Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

A Dynamic Game Formulation for Cooperative Lane Change Strategies at Highway Merges ⋆

Andres Ladino * **Meng Wang** **

* *Université Gustave Eiffel, LICIT UMR-T9401, Lyon, France. (e-mail: andres.ladino@univ-eiffel.fr).*
** *Department of Transport & Planing, Delft University of Technology, The Netherlands. (e-mail: m.wang@tudelft.nl)*

Abstract: A dynamic game framework is put forward to derive the system optimum strategy for a network of cooperative vehicles interacting at a merging bottleneck with simplified vehicle dynamics model. Merging vehicles minimize the distance travelled on the acceleration lane in addition to the same cost terms of the mainline vehicles, taking into account the predicted reaction of mainline vehicles to their merging actions. An optimum strategy is found by minimizing the joint cost of all interacting vehicles while respecting behavioral and physical constraints. The full dynamic game is cast as a set of sub-problems regularly expressed as standard optimal control problems that can be solved efficiently. Numerical examples show the feasibility of the approach in capturing the nature of conflict and cooperation during the merging process.

Keywords: Dynamic game, cooperative vehicles, optimal control, cooperative lane change, traffic flow

1. INTRODUCTION

The societal and economical impact of traffic congestion and accidents has encouraged the development of automated driving systems, where planning, design, and deployment of such systems face new challenges everyday (Hanappe et al., 2018). In particular, when multiple vehicles interact, the problem of decision-making under competition and cooperation with multiple players appears, especially at network discontinuities such as highway on-ramps (Rios-Torres and Malikopoulos, 2017b). In order to optimize the utility of the road network at merges, vehicular flow control has been proposed on the infrastructure side via ramp metering and variable speed limits strategies (Papageorgiou et al., 2003).

Several strategies were reported to deal with merging situation, most of which act on the longitudinal speed regulation. Ntousakis et al. (2016) proposed an optimal acceleration trajectory planing method for merging vehicles, relying on a passing order decided by a higher decision layer. A specific trajectory design is proposed and fuzzy controllers were used as regulation strategies in (Milanés et al., 2011). Ge and Murray (2019) used control improvisation to synthesize lane change policies for an automated vehicle in various traffic conditions. The scalability of this approach to multiple CAVs remains an open question. For a more complete literature review on this topic, we refer the reader to Rios-Torres and Malikopoulos (2017a).

⋆ This work has received funding from the EU Horizon 2020 research and innovation programme (grant agreement ID 769115).

The merge situation can be seen as a negotiation process between vehicles on the main carriageway and vehicles on the on-ramp willing to join the highway (See Fig. 1). Wang et al. (2015) proposed a game theoretical framework where interacting CAVs predictively determine discrete desired lane sequences and continuous accelerations to minimize a cost function reflecting undesirable future situations. The computational load of this approach makes the real-time application a daunting task. Fabiani and Grammatico (2018) also considered a similar approach where the constraints of the changing lane are formulated as a Mixed Logical Dynamical (MLD) model introduced by Bemporad and Morari (1999) and the final control problem is cast via Mixed Integer Linear Programming (MILP). The framework assumes non-cooperative nature of automated vehicles.

This paper puts forward a dynamic game framework to derive system optimum strategies for a network of cooperative vehicles interacting at a merging bottleneck. Cooperative vehicles on the highway mainline seek optimal strategies (i.e. whether and when to perform courtesy lane change to facilitate the merging vehicle) to minimize their cost, which penalizes deviations from their desired driving conditions while taking into account the predicted action of merging vehicles. An optimum strategy is found by minimizing the joint cost of interacting vehicles while respecting behavioral and physical constraints. Properties of the games and existence of solutions will be provided in this work.

To solve the problem, a simplified discrete formulation of longitudinal vehicle dynamics is formulated. The longitudinal model is distributed, e.g. only interacting under predecessor-follower topology, and can be easily adapted to capture platooning systems dynamics. The full dynamic game is then cast as a set of sub-problems regularly expressed as standard optimal control problems that can be solved by mixed-integer quadratic/linear programming. Several examples at simulation level show the feasibility of the approach in capturing the nature of cooperation.

The operational assumptions and problem setup are explained with more detail in Section 2, then the model including longitudinal and lateral dynamics is explained in Section 3. The lane change decision action is cast as a dynamic game in Section 4. Section 5 details the approach to solve the merging problem with numerical examples in Section 6.

2. PROBLEM FORMULATION

In this paper we consider the situation shown in Fig. 1. Let $\mathcal{V} = \{1, \dots, n\}$ be a group of CAV traveling along a road infrastructure composed by specific lanes labeled $\sigma = \{1, 2, 3\} \in \mathbb{N}$ from right two left. Let denote $\sigma_i(k)$ the lane occupied by vehicle i at a specific instant of time k. Two vehicles i, j traveling in different lanes $\sigma_i \neq \sigma_j$ are going to perform a merging negotiation at a current time k_0 in a time horizon of N steps.

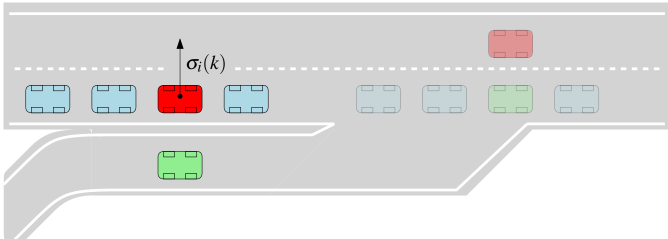
Two dimensions of maneuvers are possible in this case. First, as shown in Fig. 1a the i-th vehicle in the platoon can modify its lateral position (in discrete lanes) to a new state $\sigma_i(k) = \sigma_i(k_0)+1$, while other vehicles in the platoon will keep the same position $\sigma_{i^-}(k) = \sigma_{i^-}(k_0)\ \forall\ i^- \in \mathcal{I} \setminus i$. In this case, a *lateral* decision operates over the vehicle i. A second situation can be envisaged as shown in Fig. 1b, the decision is taken at the level of the longitudinal control where a vehicle i performs a maneuver to pass vehicle j or yields in courtesy to open a gap where the j vehicle will insert in front of vehicle i. Control maneuvers for this situation can be designed under knowledge of the state of the inserting vehicle j (Duret et al., 2019). In this case a *longitudinal* decision operates over vehicle i.

The decision-making and control system follows a hierarchical setting, where the decision-making module is placed on top of a motion control module (Duret et al., 2019). This decision-making is based on a dynamic game framework (Wang et al., 2015). It takes into account the current state information of the dynamic driving environment, which consists of surrounding cooperative/non-cooperative vehicles. The interacting vehicles negotiate and jointly decide whether and when to change lane to optimize a joint cost/payoff function, taking into account the dynamic process as a response to the lane change actions. The control problem can be cast as follows: *Determine the lateral optimal control strategy such that a joint payoff/cost for vehicle i and j is maximized/minimized.*

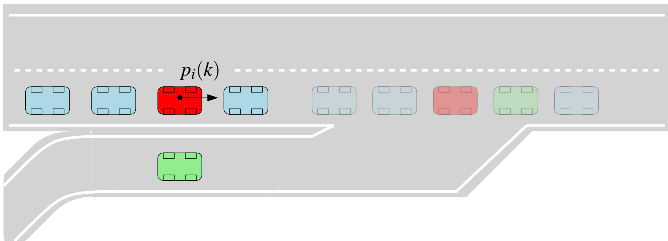
3. HIGHWAY TRAFFIC SYSTEM DYNAMICS

3.1 Longitudinal dynamics

The headway space and longitudinal position for vehicle i are considered as:



(a) Lateral control maneuver



(b) Longitudinal control maneuver

Fig. 1. Control actions for cooperative lane change maneuvers. In this case the red CAV illustrates two behaviors to open gaps for the inserting vehicle in green

$$\begin{aligned} s_i(k+1) &= s_i(k) + (v_l(k) - v_i(k))\,\Delta t \\ p_i(k+1) &= p_i(k) + v_i(k)\Delta t \end{aligned} \tag{1}$$

where $k \in \mathbb{Z}^+$ denotes the discrete time index and Δt is the time step size. The collection $\mathbf{p}, \mathbf{s}, \mathbf{v} \in \mathbb{R}^n$ denote vehicle's position, the headway space and the longitudinal speed respectively. Let define the error

$$e^v_{0,i}(k) = v_{0,i} - v_i(k) \tag{2}$$

$$e^v_{l,i}(k) = v_l(k) - v_i(k) \tag{3}$$

where $v_{0,i}$ denotes the desired speed of vehicle i and the subscript $l \in \mathcal{V} \cup \{j\}$ denotes the index of the direct leader of vehicle i.

A feedback control law can be formulated as:

$$v_i(k+1) = k_0 e^v_{0,i}(k) + k_l e^v_{l,i}(k) \tag{4}$$

k_0, k_l are feedback gains for the errors to the desired speed and the predecessor speed respectively.

The vehicle dynamics are subject to the following linear constraints:

$$a_{\min}\Delta t \leq v_i(k+1) - v_i(k) \leq a_{\max}\Delta t \tag{5}$$

$$v_{\min} \leq v_i(k) \leq v_{\max} \tag{6}$$

$$s_i(k) \geq v_i(k)t_{\min} + s_0 \tag{7}$$

where $t_{\min}$ denotes the minimum time gap between two vehicles on the same lane. s_0 denotes the minimum spacing between two vehicles. (7) states that any leader-follower space headway should keep some safe distance at any time instant k. $a_{\min}, a_{\max}, v_{\min}, v_{\max}$ represent boundaries in acceleration and speed correspondingly.

We choose (2) to capture the heterogeneous choice of desired speed by system users, while acknowledging that this is not the unique model for CAV platoons. If we use the gap error:

$$\begin{aligned} e^s_i(k+1) &= s_i(k) - v_i(k)t_d - s_0 \\ v_i(k+1) &= k_s e^s_i(k) + k_l e^v_{l,i}(k) \end{aligned}$$

where t_d denotes the desired time gap of ACC/CACC systems and k_s denotes the feedback gain. The model can describe CACC platoon dynamics with proper tuning of feedback gains (Milanés and Shladover, 2014).

3.2 Lateral dynamics

We use the discrete lane change decision δ as the control decision variable, $\delta_i \in \mathcal{D} := \{-1, 0, 1\}$ where $\{-1, 0, 1\} :=$ {change right, no lane change, change left}. In the paper we assume only one lane change during the prediction horizon, but the framework is general to include multiple lane changes in the horizon (Wang et al., 2015). This single switch aims to reduce the computational burden of the approach.

We use the travel lane of vehicle i, $\sigma_i(k)$ as the discrete state variable at time k. The dynamics of the lateral behavior are determined by:

$$\sigma_i(k+1) = \sigma_i(k) + \delta_i(k) \tag{8}$$

We assume lane change can take place as long as the gap is sufficiently large according to (7).

3.3 Lane change and dynamic communication topology

The leader-follower pair is dynamic as a result of lane changes for the group of n CAVs. Let a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, $\mathcal{V}$ represents the set nodes consisting in all CAVs within the network and $\mathcal{E} = \{\mathcal{V} \times \mathcal{V}\}$ the set of edges representing a relationship between leaders and followers. Then $\mathcal{E} = \{\varepsilon_{il} = 1\}$ if vehicle l is the leader of vehicle i at specific sample time k, 0 otherwise. The adjacency matrix of $\mathcal{G}$ is concentrated in the squared matrix $A_g = [\varepsilon_{ij}]$. In general thanks to the lane change model (8), the set $\mathcal{E}$ is dynamic in time.

3.4 Stability of the closed loop dynamics

In the following we describe a set of characteristics of the closed loop system, in particular the stability property.

Remark 1. (Stability of the longitudinal control). The control law (4) can verify the constraints (5),(6), (7) in a uniform asymptotic stable setting.

Let suppose a uniform formation where the desired speeds for all vehicles are the same and constant $v_{0,i} = \bar{v}_0$. For system (1), and combining with (4), it is possible to write the closed loop system as:

$$\begin{aligned} s_i(k+1) &= s_i(k) + (v_l(k) - v_i(k))\Delta t \\ v_i(k+1) &= k_0\left(\bar{v}_0 - v_i(k)\right) + k_l\left(v_l(k) - v_i(k)\right) \end{aligned} \tag{9}$$

Gathering all individual systems i into an algebraic equation, it can be expressed as:

$$\begin{bmatrix} \mathbf{s}(k+1) \\ \mathbf{v}(k+1) \end{bmatrix} = \underbrace{\begin{pmatrix} \mathbb{I} & (A_g - \mathbb{I})T \\ \mathbb{0} & K_l(A_g - \mathbb{I}) - K_0 \end{pmatrix}}_{\bar{A}} \begin{pmatrix} \mathbf{s}(k) \\ \mathbf{v}(k) \end{pmatrix} + \begin{pmatrix} \mathbb{0} \\ K_0 \bar{\mathbf{v}}_0 \end{pmatrix} \tag{10}$$

where K_0, K_l, T are diagonal matrices in $\mathbb{R}^{n \times n}$ with corresponding elements $k_0, k_l, \Delta t$ in their diagonal. $\mathbb{I}, \mathbb{0}$ are the identity and the zero matrices of corresponding dimensions. $\bar{\mathbf{v}}_0 \in \mathbb{R}^n$ is the constant vector containing on each element $\bar{v}_0$. A_g is the adjacency matrix of the network topology (see 3.2).

System (10) is stable if and only if the spectral radius $\rho(\bar{A}) \leq 1, \rho(A) := \{\max|\lambda| : \lambda = \text{eig}(A)\}$. This condition can be translated into $\rho(K_l\left(A_g - \mathbb{I}\right) - K_0) < 1$. For a single lane, the matrix $A_g - \mathbb{I}$ is lower triangular by construction, in particular, $\text{eig}(A_g - \mathbb{I}) = \{0, -1\}$. Given the diagonal nature of K_0, K_L, for stability it is necessary to guarantee $\rho(K_L) < 1$. Given the diagonal construction of these matrices, then the necessary condition for stability is then given by $|k_l - k_0| < 1$.

At same time by inserting (1) into (5)-(7) it is possible to construct the following system of linear matrix inequalities (LMI)s:

$$\begin{pmatrix} 0 & -F \\ 0 & F \\ 0 & \mathbb{I} \\ 0 & -\mathbb{I} \\ -t_{\min}\mathbb{I} & (A_g - \mathbb{I}) \end{pmatrix} \begin{pmatrix} \mathbf{s}(k) \\ \mathbf{v}(k) \end{pmatrix} \leq \begin{pmatrix} -a_{\min}T + K_0\bar{\mathbf{v}}_0 \\ a_{\max}T - K_0\bar{\mathbf{v}}_0 \\ V_{\max} \\ V_{\min} \\ s_0 \end{pmatrix} \tag{11}$$

Given a fixed values $\{k_l, k_0 : |k_l| < 1, |k_l - k_0| < 1\}$, and $F = K_l(A_g - \mathbb{I}) - K_0 - \mathbb{I}$. All values that satisfy the LMI (11) make the system uniformly and asymptotically stable.

4. GAME THEORETIC FORMULATION OF THE LANE CHANGE DECISION PROBLEM

In this section we propose the dynamic game formulation for the lane change control maneuver.

4.1 Dynamic lane change game formulation

Definition 1. (Lane change strategy). A vehicle lane change strategy from lane $\sigma_\ell \to \sigma_{\ell+}$ is defined as the sequence:

$$\begin{aligned} &\xi_\delta = \big\{\sigma(k_0), \sigma(k_0+1), \ldots, \sigma(k_0+N-1)\big\} \\ &\quad \sigma(k^\star) = \sigma_\ell \\ &\quad \sigma(k^\star+1) = \sigma_{\ell+} \\ &\quad \sigma(k+1) = \sigma(k) + \delta(k), \\ &\quad \sum_{k=0}^{N-1} |\delta(k)| = 1 \end{aligned} \tag{12}$$

ξ_δ represents the sequence associated to a particular lateral control $\delta(k)$ which induces the choice lane changing maneuver at $k^\star$ in the horizon N.

Consider the case of Fig. 2. The objective of the dynamic

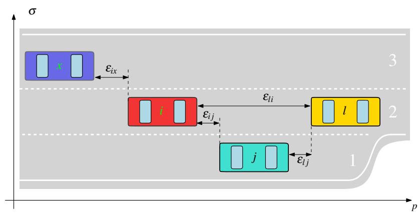


Fig. 2. Lane change dynamic game. The controlled CAV in red optimizes the decision making between yielding at the merging time and changing lane.

game is to create a decision block that considers the trade off between two possible cases. First, the situation in which in a finite time horizon the vehicle i performs a lane change maneuver to create the necessary gap for insertion as depicted in Fig. 1a and a second situation where the vehicle j should wait for the mainline vehicle to yield the necessary gap to so that the merging maneuver is performed without violating constraints. The cost for each vehicle is measured by undesirable situations:

$$L_i(\mathbf{p}(k), \mathbf{v}(k), \delta_i(k)) = \beta_1|e^v_{0,i}(k)| + \beta_2|e^v_{l,i}(k)| \quad (13a)$$
$$+ \beta_3|v_i(k+1) - v_i(k)| \quad (13b)$$
$$+ \beta_4|\sigma_i(k) - \sigma^*_i| \quad (13c)$$
$$+ \beta_5|\delta_i(k)| \quad (13d)$$
$$- \min\{0, \beta_6(p_j(k) - p_{j,end})\} \quad (13e)$$

where $\beta_g, g \in \{1, 2, 3, 4, 5, 6\}$ are the weights on different cost terms. $p_{j,end}$ denotes the position of the end of a mandatory lane change section for vehicle j. The running cost function can be interpreted as follows:

- (13a) encourages the vehicle to travel at its desired speed;
- The second term of (13a) encourages consensus on speed for each leader-follower pair;
- (13b) favors smooth speed change and hence discourage sharp acceleration and deceleration;
- (13d) penalizes deviation from desired lane σ^*_i and the fifth term penalizes lane changes.
- (13e) penalizes potential failure for mandatory lane change. It favours early mandatory lane changes and increases when the distance to the end of the merging lane p_{end} is decreasing.

The optimal control problem can be cast as an optimization of the running cost L_i for each one of the players while other players have already decided. A dynamic game can be integrated within an optimal control problem where each one of the players fixes a specific strategy in particular for the lane change by targeting the specific value σ^*_i. Notice that each player i has a finite number of strategies to choose by selecting specific δ_i. In particular, when playing the game in between vehicle i and vehicle j it is possible to write the following finite horizon problem:

$$\begin{aligned} \min_{\delta_i(\cdot)\in\mathcal{D}} \quad & \sum_{g=i,j}\sum_{k=0}^{N-1} L_g\left(\mathbf{p}(k), \delta_g(k)\right) + \Phi_g\left(\mathbf{p}(N)), \delta(N)\right) \\ \text{s.t} \quad & (1),(5),(7),(6),(8) \\ & \delta_i(k) \in \underline{\mathcal{D}} = \{0, 1\}, \text{ only allow left lane changes} \\ & \sum_{k=0}^{N-1} \delta_i(k) \leq 1, \text{ only allow one lane change} \end{aligned} \quad (14)$$

The objective of the former optimal control problem is to promote the minimization of the individual costs. This is formulated as an optimization problem, where one seeks the optimal lane change decision trajectories for each vehicle i in a prediction horizon N to maximize the payoff function of the whole group. In fact each one of the player should maximize a payoff given by:

$$J_i\left(\mathbf{p}(k), \mathbf{v}(k), \delta_i(k)\right) = -\sum_{k=0}^{N-1} L_i\left(\mathbf{p}(k), \mathbf{v}(k), \delta_i(k)\right) \quad (15)$$

The dynamic game entails prediction of the payoff over a time horizon with N steps: $[0, N]$. We consider N to be sufficiently large and therefore set the terminal cost $\Phi = 0$. The player i will select a strategy among a finite set $\underline{\mathcal{D}}$ of strategies.

Let consider the vehicle i and all the possible set of finite strategies $A = \{a_1, a_2, \ldots, a_r\}$ to be chosen for the lateral decisions. Let $B = \{b_1, b_2, \ldots, b_q\}$ the possible decisions for the j vehicle traveling in the on-ramp lane. It is worth to remark that a vehicle i, j have at most $|A^i| = |B^i| = 2^{N-1}$ possibilities to change lane during a future finite horizon.

Definition 2. (Payoff function). Let $J^A_i(\mathbf{p}(k), \mathbf{v}(k), a_\delta, b_\delta)$ be the function defining the payoff after a player decides among the set of strategies A as:

$$J^A_i(\mathbf{p}(k), \mathbf{v}(k), a_\delta, b_\delta) = \psi_i(p(N)) - \sum_{k=0}^{N-1} L_i(\mathbf{p}(k), a_\delta, b_\delta) \quad (16)$$

In definition 2 L_i is defined as the running cost while the ψ_i is called the final cost.

Assumption 1. (Available game information). System (1) is fixed for each participant of the game. The same as $J^A_i(\mathbf{p}(k), \mathbf{v}(k), a_\delta, b_\delta)$, $J^B_j(\mathbf{p}(k), \mathbf{v}(k), a_\delta, b_\delta)$ and the sample time k is considered synchronous in vehicles i, j.

4.2 Properties of the dynamic lane change game

Consider the full dynamics expressed in equation (10) jointly with (8) and enclosed in the form $\mathbf{x}(k+1) = f(\mathbf{x}(k), \delta(k)) = A\mathbf{x}(k) + M\boldsymbol{\delta}(k)$. $\mathbf{x}^T = \begin{pmatrix}\mathbf{p}^T & \mathbf{v}^T & \boldsymbol{\sigma}^T\end{pmatrix}$. In a particular case where two players are defining a game it is possible to define split dynamics and running costs as:

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + M_1\delta_1(k) + M_2\delta_2(k) \quad (17)$$
$$L_i(k) = L_{i1}(\mathbf{x}(k), \delta_1(k)) + L_{i2}(\mathbf{x}(k), \delta_2(k)) \quad (18)$$

Remark 2. (Finding equilibrium via PMP). Let consider the system (17) with associated running cost (18). Let $\mathbf{x}^*(\cdot), \delta^*_1(\cdot), \delta^*_2(\cdot)$ be respectively the trajectory and open loop controls of two players in a Nash equilibrium. By definition this two controls provide corresponding solutions to the associated optimal control problems for each player. Applying the Pontryagin Maximum Principle (PMP) the following are necessary conditions for the Nash equilibrium (Lewis et al., 2012).

$$\mathbf{x}(k+1) = \mathbf{x}(k) + M_1\delta^\natural_1(k) + M_2\delta^\natural_2(k) \quad (19)$$
$$\lambda_1(k) = \bar{A}\lambda_1(k+1) + \nabla_{\mathbf{x}}L_{11}\left(\mathbf{x}(k), \delta^\natural_1(k)\right) \quad (20)$$
$$\lambda_2(k) = \bar{A}\lambda_2(k+1) + \nabla_{\mathbf{x}}L_{22}\left(\mathbf{x}(k), \delta^\natural_2(k)\right) \quad (21)$$

where

$$\begin{aligned} \delta^\natural_1 &= \arg\max_{\delta_1\in\mathcal{D}} \lambda_1 M_1\omega - L_{11}(t, \mathbf{x}, \delta_1) \\ \delta^\natural_2 &= \arg\max_{\delta_2\in\mathcal{D}} \lambda_2 M_2\omega - L_{22}(t, \mathbf{x}, \delta_2) \end{aligned} \quad (22)$$

In order to solve the find the conditions for Nash equilibrium let define the Hamiltonian for the control problem (14) based on (17),(18):

$$\begin{aligned} H(\mathbf{x}(k), \delta_1(k), \delta_2(k)) = \sum_{i\in\mathcal{I}} \big(&L_{i1}\big(\mathbf{x}(k), \delta_1\big) \\ &L_{i2}\big(x(k), \delta_2(k)\big)\big) - \\ &\lambda_i\big(\mathbf{x}(k) + M_1\delta_1(k) + M_2\delta_2(k)\big) \end{aligned} \quad (23)$$

By considering the costate condition for $\lambda \in \mathbb{R}^n$ from the PMP (Lewis et al., 2012):

$$\lambda_i(k) = \frac{\partial H}{\partial x_i} = \left(\frac{\partial H}{\partial x_i}\right)^T \lambda_i(k+1) + \frac{\partial L(k)}{\partial x_i} \tag{24}$$

with the final condition $x(T) = 0$, then it is possible to obtain the conditions in (20),(21). The optimal condition is derived from the fact that for a fixed lateral control $\bar{\delta}_2(\cdot)$, the optimal $\delta_1(\cdot)$ can be found via

$$\delta_1^*(\cdot) = \arg\min_{\delta_1 \in \underline{\mathcal{D}}} H(\mathbf{x}(k), \delta_1(k), \bar{\delta}_2(k)) \tag{25}$$

which can be transformed into a maximization problem where the player is maximizing the payoff function similar to (16), leading to

$$\delta_1^*(\cdot) = \arg\max_{\delta_1 \in \underline{\mathcal{D}}} -H(\mathbf{x}(k), \delta_1(k), \bar{\delta}_2(k)) \tag{26}$$

The stationary condition is necessary for optimality then by introducing (23) in to (26), we obtain:

$$0 = -\frac{\partial L_{11}(\mathbf{x}(k), \delta_1(k)) + L_{12}(\mathbf{x}(k), \bar{\delta}_2(k))}{\partial \delta_1} + \lambda_1 \frac{\partial\left(x(k) + M_1\delta_1(k) + M_2\bar{\delta}_2(k)\right)}{\partial \delta_1} \tag{27}$$

leading to (22). In the same way the second equation can be obtained when the first player fixes its own strategy to a value $\bar{\delta}_1 = \delta_1^*$. The nash equilibrium is obtained when the payoff for player 1 is maximized 26 with the best reply of player 2 and vice-versa (Bressan, 2010). In other words, no player can increase his payoff by single-mindedly changing his strategy, as long as the other player sticks to the equilibrium strategy.

5. SOLUTION ALGORITHM

The problem formulation for this case brings inherent complexity to the solution of the game which in fact cannot be found in an explicit form due to the nature of the control signal $\delta(k)$. To solve the optimal control problem (14) the following algorithm 1 is proposed. In general the game here presented is a non-zero sum game, and as players in fact cooperate towards the common objective, given by the successful lane change. On the other hand, the scalability of this approach may suffer with long time horizons. In this case we propose an heuristic way to solve this algorithm. As part of future research it is desired a specific reduction of the search space via integer programming.

6. NUMERICAL EXAMPLES

6.1 Experimental setting

To test the working of the dynamic game framework, we conducted numerical examples. The scenario is set up as in Fig. 2We simulate 3 vehicles, with Vehicle 2 (red) and Vehicle 3 (turquoise) interacting with each other in the merging section. The initial conditions are: $p_1(0) = 0m, p_2(0) = -50m$ $v_{l1}(0) = v_{l2}(0) = v_1(1) = v_2(0) = 30m/s$, $\sigma_1(0) = \sigma_2(0) = 2$, $\sigma_3(0) = 1$, $\sigma_2^* = \sigma_3^* = 2$ (The desired lanes for both Vehicle 2 and Vehicle 3 are Lane 2, the right lane on the main freeway). $v_3(1) = 25m/s$, $p_3(1) = -45m$ for the scenario. $v_0 = 30m/s, v_{min} = 0m/s, v_{max} = 35m/s, a_{min} = -5m/s^2, a_{max} = 2m/s^2$ $\beta_1 = 0.2, \beta_2 = 0.2, \beta_3 = 0.5, \beta_4 = 5, \beta_5 = 5, \beta_6 = 0.05$, $t_{min} = 0.5s$.

begin
  Data: Initial condition: $p_i(0), v_i(0) \quad \forall\, i \in \mathcal{I}$
  Result: Control input: $\delta_i(\cdot)\ \forall\ i$
  begin
    Initialize $\delta_i(\cdot) = 0,\ \forall i \in \mathcal{I}, k \in [0, N-1]$
  end
  for *each* k **do**
    for *each* $i \in \mathcal{I}, k \in [0, \ldots, N-1]$ **do**
      for *each* $j \in \mathcal{I}, k \in [0, \ldots, N-1]$ **do**
        Dynamic game
        while $J_1^r < J_1^{r-1}, J_2^r < J_2^{r-1}$ **do**
          Fix $\bar{\delta}_2(\cdot) = \delta_2^*$
          for *each* $\delta_1(k) \in \underline{\mathcal{D}}$ **do**
            Optimize $\delta_1^* = \arg\max_{\delta_1(k)} -L_{ij}(\mathbf{x}, \delta_1, \bar{\delta}_2)$
          end
          Update $J_1^r \leftarrow -L_{ij}(\mathbf{x}, \delta_1^*, \bar{\delta}_2)$
          Fix $\bar{\delta}_1(\cdot) = \delta_1^*$
          for *each* $\delta_2(k) \in \underline{\mathcal{D}}$ **do**
            Optimize $\delta_2^* = \arg\max_{\delta_2(k)} -L_{ij}(\mathbf{x}, \bar{\delta}_1, \delta_2)$
          end
          Update $J_2^r \leftarrow -L_{ij}(\mathbf{x}, \bar{\delta}_1, \delta_2^*)$
        end
        Get equilibrium with $\delta_1^\natural = \delta_1^*, \delta_2^\natural = \delta_2^*$
      end
      Evolve system (17) with $\delta_1^\natural(k), \delta_2^\natural(k)$
    end
  end
end

Algorithm 1: Closed loop operation for the proposed control strategy

6.2 Scenario: delayed merge

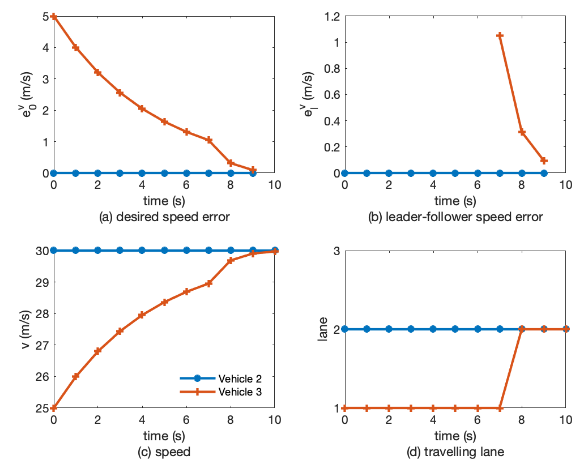


Fig. 3. Delayed merge

Vehicle 3 is 5 meters in front of Vehicle 2 but with a slower speed. The resulting cost of all vehicles and the cost of Vehicle 3 and Vehicle 2 are shown in Fig. 4. Vehicle 2, leading to a cost of 50. The overall cost is not the optimum for the whole vehicle group. From the collective system perspective, the best strategy is that Vehicle 2 stays in the

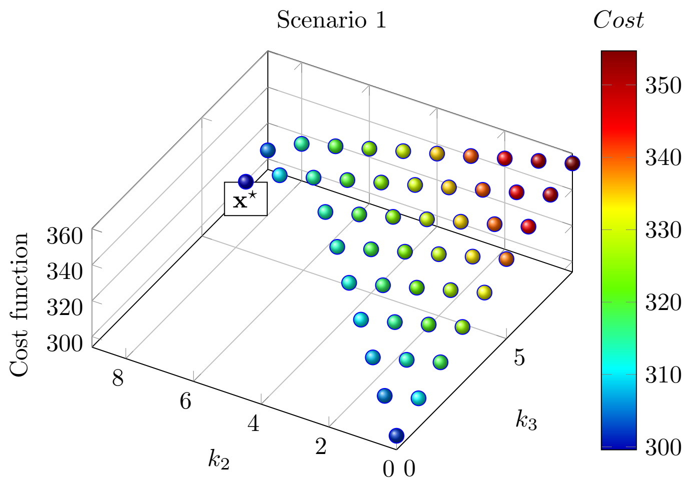


Fig. 4. Overall cost for Scenario 1. k_i represents the time to change lane of vehicle i

same lane and passes vehicle 3. Vehicle 3 waits for Vehicle 2 to pass until sufficient safety gap is developed in front, and changes lane at $k = 7$ second. Interestingly, if following a first-in-first-out strategy that is widely used in cooperative merging systems (Rios-Torres and Malikopoulos, 2017a), it leads to the feasible strategy that is best for vehicle 3, but not the best for the collective vehicle group.

Fig. 3 shows the system optimal solution, where the error on desired speed e_0^v, speed error to predecessor e_l^v, vehicle speed and lane sequence are depicted. Note that the change of increasing rate in speed for Vehicle 3 is due to fact that before the lane change, Vehicle 3 has no leader and it only accelerates towards the desired speed. When it changes lane, the both the error on desired speed and speed error to predecessor demand it to accelerate, resulting in an increase in speed change rate.

7. CONCLUSION

We proposed a dynamic game formulation for cooperative lane change maneuvers of automated vehicles at highway merges. Simplified vehicle longitudinal and lateral dynamics models are used to predict the system process under different lane change strategies. The framework captures the competitive and cooperative nature of the interactions between the merging vehicle and the mainline vehicle, and renders the design tractable to a range of mathematical tools related to optimal control and integer programming. The discrete dynamic model with control input substantially reduces the computational load for the dynamic merging game compared to previous work. Numerical examples demonstrate the potential of the approach in generating system optimum strategies as opposed to existing non-cooperative merging algorithms.

Future research is directed to the scalability analysis of the proposed framework and efficient solution algorithms to a large network of cooperative vehicles and the assessment of the effect of this framework on traffic operations.

ACKNOWLEDGEMENT

This research work has received funding from the EU project ENabling SafE Multi-Brand pLatooning for Europe - ENSEMBLE (grant agreement No. 769115).

REFERENCES

Bemporad, A. and Morari, M. (1999). Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3), 407–427.

Bressan, A. (2010). Non Cooperative Differential Games. A Tutorial. Technical report, Penn State University.

Duret, A., Wang, M., and Ladino, A. (2019). A hierarchical approach for splitting truck platoons near network discontinuities. *Transportation Research Part B: Methodological*.

Fabiani, F. and Grammatico, S. (2018). A Mixed-Logical-Dynamical model for Automated Driving on highways. In *2018 IEEE Conference on Decision and Control (CDC)*, 1011–1015. IEEE.

Ge, J.I. and Murray, R.M. (2019). Voluntary lane-change policy synthesis with control improvisation. *Proceedings of the IEEE Conference on Decision and Control*, 2018-Decem(Cdc), 3640–3647.

Hanappe, F., Hudson, A., Pelloux, P., Alba, D., and Musseau, P. (2018). Impacts and potential benefits of autonomous vehicles: From an international context to Grand Paris. Technical report, Apur.

Lewis, F.L., Vrabie, D., and Vassilis, L.S. (2012). *Optimal Control*. John Wiley & Sons, third edition.

Milanés, V., Godoy, J., Villagra, J., and Perez, J. (2011). Automated On-Ramp Merging System for Congested Traffic Situations. *IEEE Transactions on Intelligent Transportation Systems*, 12(2), 500–508.

Milanés, V. and Shladover, S.E. (2014). Modeling cooperative and autonomous adaptive cruise control dynamic responses using experimental data. *Transportation Research Part C: Emerging Technologies*, 48, 285–300.

Ntousakis, I.A., Nikolos, I.K., and Papageorgiou, M. (2016). Optimal vehicle trajectory planning in the context of cooperative merging on highways. *Transportation Research Part C: Emerging Technologies*, 71, 464–488.

Papageorgiou, M., Kiakaki, C., Dinopoulou, V., Kotsialos, A., and Yibing Wang (2003). Review of road traffic control strategies. *Proceedings of the IEEE*, 91(12), 2043–2067.

Rios-Torres, J. and Malikopoulos, A.A. (2017a). A Survey on the Coordination of Connected and Automated Vehicles at Intersections and Merging at Highway On-Ramps. *IEEE Transactions on Intelligent Transportation Systems*, 18(5), 1066–1077.

Rios-Torres, J. and Malikopoulos, A.A. (2017b). Automated and Cooperative Vehicle Merging at Highway On-Ramps. *IEEE Transactions on Intelligent Transportation Systems*, 18(4), 780–789.

Wang, M., Hoogendoorn, S.P., Daamen, W., van Arem, B., and Happee, R. (2015). Game theoretic approach for predictive lane-changing and car-following control. *Transportation Research Part C: Emerging Technologies*, 58, 73–92.