Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Decoupled feedforward-feedback periodic event-triggered control for disturbance rejection ⋆

Ernesto Aranda-Escolástico * **María Guinaldo** ** **José Luis Guzmán** *** **Sebastián Dormido** **

* *Dep. of Software and Systems Engineering, UNED (e-mail: earandae@issi.uned.es).*
** *Dep. of Computer Sciences and Automatic Control, UNED (e-mail: {mguinaldo,sdormido}@dia.uned.es)*
*** *Dep. of Informatics, ceiA3, CIESOL, University of Almería, (e-mail: joguzman@ual.es)*

Abstract: In this paper, feedforward and feedback controllers are studied considering decoupled periodic event-triggering mechanisms for output and disturbance sensors. Stability and robustness conditions for linear systems are obtained considering transportation delays and actuator saturation following the Lyapunov-Krasovskii procedure. A numerical example shows that the proposed control strategy reduces the communication between sensors and controller significantly, while the system performance is not deteriorated.

Keywords: Event-triggered control, feedforward control, Lyapunov stability, time delay, disturbance rejection, saturation

1. INTRODUCTION

In cyber-physical systems, there is a tight integration and dependence of control, communication and computation. The existence of a network to connect the set of embedded nodes implies some benefits that can be obtained with respect to lower costs, simplified installation and maintenance. However, the communication network has also an impact over the performance of the system, as it can introduce delays or packet losses limiting the transmission rate. In this context, event-triggered control (ETC) has gained much attention since it has been demonstrated to reduce communication and, hence, alleviates the effects of the network over the system performance. Different ETC strategies have been proposed , but the main idea is the following: the decision of when to exchange information with other nodes in the system is taken based on the internal state of the element rather than on time.

One possible classification of these strategies is based on *how often* the condition that decides if a transmission occurs is checked. In self-triggered control (see, e.g., (Velasco et al., 2003; Mazo et al., 2010)) a prediction of the evolution of the system is used to determine the next transmission time. Under this approach, conservative sampling intervals may be necessary to properly address unknown phenomena such as disturbances. In Continuous Event-Triggered Control (CETC), the triggering condition is checked continuously (Tabuada, 2007; Lunze and Lehmann, 2010), which can offer better performance results, though it is not implementable as such in digital platforms. Finally, Periodic Event-Triggered Control (PETC) (Heemels et al., 2013; Peng and Han, 2013; Aranda-Escolástico et al., 2016, 2018) evaluates the condition at prefixed instances of time.

Even though the benefits of event-triggering have been demonstrated, it might occur that an ETC system that performs properly in the absence of disturbances, becomes rather ineffective in the presence of disturbances, even if these are small (Borgers and Heemels, 2013). Hence, the design of control schemes that take into account disturbances and deal with them effectively is one of the open problems in ETC.

Traditionally, three control strategies are used to reduce the effects of disturbances: local feedback, direct feedforward and prediction-based feedforward (Åström and Wittenmark, 1997). The last two require precise information of the process. Whereas direct feedforward consists in supplying a complementary control signal that is computed from the current measured value of the disturbance, prediction-based feedforward estimates the value of the disturbance using an internal model. The choice of the most convenient technique depends on the characteristics of the process, e.g., whether the disturbance can be measured or is affected by a dead time. In general, feedforward control has been proved to improve the performance, especially in process control such as chemical or agricultural systems, but also in robotic manipulators, servo systems or disk drive systems (see, e.g., (Guzmán and Hägglund, 2011; Li et al., 2016), and references therein).

Another constraint of practical applications is the actuator saturation. It is known that the performance can deteriorate whenever the actuators saturate due to physical or

⋆ This work was supported by the Spanish Ministry of Science, Innovation and Universities under Projects DPI2017-84259-C2-1-R, DPI2017-84259-C2-2-R and CICYT RTI2018-094665-B-I00.

safety constraints (Tarbouriech et al., 2011). Its study is specially interesting in feedforward control, since the input depends directly on the disturbance, so the actuator saturation limits the disturbances that can be compensated.

To the best knowledge of the authors, the analysis of direct feedforward control of dead-time systems within an ETC paradigm has not been treated in literature, remaining as an interesting open line of research (Lunze, 2015). In (Sánchez et al., 2011; Beschi et al., 2014), a send-on-delta PI plus feedforward controller is developed. However, it is limited to specific processes and disturbance transfer functions. Besides, input saturation is not considered and no stability analysis is provided. In (Iwaki et al., 2018), input saturation is studied but only static feedforward control without disturbance and input delays is considered, which reduces the possibilities of disturbance compensation (Guzmán and Hägglund, 2011). Finally, all of them consider a CETC strategy, hindering its implementation in digital platforms. Rodríguez et al. (2019) does study a PETC strategy, but in a much more limited framework, i.e. only static feedforward controllers are studied and input saturation and disturbance delays are not considered. In this work, we address these issues to provide a more general and more applicable ETC strategy.

In summary, we propose an ETC strategy that is aware of four aspects that characterizes practical implementations: 1) the digital platforms do not allow to monitor signals continuously, so the proposed design is based on PETC; 2) the occurrence of disturbances, which is addressed by direct feedforward control; 3) the actuator saturation, which is studied in the design and the stability analysis; 4) the transportation delays in disturbance and input signals, which are considered in the robustness analysis.

1.1 Preliminaries

We define the set of real numbers and the set of natural numbers as $\mathbb{R}$ and $\mathbb{N}$, respectively and $\mathbb{R}_{\geq 0}$ denotes the set $\{x \in \mathbb{R} | x \geq 0\}$. The n-dimensional real space is defined by $\mathbb{R}^n$. We refer to the euclidean norm of vector $x \in \mathbb{R}^n$ as $\|x\| := \sqrt{x^T x}$. The $\mathcal{L}_2[0,\infty)$ space is the set of all real vector valued functions $f : \mathbb{R}^n \to \mathbb{R}^m$ such that $\|f(x)\|_2^2 = \int_0^\infty \|f(x)\|^2 dx < \infty$. Let $A \in \mathbb{R}^{n\times m}$, the transpose matrix of A is denoted by $A^\top$. $\lambda_M(A)$ and $\lambda_m(A)$ denote the maximum and minimum eigenvalue of A, respectively. $\|A\|$ denotes the maximum singular value of A. We denote the identity matrix of appropriate dimensions by $\mathbb{I}$. Symmetric matrices of the form $\begin{bmatrix} A & B^\top \\ B & C \end{bmatrix}$ are denoted as $\begin{bmatrix} A & \star \\ B & C \end{bmatrix}$. We further denote a symmetric positive-definite matrix $P \in \mathbb{R}^{n\times n}$ as $P \succ 0$. Matrices $P \succeq 0$, $P \prec 0$ and $P \preceq 0$ refer to symmetric positive-semidefinite, negative-definite, and negative-semidefinite matrices, respectively. We denote by $W[-\bar{\delta}, 0]$ the space of functions $\phi : [-\bar{\delta}, 0] \to \mathbb{R}$, which are absolutely continuous on $[-\bar{\delta}, 0)$, have finite limit $\lim_{s\to 0^-} \phi(s)$ and have square integrable first order derivatives with the norm $\|\phi\|_W = \max_{s\in[-\bar{\delta},0]} \|\phi(\theta)\| + \left[\int_{-h}^{0} \|\phi(s)\|^2 ds\right]^{\frac{1}{2}} + \left[\int_{-\bar{\delta}}^{0}\int_{s+\alpha}^{s} \|\dot{\phi}(\theta)\|^2 d\theta d\alpha\right]^{\frac{1}{2}}$.

Definition 1. *The saturation vector function of $u \in \mathbb{R}^m$ is defined by $sat(u) = [sat_1(u_1)\ sat_2(u_2)\ ...\ sat_m(u_m)]^\top$ and $sat_i(\cdot)$ for $i = 1, ..., m$ denotes the standard saturation function* $sat_i(u_i) = \begin{cases} \rho_{Mi}, & \text{if } u_i > \rho_{Mi} \\ u_i, & \text{if } \rho_{mi} \leq u_i \leq \rho_{Mi} \\ -\rho_{mi}, & \text{if } u_i < -\rho_{mi} \end{cases}$, *where $\rho_{Mi} \geq 0$ and $\rho_{mi} \geq 0$ are the upper and lower bounds of input u_i, respectively.*

Lemma 1 ((Sun et al., 2015)). *Let $u \in \mathbb{R}^m$, $sat(\cdot)$ from Definition 1, and*

$$\eta(u) = u - sat(u). \tag{1}$$

Then, there exists a real number $\epsilon \in (0,1)$ such that

$$\eta(u)^\top \eta(u) \leq \epsilon u^\top u, \tag{2}$$

where $\eta = [\eta_1\ \eta_2\ ...\ \eta_m]^\top \in \mathbb{R}^m$, and η_i is the dead-zone nonlinearity function for $i = 1, 2, ..., m$.

Remark 1. Note that it can be proved that $\epsilon \geq (1 - \bar{u}/u_i)^2$ for $i = 1, ..., n$ and where $\bar{u} = \max\{\rho_{M1}, \rho_{M2}, ..., \rho_{Mm}, \rho_{m1}, \rho_{m2}, ..., \rho_{mm}\}$, following the development in (Sun et al., 2015). This implies that

$$u_i \leq \bar{u}/\left(1 - \sqrt{\epsilon}\right), \tag{3}$$

and consequently, the lemma can only be applied locally.

Lemma 2 ((Jensen, 1906)). *Let $M \in \mathbb{R}^{n\times n}$ be a symmetric positive definite matrix, $a, b \in \mathbb{R}$ scalars with $b > a$, and $\omega : [a, b] \to \mathbb{R}^n$ an integrable vector function. Then, it holds for any $\beta \in [a, b]$ that $\int_a^b \omega^T(\beta) M \omega(\beta) d\beta \geq \frac{1}{b-a}\left(\int_a^b \omega(\beta) d\beta\right)^T M \left(\int_a^b \omega(\beta) d\beta\right)$.*

Lemma 3 ((Park et al., 2011)). *Let $R_1 \in \mathbb{R}^{n_1\times n_1}$,...,$R_N \in \mathbb{R}^{n_N\times n_N}$ be symmetric positive definite matrices. Then, for all $\xi_1 \in \mathbb{R}^{n_1}$,...,$\xi_N \in \mathbb{R}^{n_N}$, for all $\alpha_i > 0$ with $\sum_{i=0}^{N} \alpha_i = 1$ and for all $S_{ij} \in \mathbb{R}^{n_i\times n_j}$ with $i = 1, ..., N$, $j = 1, ..., i-1$ such that $\begin{bmatrix} R_i & \star \\ S_{ij} & R_j \end{bmatrix} \succeq 0$, the following inequality holds*

$$\sum_{i=0}^{N} \alpha_i^{-1} \xi_i^\top R_i \xi_i \geq \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_N \end{bmatrix}^\top \begin{bmatrix} R_1 & \star & \star & \star \\ S_{21} & R_2 & \star & \star \\ \vdots & \vdots & \ddots & \vdots \\ S_{N1} & S_{N2} & \cdots & R_N \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_N \end{bmatrix}.$$

2. PROCESS AND DISTURBANCE DESCRIPTION

In this section, we describe the process and disturbance models. We consider a process affected by an external disturbance, which is compensated with periodic event-triggered feedforward and feedback controllers, as shown in Figure 1. The process and disturbance outputs are periodically sampled with sampling period $h > 0$ but transmitted only when the corresponding decoupled event-triggering conditions are satisfied.

2.1 Process model

The process is given by the continuous linear time-invariant (LTI) system

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p \hat{u}(t - \delta_u) + B_w w(t), \\ y_p(t) &= C_p x_p(t), \quad x_p(t_0) = x_{p0}, \end{aligned} \tag{4}$$

where $x_p(t) \in \mathbb{R}^{n_p}$ is the state of the process, $x_{p0} \in \mathbb{R}^{n_p}$ the initial condition, $\hat{u}(t) \in \mathbb{R}^m$ the control input given

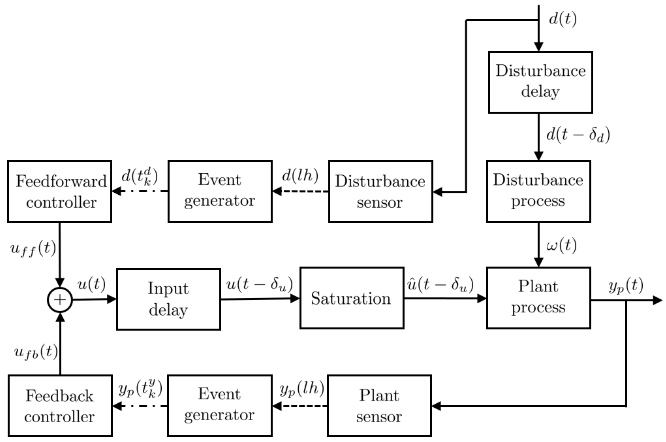

Fig. 1: Block diagram of the control system. (Solid) Continuous signals. (Dashed) Periodic time-triggered signals. (Dash-dotted) Event-triggered signals.

by $\hat{u}(t) = \text{sat}\,(u(t))$, $\delta_u \geq 0$ a constant transportation delay, $w(t) \in \mathbb{R}^q$ the disturbance, $y(t) \in \mathbb{R}^{r_p}$ the output and $t \in \mathbb{R}_{\geq 0}$. It is assumed that A_p, B_p, B_w and C_p are real matrices of appropriate dimensions, and that the pair (A_p, B_p) is controllable and (A_p, C_p) is observable.

2.2 Disturbance model

In most cases, the process is not directly affected by the disturbance. For example, it might have a transportation delay (Guzmán and Hägglund, 2011). Hence, we consider that the influence of the disturbance is modeled by the following continuous LTI system

$$\begin{aligned} \dot{x}_d(t) &= A_d x_d(t) + B_d d(t-\delta_d), \\ w(t) &= C_w x_d(t) + D_w d(t-\delta_d), \\ x_d(t_0) &= x_{d0}, \end{aligned} \tag{5}$$

where $x_d(t) \in \mathbb{R}^{n_d}$ is the state of the disturbance, $x_{d0} \in \mathbb{R}^{n_p}$ the initial condition, $d(t) \in \mathbb{R}^s$ the original disturbance and $\delta_d \geq 0$ the disturbance dead time. As before, it is assumed that A_d, B_d, C_w and D_w are real matrices of appropriate dimensions. We assume also that A_d is Hurwitz, i.e. the disturbance model (5) is stable, which is logical because if $d(t) \to 0$, then the effect of the disturbance over the process should tend to zero also. Besides, we consider two other assumptions with respect to $d(t)$. First, we assume that it is observable such that

$$y_d(t) = C_d d(t), \tag{6}$$

with $y_d(t) \in \mathbb{R}^{r_d}$ and C_d a real matrix of appropriate dimensions. Second, we assume that $d(t) \in L_2[t_0, \infty)$, which is used in the following to guarantee the robustness. Note that this assumption implies that there is a d_M such that $d(t) \leq d_M$ $\forall t$, and that $\|w(t)\|_2 \leq \tilde{\gamma}\|d(t)\|_2$ for some $\tilde{\gamma} > 0$. Logically, it follows that $w(t) \in L_2[t_0, \infty)$.

3. DECOUPLED PERIODIC EVENT-TRIGGERED CONTROLLERS

In this section, we introduce the controllers designed following an emulation-based approach.

3.1 Feedback controller

We consider the dynamic output feedback controller

$$\begin{aligned} \dot{x}_{fb}(t) &= A_{fb} x_{fb}(t) + B_{fb}\hat{y}_p(t), \\ u_{fb}(t) &= C_{fb} x_{fb}(t) + D_{fb}\hat{y}_p(t), \\ x_{fb}(t_0) &= x_{fb0}, \end{aligned} \tag{7}$$

where $x_{fb}(t) \in \mathbb{R}^{n_{fb}}$ is the state of the feedback controller, $x_{fb0} \in \mathbb{R}^{n_{fb}}$ the initial condition, $u_{fb}(t) \in \mathbb{R}^m$ the feedback control input and $\hat{y}_p(t) = y_p(t_k^p)$ for $t \in [t_k^p, t_{k+1}^p)$, where t_k^p is the last transmission instant of y_p. A_{fb}, B_{fb}, C_{fb} and D_{fb} are real matrices of appropriate dimensions.

3.2 Feedforward controller

Based on the output disturbance (6), we consider the following dynamic feedforward controller

$$\begin{aligned} \dot{x}_{ff}(t) &= A_{ff} x_{ff}(t) + B_{ff}\hat{y}_d(t), \\ u_{ff}(t) &= C_{ff} x_{ff}(t) + D_{ff}\hat{y}_d(t), \\ x_{ff}(t_0) &= x_{ff0}, \end{aligned} \tag{8}$$

where $x_{ff}(t) \in \mathbb{R}^{n_{ff}}$ is the state of the feedforward controller, $x_{ff0} \in \mathbb{R}^{n_{ff}}$ the initial condition, $u_{ff}(t) \in \mathbb{R}^m$ the feedforward input and $\hat{y}_d(t) = y_d(t_k^d)$ for $t \in [t_k^d, t_{k+1}^d)$, where t_k^d is the last transmission instant of y_d. A_{ff}, B_{ff}, C_{ff} and D_{ff} are real matrices of appropriate dimensions.

3.3 Event-triggering mechanisms

In this work, we use two different event-triggering mechanisms (ETMs) to transmit the output and disturbance signals. We denote the process error vector as

$$e_p(t) := \hat{y}_p(t) - y_p(t), \tag{9}$$

which is reset to zero at each transmission instant t_k^p. Similarly, we denote the disturbance error vector as

$$e_d(t) := \hat{y}_d(t) - y_d(t), \tag{10}$$

which is reset to zero at each t_k^d. Finally, we define

$$\begin{aligned} t_{k+1}^p &= \inf\left\{lh > t_k^p \mid \mathcal{C}_p\left(e_p(lh), y_p(lh)\right) > 0,\ l \in \mathbb{N}\right\}, \\ t_{k+1}^d &= \inf\left\{lh > t_k^d \mid \mathcal{C}_d\left(e_d(lh), y_d(lh)\right) > 0,\ l \in \mathbb{N}\right\}, \end{aligned} \tag{11}$$

where $h > 0$ is the sampling period and $\mathcal{C}_p(e_p, y_p) = e_p^\top \Omega_p e_p - \sigma_p^2 y_p^\top \Omega_p y_p$, $\mathcal{C}_d(e_d, y_d) = e_d^\top \Omega_d e_d - \sigma_d^2 y_d^\top \Omega_d y_d$, with $\sigma_p \geq 0$ and $\sigma_d \geq 0$. If $\sigma_p = 0$ ($\sigma_d = 0$), then the process (disturbance) output is periodically transmitted.

3.4 Closed-loop system

We introduce an artificial delay $\delta(t)$, which is the difference between the current instant and the last sampling instant, i.e. $\delta(t) = t - lh \leq h$, and the augmented state vector $x(t) = [x_p(t)\ x_d(t)\ x_{fb}(t)\ x_{ff}(t)]^\top$, such that $x(t) \in \mathbb{R}^n$, where $n = n_p + n_d + n_{fb} + n_{ff}$, and the augmented error vector $e(t) = [e_p(t)\ e_d(t)]^\top$, such that $e(t) \in \mathbb{R}^r$, where $r = r_p + r_d$. Now, we define the control input $u(t)$ as $u(t) = u_{fb}(t) + u_{ff}(t)$. Using (9)-(10), we have

$$\begin{aligned} u(t) =& K_1 x(t) + K_2 x(t-\delta(t)) \\ &+ K_3 d(t-\delta(t)) + K_4 e(t-\delta(t)). \end{aligned} \tag{12}$$

for $t \in [lh, (l+1)h)$, where $K_1 = [0\ 0\ C_{fb}\ C_{ff}]$, $K_2 = [D_{fb}C_p\ 0\ 0\ 0]$. $K_3 = D_{ff}C_d$ and $K_4 = [D_{fb}\ D_{ff}]$. Finally, combining (4), (5), (7), (8), (12) and (1), we obtain

$$\begin{aligned} \dot{x}(t) =& A_1 x(t) + A_2 x(t_u) + A_3 x(t_u - \delta(t_u)) \\ &+ A_4 d(t_d) + A_5 d(t_u - \delta(t_u)) \\ &+ A_6 e(t_u - \delta(t_u)) + A_7 \eta(u(t-\delta_u)), \\ y(t) =& y_p(t) = Cx(t) \end{aligned} \tag{13}$$

where $t_u = t - \delta_u$, $t_d = t - \delta_d$, and

$$A_1 = \begin{bmatrix} A_p & 0 & 0 & 0 \\ 0 & A_d & 0 & 0 \\ 0 & 0 & A_{fb} & 0 \\ 0 & 0 & 0 & A_{ff} \end{bmatrix},\; A_2 = \begin{bmatrix} 0 & B_w C_w & B_p C_{fb} & B_p C_{ff} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} B_p D_{fb} C_p & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ B_{fb} C_p & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\; A_4 = \left[(B_w D_w)^\top \; B_d^\top \; 0 \; 0\right]^\top$$

$$A_5 = \left[(B_p D_{ff} C_d)^\top \; 0 \; 0 \; (B_{ff} C_d)^\top\right]^\top,\; C = [C_p \; 0 \; 0 \; 0],$$

$$A_6 = \begin{bmatrix} B_p D_{fb} & B_p D_{ff} \\ 0 & 0 \\ B_{fb} & 0 \\ 0 & B_{ff} \end{bmatrix},\; A_7 = \left[B_p^\top \; 0 \; 0 \; 0\right]^\top.$$

4. STABILITY ANALYSIS

The stability analysis of system (13) under the ETM (11) is carried out following the Lyapunov-Krasovskii approach (Fridman, 2014). For a given disturbance attenuation level γ, a maximum disturbance d_M and an input saturation $\bar{u}$, we achieve the following control objectives:

(i) The state trajectories of the closed-loop system (13) under (11) that start from a region $\mathcal{E} = \{x(t) : \|x(t)\|_W \leq \zeta\}$, with ζ a positive constant bound to be estimated later on, will remain in $\mathcal{E}$ for any disturbance such that $\|K_3 + 2D_{ff}\|\|d_M\| < \bar{u}/\left(1 - \sqrt{(\epsilon)}\right)$, with ϵ from Lemma 1. In addition, the closed is locally asymptotically stable in absence of disturbances and globally asymptotically stable in absence of disturbances and input saturation.

(ii) Under zero initial condition, the controlled process output $y_p(t)$ meets $\|y_p(t)\| \leq \gamma\|d(t)\|_2$ for any nonzero $d(t) \in \mathcal{L}_2[0, \infty)$ and $d(t - \delta(t)) \in \mathcal{L}_2[0, \infty)$ such that $\|K_3 + 2D_{ff}\|\|d_M\| < \bar{u}/\left(1 - \sqrt{(\epsilon)}\right)$.

Theorem 1. *For given σ_p, σ_u, $\bar{\delta} = \delta_u + h$, γ, β and ϵ, if there exists real matrices $P \succ 0$, $Q \succ$, $R \succ 0$, S_{21}, S_{31}, S_{32}, $\Omega_p \succ 0$ and $\Omega_d \succ 0$ of appropriate dimensions, and real numbers $\mu_1 > 0$ and $\mu_2 > 0$ such that*

$$\Pi \prec 0, \quad \begin{bmatrix} R & \star & \star \\ S_{21} & R & \star \\ S_{31} & S_{32} & R \end{bmatrix} \succ 0, \tag{14}$$

where Π is defined in Box 1. Then, the control objectives (i) and (ii) are satisfied for the closed loop system (13) with ETM (11) for the region $\mathcal{E}$ with $\zeta = \zeta_1\lambda_m(P)/\zeta_2$, where $\zeta_1 = \bar{u}/\left(1 - \sqrt{(\epsilon)}\right) - \|K_3 + 2D_{ff}\|\|d_M\|/\|K_1 + 3K_2\|$ and $\zeta_2 = \max\{\lambda_M(P), \lambda_M(Q_1 + Q_2), \bar{\delta}^2\lambda_M(R)\}$.

Proof. Construct the Lyapunov functional

$$V(t) = x^\top(t)Px(t) + \bar{\delta}\int_{-\bar{\delta}}^{0}\int_{t+\alpha}^{t} \dot{x}^\top(\theta)R\dot{x}(\theta)d\theta d\alpha + \int_{t-\bar{\delta}}^{t} x^\top(\alpha)Q_1x(\alpha)d\alpha + \int_{t-\delta_u}^{t} x^\top(\alpha)Q_2x(\alpha)d\alpha, \tag{15}$$

where P, Q and R are symmetric positive-definite matrices. The time derivative of (15) results

$$\begin{aligned}\dot{V}(t) =& 2\dot{x}^\top(t)Px(t) + x^\top(t)Q_1x(t) + x^\top(t)Q_2x(t) \\ &- x^\top(t - \bar{\delta})Q_1x(t - \bar{\delta}) - x^\top(t_u)Q_2x(t_u) \\ &+ \bar{\delta}^2\dot{x}^\top(t)R\dot{x}(t) - \bar{\delta}\int_{t-\bar{\delta}}^{t} \dot{x}^\top(\alpha)R\dot{x}(\alpha)d\alpha.\end{aligned} \tag{16}$$

Applying Lemmas 2 and 3, we bound the integral term by

$$-\bar{\delta}\int_{t-\bar{\delta}}^{t} \dot{x}^\top(\alpha)R\dot{x}(\alpha)d\alpha \leq -\begin{bmatrix}\xi_1 \\ \xi_2 \\ \xi_3\end{bmatrix}^\top \begin{bmatrix} R & \star & \star \\ S_{21} & R & \star \\ S_{31} & S_{32} & R \end{bmatrix} \begin{bmatrix}\xi_1 \\ \xi_2 \\ \xi_3\end{bmatrix}, \tag{17}$$

where $\xi_1 = x(t) - x(t_u)$, $\xi_2 = x(t_u) - x(t_u - \delta(t_u))$ and $\xi_3 = x(t_u - \delta(t_u)) - x(t - \bar{\delta})$. We add now to (16) the null terms $0 = e^\top(t_u - \delta(t_u))\Omega_2 e(t_u - \delta(t_u)) - e^\top(t_u - \delta(t_u))\Omega_2 e(t_u - \delta(t_u))$, $0 = x^\top(t)C^\top Cx(t) - y_p^\top(t)y_p(t)$, $0 = \gamma^2(1 - \mu_1\beta^2)d^\top(t - \delta_d)d(t_d) - \gamma^2(1 - \mu_1\beta^2)d^\top(t - \delta_d)d(t_d)$, $0 = \gamma^2\mu_1 d^\top(t - \delta(t))d(t - \delta(t)) - \gamma^2\mu_1 d^\top(t - \delta(t))d(t - \delta(t))$, $0 = \mu_2\eta^\top(u(t_u))\eta(u(t_u)) - \mu_2\eta^\top(u(t_u))\eta(u(t_u))$. Finally, using inequalities (2), (11) and (17), we obtain

$$\begin{aligned}\dot{V}(t) \leq& \upsilon^\top\Pi\upsilon + \gamma^2(1 - \mu_1\beta^2)d^\top(t_d)d(t_d) \\ &+ \gamma^2\mu_1 d^\top(t - \delta(t))d(t - \delta(t)) - y_p^\top(t)y_p(t),\end{aligned} \tag{18}$$

where $\upsilon^\top = [x^\top(t) \; x^\top(t_u - \delta(t_u)) \; d^\top(t_d) \; d^\top(t - \delta(t)) \; e^\top(t_u - \delta(t_u)) \; \eta^\top(u(t_u)) \; x^\top(t - t_u) \; x^\top(t - \bar{\delta})]$. In absence of disturbances, $d(t) = d(t - \delta(t)) = d(t_d) = 0$ for $t \geq 0$, and thus, $\dot{V}(t) \leq 0$ if (14) is satisfied, and the system is asymptotically stable.

In presence of disturbances and considering (14), we can integrate (18). Now, observe that if $d(t) \in \mathcal{L}_2[0, \infty)$, then $\|d(t_d)\|_2 = \|d(t)\|_2 < \infty$, because δ_d is a constant transportation delay, and that if $d(t - \delta(t)) \in \mathcal{L}_2[0, \infty)$, then $\|d(t - \delta(t))\|_2 < \infty$. Consequently, there exists $\beta > 0$ such that $\|d(t - \delta(t))\|_2 \leq \beta\|d(t)\|_2$. Thus, letting $t \to \infty$, and taking into account that $V(t_0) = 0$ under zero initial conditions and that $V(t) \geq 0$, it is obtained $\int_{t_0}^{\infty} y_p^\top(\alpha)y_p(\alpha)d\alpha \leq \gamma^2\int_{t_0}^{\infty} d^\top(\alpha)d(\alpha)d\alpha$ if (14) is satisfied. Note now that Π depends on ϵ. This implies that (3) should be verified for that value of ϵ, and therefore, the result is only applicable if the state and the disturbance are bounded, i.e. it is only locally valid. To obtain an estimation of the region, we make it two steps. First, we find the maximum value that $x(t)$ can reach to preserve (3) if the disturbance takes its maximum value; and secondly, we obtain, using (15), an upper bound for $x(t)$ depending on the initial conditions. So, if this second bound satisfies the first condition, then we can ensure that $x(t)$ remains bounded for all time. So, using (3) and (12),

$$\|x(t)\| \leq \frac{\bar{u}/\left(1 - \sqrt{(\epsilon)}\right) - \|K_3 + 2D_{ff}\|\|d_M\|}{\|K_1 + 3K_2\|} = \zeta_1 \tag{19}$$

for all t. Finally, to ensure that $\|x(t)\|$ satisfies this bound, we use the Lyapunov functional (15), to guarantee that $\lambda_m(P)\|x(t)\| \leq V(t) \leq V(0) \leq \zeta_2\|x(0)\|_W$, if (3) is satisfied. Thus, if the initial conditions satisfy $\|x(0)\|_W \leq \zeta_1\lambda_m(P)/\zeta_2 = \zeta$, then it is guaranteed that (19) is satisfied for all t and the proof is completed. □

$$\Pi = \begin{bmatrix} \Pi_{11} & \star & \star & \star & \star & \star & \star & \star \\ \Pi_{21} & \Pi_{22} & \star & \star & \star & \star & \star & \star \\ A_3^\top P + \Psi_{31} & \Psi_{32} & \Pi_{33} & \star & \star & \star & \star & \star \\ \Pi_{41} & \Psi_{42} + \epsilon\mu_2 K_3^\top K_2 & \Psi_{43} & \Pi_{44} & \star & \star & \star & \star \\ \Pi_{51} & \Psi_{52} + \epsilon\mu_2 K_4^\top K_2 & \Psi_{53} & \Psi_{54} + \epsilon\mu_2 K_4^\top K_3 & \Pi_{55} & \star & \star & \star \\ A_6^\top P + \Psi_{61} & \Psi_{62} & \Psi_{63} & \Psi_{64} & \Psi_{65} & \Psi_{66} - \mu_2 \mathbb{I} & \star & \star \\ R - S_{32} & \Pi_{72} & 0 & 0 & 0 & 0 & \Pi_{77} & \star \\ \Pi_{71} & R - S_{32} & 0 & 0 & 0 & 0 & S_{32} - S_{31} & -R - Q_1 \end{bmatrix},$$

where

$$\Pi_{11} = PA_1 + A_1^\top P + \Psi_{11} + Q_1 + Q_2 - R + C^\top C + \epsilon\mu_2 K_1^\top K_1 \quad \Pi_{21} = A_2^\top P + \Psi_{21} + S_{21} - S_{31} + \epsilon\mu_2 K_2^\top K_1$$
$$\Pi_{22} = \Psi_{22} - 2R + S_{32} + S_{32}^\top + \sigma_y^2 C^\top \Omega_1 C + \epsilon\mu_2 K_2^\top K_2 \quad \Pi_{33} = \Psi_{33} + \gamma^2\left(\mu_1\beta^2 - 1\right)\mathbb{I}$$
$$\Pi_{41} = A_4^\top P + \Psi_{41} + \epsilon\mu_2 K_3^\top K_1 \quad \Pi_{44} = \Psi_{44} - \gamma^2\mathbb{I} + \epsilon\mu_2 K_3^\top K_3 + \sigma_d^2\Omega_d$$
$$\Pi_{51} = A_5^\top P + \Psi_{51} + \epsilon\mu_2 K_4^\top K_1 \quad \Pi_{55} = \Psi_{55} - \Omega_2 + \epsilon\mu_2 K_4^\top K_4$$
$$\Pi_{71} = R + S_{31} - S_{21} - S_{32} \quad \Pi_{77} = -Q_2 - 2R + S_{21} + S_{21}^\top$$
$$\Omega_1 = \begin{bmatrix} \Omega_y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \Omega_2 = \begin{bmatrix} \Omega_y & 0 \\ 0 & \Omega_d \end{bmatrix} \qquad \Psi_{ij} = \bar{\delta} A_i^\top R A_j$$

Box 1

Remark 2. Note that if there is no saturation, then $\bar{u} \to \infty$, $\zeta_1 \to \infty$, and Theorem 1 can be applied globally.

Remark 3. Note that we assume not only that $d(t) \in \mathcal{L}_2$, but also that $d(t - \delta(t)) \in \mathcal{L}_2$, i.e. the signal formed by the sampled measurements has 2-norm bounded. This is an acceptable assumption, which in fact is always satisfied for disturbances which are zero after a certain time.

Remark 4. The LMI (14) depends on σ_p, σ_u, $\bar{\delta}$, γ and ϵ. The larger σ_p and σ_u are, the less events are triggered and the less communication resources are wasted. The larger $\bar{\delta}$ is, the larger delays are allowed. The smaller γ is, the better disturbance attenuation level is achieved. The larger ϵ is, the larger the region of attraction $\mathcal{E}$ is. Therefore, there is a logical trade-off in (14), where the different performance parameters of the system are involved.

5. SIMULATIONS RESULTS

Consider the process described in (Liu and Yang, 2018):

$$A_p = \begin{bmatrix} 0.1 & 0.6 \\ 0 & -0.1 \end{bmatrix}, \ B_p = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_w = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \ C_p = [1 \ 1].$$

Consider also that the disturbance is $d(t) = 100\sin(t)$ for $t \in (4\pi, 8\pi)$ and $d(t) = 0$ otherwise. It is easy to check that we can obtain $\beta = 2$ independently of h. In addition, we consider that the disturbance affects to the system, through a first order process such that $A_d = -1$, $B_p = 1$, $C_w = 1$, $D_w = 0$, and that the measurable disturbance is defined by $C_d = 1$. A PI controller is used for feedback control, while the feedforward controller is designed following the guidelines in (Guzmán and Hägglund, 2011). The corresponding matrices are $A_{fb} = 0$, $B_{fb} = -1$, $C_{fb} = 6.8$, $D_{fb} = -2.4, A_{ff} = -0.5$, $B_{ff} = 1$, $C_{ff} = -0.025$, $D_{ff} = -0.05$.

Inequalities (14) depends on the parameters σ_p, σ_d, $\bar{\delta}$, γ and ϵ, i.e there is a trade-off between the event generation, the sampling period, the delays, the disturbance attenuation level and the input saturation. For the simulation, we have chosen $h = 0.01$, $\delta_u = 0.02$, $\delta_d = 0.01$, $\sigma_p = 0.1$, $\sigma_d = 0.3$, $\epsilon = 0.35$, which implies a minimum disturbance attenuation level $\gamma = 1.52$ and matrices $\Omega_p = 642.46$ and $\Omega_d = 1.31$ to satisfy (14). We set $\rho_M = \rho_m = 8$ and initial conditions $x(0) = [1 \ -2 \ 0 \ 0 \ 0]$. We have compared the periodic event-triggered feedforward control with respect to the control without feedforward and with respect to the periodic feedforward control. In all cases, we consider the same periodic event-triggered feedback controller. We verify that the feedforward controller improves considerably the disturbance compensation. In addition, the PETC strategy reduces considerably the communication from sensor to controller but maintaining a good performance, as shown in Table 1. In Figures 2-3, the output and the input of the process are depicted for the three cases.

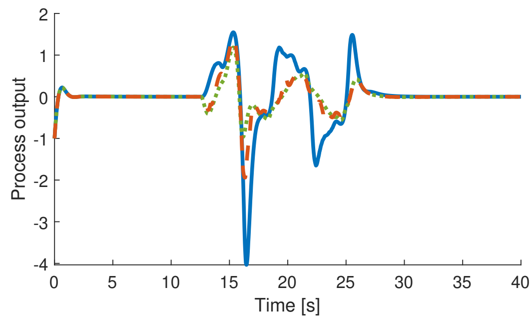

Fig. 2: Output signal of the process. (Solid blue) Without feedforward. (Dotted green) Periodic feedforward. (Dashed red) Periodic event-triggered feedforward.

6. CONCLUSIONS AND FUTURE WORKS

We propose a generalized framework of periodic event-triggered feedback-feedforward control. It enables to obtain a trade-off between disturbance compensation and waste of communication resources. Stability and robustness analysis are provided taking into account dynamical controllers. These analyses show that there exists a compromise between the parameters of closed control loop, i.e.

Method	ISE (y_p)	IAE (y_p)	ITAE (y_p)	Events generated at the output sensor	Events generated at the disturbance sensor
Without feedforward	18.12	13.18	249.98	399	–
With periodic feedforward	2.91	5.62	102.52	458	2000
With periodic event-triggered feedforward	4.57	6.32	112.31	435	82

Table 1: Comparison of performance index and number of transmissions.

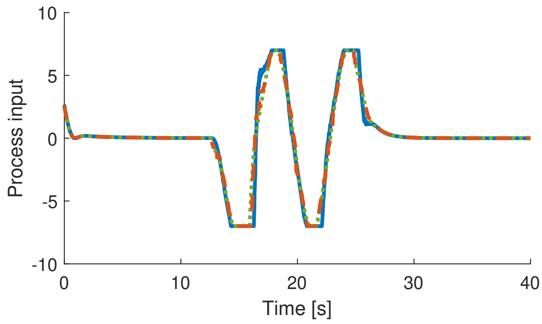

Fig. 3: Input signal of the process. (Solid blue) Without feedforward. (Dotted green) Periodic feedforward. (Dashed red) Periodic event-triggered feedforward.

between the parameters of the event-triggering condition, the sampling period, the delays, the actuator saturation and the disturbance attenuation level.

Feedback and feedforward controllers are assumed to be known, and then, stability and robustness are proved for PETC. When the controller is static, it is possible to co-design it with the ETM. However, this leads to a new research line for dynamical controllers. Obtaining a design of dynamical controllers taking into account the ETC strategy (or at least some tuning rules) might improve the system performance maintaining a reduced communication. Additionally, an extension to consider estimated disturbances would make the method applicable.

REFERENCES

Aranda-Escolástico, E., Guinaldo, M., Gordillo, F., and Dormido, S. (2016). A novel approach to periodic event-triggered control: Design and application to the inverted pendulum. *ISA Trans.*, 65, 327–338.

Aranda-Escolástico, E., Rodríguez, C., Guinaldo, M., Guzmán, J.L., and Dormido, S. (2018). Asynchronous periodic event-triggered control with dynamical controllers. *J. Frankl. Inst.*, 355(8), 3455–3469.

Åström, K.J. and Wittenmark, B. (1997). *Computer-controlled systems: Theory and design (3rd ed.).* Prentice Hall.

Beschi, M., Dormido, S., Sanchez, J., Visioli, A., and Yebra, L.J. (2014). Event-based PI plus feedforward control strategies for a distributed solar collector field. *IEEE Trans. Control Syst. Tech.*, 22(4), 1615–1622.

Borgers, D.P. and Heemels, W.P.M.H. (2013). On minimum inter-event times in event-triggered control. In *52nd IEEE Conference on Decision and Control (CDC)*, 7370–7375.

Fridman, E. (2014). Introduction to time-delay and sampled-data systems. In *European Control Conference (ECC)*, 1428–1433.

Guzmán, J.L. and Hägglund, T. (2011). Simple tuning rules for feedforward compensators. *J. Process Control*, 21(1), 92–102.

Heemels, W.P.M.H., Donkers, M.C.F., and Teel, A.R. (2013). Periodic event-triggered control for linear systems. *IEEE Trans. Autom. Control*, 58(4), 847–861.

Iwaki, T., Wu, J., and Johansson, K.H. (2018). Event-triggered feedforward control subject to actuator saturation for disturbance compensation. In *European Control Conference (ECC)*, 501–506.

Jensen, J.L.W.V. (1906). Sur les fonctions convexes et les inégalités entre les valeurs moyennes. *Acta Math.*, 30(1), 175–193.

Li, S., Yang, J., Chen, W.H., and Chen, X. (2016). *Disturbance observer-based control: methods and applications.* CRC press.

Liu, D. and Yang, G.H. (2018). Event-triggered non-fragile control for linear systems with actuator saturation and disturbances. *Inf. Sci.*, 429, 1–11.

Lunze, J. (2015). *Event-based Control and Signal Processing*, chapter Event-based Control: Introduction and Survey, 3–20. CRC Press.

Lunze, J. and Lehmann, D. (2010). A state-feedback approach to event-based control. *Autom.*, 46, 211–215.

Mazo, M., Anta, A., and Tabuada, P. (2010). An iss self-triggered implementation for linear controllers. *Autom.*, 46(8), 1310–1314.

Park, P., Ko, J.W., and Jeong, C. (2011). Reciprocally convex approach to stability of systems with time-varying delays. *Autom.*, 47(1), 235–238.

Peng, C. and Han, Q.L. (2013). A novel event-triggered transmission scheme and l_2 control co-design for sampled-data control systems. *IEEE Trans. Autom. Control*, 58(10), 2620–2626.

Rodríguez, C., Aranda-Escolástico, E., Guinaldo, M., Guzmán, J.L., and Dormido, S. (2019). Event–based feedforward control of linear systems with input time–delay. *Int. J. Appl. Math. Comput. Sci.*, 29(3), 541–553.

Sánchez, J., Visioli, A., and Dormido, S. (2011). A two-degree-of-freedom PI controller based on events. *J. Process Control*, 21(4), 639–651.

Sun, L., Wang, Y., and Feng, G. (2015). Control design for a class of affine nonlinear descriptor systems with actuator saturation. *IEEE Trans. Autom. Control*, 60(8), 2195–2200.

Tabuada, P. (2007). Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Trans. Autom. Control*, 52(9), 1680–1685.

Tarbouriech, S., Garcia, G., Gomes da Silva Jr, J., and Queinnec, I. (2011). *Stability and stabilization of linear systems with saturating actuators.* Springer, London.

Velasco, M., Fuertes, J., and Marti, P. (2003). The self triggered task model for real-time control systems. In *24th IEEE Real-Time Systems Symposium*, volume 384.