Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Design of Prediction-Based Estimator for Time-Varying Networks subject to Communication Delays and Missing Data [⋆]

Jun Hu [*,**] **Guo-Ping Liu** [*,***]

[*] *School of Engineering, University of South Wales, Pontypridd CF37 1DL, United Kingdom. (e-mail: hujun2013@gmail.com; guoping.liu@southwales.ac.uk).*
[**] *School of Science, Harbin University of Science and Technology, Harbin 150080, China.*
[***] *Department of Artificial Intelligence and Automation, Wuhan University, Wuhan 430072, China.*

Abstract: This paper is concerned with the robust optimal estimation problem based on the prediction compensation mechanism for dynamical networks with time-varying parameters, where communication delays and degraded measurements are considered. The missing measurements are characterized by some random variables governed by Bernoulli distribution, where each sensor having individual missing probability is reflected. During the signal transmissions through the communication networks, the network-induced communication delays commonly exist among the adjacent nodes transmissions and a prediction updating method is given to compensate the caused impacts. Accordingly, a time-varying state estimator with hybrid compensation scheme is constructed such that, for both the communication delays and missing measurements, a minimized upper bound matrix with regards to the estimation error covariance matrix is found and an explicit estimator parameter matrix is designed at each sampling step accordingly. Finally, the comparative simulations are given to validate the advantages of main results.

Keywords: Time-varying dynamical networks; Prediction estimation; Communication delays; Missing measurements.

1. INTRODUCTION

The complex networks with non-trivial topological structures have successful applications in the system modelling such as technological networks, computer networks, social networks and so on Boccaletti et al. (2006); Yu et al. (2017); Li and Yang (2016). Generally, the node state information can not be always available due to various reasons Huang et al. (2012). Hence, there is a need to design estimation methods so as to provide efficient estimates for the node state. For example, a state estimation scheme with guaranteed H_∞ performance has been given in Shen et al. (2013) for time-invariant complex networks in order to estimate the node state and discuss the uncertain inner coupling influences. To save the communication resources, an event-based H_∞ estimation method has been given in Li, Shen, et al. (2018) for time-invariant complex networks to attenuate the effects caused by state saturation and quantized measurements. For time-varying complex networks, some estimation methods under different performance indices have been reported Dong et al. (2018); Li, Jia, et al. (2018). In Dong et al. (2018), a new recursive estimation scheme has been presented for time-varying networks subject to varying topological features, where H_∞ performance criterion and variance constraint have been considered. To address the bounded constraint, a set-membership H_∞ estimation method has been developed in Wang et al. (2018) for uncertain time-varying networks, where major effort has been devoted to handle the randomly occurring nonlinearities and fading measurements. By considering the variance constraint, a recursive estimation algorithm has been established in Li, Jia, et al. (2018) for coupled stochastic networks with time-varying characteristics.

During the data transmissions, the information might be lost especially in the unreliable communication environment Duan and Shen (2019). As such, special effort has been devoted and some estimation schemes have been given for complex dynamical networks to deal with the missing data Asif et al. (2016); Hu et al. (2016). In Asif et al. (2016), the matrix and tensor based methods have been proposed to estimate the missing values in the intelligent transportation systems, where the algorithm performance has been discussed by evaluating the estimation accuracy, the variance of the data set as well as the bias. In Hu et al. (2016), a recursive estimation algorithm combining the variance constraint with local optimal error criterion has been given to discuss the time-

[⋆] This work was supported in part by the European Regional Development Fund and Sêr Cymru Fellowship under Grant 80761-USW-059, the National Natural Science Foundation of China under Grants 61673141 and 61773144, and the Alexander von Humboldt Foundation of Germany.

varying case, where new estimation scheme possessed the recursive feature has been given for purpose of real-time applications. On the other hand, it should be pointed out that the delays would affect the estimation accuracy if not handled properly. Recently, a great number of analysis techniques have been proposed to deal with the impacts from different types of delays onto the behaviours of dynamical networks. For instance, the sampled-data schemes have been proposed in Ali et al. (2019); Rakkiyappan and Sivaranjani (2016) for delayed dynamical networks. In Wang et al. (2016); Sasirekha and Rakkiyappan (2017), some efficient estimation results have been reported to handle the mixed time-delays. However, it should be noticed that most of the existing state estimation approaches dealing with the time delays relied on the delay information only, which inevitably sacrifice certain estimation accuracy. Consequently, it is of significant importance to properly tackle the effects from communication delays and missing measurements and then provide a new estimation method based on the prediction schemes in Liu (2017); Li et al. (2019).

In this paper, we aim to handle the state estimation problem with predication compensation ability for dynamical networks, where both communication delays and missing measurements are considered within the time-varying framework. To fulfill the major objective, a hybrid compensation approach is given to attenuate the influences from communication delays and missing measurements onto estimation performance. In particular, the efficient information of the delay upper bounds and missing probability is introduced when constructing the time-varying estimator. Accordingly, a new recursive estimation algorithm is developed to determine the optimal estimator parameter and then propose the state estimate.

2. STATEMENT OF THE PROBLEM

In this paper, the following class of time-varying uncertain coupled networks with missing measurements is considered:

$$\begin{aligned} x_{i,k+1} =&(A_{i,k}+\Delta A_{i,k})x_{i,k}+\sum_{j=1}^{N}\omega_{ij,k}\Gamma x_{j,k} \\ &+B_{i,k}w_{i,k},\ \ i=1,2,\cdots,N \qquad (1)\\ y_{i,k} =&\lambda_{i,k}C_{i,k}x_{i,k}+v_{i,k}, \qquad (2)\end{aligned}$$

where $x_{i,k}\in\mathbb{R}^n$ depicts the state of the i-th node to be estimated, the initial value is $x_{i,0}$ with mean $\bar{x}_{i,0}$, $y_{i,k}\in\mathbb{R}^m$ stands for the measurement output of the i-th node, $\Omega_k=[\omega_{ij,k}]_{N\times N}$ is the coupling strength matrix, and Γ is a known inner-coupling matrix. $w_{i,k}\in\mathbb{R}^p$ represents the process noise with zero mean and covariance $W_{i,k}$. $v_{i,k}\in\mathbb{R}^m$ denotes the zero mean measurement noise with covariance $V_{i,k}>0$. $\Delta A_{i,k}=M_{i,k}\mathcal{F}_{i,k}H_{i,k}$ represents the parameter uncertainty with $\mathcal{F}_{i,k}\mathcal{F}_{i,k}^T\le I$. $\lambda_{i,k}$ $(i=1,2,\cdots,N)$ are the Bernoulli distributed random variables. $A_{i,k}$, $H_{i,k}$, $M_{i,k}$, $B_{i,k}$, Γ and $C_{i,k}$ are known matrices.

A set of Bernoulli distributed random variables $\lambda_{i,k}$ $(i=1,2,\cdots,N)$ is employed to depict the missing measurements, which satisfies

$$\mathrm{Prob}\{\lambda_{i,k}=1\}=\bar{\lambda}_{i,k}, \qquad (3)$$

$$\mathrm{Prob}\{\lambda_{i,k}=0\}=1-\bar{\lambda}_{i,k}, \qquad (4)$$

where $\bar{\lambda}_{i,k}\in[0,1]$ are known constants. In the sequel, we assume that $\lambda_{i,k}$, $w_{i,k}$, $v_{i,k}$ and $x_{i,0}$ are mutually independent.

In what follows, it should be mentioned that the communication delays are inevitably occurred among the communications of the state estimation, that is, there exist the communication delays d_{ij} between the node j and the node i during the transmissions. Thus, the following updating rule is utilized:

$$\begin{aligned} \hat{x}_{j,k-d_{ij}+1|k-d_{ij}} &= A_{j,k-d_{ij}}\hat{x}_{j,k-d_{ij}},\\ \hat{x}_{j,k-d_{ij}+2|k-d_{ij}} &= A_{j,k-d_{ij}+1}\hat{x}_{j,k-d_{ij}+1|k-d_{ij}},\\ &\vdots\\ \hat{x}_{j,k|k-d_{ij}} &= A_{j,k-1}\hat{x}_{j,k-1|k-d_{ij}}. \qquad (5)\end{aligned}$$

Then, from (5), it further yields that $\hat{x}_{j,k|k-d_{ij}}=A_{j,d_{ij}}\hat{x}_{j,k-d_{ij}}$ with $A_{j,d_{ij}}=\prod_{s=1}^{d_{ij}}A_{j,k-s}$.

For the i-th node, we design time-varying estimator of the following form:

$$\begin{aligned}\hat{x}_{i,k+1}=&A_{i,k}\hat{x}_{i,k}+\omega_{ii,k}\Gamma\hat{x}_{i,k}+\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma\hat{x}_{j,k|k-d_{ij}}\\ &+\mathcal{K}_{i,k}(y_{i,k}-\bar{\lambda}_{i,k}C_{i,k}\hat{x}_{i,k}), \qquad (6)\end{aligned}$$

where $\hat{x}_{i,k}$ denotes the state estimation of $x_{i,k}$ at the instant k, and $\mathcal{K}_{i,k}$ represents the desired estimator parameter matrix to be determined later.

Remark 1: The prediction method is given in (5) to actively compensate the communication delays with a prediction updating rule, where the predictive state estimation is obtained in terms of the delayed estimations. Besides, the statistical information of incomplete measurements is utilized during the estimator design. In fact, the estimator (6) is a Kalman-like estimator for dynamical networks, where the estimations from all nodes at the last instant and the innovations measurements are utilized. However, it should be pointed that a predictive-compensation term (i.e., the third term of (6)) is introduced due to the existence of communication delays d_{ij}, which is indeed new yet different compared with existing results. Later, a hybrid estimation scheme will be proposed to jointly improve the estimation accuracy. In particular, major effort will be devoted to deal with and compensate the impacts of communication delays between the node's transmissions and the degraded measurements collected by sensors.

Define $\tilde{x}_{i,k+1}=x_{i,k+1}-\hat{x}_{i,k+1}$ as the estimation error and $Q_{i,k+1}=\mathbb{E}\{\tilde{x}_{i,k+1}\tilde{x}_{i,k+1}^T\}$ as the estimation error covariance matrix. Now, we are ready to summarize the objectives of this paper. 1) Construct a state estimator of recursive form (6) such that there exists an upper bound covariance matrix $\mathcal{Q}_{i,k+1}$ of $Q_{i,k+1}$. 2) Design the estimator gain $\mathcal{K}_{i,k+1}$ to minimize the trace of the upper bound covariance matrix $\mathcal{Q}_{i,k+1}$. 3) Provide the monotonicity analysis between the occurrence probabilities and the upper bound covariance matrix.

3. DESIGN OF TIME-VARYING ESTIMATION SCHEME

In this section, the covariance matrices of state and estimation error are firstly calculated. Next, the recursion equation of an optimal upper bound matrix is derived by choosing the state estimator gain properly.

To begin with, introduce the notation $\tilde{\lambda}_{i,k} = \lambda_{i,k} - \bar{\lambda}_{i,k}$. From (1) and (6), $\tilde{x}_{i,k+1}$ can be calculated as:

$$\begin{aligned}\tilde{x}_{i,k+1} =& (A_{i,k} + \omega_{ii,k}\Gamma - \bar{\lambda}_{i,k}\mathcal{K}_{i,k}C_{i,k})\tilde{x}_{i,k} \\ &+ \sum_{j=1,j\neq i}^{N} \omega_{ij,k}\Gamma(x_{j,k} - \hat{x}_{j,k|k-d_{ij}}) \\ &+\Delta A_{i,k}x_{i,k} - \tilde{\lambda}_{i,k}\mathcal{K}_{i,k}C_{i,k}x_{i,k} - \mathcal{K}_{i,k}v_{i,k}. \quad (7)\end{aligned}$$

To proceed, let us calculate the state covariance and estimation error covariance. Here, the related proofs are omitted due to the space limitation.

Theorem 1. Consider the time-varying uncertain coupled networks (1)-(2) with the recursive estimator (6). For a scalar $\delta > 0$, the state covariance matrix $X_{i,k+1}$ has the following upper bound:

$$\begin{aligned}&X_{i,k+1} \\ &\leq 2(1+\delta)\big[A_{i,k}X_{i,k}A_{i,k}^T + \mathrm{tr}(H_{i,k}X_{i,k}H_{i,k}^T)M_{i,k}M_{i,k}^T\big] \\ &\quad +N(1+\delta^{-1})\sum_{j=1}^{N}\omega_{ij,k}^2\Gamma X_{j,k}\Gamma^T + B_{i,k}W_{i,k}B_{i,k}^T \\ &\triangleq \mathcal{X}_{i,k+1}. \quad (8)\end{aligned}$$

Theorem 2. Consider the time-varying uncertain coupled networks (1)-(2) with the recursive estimator (6). The recursive equation of estimation error covariance matrix $Q_{i,k+1}$ is described as follows:

$$\begin{aligned}&Q_{i,k+1} \\ &= (A_{i,k} + \omega_{ii,k}\Gamma - \bar{\lambda}_{i,k}\mathcal{K}_{i,k}C_{i,k})Q_{i,k}(A_{i,k} + \omega_{ii,k}\Gamma \\ &\quad -\bar{\lambda}_{i,k}\mathcal{K}_{i,k}C_{i,k})^T + \mathcal{A}_{1,k} + \mathcal{A}_{1,k}^T + \mathcal{A}_{2,k} + \mathcal{A}_{2,k}^T \\ &\quad +\mathbb{E}\Bigg\{\Bigg[\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(x_{j,k} - \hat{x}_{j,k|k-d_{ij}})\Bigg] \\ &\quad \times\Bigg[\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(x_{j,k} - \hat{x}_{j,k|k-d_{ij}})\Bigg]^T\Bigg\} \\ &\quad +\mathcal{A}_{3,k} + \mathcal{A}_{3,k}^T + \Delta A_{i,k}X_{i,k}\Delta A_{i,k}^T \\ &\quad +\bar{\lambda}_{i,k}(1-\bar{\lambda}_{i,k})\mathcal{K}_{i,k}C_{i,k}X_{i,k}C_{i,k}^T\mathcal{K}_{i,k}^T \\ &\quad +B_{i,k}W_{i,k}B_{i,k}^T + \mathcal{K}_{i,k}V_{i,k}\mathcal{K}_{i,k}^T \quad (9)\end{aligned}$$

where

$$\begin{aligned}\mathcal{A}_{1,k} =& (A_{i,k} + \omega_{ii,k}\Gamma - \bar{\lambda}_{i,k}\mathcal{K}_{i,k}C_{i,k}) \\ &\times\mathbb{E}\Bigg\{\tilde{x}_{i,k}\Bigg[\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(x_{j,k} - \hat{x}_{j,k|k-d_{ij}})\Bigg]^T\Bigg\}, \\ \mathcal{A}_{2,k} =& (A_{i,k} + \omega_{ii,k}\Gamma - \bar{\lambda}_{i,k}\mathcal{K}_{i,k}C_{i,k}) \\ &\times\mathbb{E}\{\tilde{x}_{i,k}x_{i,k}^T\}\Delta A_{i,k}^T, \\ \mathcal{A}_{3,k} =& \mathbb{E}\Bigg\{\Bigg[\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(x_{j,k} - \hat{x}_{j,k|k-d_{ij}})\Bigg]x_{i,k}^T\Bigg\}\Delta A_{i,k}^T.\end{aligned}$$

Now, we are in a position to provide the recursion of the upper bound matrix $\mathcal{Q}_{i,k+1}$ regarding $Q_{i,k+1}$.

Theorem 3. Consider the estimation error covariance matrix in (9). Let $\delta > 0$ and $\epsilon_t > 0$ $(t = 1, 2, 3)$ be constant scalars. Assume that the following matrix difference equation with $\mathcal{Q}_{i,0} = \mathcal{X}_{i,0} > 0$

$$\begin{aligned}&\mathcal{Q}_{i,k+1} \\ &= (1+\epsilon_1+\epsilon_2)(A_{i,k} + \omega_{ii,k}\Gamma - \bar{\lambda}_{i,k}\mathcal{K}_{i,k}C_{i,k})\mathcal{Q}_{i,k} \\ &\quad \times(A_{i,k} + \omega_{ii,k}\Gamma - \bar{\lambda}_{i,k}\mathcal{K}_{i,k}C_{i,k})^T \\ &\quad +2(1+\epsilon_1^{-1}+\epsilon_3)(N-1)\sum_{j=1,j\neq i}^{N}\omega_{ij,k}^2 \\ &\quad \times\Gamma(\mathcal{X}_{j,k} + \hat{x}_{j,k|k-d_{ij}}\hat{x}_{j,k|k-d_{ij}}^T) \\ &\quad +(1+\epsilon_2^{-1}+\epsilon_3^{-1})\mathrm{tr}(H_{i,k}\mathcal{X}_{i,k}H_{i,k}^T)M_{i,k}M_{i,k}^T \\ &\quad +\bar{\lambda}_{i,k}(1-\bar{\lambda}_{i,k})\mathcal{K}_{i,k}C_{i,k}\mathcal{X}_{i,k}C_{i,k}^T\mathcal{K}_{i,k}^T \\ &\quad +B_{i,k}W_{i,k}B_{i,k}^T + \mathcal{K}_{i,k}V_{i,k}\mathcal{K}_{i,k}^T \quad (10)\end{aligned}$$

has the solution $\mathcal{X}_{i,k+1} > 0$. Then, it can be concluded that

$$Q_{i,k+1} \leq \mathcal{Q}_{i,k+1}.$$

Furthermore, if we choose $\mathcal{K}_{i,k}$ as

$$\mathcal{K}_{i,k} = (1+\epsilon_1+\epsilon_2)\bar{\lambda}_{i,k}(A_{i,k} + \omega_{ii,k}\Gamma)\mathcal{Q}_{i,k}C_{i,k}^T\Sigma_{i,k}^{-1} \quad (11)$$

with

$$\begin{aligned}\Sigma_{i,k} =& (1+\epsilon_1+\epsilon_2)\bar{\lambda}_{i,k}^2C_{i,k}\mathcal{Q}_{i,k}C_{i,k}^T \\ &+\bar{\lambda}_{i,k}(1-\bar{\lambda}_{i,k})C_{i,k}\mathcal{X}_{i,k}C_{i,k}^T + V_{i,k}, \quad (12)\end{aligned}$$

then $\mathrm{tr}(\mathcal{Q}_{i,k+1})$ is minimized at every sampling step.

Proof. The main results can be verified by using the mathematical induction method and completing square technique. First, let us deal with the cross and unknown terms in (9). From simple computation, it yields

$$\begin{aligned}&\mathcal{A}_{1,k} + \mathcal{A}_{1,k}^T \\ &\leq \epsilon_1(A_{i,k} + \omega_{ii,k}\Gamma - \bar{\lambda}_{i,k}\mathcal{K}_{i,k}C_{i,k})Q_{i,k} \\ &\quad \times(A_{i,k} + \omega_{ii,k}\Gamma - \bar{\lambda}_{i,k}\mathcal{K}_{i,k}C_{i,k})^T \\ &\quad +\epsilon_1^{-1}\mathbb{E}\Bigg\{\Bigg[\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(x_{j,k} - \hat{x}_{j,k|k-d_{ij}})\Bigg] \\ &\quad \times\Bigg[\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(x_{j,k} - \hat{x}_{j,k|k-d_{ij}})\Bigg]^T\Bigg\}, \quad (13) \\ &\mathcal{A}_{2,k} + \mathcal{A}_{2,k}^T \\ &\leq \epsilon_2(A_{i,k} + \omega_{ii,k}\Gamma - \bar{\lambda}_{i,k}\mathcal{K}_{i,k}C_{i,k})Q_{i,k} \\ &\quad \times(A_{i,k} + \omega_{ii,k}\Gamma - \bar{\lambda}_{i,k}\mathcal{K}_{i,k}C_{i,k})^T \\ &\quad +\epsilon_2^{-1}\Delta A_{i,k}\mathbb{E}\{x_{i,k}x_{i,k}^T\}\Delta A_{i,k}^T, \quad (14) \\ &\mathcal{A}_{3,k} + \mathcal{A}_{3,k}^T\end{aligned}$$

$$\leq \epsilon_3 \mathbb{E}\Bigg\{\Bigg[\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(x_{j,k}-\hat{x}_{j,k|k-d_{ij}})\Bigg] \times\Bigg[\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(x_{j,k}-\hat{x}_{j,k|k-d_{ij}})\Bigg]^T\Bigg\} +\epsilon_3^{-1}\Delta A_{i,k}\mathbb{E}\{x_{i,k}x_{i,k}^T\}\Delta A_{i,k}^T, \quad (15)$$

where $\epsilon_t > 0$ $(t = 1, 2, 3)$. Then, it follows from (13)-(15) that (9) can be rewritten as

$$\begin{aligned} &Q_{i,k+1} \\ &\leq (1+\epsilon_1+\epsilon_2)(A_{i,k}+\omega_{ii,k}\Gamma-\bar{\lambda}_{i,k}\mathcal{K}_{i,k+1}C_{i,k})Q_{i,k} \\ &\times(A_{i,k}+\omega_{ii,k}\Gamma-\bar{\lambda}_{i,k}\mathcal{K}_{i,k+1}C_{i,k})^T \\ &+(1+\epsilon_1^{-1}+\epsilon_3)\mathbb{E}\Bigg\{\Bigg[\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(x_{j,k}-\hat{x}_{j,k|k-d_{ij}})\Bigg] \\ &\times\Bigg[\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(x_{j,k}-\hat{x}_{j,k|k-d_{ij}})\Bigg]^T\Bigg\} \\ &+(1+\epsilon_2^{-1}+\epsilon_3^{-1})\Delta A_{i,k}X_{i,k}\Delta A_{i,k}^T \\ &+\bar{\lambda}_{i,k}(1-\bar{\lambda}_{i,k})\mathcal{K}_{i,k}C_{i,k}X_{i,k}C_{i,k}^T\mathcal{K}_{i,k}^T \\ &+\mathcal{K}_{i,k}V_{i,k}\mathcal{K}_{i,k}^T. \end{aligned} \quad (16)$$

Next, the second term in (16) is handled as

$$\begin{aligned} &\mathbb{E}\Bigg\{\Bigg[\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(x_{j,k}-\hat{x}_{j,k|k-d_{ij}})\Bigg] \\ &\times\Bigg[\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(x_{j,k}-\hat{x}_{j,k|k-d_{ij}})\Bigg]^T\Bigg\} \\ &\leq (N-1)\mathbb{E}\Bigg\{\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(x_{j,k}x_{j,k}^T \\ &+\hat{x}_{j,k|k-d_{ij}}\hat{x}_{j,k|k-d_{ij}}^T)\Bigg\} \\ &=(N-1)\sum_{j=1,j\neq i}^{N}\omega_{ij,k}\Gamma(X_{j,k}+\hat{x}_{j,k|k-d_{ij}}\hat{x}_{j,k|k-d_{ij}}^T). \end{aligned} \quad (17)$$

Moreover, the third term in (16) with parameter uncertainty is described by:

$$\begin{aligned} &(1+\epsilon_2^{-1}+\epsilon_3^{-1})\Delta A_{i,k}X_{i,k}\Delta A_{i,k}^T \\ &\leq(1+\epsilon_2^{-1}+\epsilon_3^{-1})\mathrm{tr}(H_{i,k}X_{i,k}H_{i,k}^T)M_{i,k}M_{i,k}^T. \end{aligned} \quad (18)$$

Together with (16)-(18), we can get

$$\begin{aligned} &Q_{i,k+1} \\ &\leq (1+\epsilon_1+\epsilon_2)(A_{i,k}+\omega_{ii,k}\Gamma-\bar{\lambda}_{i,k}\mathcal{K}_{i,k+1}C_{i,k})Q_{i,k} \\ &\times(A_{i,k}+\omega_{ii,k}\Gamma-\bar{\lambda}_{i,k}\mathcal{K}_{i,k+1}C_{i,k})^T \\ &+(1+\epsilon_1^{-1}+\epsilon_3)(N-1)\sum_{j=1,j\neq i}^{N}\omega_{ij,k} \\ &\times\Gamma(X_{j,k}+\hat{x}_{j,k|k-d_{ij}}\hat{x}_{j,k|k-d_{ij}}^T) \\ &+(1+\epsilon_2^{-1}+\epsilon_3^{-1})\mathrm{tr}(H_{i,k}X_{i,k}H_{i,k}^T)M_{i,k}M_{i,k}^T \\ &+\bar{\lambda}_{i,k}(1-\bar{\lambda}_{i,k})\mathcal{K}_{i,k}C_{i,k}X_{i,k}C_{i,k}^T\mathcal{K}_{i,k}^T \\ &+\mathcal{K}_{i,k}V_{i,k}\mathcal{K}_{i,k}^T. \end{aligned} \quad (19)$$

Then, the above results lead to

$$Q_{i,k+1} \leq \mathcal{Q}_{i,k+1}.$$

Finally, we are ready to design the desired $\mathcal{K}_{i,k}$ in order to minimize $\mathcal{Q}_{i,k+1}$. From the completing square method, rewrite the upper matrix $\mathcal{Q}_{i,k+1}$ with regards to the estimator gain matrix $\mathcal{K}_{i,k}$. Then, we have

$$\begin{aligned} &\mathcal{Q}_{i,k+1} \\ &=(1+\epsilon_1+\epsilon_2)(A_{i,k}+\omega_{ii,k}\Gamma)\mathcal{Q}_{i,k}(A_{i,k}+\omega_{ii,k}\Gamma)^T \\ &+(1+\epsilon_1^{-1}+\epsilon_3)(N-1)\sum_{j=1,j\neq i}^{N}\omega_{ij,k} \\ &\times\Gamma(\mathcal{X}_{j,k}+\hat{x}_{j,k|k-d_{ij}}\hat{x}_{j,k|k-d_{ij}}^T) \\ &+(1+\epsilon_2^{-1}+\epsilon_3^{-1})\mathrm{tr}(H_{i,k}\mathcal{X}_{i,k}H_{i,k}^T)M_{i,k}M_{i,k}^T \\ &+[\mathcal{K}_{i,k}-(1+\epsilon_1+\epsilon_2)\bar{\lambda}_{i,k}(A_{i,k}+\omega_{ii,k}\Gamma)\mathcal{Q}_{i,k}C_{i,k}^T\Sigma_{i,k}^{-1}] \\ &\times\Sigma_{i,k}[\mathcal{K}_{i,k}-(1+\epsilon_1+\epsilon_2)\bar{\lambda}_{i,k}(A_{i,k}+\omega_{ii,k}\Gamma)\mathcal{Q}_{i,k} \\ &\times C_{i,k}^T\Sigma_{i,k}^{-1}]^T-(1+\epsilon_1+\epsilon_2)^2\bar{\lambda}_{i,k}^2(A_{i,k}+\omega_{ii,k}\Gamma) \\ &\times\mathcal{Q}_{i,k}C_{i,k}^T\Sigma_{i,k}^{-1}C_{i,k}\mathcal{Q}_{i,k}(A_{i,k}+\omega_{ii,k}\Gamma)^T \end{aligned} \quad (20)$$

with $\Sigma_{i,k}$ defined in (12). Now, select $\mathcal{K}_{i,k}$ as

$$\mathcal{K}_{i,k}=(1+\epsilon_1+\epsilon_2)\bar{\lambda}_{i,k}(A_{i,k}+\omega_{ii,k}\Gamma)\mathcal{Q}_{i,k}C_{i,k}^T\Sigma_{i,k}^{-1}.$$

Thus, the proof is complete.

To end this section, the newly prediction-based state estimation (PBSE) algorithm is listed to facilitate future applications.

PBST Algorithm :
S1: Initialize the related values.
S2: Calculate the estimator parameter matrix $\mathcal{K}_{i,k}$ via (11).
S3: Update the prediction estimation $\hat{x}_{j,k|k-d_{ij}}$ in view of (5).
S4: Obtain the state estimation $\hat{x}_{i,k+1}$ according to (6).
S5: Derive the estimation error covariance $\mathcal{Q}_{i,k+1}$ by utilizing (10).
S6: Let $k = k + 1$. Go back to S2.

Remark 2: Notice that the PBSE problem has been solved for uncertain dynamical networks with communication delays and missing measurements. The communication delays during the data transmissions between the adjacent network nodes have been taken into account and the prediction state compensation method has been given accordingly. Both the probability of the missing measurements and the predictive state estimations have been utilized when designing the time-varying estimator and proposing the estimation algorithm, which have been explicitly reflected in the new PBSE algorithm mentioned above.

4. AN ILLUSTRATIVE SIMULATION

In this section, the following simulation experiment is provided to show the validity of new prediction-based state estimation scheme.

Consider the uncertain coupled networks (1)-(2) with the following parameters:

$$A_{1,k} = \begin{bmatrix} -0.56 & -0.46 \\ 0.18 & -0.15 + 0.5\cos(2k) \end{bmatrix},$$
$$A_{2,k} = \begin{bmatrix} 0.76 & -0.72 + 0.12\cos(2k) \\ 0.62 & 0.58 + 0.45\cos(2k) \end{bmatrix},$$
$$A_{3,k} = \begin{bmatrix} 0 & -1.6 \\ 0.66 & -0.06 + 0.05\sin(k) \end{bmatrix}$$
$$B_{1,k} = \begin{bmatrix} 0.48 \\ 0.5 \end{bmatrix},\ B_{2,k} = \begin{bmatrix} 0.35 \\ 0.89 \end{bmatrix},\ B_{3,k} = \begin{bmatrix} 0.3 \\ 0.37 \end{bmatrix},$$
$$C_{1,k} = [\,1.6\ \ 1.5\,],\ C_{2,k} = [\,1.6\ \ 2.1\,],$$
$$C_{3,k} = [\,1.9\ \ 1.4\,],\ M_{i,k} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},$$
$$H_{i,k} = [\,-0.3\ \ 0.2\,],\ i = 1, 2, 3.$$

Moreover, let $\Gamma = \mathrm{diag}\{0.08, 0.08\}$ and $w_{ij} = 0.5$ $(i \neq j)$ and $w_{ii} = 1$ $(i = 1, 2, 3)$.

During the simulation experiment, choose the initial conditions $\bar{x}_{1,0} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $\bar{x}_{2,0} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$, $\bar{x}_{3,0} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, $\mathcal{Q}_{i,0} = 5I_2$, $\mathcal{X}_{i,0} = \bar{x}_{i,0}\bar{x}_{i,0}^T$, $\epsilon_i = 1$ $(i = 1, 2, 3)$, $\delta = 0.1$, $W_{1,k} = W_{2,k} = 0.15$, $W_{3,k} = 0.2$, $V_{1,k} = V_{2,k} = 0.2$, $V_{3,k} = 0.15$, $\hat{x}_{i,0} = \bar{x}_{i,0}$, $\hat{x}_{i,k} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$ $(k < 0)$ $(i = 1, 2, 3)$.

To validate the efficiency and advantages of the PBSE algorithm, we compare the cases with/without prediction compensation, i.e., Case I: estimation with PBSE algorithm; Case II: estimation without the updating rule (5). Simulation results under $\bar{\lambda}_{i,k} = 0.88$ and $d_{ij} = 6$ are presented in Figs. 1-5, where Figs. 1-3 are the state trajectories of 3 nodes and their estimations with/without prediction compensation. Fig. 4 depicts the log(MSEs) of 3 nodes and their upper bounds under Case I. The comparisons of MSE with/without delay compensation under 500 iterations are shown in Fig. 5. It can be seen from the simulations that the estimation performance with the PBSE algorithm performs well than the one without using delay prediction estimation.

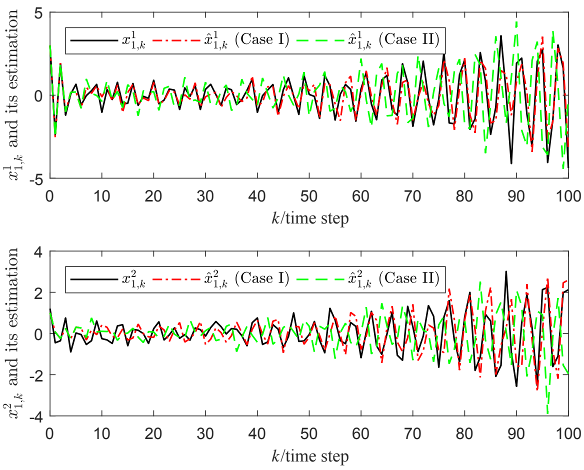

Fig. 1. The curves of $x_{1,k}$ and $\hat{x}_{1,k}$.

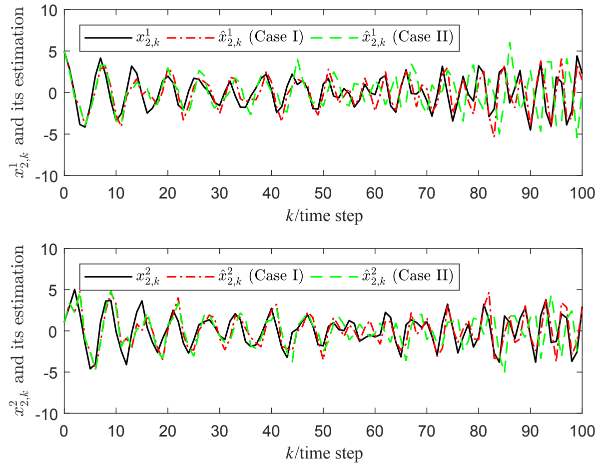

Fig. 2. The curves of $x_{2,k}$ and $\hat{x}_{2,k}$.

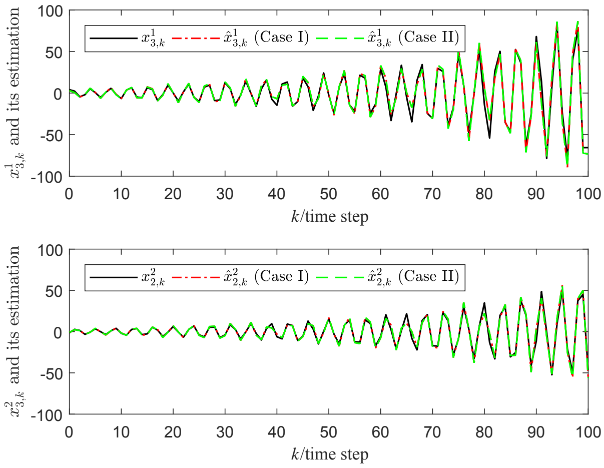

Fig. 3. The curves of $x_{3,k}$ and $\hat{x}_{3,k}$.

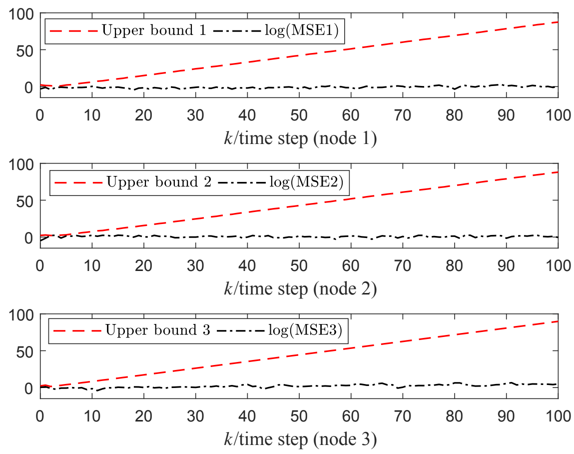

Fig. 4. log(MSE) and upper bounds.

5. CONCLUSIONS

In this paper, we have tackled the prediction-based estimation problem for time-varying uncertain dynamical

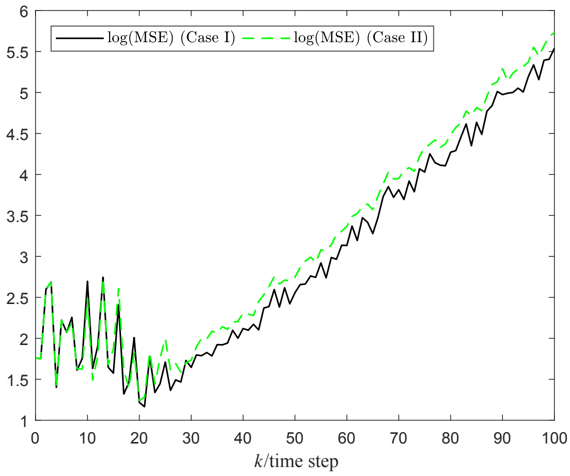

Fig. 5. log(MSE) with/without compensation (500 iterations).

networks with communication delays and missing measurements. The proposed main result has the following features: 1) a hybrid compensation estimation has been presented, where both the communication delays and the missing measurements have been taken into account; 2) a local optimal upper bound matrix has been found for the estimation error covariance and the estimator gain matrix has been parameterized accordingly, hence a recursive algorithm suitable for online application has been established.

REFERENCES

Ali, M.S., Usha, M., Orman, Z., and Arik, S. (2019). Improved result on state estimation for complex dynamical networks with time varying delays and stochastic sampling via sampled-data control. *Neural networks*, 114:28–37.

Asif, M.T., Mitrovic, N., Dauwels, J., and Jaillet, P. (2016). Matrix and tensor based methods for missing data estimation in large traffic networks. *IEEE Transactions on Intelligent Transportation Systems*, 17(7):1816–1825.

Boccaletti, S., Latora, V., Moreno, Y., Chavez, M., and Hwang, D.-U. (2006). Complex networks: Structure and dynamics. *Physics reports*, 424(4-5):175–308.

Dong, H., Hou, N., Wang, Z., and Ren, W. (2018). Variance-constrained state estimation for complex networks with randomly varying topologies. *IEEE Transactions on Neural Networks and Learning Systems*, 29(7): 2757–2768.

Duan, Z. and Shen, J. (2019). Synchronization problem of 2-D coupled dynamical networks with communication delays and missing measurements. *Multidimensional Systems and Signal Processing*, 30(1):39–67.

Hu, J., Wang, Z., Liu, S., and Gao, H. (2016). A variance-constrained approach to recursive state estimation for time-varying complex networks with missing measurements. *Automatica*, 64:155–162.

Huang, Y.F., Werner, S., Huang, J., Kashyap, N., and Gupta, V. (2012). State estimation in electric power grids: Meeting new challenges presented by the requirements of the future grid. *IEEE Signal Processing Magazine*, 29(5):33–43.

Li, Q., Shen, B., Wang, Z., and Alsaadi, F.E. (2018). Event-triggered H_∞ state estimation for state-saturated complex networks subject to quantization effects and distributed delays. *Journal of the Franklin Institute*, 355(5):2874–2891.

Li, W., Jia, Y., and Du, J. (2018). Variance-constrained state estimation for complex networks with randomly varying topologies. *IEEE Transactions on Cybernetics*, 48(2):818–824.

Li. X. and Yang, G. (2016). FLS-based adaptive synchronization control of complex dynamical networks with nonlinear couplings and state-dependent uncertainties. *IEEE Transactions on Cybernetics*, 46(1):171–180.

Li, Y., Liu, G.-P., Sun, S., and Tan, C. (2019). Prediction-based approach to finite-time stabilization of networked control systems with time delays and data packet dropouts. *Neurocomputing*, 329:320–328.

Liu, G.-P. (2017). Consensus and stability analysis of networked multiagent predictive control systems. *IEEE Transactions Cybernetics*, 47(4):1114–1119.

Rakkiyappan, R. and Sivaranjani, K. (2016). Sampled-data synchronization and state estimation for nonlinear singularly perturbed complex networks with time-delays. *Nonlinear Dynamics*, 84(3):1623–1636.

Sasirekha, R. and Rakkiyappan, R. (2017). Extended dissipativity state estimation for switched discrete-time complex dynamical networks with multiple communication channels: A sojourn probability dependent approach. *Neurocomputing*, 267:55–68.

Shen, B., Wang, Z., Ding, D., and Shu, H. (2013). H_∞ State estimation for complex networks with uncertain inner coupling and incomplete measurements. *IEEE Transactions on Neural Networks and Learning Systems*, 24(12):2027–2037.

Wang, F. and Liang, J. (2018). Constrained H_∞ estimation for time-varying networks with hybrid incomplete information. *International Journal of Robust and Nonlinear Control*, 28(2):699–715.

Wang, L., Wang, Z., Huang, T., and Wei, G. (2016). An event-triggered approach to state estimation for a class of complex networks with mixed time delays and nonlinearities. *IEEE Transactions on Cybernetics*, 46(11):2497–2508.

Yu, Y., Feng, J., Meng, M., and Wang, B. (2017). Topological structure of implicit Boolean networks. *IET Control Theory & Applications*, 11(13):2058–2064.