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Control of Vehicular Platoons: Stochastic Robustness Against Jamming Attacks ⋆

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Abstract: Control of a platoon of vehicles subject to jamming attacks is addressed in this paper. Because of jamming attacks, some communicated information and radio data are assumed to be lost or delayed in a stochastic manner. By considering the constant time-gap spacing policy, we propose a control strategy which under certain conditions guarantees the almost sure regulation of the vehicles in desired relative distances. Accordingly, depending on the control gains, the robustness of the platoon against a wide range of jamming attacks is guaranteed. The main contribution of the paper is that the proposed control scheme is robust against jamming attacks on both the communication network and vehicles radars. Simulation results illustrate the performance of the proposed control strategy.

Keywords: Adaptive cruise control, jamming attack, stochastic systems, vehicular platoons.

1. INTRODUCTION

Reducing fuel consumption and the risk of accidents and increasing the rate of transportation are the main issues in urban traffic systems. The cooperative adaptive cruise control (CACC) has appeared as an idea to address these issues in modern traffic systems. Based on such technology, each vehicle is equipped with a cruise control system to adjust its speed to keep a safe distance from the preceding vehicle in a cooperative manner. Accordingly, a vehicle (driven manually by a human, or semi-autonomously, or autonomously) will be a leader for a platoon of vehicles, while all the vehicles are equipped with the required sensors and vehicle-to-vehicle communication platforms to obtain the necessary information for CACC (Xu et al., 2019; Alipour-Fanid et al., 2017).

Due to the use of radars and communication networks, vehicular platoons are vulnerable to several sources of attacks. Among various sources of attacks, jamming attack is one of main concerns due to the ease of carrying out by using simple electronic and communication devices. Jamming is a type of denial of service (DoS) attack defined as the intentional emission of random radio signals to saturate communication devices. Such attacks may lead to data loss or delays in receiving data, and can affect the performance of closed-loop control systems (Tanis, 2018; Laurendeau and Barbeau, 2006).

Studies in the area of the security of vehicular platoons mainly have been devoted to the detection of attacks in which developing strategies for the detection and estimation of various types of attacks are considered (Merco et al., 2018; Sargolzaei et al., 2016; Biron et al., 2018). Moreover, recently some efforts have been done on robust and resilient control of vehicular platoons in which robustness and reconfigurability against attacks are addressed. For instance, in (Alipour-Fanid et al., 2017), control of a platoon of vehicles in the presence of jamming attacks is investigated in which the jamming attack is modeled as random data loss in communication among the vehicles. In (Biron et al., 2017), a resilient control strategy is proposed for vehicular platoons in the presence of DoS attacks, where the DoS is modeled as delays in communication among the vehicles. In (Tamba and Nazaruddin, 2017), resilient control of vehicular platoons subject to DoS attacks in communication among controllers and actuators is studied, in which the DoS is modeled as communication loss. Moreover, some studies also have been devoted to increasing the robustness of vehicular platoons against communication loss and delays, which are the main outcomes of jamming attacks. For instance, in (Acciani et al., 2018), an observation strategy to increase the robustness of a platoon of vehicles subject to communication loss is proposed. In (Harfouch et al., 2018), a switching adaptive strategy for control of a platoon of vehicles with communication losses is proposed. In (Guo and Wen, 2016), control of a platoon of vehicles in the presence of random data loss is investigated. The stochastic $\mathcal{L}_2$ stability of platoons of vehicles in the presence of random packet loss is addressed in (Li et al., 2019). In (Van Nunen et al., 2019), model predictive control is used to increase the robustness of vehicular platoons against data loss, and in (Ge and Orosz, 2017) and (Ploeg et al., 2015), control of connected vehicles in the presence of communication delays is studied.

Based on the existing literature on control of vehicular platoons, it can be said that since the distances among the

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vehicles are the variables to be regulated, the performance of the existing results relies on the accurate measurement of the intervehicle distances. In other words, whereas various sources of uncertainties are considered in the existing results, since intervehicle distances are the variables to be regulated, the accurate performance of the radars in measurement of intervehicle distances is a main assumption. Thus, in those studies, only cyber/jamming attacks on communication networks are considered, whereas radar jamming is one of main concerns in control of connected vehicles. Therefore, control of vehicular platoons in the presence of radar jamming attacks requires further investigation.

By considering the constant time-gap spacing policy, a control strategy for a vehicular platoon is proposed in this paper. We assume that the platoon is subject to jamming attacks such that some vehicles at some time instants lose communicated/radar information or receive information with delays. The main contribution of this paper is designing a control strategy that increases the robustness of the platoon against both the communication and radar jamming attacks. Specifically, we develop a control strategy such that under some conditions on the control gains and the jammed signals, guarantees the almost sure convergence of the intervehicle distances to desired values. Hence, depending on the control gains, the robustness of the platoon against a wide range of jamming attacks is guaranteed.

The paper organization is as follows. Preliminaries are presented in the next section. The problem is stated in Section 3. The proposed control strategy is presented in Section 4. Simulation results are given in Section 5, and the paper ends with conclusions in Section 6.

2. PRELIMINARIES

Notation and some concepts on stochastic processes are provided in this section.

2.1 Notation

$\mathbb{R}$ denotes the set of real numbers. $\mathbb{E}\{\cdot\}$ and $\mathbb{P}\{\cdot\}$ denote the expected value and probability of a stochastic variable, respectively. $\mathbb{E}\{X|E\}$ expresses the conditional expected value of X given an event E. For a scalar, $|\cdot|$ denotes the absolute value. $\text{sgn}(\cdot)$ denotes the sign function. Moreover, 'n.a.' means 'not available', 'a.s.' means 'almost surely', and 'w.p.' means 'with probability'.

2.2 Stochastic Processes

A stochastic process is described by the triple $(\Omega, \mathcal{F}, \mathbb{P})$ where Ω is the space of events, $\mathcal{F}$ is a σ-algebra on Ω, and $\mathbb{P}$ is a probability measure on $(\Omega, \mathcal{F})$ where $0 \leq \mathbb{P}\{\cdot\} \leq 1$ and $\mathbb{P}\{\Omega\} = 1$ (Williams, 1991). A filtration $\{\mathcal{F}_t, t \geq 0\}$ on $(\Omega, \mathcal{F}, \mathbb{P})$ is a family of sub-σ-algebras of $\mathcal{F}$ where $\mathcal{F}_q \subset \mathcal{F}_t, q < t$. A stochastic process $X = \{X(t), t \geq 0\}$ is adapted to the filtration $\{\mathcal{F}_t\}$ if $X(t)$ is $\mathcal{F}_t$-measurable for each $t \geq 0$. Now, a process X is a *super-martingale* relative to $\{\mathcal{F}_t\}$ and $\mathbb{P}$ if (Mahmoud et al., 2003; Williams, 1991):

i) X is adapted to the filtration $\{\mathcal{F}_t\}$,
ii) $\mathbb{E}\{|X(t)|\} < \infty, \forall t$,
iii) $\mathbb{E}\{X(t)|\mathcal{F}_q\} \leq X(q), t > q$.

We say $X(t)$ *almost surely* converges to X_f if

$$\mathbb{P}\{\lim_{t\to\infty} X(t) = X_f\} = 1,$$

and we write

$$\lim_{t\to\infty} X(t) \xrightarrow{\text{a.s.}} X_f.$$

In general, we say that an event happens almost surely if it happens with probability 1 (Mahmoud et al., 2003).

3. PROBLEM STATEMENT

Consider a platoon of connected vehicles comprising of a leader indexed by $i = 0$ and N followers indexed by $i \in \mathcal{S} = \{1, 2, \ldots, N\}$, where Vehicle $i-1, i \in \mathcal{S}$, is the preceding vehicle of Vehicle i. The mathematical model of the longitudinal dynamics of the ith vehicle is considered as follows (Santhanakrishnan and Rajamani, 2003):

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t), \end{aligned} \tag{1}$$

where $x_i(t)$ denotes the displacement of the rear-bumper, $v_i(t)$ is the speed, and $u_i(t)$ is the control command (obtained via the engine and braking forces). Note that $x_i(t)$ may not be an accessible information, and thus we assume that just the information of the distance from the preceding vehicle is available for each vehicle.

The objective of CACC for Vehicle $i, i \in \mathcal{S}$, is to reach a desired distance from Vehicle $i-1$ by receiving required state information from Vehicle $i-1$. Let us define the distance of Vehicle i from Vehicle $i-1$ as follows:

$$d_i(t) = x_{i-1}(t) - L_i - x_i(t), \tag{2}$$

where L_i is the length of Vehicle i. A common policy to determine the desired value of $d_i(t)$ is the *constant time-gap spacing policy* (Santhanakrishnan and Rajamani, 2003). According to this policy, the desired distance of Vehicle i from Vehicle $i-1$ is determined based on the speed of Vehicle i as follows:

$$d_{\text{des},i}(t) = r_i + h_i v_i(t), i \in \mathcal{S}, \tag{3}$$

where r_i is the standstill distance and h_i is the time headway which are constant. Therefore, the *spacing error* of the ith vehicle can be expressed as follows:

$$e_i(t) = d_{\text{des},i}(t) - d_i(t), i \in \mathcal{S}. \tag{4}$$

To control the spacing errors (4) such that Vehicle i keeps the desired distance $d_{\text{des},i}(t)$ from Vehicle $i-1$, some information of the preceding vehicle such as distance is required. We assume that the distance $d_i(t)$ is measured by a local radar, and more required information from the preceding vehicle can be achieved via the communication network under which the vehicles exchange their states information. A platoon of vehicles equipped with radars and a communication network is depicted in Fig. 1.

However, the radars and the communication network may be subject to jamming attacks such that some communication or radio data received by some vehicles may be lost or delayed. Therefore, by defining $\bar{\vartheta}(t)$ as the value of the data $\vartheta(t)$ after attack, we have

$$\bar{\vartheta}(t) = \begin{cases} \text{n.a.} & \text{w.p.} \quad p_{\ell\vartheta}(t) \\ \vartheta(t - \tau_\vartheta(t)) & \text{w.p.} \quad p_{d\vartheta}(t) \\ \vartheta(t) & \text{w.p.} \quad p_\vartheta(t) \end{cases} \tag{5}$$

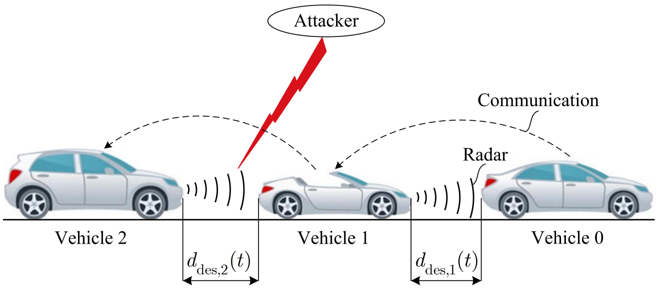

Fig. 1. A platoon of three vehicles.

where $p_{\ell\vartheta}(t) + p_{d\vartheta}(t) + p_{\vartheta}(t) = 1$. In (5), $\bar{\vartheta}(t) =$ n.a. implies data loss with probability $p_{\ell\vartheta}(t)$, $\bar{\vartheta}(t) = \vartheta(t-\tau_{\vartheta}(t))$ implies receiving the information of $\vartheta(t)$ with delays with probability $p_{d\vartheta}(t)$, where $\tau_{\vartheta}(t) > 0$ denotes the delay, and $\bar{\vartheta}(t) = \vartheta(t)$ implies receiving the correct data with probability $p_{\vartheta}(t)$.

In this condition, the objective is to develop a control strategy such that the spacing errors almost surely converge to zero, when the communication network and the radars are under jamming attacks.

4. MAIN RESULTS

To achieve the CACC objective for the vehicular platoon introduced in the previous section, we propose the following control law for each follower:

$$u_i(t) = \xi_{3i}(t) - \lambda_{1i}s_i(t) - \lambda_{2i}\dot{s}_i(t), i \in \mathcal{S}, \tag{6}$$

where λ_{1i} and λ_{2i} are positive constant gains, and $s_i(t)$ and $\xi_{3i}(t)$ are designed as follows:

$$\begin{aligned} s_i(t) =& \xi_{0i}(t) - \xi_{1i}(t), \\ \dot{\xi}_{0i}(t) =& v_i(t), \\ \dot{\xi}_{1i}(t) =& \xi_{2i}(t), \\ \dot{\xi}_{2i}(t) =& \xi_{3i}(t), \\ \dot{\xi}_{3i}(t) =& \frac{1}{h_i}\Big(-\xi_{3i}(t) - k_i(\xi_{2i}(t) + h_i\xi_{3i}(t)) \\ & - \chi_i(t)a_i(t)\mathrm{sgn}(\zeta_i(t))\Big), \end{aligned} \tag{7}$$

in which $\xi_{0i}(t)$, $\xi_{1i}(t)$, $\xi_{2i}(t)$, and $\xi_{3i}(t)$ are auxiliary states with arbitrary initial values, k_i is a positive constant gain, and $\zeta_i(t)$ is as follows:

$$\begin{aligned} \zeta_i(t) =& \xi_{2,i}(t) - \xi_{2,i-1}(t) + h_i\xi_{3i}(t) \\ & + k_i(-d_i(t) + d_{\mathrm{des},i}(t)). \end{aligned} \tag{8}$$

Moreover, $a_i(t) \in \{0, 1\}$ and $0 < \chi_i(t)$ are control gains designed later. Note that as the discontinuous term $a_i(t)\mathrm{sgn}(\zeta_i(t))$ is measurable and locally bounded, Filippov solutions for (7) exist (Filippov, 1988). Moreover, since the leader does not follow other vehicles, $s_0(t)$ and $\xi_{00}(t)$, $\xi_{10}(t)$, $\xi_{20}(t)$, and $\xi_{30}(t)$ are as follows:

$$\begin{aligned} s_0(t) =& 0, \\ \xi_{00}(t) =& x_0(t), \xi_{10}(t) = x_0(t), \\ \xi_{20}(t) =& v_0(t), \xi_{30}(t) = \dot{v}_0(t). \end{aligned} \tag{9}$$

In the presence of jamming attacks on Vehicle i, in (8), $d_i(t)$ obtained from the radar should be replaced by $\bar{d}_i(t)$, and $\xi_{2,i-1}(t)$ obtained from the communication network should be replaced by $\bar{\xi}_{2,i-1}(t)$. Therefore, in the presence of attacks, $\zeta_i(t)$ is as

$$\begin{aligned} \zeta_i(t) =& \xi_{2,i}(t) - \bar{\xi}_{2,i-1}(t) + h_i\xi_{3i}(t) \\ & + k_i(-\bar{d}_i(t) + d_{\mathrm{des},i}(t)). \end{aligned} \tag{10}$$

In this condition, $a_i(t)$ is set to zero if at time t, Vehicle i does not receive some data via the communication network or its radar, and $a_i(t) = 1$ when there is no data loss. For simplicity in the formulation, by considering $p_{\vartheta_i}(t) = 1, \vartheta_i \in \{d_i(t), \xi_{2,i-1}(t)\}$, as the case when Vehicle i is not probable to be under attacks, we replace $d_i(t)$ and $\xi_{2,i-1}(t)$ respectively by $\bar{d}_i(t)$ and $\bar{\xi}_{2,i-1}(t)$ for all the follower vehicles. Moreover, as the performance of the proposed strategy relies on the accuracy of all the received data, for Vehicle $i, i \in \mathcal{S}$, we define

$$\begin{aligned} p_{\ell i}(t) &= \mathbb{P}\{\text{losing radio or communication data}\}, \\ p_{di}(t) &= \mathbb{P}\{\text{delay in radio or communication data}\}, \\ p_i(t) &= \mathbb{P}\{\text{all the information being correct}\}. \end{aligned} \tag{11}$$

***Remark** 1.* Note that according to (6) and (7), $\mathrm{sgn}(\zeta_i(t))$ does not appear in $u_i(t)$, and therefore there is no chattering in $u_i(t)$. Indeed, $\dot{\xi}_{3i}(t)$ does not appear in $u_i(t)$, while the existing integrator between $\dot{\xi}_{3i}(t)$ and $\xi_{3i}(t)$ filters the chattering in all the states.

***Assumption** 1.* The leader displacement and speed are bounded.

***Assumption** 2.* While the time delays can be any bounded value, we assume that $p_i(t) > p_{di}(t)$ which implies that the probability of receiving data with delays should be less than the probability of receiving data correctly.

***Theorem** 1.* Consider the vehicular platoon described in (1) under the control law (6). Under these conditions, if for a deterministic control gain $\chi_i(t)$,

$$\frac{|\xi_{3,i-1}(t) + k_i\xi_{2,i-1}(t)|}{p_i(t) - p_{di}(t)} < \chi_i(t), i \in \mathcal{S}, \tag{12}$$

we will have, $\lim_{t\to\infty} e_i(t) \xrightarrow{\text{a.s.}} 0$. Moreover, all the auxiliary states $\xi_{0i}(t)$, $\xi_{1i}(t)$, $\xi_{2i}(t)$, and $\xi_{3i}(t)$ and the states $d_i(t)$ and $v_i(t)$ almost surely remain bounded.

Proof. From (1) and (7), it follows that

$$\ddot{s}_i(t) = u_i(t) - \xi_{3i}(t), i \in \mathcal{S}. \tag{13}$$

By substituting $u_i(t)$ from (6) into (13), one gets $\ddot{s}_i(t) = -\lambda_{1i}s_i(t) - \lambda_{2i}\dot{s}_i(t)$ which implies that $s_i(t)$ and $\dot{s}_i(t)$ remain bounded and converge to zero. Moreover, from (1) and (7), it follows that $\dot{x}_i(t) = \dot{\xi}_{0i}(t) = v_i(t)$. Therefore, there exists a constant c_i such that $x_i(t) = \xi_{0i}(t) + c_i$, and since $\xi_{0i}(t) = s_i(t) + \xi_{1i}(t)$, it implies that

$$x_i(t) = s_i(t) + \xi_{1i}(t) + c_i. \tag{14}$$

Considering (7), one gets

$$v_i(t) = \dot{s}_i(t) + \xi_{2i}(t). \tag{15}$$

Thus, according to (14) and (15), $d_i(t)$ and $d_{\mathrm{des},i}(t)$ defined in (2) and (3) can be restated as follows:

$$\begin{aligned} d_i(t) =& s_{i-1}(t) + \xi_{1,i-1}(t) + c_{i-1} - L_i - s_i(t) \\ & - \xi_{1i}(t) - c_i, \\ d_{\mathrm{des},i}(t) =& r_i + h_i\dot{s}_i(t) + h_i\xi_{2i}(t). \end{aligned} \tag{16}$$

Accordingly, we consider the following modified error:

$$\begin{aligned} \varepsilon_i(t) =& r_i + h_i\xi_{2i}(t) - \xi_{1,i-1}(t) - c_{i-1} + L_i \\ & + \xi_{1i}(t) + c_i. \end{aligned} \tag{17}$$

We will show that $\lim_{t\to\infty} \varepsilon_i(t) \xrightarrow{\text{a.s.}} 0$, and since $s_i(t)$, $s_{i-1}(t)$, and $\dot{s}_i(t)$ converge to zero, according to (4) and (16), we will conclude that $\lim_{t\to\infty} e_i(t) \xrightarrow{\text{a.s.}} 0$. By letting

$$z_i(t) = \dot{\varepsilon}_i(t) + k_i\varepsilon_i(t),$$

we consider the following Lyapunov candidate:

$$V_i(t) = \frac{1}{2}z_i(t)^2. \tag{18}$$

From (7), we recall that

$$\begin{aligned}\dot{\xi}_{3i}(t) =& \frac{1}{h_i}\Big(- \xi_{3i}(t) - k_i(\xi_{2i}(t) + h_i\xi_{3i}(t)) \\ & - \chi_i(t)a_i(t)\mathrm{sgn}(\zeta_i(t))\Big).\end{aligned} \tag{19}$$

Let us define

$$\Xi_i(t) = \frac{1}{h_i}(\xi_{3,i-1}(t) + k_i\xi_{2,i-1}(t)). \tag{20}$$

If we add and subtract the right-hand side of (19) by $\Xi_i(t)$, from (7) and (17), one gets

$$\ddot{\varepsilon}_i(t) = - h_i\Xi_i(t) - k_i\dot{\varepsilon}_i(t) - \chi_i(t)a_i(t)\mathrm{sgn}(\zeta_i(t)). \tag{21}$$

Then, according to the definition of $z_i(t)$, (21) yields

$$\dot{z}_i(t) = -h_i\Xi_i(t) - \chi_i(t)a_i(t)\mathrm{sgn}(\zeta_i(t)). \tag{22}$$

Thus, the conditional expected value of the time derivative of $V_i(t)$ along (22) is as follows:

$$\begin{aligned}\mathbb{E}\{\dot{V}_i(t)|\mathcal{F}_t\} = & z_i(t)\Big(- h_i\Xi_i(t) \\ & - \chi_i(t)\mathbb{E}\big\{a_i(t)\mathrm{sgn}(\zeta_i(t))\big|\mathcal{F}_t\big\}\Big),\end{aligned} \tag{23}$$

where $\mathcal{F}_t$ is a filtration as follows:

$$\mathcal{F}_t = \big\{\digamma_i(\varrho), i \in \mathcal{S} \cup \{0\}, 0 \le \varrho \le t\big\},$$

in which $\digamma_i(\varrho) = \{x_i(\varrho), v_i(\varrho), \xi_{0i}(\varrho), \xi_{1i}(\varrho), \xi_{2i}(\varrho), \xi_{3i}(\varrho)\}$. Now, we analyze $z_i(t)\chi_i(t)\mathbb{E}\big\{a_i(t)\mathrm{sgn}(\zeta_i(t))\big|\mathcal{F}_t\big\}$. Since in the case of data loss, $a_i(t) = 0$, from (11), it follows that

$$a_i(t) = 0 \quad \text{w.p.} \quad p_{\ell i}(t). \tag{24}$$

By considering (10) and (11), one gets

$$\begin{aligned}\zeta_i(t) =& \xi_{2,i}(t) - \xi_{2,i-1}(t) + h_i\xi_{3i}(t) \\ & + k_i(-d_i(t) + d_{\mathrm{des},i}(t)) \quad \text{w.p.} \quad p_i(t).\end{aligned} \tag{25}$$

From (7) and (17), we have

$$\xi_{2,i}(t) - \xi_{2,i-1}(t) + h_i\xi_{3i}(t) = \dot{\varepsilon}_i(t), \tag{26}$$

and from (16) and (17), one can say that

$$-d_i(t) + d_{\mathrm{des},i}(t) = \varepsilon_i(t) + s_i(t) - s_{i-1}(t) + h_i\dot{s}_i(t). \tag{27}$$

As a result, from (26) and (27) and according to the definition of $z_i(t)$, (25) yields

$$\zeta_i(t) = z_i(t) + k_i(s_i(t) - s_{i-1}(t) + h_i\dot{s}_i(t)) \quad \text{w.p.} \quad p_i(t).$$

Thus, as $s_i(t)$, $s_{i-1}(t)$, and $\dot{s}_i(t)$ are bounded and converge to zero, for a nonzero $z_i(t)$, there exists a finite time t_f such that for $t \ge t_f$,

$$\mathrm{sgn}(\zeta_i(t)) = \mathrm{sgn}(z_i(t)) \quad \text{w.p.} \quad p_i(t). \tag{28}$$

Therefore, by considering (24) and (28), for $t \ge t_f$, one gets

$$\begin{aligned}& z_i(t)\chi_i(t)\mathbb{E}\Big\{a_i(t)\mathrm{sgn}(\zeta_i(t))\Big|\mathcal{F}_t\Big\} \\ & = z_i(t)\chi_i(t)\delta_i(t)\mathrm{sgn}(z_i(t)),\end{aligned} \tag{29}$$

where $0 < p_i(t) - p_{di}(t) \le \delta_i(t) \le 1$ (see Assumption 2). Note that $\delta_i(t) = p_i(t) - p_{di}(t)$ implies the worst case when the delays always inverse the sign of $\zeta_i(t)$. Now, from (23) and (29), it follows that

$$\begin{aligned}\mathbb{E}\{\dot{V}_i(t)|\mathcal{F}_t\} =& z_i(t)\Big(- h_i\Xi_i(t) \\ & - \chi_i(t)\delta_i(t)\mathrm{sgn}(z_i(t))\Big).\end{aligned} \tag{30}$$

By considering (12), one gets

$$\chi_i(t)\delta_i(t) - |\xi_{3,i-1}(t) + k_i\xi_{2,i-1}(t)| > 0. \tag{31}$$

Therefore, by letting

$$\Upsilon_i(t) = \chi_i(t)\delta_i(t) - |\xi_{3,i-1}(t) + k_i\xi_{2,i-1}(t)| > 0,$$

from (20) and (31), it follows that

$$\begin{aligned}-h_i\Xi_i(t)z_i(t) - \chi_i(t)\delta_i(t)z_i(t)\mathrm{sgn}(z_i(t)) \\ \le -\Upsilon_i(t)z_i(t)\mathrm{sgn}(z_i(t)).\end{aligned} \tag{32}$$

Now, from (30) and (32), one gets

$$\mathbb{E}\{\dot{V}_i(t)|\mathcal{F}_t\} \le - \Upsilon_i(t)z_i(t)\mathrm{sgn}(z_i(t)).$$

Based on all the above-mentioned issues, it can be said that for $t \ge t_f$, $\mathbb{E}\{\dot{V}_i(t)|\mathcal{F}_t\}$ is negative definite. According to (7), $z_i(t)$ remains bounded in the finite time $t < t_f$. Thus, as $\mathbb{E}\{\dot{V}_i(t)|\mathcal{F}_t\}$ is negative definite for $t \ge t_f$, $\mathbb{E}\{V_i(t)\}$ is bounded implying the almost sure boundedness of $V_i(t)$ as well (if the boundedness is not almost surely, there exist nonzero probabilities for unboundedness which has contradiction with the boundedness of $\mathbb{E}\{V_i(t)\}$). Thus, $V_i(t)$ satisfies the second and third conditions of super-martingales given in Section 2.2, and if we consider the filtration $\mathcal{F}_t$, it satisfies all the conditions of super-martingales. Therefore, by invoking the super-martingales convergence theorem (Mahmoud et al., 2003), there exists $V_{if} \ge 0$ such that

$$\lim_{t\to\infty} V_i(t) \xrightarrow{\text{a.s.}} V_{if}. \tag{33}$$

Furthermore, we can conclude that

$$\lim_{t\to\infty} \mathbb{E}\{V_i(t)\} = 0. \tag{34}$$

Thus, from (33) and (34), we have $V_{if} = 0$, which from (18) implies that

$$\lim_{t\to\infty} z_i(t) \xrightarrow{\text{a.s.}} 0.$$

Moreover, the almost sure boundedness of $V_i(t)$ implies the almost sure boundedness of $z_i(t)$. In this condition, since $z_i(t)$ is a Hurwitz polynomial of $\varepsilon_i(t)$ as $z_i(t) = \dot{\varepsilon}_i(t) + k_i\varepsilon_i(t)$, $\varepsilon_i(t)$ and $\dot{\varepsilon}_i(t)$ almost surely remain bounded (due to the input to state stability of Hurwitz linear systems (Khalil, 2002)) and converge to zero. By considering (4) and (27), it can be said that

$$e_i(t) = \varepsilon_i(t) + s_i(t) - s_{i-1}(t) + h_i\dot{s}_i(t). \tag{35}$$

Since $\varepsilon_i(t)$ almost surely remains bounded and converges to zero, and $s_i(t) - s_{i-1}(t) + h_i\dot{s}_i(t)$ is bounded and converges to zero, according to (35), $e_i(t)$ remains bounded and almost surely converges to zero. To complete the proof, we show that $\xi_{0i}(t)$, $\xi_{1i}(t)$, $\xi_{2i}(t)$, $\xi_{3i}(t)$, $v_i(t)$, and $d_i(t)$ almost surely remain bounded. From (17), one gets

$$\begin{aligned}\xi_{2i}(t) =& \frac{1}{h_i}\big(- \xi_{1i}(t) + \xi_{1,i-1}(t) + \varepsilon_i(t) - r_i + c_{i-1} \\ & - L_i - c_i\big).\end{aligned} \tag{36}$$

By considering (7), (36) and its time derivative yield

$$\begin{aligned}\dot{\xi}_{1i}(t) =& \frac{1}{h_i}\big(- \xi_{1i}(t) + \xi_{1,i-1}(t) + \varepsilon_i(t) - r_i + c_{i-1} \\ & - L_i - c_i\big), \\ \dot{\xi}_{2i}(t) =& \frac{1}{h_i}\big(- \xi_{2i}(t) + \xi_{2,i-1}(t) + \dot{\varepsilon}_i(t)\big).\end{aligned} \tag{37}$$

Since $\xi_{10}(t)$ and $\xi_{20}(t)$ are bounded (due to (9) and Assumption 1) and $\varepsilon_i(t)$ and $\dot{\varepsilon}_i(t), i \in \mathcal{S}$, almost surely remain bounded, from (37), the almost sure boundedness of $\xi_{1i}(t)$, $\xi_{2i}(t)$, and $\dot{\xi}_{2i}(t)$ can be concluded (due to the input to state stability of Hurwitz linear systems) which from (7), they imply the almost sure boundedness of $\xi_{3i}(t)$. Since $s_i(t)$ and $\xi_{1i}(t)$ are almost surely bounded, from (7), the almost sure boundedness of $\xi_{0i}(t)$ can be concluded. Then, from (15) and (16), the almost sure boundedness of $v_i(t)$ and $d_i(t)$ is concluded as well, and the proof is completed. ■

Remark 2. According to Theorem 1, certain conditions for the vehicular platoon in the presence of jamming attacks are derived such that the spacing errors almost surely converge to zero. However, as $p_{di}(t)$ and $p_i(t)$ are unknown and the exact values of $\xi_{2,i-1}(t)$ and $\xi_{3,i-1}(t)$ may be unknown (due to attacks), it is not straightforward to guarantee (12). Indeed, Theorem 1 implies that larger $\chi_i(t)$ leads to more robustness against jamming attacks. Next, we propose to design $\chi_i(t)$ as a function of $\{\mathcal{F}_q\}, q < t$, as follows (if $\chi_i(t)$ is a function of $\{\mathcal{F}_q\}$, it is deterministic and the proof of Theorem 1 is still valid):

$$\chi_i(t) = \kappa_{1i}|\bar{\xi}_{3,i-1}(q) + k_i\bar{\xi}_{2,i-1}(q)| + \kappa_{2i}, \qquad (38)$$

where κ_{1i} and κ_{2i} are positive constants, and $\bar{\xi}_{3,i-1}(q)$ and $\bar{\xi}_{2,i-1}(q)$ imply any 'available' (deterministic) information before time t. Therefore, (12) is guaranteed if

$$\frac{|\xi_{3,i-1}(t) + k_i\xi_{2,i-1}(t)|}{p_i(t) - p_{di}(t)} < \kappa_{1i}|\bar{\xi}_{3,i-1}(q) + k_i\bar{\xi}_{2,i-1}(q)| + \kappa_{2i}. \qquad (39)$$

Hence, Theorem 1 provides a control strategy with a criterion to increase the robustness against jamming attacks such that depending on the magnitudes of κ_{1i} and κ_{2i}, the robustness of the performance of the vehicular platoon against a range of jammed signals is guaranteed. Accordingly, larger κ_{1i} and κ_{2i} lead to more robustness against jamming attacks. However, larger κ_{1i} and κ_{2i} may lead to larger control efforts. Indeed, increasing the robustness may be corresponding to larger costs (more energy consumption). These issues are illustrated by numerical examples in the next section.

Remark 3. Note that in (38), $\xi_{30}(t) = \dot{v}_0(t)$, and such information can be provided via an accelerometer; otherwise, to satisfy the inequality (39), instead of $\bar{\xi}_{30}(t)$, we can use the change of $\bar{v}_0(t)$ over a short period of time (by considering the robustifying effect of κ_{1i} and κ_{2i}).

5. SIMULATION RESULTS

We consider a platoon of five vehicles comprising of a leader and four followers. We assume that the leader is moving with a speed of 20m/s, and the followers initial states, $(d_i(0)\text{m}, v_i(0)\text{m/s})$, are $(5, 18)$, $(4.5, 17)$, $(7, 18)$, and $(9, 21)$, respectively. The objective is to achieve a platoon of vehicles such that each follower keeps the desired distance (3) from the preceding vehicle by employing the CACC law proposed in Theorem 1. We assume that $L_1 =$ 4m, $L_2 = 3.5$m, $L_3 = 3$m, and $L_4 = 3.5$m, and let $r_i = 2$m and $h_i = 0.2$s, $i \in \mathcal{S}$. Moreover, for $i \in \mathcal{S}$, the control gains are set to $k_i = \lambda_{1i} = \lambda_{2i} = 1$. We assume that Vehicles 1 and 2 are under jamming attacks after $t = 10$s, and three scenarios are considered discussed below.

Scenario 1. Without loss of generality and for simplicity, for each follower vehicle, the effects of the attack on the radar and the communication network are assumed to be the same. For Vehicles 1 and 2, the probabilities of data loss are supposed to be 0.1 and $0.1 + 0.05\cos(0.2t)$, respectively, the probabilities of receiving information with delays are considered to be $0.2|\sin(t)|$ and 0.3, respectively, and we have used positive sinusoidal functions with a bound of 2 to model time delays. Moreover, to design $\chi_i(t)$ in (38), we have set $\kappa_{1i} = 1$ and $\kappa_{2i} = 5$. Under these conditions, the distance of each follower vehicle from the preceding vehicle is depicted in Fig. 2. According to the figure, the vehicles reach the desired distances (3), while the radars and the communication network associated with Vehicles 1 and 2 are under jamming attacks. Moreover, the control efforts of the follower vehicles are depicted in Fig. 3.

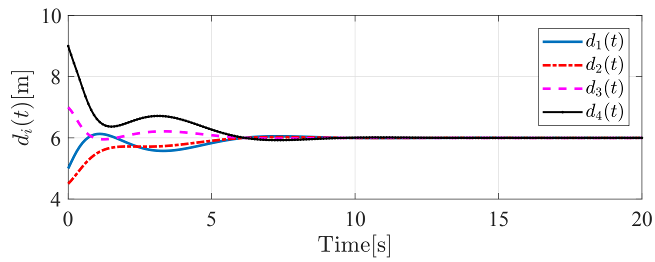

Fig. 2. Distance of each follower vehicle from the preceding vehicle in Scenario 1.

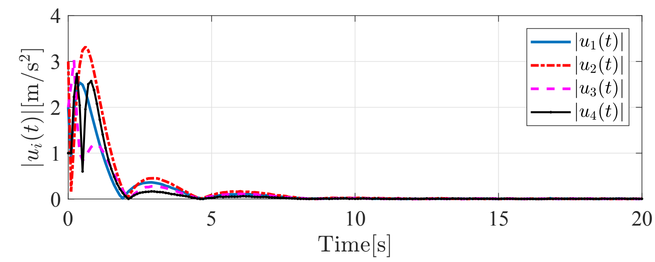

Fig. 3. Control efforts of the follower vehicles in Scenario 1.

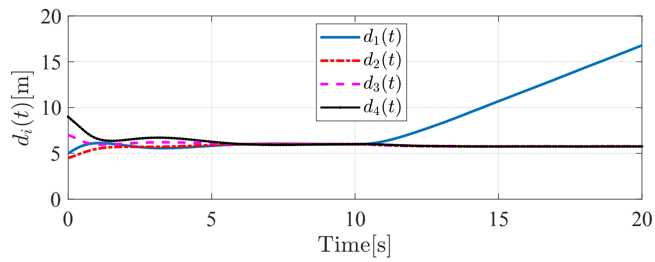

Fig. 4. Deterioration of the performance of the platoon in Scenario 2 by increasing the probability of the jammed signals.

Scenario 2. We repeat Scenario 1, and just for Vehicle 1, we increase the probability of data loss to 0.25. As shown in Fig. 4, the controller of Scenario 1 cannot handle such jammed signals. Thus, as depicted in Fig. 4, the distance among Vehicle 1 and Vehicle 0 will be increased. Moreover, the follower vehicles control efforts are depicted in Fig. 5.

Scenario 3. To show the effect of increasing $\chi_i(t)$ in increasing the robustness of the platoon against jamming attacks, we have repeated Scenario 2, and we increase κ_{1i} and $\kappa_{2i}, i \in \mathcal{S}$, to $\kappa_{1i} = 1.4$ and $\kappa_{2i} = 7$. As depicted in Fig. 6, by employing these control gains, the robustness

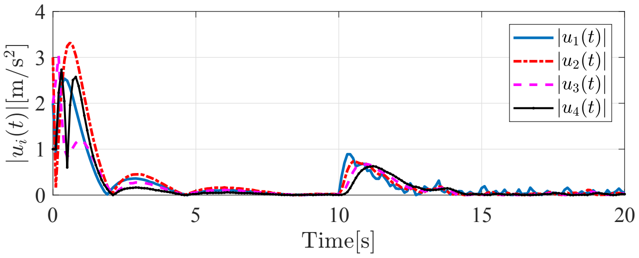

Fig. 5. Control efforts of the follower vehicles in Scenario 2.

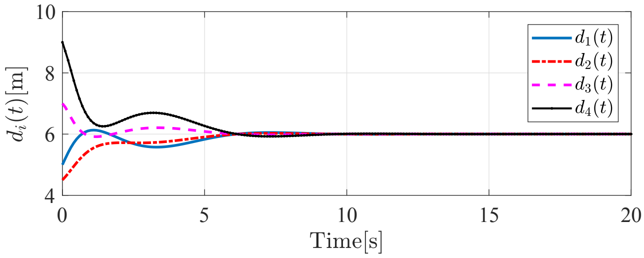

Fig. 6. Distance of each follower vehicle from the preceding vehicle in Scenario 3 by increasing the probability of the jammed signals and increasing the robustness of the control law.

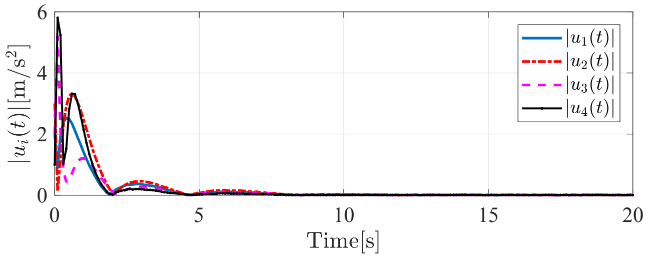

Fig. 7. Control efforts of the follower vehicles in Scenario 3 by increasing the robustness of the control law.

of the platoon is increased such that in the presence of the jamming attacks considered in Scenario 2, the follower vehicles reach the desired distances (3). Moreover, as depicted in Fig. 7, by increasing the robustness of the control law, the control efforts of most follower vehicles are increased.

6. CONCLUSIONS AND FUTURE WORK

A control strategy to increase the stochastic robustness of a vehicular platoon against jamming attacks on the communication network and the vehicles radars was addressed in this paper. We proposed a framework such that depending on the control gains, the compensation of the effect of the jammed signals was realized such that the spacing errors almost surely converged to zero. Accordingly, based on the magnitudes of the control gains, the robustness of the vehicular platoon against a range of jammed signals was guaranteed. Control of nonlinear vehicular platoons with unknown model parameters in the presence of jamming attacks is another problem to be investigated as future work.

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