receiving data from node i is equivalent to $(i,j) \in \mathcal{E}$. Furthermore, for each node $i \in \mathcal{N}$, $\mathcal{N}_i = \{j|(j,i) \in \mathcal{E}\} \cup \{i\}$ denotes the adjacent nodes of node i and itself.

The tracking problem investigated in this work is to estimate the state $\boldsymbol{x}_k$ from multiple sensor observations $\boldsymbol{z}_k^i$ using the diffusion strategy.

2.3 Transformation of UKF

The UKF algorithm was summarized in Li et al. (2016). The equations will be used directly in this paper. In order to derive the proper form that can be used in the diffusion framework, we introduce the pseudo measurement matrix $\mathcal{H}_k^i$ Li et al. (2016) defined by

$$\mathcal{H}_k^i = \left(P_{\boldsymbol{x}_k \boldsymbol{z}_k}^i\right)^T \left(P_{k|k-1}^i\right)^{-1} \tag{2}$$

According to matrix inversion lemma, it can be derived that

$$\begin{aligned}(P_k^i)^{-1} =& (P_{k|k-1}^i)^{-1} + (P_{k|k-1}^i)^{-1} K_k^i ((P_{\boldsymbol{z}_k \boldsymbol{z}_k}^i)^{-1} - \\ & (K_k^i)^T (P_{k|k-1}^i)^{-1} K_k^i)^{-1} (K_k^i)^T (P_{k|k-1}^i)^{-1} \\ =& (P_{k|k-1}^i)^{-1} + (\mathcal{H}_k^i)^T (P_{\boldsymbol{z}_k \boldsymbol{z}_k}^i - \mathcal{H}_k^i P_{\boldsymbol{x}_k \boldsymbol{z}_k}^i)^{-1} \mathcal{H}_k^i \\ =& (P_{k|k-1}^i)^{-1} + (\mathcal{H}_k^i)^T (\mathcal{R}_k^i)^{-1} \mathcal{H}_k^i \end{aligned} \tag{3}$$

where

$$\mathcal{R}_k^i = P_{\boldsymbol{z}_k \boldsymbol{z}_k}^i - \mathcal{H}_k^i P_{\boldsymbol{x}_k \boldsymbol{z}_k}^i \tag{4}$$

Next, the equivalent form of K_k^i is given as

$$\begin{aligned} K_k^i =& P_{\boldsymbol{x}_k \boldsymbol{z}_k}^i (P_{\boldsymbol{z}_k \boldsymbol{z}_k}^i)^{-1} \\ =& P_k^i ((P_{k|k-1}^i)^{-1} + (\mathcal{H}_k^i)^T (P_{\boldsymbol{z}_k \boldsymbol{z}_k}^i - \mathcal{H}_k^i P_{\boldsymbol{x}_k \boldsymbol{z}_k}^i)^{-1} \mathcal{H}_k^i) \\ & P_{\boldsymbol{x}_k \boldsymbol{z}_k}^i (P_{\boldsymbol{z}_k \boldsymbol{z}_k}^i)^{-1} \\ =& P_k^i ((\mathcal{H}_k^i)^T + (\mathcal{H}_k^i)^T (P_{\boldsymbol{z}_k \boldsymbol{z}_k}^i - \mathcal{H}_k^i P_{\boldsymbol{x}_k \boldsymbol{z}_k}^i)^{-1} \mathcal{H}_k^i P_{\boldsymbol{x}_k \boldsymbol{z}_k}^i) \\ & (P_{\boldsymbol{z}_k \boldsymbol{z}_k}^i)^{-1} \\ =& P_k^i (\mathcal{H}_k^i)^T (P_{\boldsymbol{z}_k \boldsymbol{z}_k}^i - \mathcal{H}_k^i P_{\boldsymbol{x}_k \boldsymbol{z}_k}^i)^{-1} (P_{\boldsymbol{z}_k \boldsymbol{z}_k}^i - \mathcal{H}_k^i P_{\boldsymbol{x}_k \boldsymbol{z}_k} \\ & + \mathcal{H}_k^i P_{\boldsymbol{x}_k \boldsymbol{z}_k}^i)(P_{\boldsymbol{z}_k \boldsymbol{z}_k}^i)^{-1} \\ =& P_k^i (\mathcal{H}_k^i)^T (\mathcal{R}_k^i)^{-1} \end{aligned} \tag{5}$$

Then the estimate can be calculated as

$$\hat{\boldsymbol{x}}_k^i = \hat{\boldsymbol{x}}_{k|k-1}^i + P_k^i (\mathcal{H}_k^i)^T (\mathcal{R}_k^i)^{-1} (\boldsymbol{z}_k^i - \hat{\boldsymbol{z}}_k^i) \tag{6}$$

Remark 1. The purpose of this transformation is to make the UKF possess the same structure as the linear Kalman filter used in Cattivelli and Sayed (2010) so as to determine the expression of the quantities that should be exchanged between neighbours and utilize the diffusion strategy in the nonlinear system.

3. DISTRIBUTED DIFFUSION UNSCENTED KALMAN FILTERING BASED ON CI

In this section, the distributed diffusion unscented Kalman filtering based on CI (DDUKF-CI) is introduced to address the DSE problem. Before describing the algorithm, the introduction of a diffusion matrix $C \in \mathbb{R}^{n \times n}$ is necessary.

$$C\mathbf{1} = \mathbf{1} \quad C^{i,j} = 0 \text{ if } j \neq \mathcal{N}_i \quad C^{i,j} \geq 0 \tag{7}$$

where $\mathbf{1}$ is an $n \times 1$ column vector with unit entries and $C^{i,j}$ denotes the (i,j) element of matrix C.

Remark 2. The elements of diffusion matrix C represent the weights used to fuse the estimates and error covariance matrices among neighboring sensor nodes in the diffusion strategy.

Based on the transformation of UKF, the DDUKF-CI algorithm is described below.

4. PERFORMANCE ANALYSIS

4.1 Approximation of UKF

In order to analyze the performance of UKF, we use the technique in Li et al. (2016) to derive a pseudo system matrix $\mathcal{F}_{k-1}^i$ and measurement matrix $\mathcal{H}_k^i$ for each filter node. $\mathcal{H}_k^i$ is defined by Eq.(2) and $\mathcal{F}_{k-1}^i$ is defined by

$$\mathcal{F}_{k-1}^i \triangleq (P_{\boldsymbol{x}_{k-1} \boldsymbol{x}_{k|k-1}}^i)^T (P_{k-1}^i)^{-1} \tag{8}$$

where $P_{\boldsymbol{x}_{k-1} \boldsymbol{x}_{k|k-1}}^i = \sum_{s=0}^{2n} W^s (\boldsymbol{\chi}_{k-1}^{i,s} - \hat{\boldsymbol{x}}_{k-1})(\boldsymbol{\chi}_{k|k-1}^{i,s} - \hat{\boldsymbol{x}}_{k|k-1})^T$.

Introduce the compensation instrumental diagonal matrices $\alpha_k^i = \mathrm{diag}(\alpha_k^{1,i}, \alpha_k^{2,i}, \cdots, \alpha_k^{n,i})$ and $\beta_k^i = \mathrm{diag}(\beta_k^{1,i}, \beta_k^{2,i}, \cdots, \beta_k^{m,i})$ to neutralize the approximation error. Then Eq.(1) can be rewritten as Li et al. (2016)

$$\begin{cases} \boldsymbol{x}_k = \alpha_{k-1}^i \mathcal{F}_{k-1}^i \boldsymbol{x}_{k-1} + \boldsymbol{w}_{k-1} \\ \boldsymbol{z}_k^i = \beta_k^i \mathcal{H}_k^i \boldsymbol{x}_k + \boldsymbol{v}_k^i, \quad i \in \mathcal{N} \end{cases} \tag{9}$$

Algorithm 1 DDUKF-CI

Step 1: Initialization

Consider the nonlinear state-space model (1). Start with $\hat{\boldsymbol{x}}_0^i = E(x_0)$ and $P_0^i = P_0$ for all sensors $i = 1, \dots, N$.

Step 2: Prediction and update steps at each node follow the standard UKF.

For $i \in \mathcal{N}$, calculate the $P_{k|k-1}^i$, $\hat{\boldsymbol{x}}_{k|k-1}$, $\hat{\boldsymbol{z}}_k^i$, $P_{\boldsymbol{z}_k\boldsymbol{z}_k}^i$ and $P_{\boldsymbol{x}_k\boldsymbol{z}_k}^i$ by the corresponding procedures of standard UKF, respectively.

Step 3: Calculate the $\mathcal{H}_k^i$ and $\mathcal{R}_k^i$ by Eq.(2) and Eq.(4), respectively.

Step 4: Incremental update

$$\begin{aligned}
S_k^i &= \textstyle\sum_{j\in\mathcal{N}_i} (\mathcal{H}_k^j)^T (\mathcal{R}_k^j)^{-1} \mathcal{H}_k^j \\
\boldsymbol{q}_k^i &= \textstyle\sum_{j\in\mathcal{N}_i} (\mathcal{H}_k^j)^T (\mathcal{R}_k^j)^{-1} \boldsymbol{z}_k^j \\
\boldsymbol{r}_k^i &= \textstyle\sum_{j\in\mathcal{N}_i} (\mathcal{H}_k^j)^T (\mathcal{R}_k^j)^{-1} \hat{\boldsymbol{z}}_k^j \\
(P_{k,loc}^i)^{-1} &= (P_{k|k-1}^i)^{-1} + S_k^i \\
\hat{\boldsymbol{x}}_{k,loc}^i &= \hat{\boldsymbol{x}}_{k|k-1}^i + P_{k,loc}^i (\boldsymbol{q}_k^i - \boldsymbol{r}_k^i)
\end{aligned}$$

Step 5: Diffusion update with CI

$$\begin{aligned}
(P_k^i)^{-1} &= \textstyle\sum_{j\in\mathcal{N}_i} C^{i,j} (P_{k,loc}^j)^{-1} \\
\boldsymbol{x}_k^i &= P_k^i \textstyle\sum_{j\in\mathcal{N}_i} C^{i,j} (P_{k,loc}^j)^{-1} \hat{\boldsymbol{x}}_{k,loc}^j
\end{aligned}$$

According to the above formulas, there exist the following relationships in UKF:

$$P_{k|k-1}^i = \alpha_{k-1}^i \mathcal{F}_{k-1}^i P_{k-1}^i (\mathcal{F}_{k-1}^i)^T \alpha_{k-1}^i + Q_{k-1} \tag{10}$$

$$\hat{\boldsymbol{x}}_{k|k-1}^i = \alpha_{k-1}^i \mathcal{F}_{k-1}^i \hat{\boldsymbol{x}}_{k-1}^i \tag{11}$$

In the fusing procedure, the equations become

$$S_k^i = \textstyle\sum_{j\in\mathcal{N}_i} (\beta_k^j \mathcal{H}_k^j)^T (R_k^j)^{-1} \beta_k^j \mathcal{H}_k^j \tag{12}$$

$$\boldsymbol{q}_k^i = \textstyle\sum_{j\in\mathcal{N}_i} (\beta_k^j \mathcal{H}_k^j)^T (R_k^j)^{-1} \boldsymbol{z}_k^j \tag{13}$$

$$(P_{k,loc}^i)^{-1} = (P_{k|k-1}^i)^{-1} + S_k^i \tag{14}$$

$$\hat{\boldsymbol{x}}_{k,loc}^i = \hat{\boldsymbol{x}}_{k|k-1}^i + P_{k,loc}^i (\boldsymbol{q}_k^i - S_k^i \hat{\boldsymbol{x}}_{k|k-1}^i) \tag{15}$$

$$(P_k^i)^{-1} = \textstyle\sum_{j\in\mathcal{N}_i} C^{i,j} (P_{k,loc}^j)^{-1} \tag{16}$$

$$\hat{\boldsymbol{x}}_k^i = P_k^i \textstyle\sum_{j\in\mathcal{N}_i} C^{i,j} (P_{k,loc}^j)^{-1} \hat{\boldsymbol{x}}_{k,loc}^j \tag{17}$$

Remark 3. The approximation is merely used to analyze the performance of DDUKF-CI. Algorithm 1 is actually used in the simulations.

4.2 Preliminaries

Proposition 1. Consider the nonlinear stochastic system (1). The estimate error $\tilde{\boldsymbol{x}}_k^i = \boldsymbol{x}_k - \hat{\boldsymbol{x}}_k^i$ of DDUKF-CI follows the recursion

$$\tilde{\boldsymbol{x}}_k^i = \textstyle\sum_{j\in\mathcal{N}_i} \Xi_k^{i,j} \tilde{\boldsymbol{x}}_{k-1}^j + \sum_{j\in\mathcal{N}_i} \Phi_k^{i,j} \boldsymbol{w}_{k-1} + \sum_{j\in\mathcal{N}_i} \boldsymbol{\Psi}_k^{i,j} \tag{18}$$

where

$$\Xi_k^{i,j} = C^{i,j} P_k^i (P_{k,loc}^j)^{-1} (I - P_{k,loc}^j S_k^j) \alpha_{k-1}^j \mathcal{F}_{k-1}^j \tag{19}$$

$$\Phi_k^{i,j} = C^{i,j} P_k^i (P_{k,loc}^j)^{-1} (I - P_{k,loc}^j S_k^j) \tag{20}$$

$$\boldsymbol{\Psi}_k^{i,j} = C^{i,j} P_k^i \textstyle\sum_{l\in\mathcal{N}_j} (\beta_k^l \mathcal{H}_k^l)^T (R_k^l)^{-1} \boldsymbol{v}_k^l \tag{21}$$

Proof 1. Firstly, Eq.(15) yields

$$\begin{aligned}
\tilde{\boldsymbol{x}}_{k,loc}^i &= \boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k,loc}^i \\
&= (I - P_{k,loc}^i S_k^i) \tilde{\boldsymbol{x}}_{k|k-1}^i + P_{k,loc}^i S_k^i \boldsymbol{x}_k - P_{k,loc}^i \boldsymbol{q}_k^i
\end{aligned} \tag{22}$$

Substituting Eq.(12) and Eq.(13) into Eq.(22) leads to

$$\begin{aligned}
\tilde{\boldsymbol{x}}_{k,loc}^i = &(I - P_{k,loc}^i S_k^i) \tilde{\boldsymbol{x}}_{k|k-1}^i - P_{k,loc}^i \textstyle\sum_{j\in\mathcal{N}_i} (\beta_k^j \mathcal{H}_k^j)^T \\
&(R_k^j)^{-1} \boldsymbol{v}_k^j
\end{aligned} \tag{23}$$

According to Eq.(16) and Eq.(17), one has

$$\begin{aligned}
\tilde{\boldsymbol{x}}_k^i &= \boldsymbol{x}_k - \hat{\boldsymbol{x}}_k^i \\
&= \boldsymbol{x}_k - P_k^i \textstyle\sum_{j\in\mathcal{N}_i} C^{i,j} (P_{k,loc}^j)^{-1} \hat{\boldsymbol{x}}_{k,loc}^j \\
&= P_k^i \textstyle\sum_{j\in\mathcal{N}_i} C^{i,j} (P_{k,loc}^j)^{-1} (\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k,loc}^j) \\
&\quad + [I - P_k^i \textstyle\sum_{j\in\mathcal{N}_i} C^{i,j} (P_{k,loc}^j)^{-1}] \boldsymbol{x}_k \\
&= P_k^i \textstyle\sum_{j\in\mathcal{N}_i} C^{i,j} (P_{k,loc}^j)^{-1} \tilde{\boldsymbol{x}}_{k,loc}^j
\end{aligned} \tag{24}$$

Substituting Eq.(23) to Eq.(24), we have

$$\begin{aligned}
\tilde{\boldsymbol{x}}_k^i = &P_k^i \textstyle\sum_{j\in\mathcal{N}_i} C^{i,j} (P_{k,loc}^j)^{-1} [(I - P_{k,loc}^j S_k^j) \tilde{\boldsymbol{x}}_{k|k-1}^j \\
&- P_{k,loc}^j \textstyle\sum_{l\in\mathcal{N}_j} (\beta_k^l \mathcal{H}_k^l)^T (R_k^l)^{-1} \boldsymbol{v}_k^l]
\end{aligned} \tag{25}$$

Noting Eq.(11), it can be derived that

$$\tilde{\boldsymbol{x}}_{k|k-1}^j = \boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k-1}^j = \alpha_{k-1}^j \mathcal{F}_{k-1}^j \tilde{\boldsymbol{x}}_{k-1}^j + \boldsymbol{w}_{k-1} \tag{26}$$

Thus,

$$\begin{aligned}
\tilde{\boldsymbol{x}}_k^i = &\textstyle\sum_{j\in\mathcal{N}_i} C^{i,j} P_k^i (P_{k,loc}^j)^{-1} (I - P_{k,loc}^j S_k^j) \alpha_{k-1}^j \mathcal{F}_{k-1}^j \\
&\tilde{\boldsymbol{x}}_{k-1}^j + \textstyle\sum_{j\in\mathcal{N}_i} C^{i,j} P_k^i (P_{k,loc}^j)^{-1} (I - P_{k,loc}^j S_k^j) \\
&\boldsymbol{w}_{k-1} - \textstyle\sum_{j\in\mathcal{N}_i} C^{i,j} P_k^i \sum_{l\in\mathcal{N}_j} (\beta_k^l \mathcal{H}_k^l)^T (R_k^l)^{-1} \boldsymbol{v}_k^l
\end{aligned} \tag{27}$$

4.3 Mean performance

Theorem 1. The estimates of DDUKF-CI are unbiased, i.e., for all $k = 0, 1, \dots$ and $i = 1, \dots, N$,

$$E(\tilde{\boldsymbol{x}}_k^i) = 0 \tag{28}$$

Proof 2. Taking expectation of both sides of Eq.(27), we can immediately get

$$\begin{aligned}
E(\tilde{\boldsymbol{x}}_k^i) = &\textstyle\sum_{j\in\mathcal{N}_i} C^{i,j} P_k^i (P_{k,loc}^j)^{-1} (I - P_{k,loc}^j S_k^j) \alpha_{k-1}^j \mathcal{F}_{k-1}^j \\
&E(\tilde{\boldsymbol{x}}_{k-1}^j)
\end{aligned} \tag{29}$$

Since $E(\tilde{x}_0^i) = 0$, we can conclude by the iteration Eq.(29) that the estimates of DDUKF-CI are unbiased.

4.4 Stability of DDUKF-CI

Prior to analysis, the following assumptions are needed.

Assumption 1. Li et al. (2016) There exist real numbers $\underline{\beta}$, $\underline{f}$, $\underline{\eta}$, $\underline{h}$, $\underline{q}$, $\underline{r}$ and $\overline{\beta}$, $\overline{f}$, $\overline{\eta}$, $\overline{h}$, $\overline{q}$, $\overline{r}$ such that the following inequalities are satisfied for every $k \geq 0$, $i \in \mathcal{N}$:

$$\begin{aligned}
&\underline{f}^2 I \leq \mathcal{F}_k^i (\mathcal{F}_k^i)^T \leq \overline{f}^2 I, \quad \underline{h}^2 I \leq \mathcal{H}_k^i (\mathcal{H}_k^i)^T \leq \overline{h}^2 I \\
&\underline{\alpha}^2 I \leq \alpha_k^i (\alpha_k^i)^T \leq \overline{\alpha}^2 I, \quad \underline{\beta}^2 I \leq \beta_k^i (\beta_k^i)^T \leq \overline{\beta}^2 I \\
&\underline{q} I \leq Q_k \leq \overline{q} I, \quad \underline{r} I \leq R_k^i \leq \overline{r} I
\end{aligned} \tag{30}$$

Assumption 2. The error covariances $P_{k,loc}^i$, P_k^i given by Alg. 1 are uniformly bounded for all $k \geq 0$, i.e., there exist positive scalars $\overline{p}_{loc}$, $\underline{p}_{loc}$, $\overline{p}$ and $\underline{p}$ such that

$$\underline{p} I_n \leq P_k^i \leq \overline{p} I_n, \quad \underline{p}_{loc} I_n \leq P_{k,loc}^i \leq \overline{p}_{loc} I_n \tag{31}$$

Lemma 1. Reif et al. (1999) Assume there is a stochastic process $V_k(\boldsymbol{\zeta}_k)$ as well as real numbers $\underline{a}$,$\overline{a}$,$\mu > 0$ and $0 < \alpha \leq 1$ such that

$$\underline{a}\|\boldsymbol{\zeta}_k\|^2 \leq V_k(\boldsymbol{\zeta}_k) \leq \overline{a}\|\boldsymbol{\zeta}_k\|^2 \tag{32}$$

and

$$E\{V_{k+1}(\boldsymbol{\zeta}_{k+1})|\boldsymbol{\zeta}_k\} - V_k(\boldsymbol{\zeta}_k) \leq \mu - \alpha V_k(\boldsymbol{\zeta}_k) \tag{33}$$

are fulfilled. The stochastic process is exponentially bounded in mean square, i.e.,

$$E\{\|\boldsymbol{\zeta}_k\|^2\} \leq \frac{\overline{a}}{\underline{a}}E\{\|\boldsymbol{\zeta}_0\|^2\}(1-\alpha)^k + \frac{\mu}{\underline{a}}\textstyle\sum_{i=1}^{k-1}(1-\alpha)^i \tag{34}$$

Lemma 2. Battistelli and Chisci (2014) Given an integer $N \geq 2$, N positive definite matrices $M_1, \cdots, M_N$, and N vectors $\boldsymbol{u}_1, \cdots, \boldsymbol{u}_N$, the following inequality holds

$$\begin{aligned}&\left(\textstyle\sum_{i=1}^N M_i\boldsymbol{u}_i\right)^T \left(\textstyle\sum_{i=1}^N M_i\right)^{-1} \left(\textstyle\sum_{i=1}^N M_i\boldsymbol{u}_i\right)\\ &\leq \textstyle\sum_{i=1}^N \boldsymbol{u}_i^T M_i \boldsymbol{u}_i\end{aligned} \tag{35}$$

Lemma 3. Assume that for $k = 0, 1, \ldots,$ $A_k, B_k \in \mathbb{R}^{n\times n}$ are invertible matrices sequence, $A_k, B_k > 0$ and A_k, B_k are bounded by $\underline{a}I_n \leq A_k \leq \overline{a}I_n$, $\underline{b}I_n \leq B_k \leq \overline{b}I_n$ then there exists a positive scalar $0 < \eta < 1$ such that

$$(A_k + B_k)^{-1} \leq \eta A_k^{-1} \tag{36}$$

Proof 3. According to matrix inversion lemma, the following equality is satisfied

$$(A_k + B_k)^{-1} = A_k^{-1} - A_k^{-1}(A_k^{-1} + B_k^{-1})^{-1}A_k^{-1} \tag{37}$$

Assume Eq.(36) holds, i.e.

$$A_k^{-1} - A_k^{-1}(A_k^{-1} + B_k^{-1})^{-1}A_k^{-1} \leq \eta A_k^{-1} \tag{38}$$

Then we can derive the following inequality

$$\eta I_n \geq I_n - (B_k^{-1}A_k + I_n)^{-1} \tag{39}$$

Noting that

$$I_n - (B_k^{-1}A_k + I_n)^{-1} \leq I_n - (\underline{b}^{-1}\overline{a} + 1)^{-1}I_n \leq I_n \tag{40}$$

Therefore, we can choose $1 - (\underline{b}^{-1}\overline{a} + 1)^{-1} \leq \eta \leq 1$ to satisfy Eq.(36).

Theorem 2. Consider the nonlinear stochastic system (1). When the Assumptions 1-2 hold, the estimate error $\tilde{\boldsymbol{x}}_k^i = \boldsymbol{x}_k - \hat{\boldsymbol{x}}_k^i$ of DDUKF-CI is exponentially bounded in mean square for any $i \in \mathcal{N}$.

Proof 4. Let $\boldsymbol{p} = [p^1, p^2, \cdots, p^n]^T$ denote the Perron-Frobenius left eigenvector of the matrix C, p^i is a strictly positive component and $p_{\min} \leq p^i \leq p_{\max}$. Then, we have

$$\boldsymbol{p}^T C = \boldsymbol{p}^T \tag{41}$$

i.e.

$$\textstyle\sum_{j\in\mathcal{N}_i} p^j C^{j,i} = p^i \tag{42}$$

Consider the following stochastic process:

$$V_k(\tilde{\boldsymbol{x}}_k) = \textstyle\sum_{i\in\mathcal{N}} p^i(\tilde{\boldsymbol{x}}_k^i)^T(P_k^i)^{-1}\tilde{\boldsymbol{x}}_k^i \tag{43}$$

Substituting Eq.(27) to Eq.(43), we have

$$E\{V_k(\tilde{\boldsymbol{x}}_k)|\tilde{\boldsymbol{x}}_{k-1}\} = \aleph_k + \daleth_k + \gimel_k \tag{44}$$

where

$$\begin{aligned}\aleph_k =& E\{\textstyle\sum_{i\in\mathcal{N}} p^i[P_k^i \sum_{j\in\mathcal{N}} C^{i,j}(P_{k,loc}^j)^{-1}(I_n - P_{k,loc}^j S_k^j)\\ &\alpha_{k-1}^j\mathcal{F}_{k-1}^j\tilde{\boldsymbol{x}}_{k-1}^j]^T(P_k^i)^{-1}[P_k^i \textstyle\sum_{j\in\mathcal{N}} C^{i,j}(P_{k,loc}^j)^{-1}\\ &(I_n - P_{k,loc}^j S_k^j)\alpha_{k-1}^j\mathcal{F}_{k-1}^j\tilde{\boldsymbol{x}}_{k-1}^j]|\tilde{\boldsymbol{x}}_{k-1}\}\\ \daleth_k =& E\{\textstyle\sum_{i\in\mathcal{N}} p^i[P_k^i \sum_{j\in\mathcal{N}} C^{i,j}(P_{k,loc}^j)^{-1}(I_n - P_{k,loc}^j S_k^j)\\ &\boldsymbol{w}_{k-1}]^T(P_k^i)^{-1}[P_k^i \textstyle\sum_{j\in\mathcal{N}} C^{i,j}(P_{k,loc}^j)^{-1}\\ &(I_n - P_{k,loc}^j S_k^j)\boldsymbol{w}_{k-1}]|\tilde{\boldsymbol{x}}_{k-1}\}\\ \gimel_k =& E\{\textstyle\sum_{i\in\mathcal{N}} p^i[P_k^i \sum_{j\in\mathcal{N}} C^{i,j}\sum_{l\in\mathcal{N}_j}(\beta_k^l\mathcal{H}_k^l)^T(R_k^l)^{-1}\\ &\boldsymbol{v}_k^l]^T(P_k^i)^{-1}[P_k^i \textstyle\sum_{j\in\mathcal{N}} C^{i,j}\sum_{l\in\mathcal{N}_j}(\beta_k^l\mathcal{H}_k^l)^T(R_k^l)^{-1}\boldsymbol{v}_k^l]\\ &|\tilde{\boldsymbol{x}}_{k-1}\}\end{aligned} \tag{45}$$

Considering the homogeneous recursion has a great influence on the stability, we focus on $\aleph_k$ firstly. Substituting Eq.(16) to $\aleph_k$, one gets

$$\begin{aligned}\aleph_k =& E\{\textstyle\sum_{i\in\mathcal{N}} p^i[\sum_{j\in\mathcal{N}} C^{i,j}(P_{k,loc}^j)^{-1}(I_n - P_{k,loc}^j S_k^j)\alpha_{k-1}^j\\ &\mathcal{F}_{k-1}^j\tilde{\boldsymbol{x}}_{k-1}^j]^T[\textstyle\sum_{j\in\mathcal{N}} C^{i,j}(P_{k,loc}^j)^{-1}]^{-1}[\sum_{j\in\mathcal{N}} C^{i,j}\\ &(P_{k,loc}^j)^{-1}(I_n - P_{k,loc}^j S_k^j)\alpha_{k-1}^j\mathcal{F}_{k-1}^j\tilde{\boldsymbol{x}}_{k-1}^j]|\tilde{\boldsymbol{x}}_{k-1}\}\end{aligned} \tag{46}$$

According to Lemma 2, one has

$$\begin{aligned}\aleph_k \leq& E\{\textstyle\sum_{i\in\mathcal{N}} p^i \sum_{j\in\mathcal{N}}(\tilde{\boldsymbol{x}}_{k-1}^j)^T(\alpha_{k-1}^j\mathcal{F}_{k-1}^j)^T(I_n\\ &- P_{k,loc}^j S_k^j)^T C^{i,j}(P_{k,loc}^j)^{-1}(I_n - P_{k,loc}^j S_k^j)\\ &(\alpha_{k-1}^j\mathcal{F}_{k-1}^j)\tilde{\boldsymbol{x}}_{k-1}^j|\tilde{\boldsymbol{x}}_{k-1}\}\\ =& E\{\textstyle\sum_{i\in\mathcal{N}} p^i \sum_{j\in\mathcal{N}} C^{i,j}(\tilde{\boldsymbol{x}}_{k-1}^j)^T(\alpha_{k-1}^j\mathcal{F}_{k-1}^j)^T(I_n\\ &- S_k^j P_{k,loc}^j)[(P_{k,loc}^j)^{-1} - S_k^j]\alpha_{k-1}^j\mathcal{F}_{k-1}^j\tilde{\boldsymbol{x}}_{k-1}^j|\tilde{\boldsymbol{x}}_{k-1}\}\\ =& E\{\textstyle\sum_{i\in\mathcal{N}} p^i \sum_{j\in\mathcal{N}} C^{i,j}(\tilde{\boldsymbol{x}}_{k-1}^j)^T(\alpha_{k-1}^j\mathcal{F}_{k-1}^j)^T(I_n\\ &- S_k^j P_{k,loc}^j)(P_{k|k-1}^j)^{-1}\alpha_{k-1}^j\mathcal{F}_{k-1}^j\tilde{\boldsymbol{x}}_{k-1}^j|\tilde{\boldsymbol{x}}_{k-1}\}\\ \leq& E\{\textstyle\sum_{i\in\mathcal{N}} p^i \sum_{j\in\mathcal{N}} C^{i,j}(\tilde{\boldsymbol{x}}_{k-1}^j)^T(\alpha_{k-1}^j\mathcal{F}_{k-1}^j)^T\\ &(P_{k|k-1}^j)^{-1}\alpha_{k-1}^j\mathcal{F}_{k-1}^j\tilde{\boldsymbol{x}}_{k-1}^j|\tilde{\boldsymbol{x}}_{k-1}\}\end{aligned} \tag{47}$$

According to Lemma 3, there exist a positive real number $0 < \eta < 1$ satisfying

$$(P_{k|k-1}^j)^{-1} \leq \eta(\alpha_{k-1}^j\mathcal{F}_{k-1}^j)^{-T}(P_{k-1}^j)^{-1}(\alpha_{k-1}^j\mathcal{F}_{k-1}^j)^{-1} \tag{48}$$

Substituting the above inequality into the inequality (47) yields

$$\begin{aligned}\aleph_k \leq& \eta E\{\textstyle\sum_{i\in\mathcal{N}} p^i \sum_{j\in\mathcal{N}} C^{i,j}(\tilde{\boldsymbol{x}}_{k-1}^j)^T(P_{k-1}^j)^{-1}\tilde{\boldsymbol{x}}_{k-1}^j|\tilde{\boldsymbol{x}}_{k-1}\}\\ =& \eta E\{\textstyle\sum_{j\in\mathcal{N}} p^j(\tilde{\boldsymbol{x}}_{k-1}^j)^T(P_{k-1}^j)^{-1}\tilde{\boldsymbol{x}}_{k-1}^j|\tilde{\boldsymbol{x}}_{k-1}\}\\ =& \eta V_k(\tilde{\boldsymbol{x}}_{k-1})\end{aligned} \tag{49}$$

Next, the boundedness of $\daleth_k$, $\gimel_k$ is shown. According to the expression in Eq.(45), we have

$$\begin{aligned}\daleth_k &\leq Np_{max}\overline{p}\underline{p}_{loc}^{-2}\overline{q} \triangleq \sigma\\ \gimel_k &\leq N^2 p_{max}\overline{\beta h}\overline{r}\underline{r}^{-2} \triangleq \tau\end{aligned} \tag{50}$$

Combining Eq.(50) and Eq.(49), we finally deduce

$$E\{V_k(\tilde{\boldsymbol{x}}_k)|\tilde{\boldsymbol{x}}_{k-1}\} \leq \eta V_{k-1}(\tilde{\boldsymbol{x}}_{k-1}) + \mu \tag{51}$$

where $\mu = \sigma + \tau$.

According to Lemma 1, the estimation error of DDUKF-CI is exponentially bounded in mean square.

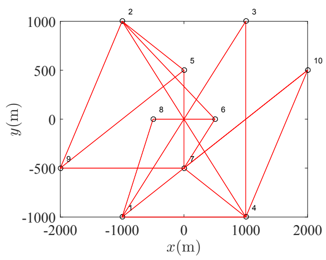


Fig. 1. Topology of Sensor Network

5. SIMULATION RESULTS AND ANALYSIS

In this section, a target localization problem using a sensor network is simulated to demonstrate the performance of DDUKF-CI. It aims to locate a target executing a maneuvering turn in a two-dimensional space with unknown and time-varying turn rate Jia et al. (2017) using 10 sensors As a comparison, CUKF Li et al. (2016) algorithm is also employed in the simulation. The target dynamics is highly nonlinear due to the unknown turn rate and trigonometric terms. The sensor communication topology is shown in Fig. 1. The kinematics of the target motion can be modeled by the following nonlinear process equation

$$\boldsymbol{X}_{k+1} = \begin{bmatrix} 1 & \dfrac{\sin(\Omega_k T)}{\Omega_k} & 0 & \dfrac{\cos(\Omega_k T)-1}{\Omega_k} & 0 \\ 0 & \cos(\Omega_k T) & 0 & -\sin(\Omega_k T) & 0 \\ 0 & \dfrac{1-\cos(\Omega_k T)}{\Omega_k} & 1 & \dfrac{\sin(\Omega_k T)}{\Omega_k} & 0 \\ 0 & \sin(\Omega_k T) & 0 & \cos(\Omega_k T) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \boldsymbol{X}_k + \boldsymbol{w}_k \tag{52}$$

where $\boldsymbol{X}_k = [x_k, v_{x,k}, y_k, v_{y,k}, \Omega_k]^T$ denotes the state vector at time k. $\mathbf{r}_k = [x_k, y_k]^T$ and $\mathbf{v}_k = [v_{x,k}, v_{y,k}]^T$ are the position and velocity at time k, respectively, Ω_k is the turn rate at time k, T is the measurement interval. The initial state is $\boldsymbol{X}_0 = [1000\text{m}; 300\text{m/s}; 1000\text{m}; 0\text{m/s}; -\pi/60\text{rad/s}]^T$. $\boldsymbol{w}_k$ is the process noise with mean zero and covariance Q_k, where

$$Q_k = \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 & 0 \\ T^2/2 & T & 0 & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 & 0 \\ 0 & 0 & T^2/2 & T & 0 \\ 0 & 0 & 0 & 0 & 1.75\times 10^{-4}T \end{bmatrix}$$

The diffusion matrix C in DDUKF-CI is chosen by the Metropolis rule Li et al. (2016).

$$C^{i,j} = \begin{cases} \dfrac{1}{1+\max\{d_i, d_j\}} & \text{if} \quad \{i,j\} \in \mathcal{E} \\ 1 - \sum_{\{i,j\}\in\mathcal{E}} C^{i,j} & \text{if} \quad i = j \\ 0 & \text{otherwise} \end{cases} \tag{53}$$

where d_i is the degree of node i.

The measurements are the distance ρ and angle ϑ given by

$$\begin{bmatrix} \rho_k^i \\ \vartheta_k^i \end{bmatrix} = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \arctan\frac{y_k}{x_k} \end{bmatrix} + \boldsymbol{v}_k^i \tag{54}$$

$\boldsymbol{v}_k^i$ is the measurement noise of the ith node at time k and the covariances of $\boldsymbol{v}_k^i$ are given as

$$R_k^1 = R_k^2 = R_k^3 = 100 \begin{bmatrix} 100\text{m}^2 & 0 \\ 0 & 10^{-5}\text{rad}^2 \end{bmatrix},$$

$R_k^4 = 0.01R_k^1$, $R_k^5 = 0.7R_k^1$, $R_k^6 = 0.5R_k^1$, $R_k^7 = 0.4R_k^1$, $R_k^8 = 0.1R_k^1$, $R_k^9 = 2R_k^1$ and $R_k^{10} = 1.5R_k^1$. The initial locations and estimates of the 10 sensors are listed in Table 1 and Table 2, respectively.

Table 1. Locations of 10 sensors

Sensors	Locations
Sensor1	$[-1000, -1000]^T$m
Sensor2	$[-1000, 1000]^T$m
Sensor3	$[1000, 1000]^T$m
Sensor4	$[1000, -1000]^T$m
Sensor5	$[0, 500]^T$m
Sensor6	$[500, 0]^T$m
Sensor7	$[0, -500]^T$m
Sensor8	$[-500, 0]^T$m
Sensor9	$[-2000, -500]^T$m
Sensor10	$[2000, 500]^T$m

Table 2. Initial estimates

Sensors	Initial estimates
Sensor1	$\mathbf{r} = [1500, 1500]^T$m $\mathbf{v} = [400, 40]^T$m/s,$\Omega = -\pi/20$rad/s
Sensor2	$\mathbf{r} = [900, 800]^T$m $\mathbf{v} = [200, 70]^T$m/s,$\Omega = -\pi/30$rad/s
Sensor3	$\mathbf{r} = [1200, 700]^T$m $\mathbf{v} = [500, 50]^T$m/s,$\Omega = -\pi/10$rad/s
Sensor4	$\mathbf{r} = [700, 900]^T$m $\mathbf{v} = [100, 60]^T$m/s,$\Omega = -\pi/80$rad/s
Sensor5	$\mathbf{r} = [850, 1600]^T$m $\mathbf{v} = [100, 60]^T$m/s,$\Omega = -\pi/50$rad/s
Sensor6	$\mathbf{r} = [600, 500]^T$m $\mathbf{v} = [540, 0]^T$m/s,$\Omega = -\pi/80$rad/s
Sensor7	$\mathbf{r} = [1100, 400]^T$m $\mathbf{v} = [350, -30]^T$m/s,$\Omega = -\pi/30$rad/s
Sensor8	$\mathbf{r} = [830, 100]^T$m $\mathbf{v} = [100, 60]^T$m/s,$\Omega = -\pi/40$rad/s
Sensor9	$\mathbf{r} = [1000, 1200]^T$m $\mathbf{v} = [600, 20]^T$m/s,$\Omega = -\pi/10$rad/s
Sensor10	$\mathbf{r} = [900, 1000]^T$m $\mathbf{v} = [100, 100]^T$m/s,$\Omega = -\pi/70$rad/s

In the simulation, the sampling interval T is 1s and the simulation time is 300s. The above same set of parameters are used for both CUKF and DDUKF-CI. 50 independent Monte Carlo simulations with the same conditions are performed.

The root mean square error (RMSE) of position (PRMSE) and root mean square error of velocity (VRMSE) are used as the performance metrics:

$$\begin{aligned} &\text{PRMSE}_k \\ &= \sqrt{\frac{1}{M}\sum_{m=1}^{M}\sum_{i=1}^{N}\frac{(\tilde{x}_k^i)_m^2 + (\tilde{y}_k^i)_m^2}{N}} \\ &\text{VRMSE}_k \\ &= \sqrt{\frac{1}{M}\sum_{m=1}^{M}\sum_{i=1}^{N}\frac{(\tilde{v}_{x,k}^i)_m^2 + (\tilde{v}_{y,k}^i)_m^2}{N}} \end{aligned} \tag{55}$$

where $M = 50$ is the number of Monte Carlo runs. The subscript m and k denote the mth Monte Carlo run and time step k, respectively. The superscript i represents the ith node. Moreover, the symbol $\sim$ means the deviation between the true state and the estimate.

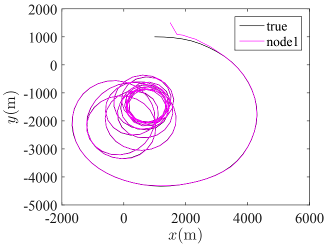
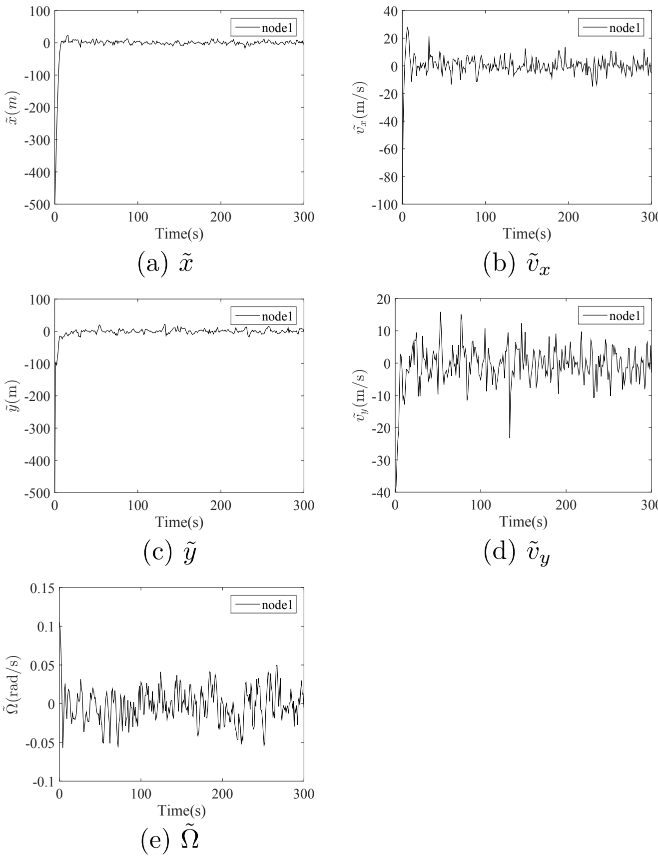


Fig. 2. True and estimated trajectories

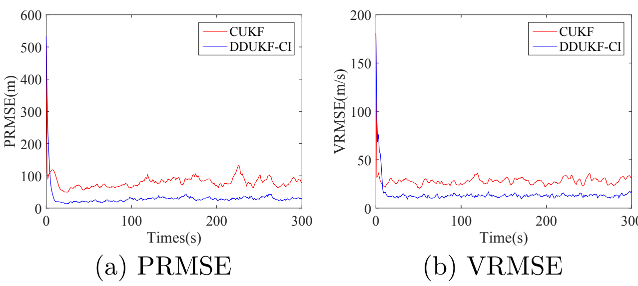


(a) $\tilde{x}$ (b) $\tilde{v}_x$

(c) $\tilde{y}$ (d) $\tilde{v}_y$

(e) $\tilde{\Omega}$

Fig. 3. The errors between true and estimated state



(a) PRMSE (b) VRMSE

Fig. 4. RMSE of position and velocity

Fig. 2 shows the true trajectory and the estimated trajectory. It can be seen that the proposed algorithm can achieve satisfying tracking performance. Fig. 3 depicts the estimation error between the true state $\boldsymbol{x}_k$ and the estimate $\hat{\boldsymbol{x}}_k$ of node 1 as a representative (other nodes perform similarly). Fig. 4 shows the PRMSE and VRMSE. From Fig. 4, it can be seen that the RMSEs of CUKF are much larger than DDUKF-CI, and CUKF exhibits relatively large estimation errors at some time steps.

6. CONCLUSION

This paper proposed a new distributed filtering algorithm in a sensor network by modifying the UKF in the framework of diffusion and covariance intersection. The stochastic stability and exponential boundedness of the estimation error are proved. A target tracking problem is utilized to demonstrate the performance of the DDUKF-CI. Compared with the consensus based distributed UKF, the proposed algorithm can achieve much more accurate results.

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