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Adaptive Second-Order Sliding Mode Control for a Tilting Quadcopter with Input Saturations ⋆

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Abstract: In this paper, we address the problem of tracking control for a novel tilting quadcopter in the presence of uncertainties and disturbance. To handle these effects, an adaptive second-order sliding mode control (ASOSMC) is proposed to obtain fast convergence rate and high precision with alleviating chattering effect. Considering the input saturations, a new scheme is designed toward the time derivative of the control inputs where an auxiliary system is adopted to the controller. Moreover, the overall uncertainties are expressed in a linearly parametric form without any prior knowledge. It can be proved that the sliding mode manifolds can fast converge to a small neighborhood around zero within finite-time, and then tracking errors exponentially. Finally, simulation results are carried out to demonstrate the effectiveness and robustness of the proposed controller.

Keywords: Adaptive second-order fast nonsingular terminal sliding mode control, tilting quadcopter, input saturations, robust control, trajectory tracking.

1. INTRODUCTION

Quadcopter has seen a boost in popularity and been an active research topic in various applications, such as exploration, search, rescue, and monitor Tayebi and McGilvray (2006); Hoffmann et al. (2007); Bouabdallah and Siegwart (2005); Ma and Ji (2016). The development of quadcopter is toward the trend of interacting with hostile and cluttered environments Jiang et al. (2018). Developing fully actuated flying vehicle is of significant for operations in ruins and narrow space Ji et al. (2019,). The performance of the conventional quadcopter is limited due to the generated force only in a single plane. Namely, the translational and rotational movements cannot be controlled independently.

Motivated by these requirements, several actuation strategies have been developed to obtain fully control ability over 6 degrees of freedom (DOF). Brescianini and D'Andrea (2016) derive an omni-directional quadcopter, which makes the vehicle a novel set of maneuvers feasible. Long and Cappelleri (2013) adopt two sets of propeller: central-rotating coaxial propellers and perimeter-mounted ducted fans to provide lift and lateral force respectively. Ryll et al. (2016) proposes a novel concept with six propellers tilting along corresponding axes. Considering the volume and maneuverability, a tilting quadcopter is designed and first flight tests, although preliminary, clearly demonstrate its superiority over the conventional quadcopter Ryll et al. (2013). To further improve the control bandwidth, a dual-axes tilting quadcopter including Servo and Push parts is designed, where the thrust and gyroscopic torque are integrated to drive the vehicle. It is a remarkable fact that the mass and the skeleton of the vehicle is time-varying while object grasping and the tilting mechanisms working. Moreover, the aerodynamic damping would degrade the tracking performance when outdoor flights, especially. However, we cannot have an exact compensation because the parameters are obtained based on some special conditions without covering some unpredictable environments, which has not been fully considered before.

The sliding mode control (SMC) has been a powerful tool to handle uncertainties and external disturbance Zheng et al. (2014). There exist two serious problems: singularity problem and the chattering effect. For the first problem, a fast nonsingular terminal sliding mode control (FNTSMC) is developed in Boukattaya et al. (2018), which not only guarantees the system states fast converge both at a distance and a close of the origin, but also avoids the singularity problem. However, the above-mentioned literature always consider the upper bound of uncertainties is known in advance, which puts constrains on the system states to some extend. To alleviate the second effect, boundary layer techniques, continuous saturation function and sigmoid function are used to replace the discontinuous function Huang et al. (2008); Jin and Sun (2008); Yao et al. (2018). The adverse impact is that the high precision cannot be guaranteed anymore, and the selection of parameters in these functions seriously affects the tracking performance. Another method drawn intensive attention is the high-order sliding mode control

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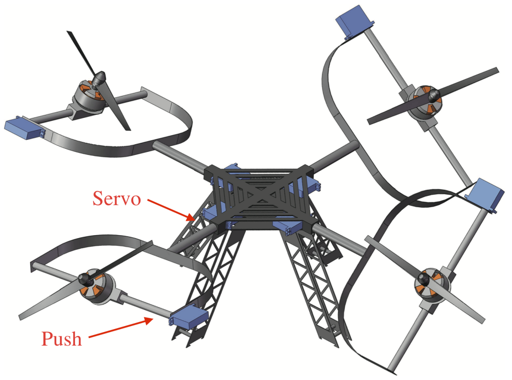

Fig. 1. The tilting quadcopter Vehicle.

(HOSMC), such as twisting algorithm Torres-González et al. (2017), super twisting Moreno and Osorio (2012), and second-order SMC (SOSMC) Ding and Li (2017); Qiao and Zhang (2018); Mondal and Mahanta (2014). The fundamental of SOSMC is that the discontinuous function is used in the time derivative of the controller inputs, and after integrating we can obtain a continuous function. Namely, we can achieve high tracking precision without losing robustness properties. However, most of the existing SOSMC does not consider the input saturations effects, which is of significant from a practical point of view. And to our best knowledge, the result is still scarce about these constrains. Motivated by the above discussion, the main contributions of this paper include:

(1) We first propose a novel tilting quadcopter conception, where the Push mechanism can be arbitrary within $[-\frac{\pi}{2}, \frac{\pi}{2}]$ to further improve the reliability and the agility of the vehicle.
(2) Considering the uncertain effects, a new form of ASOSMC is proposed, which can guarantee fast finite-time convergence and high precision with alleviating the chattering effect. Moreover, the overall uncertainties are expressed in a linearly parametric form without any prior knowledge.
(3) A new scheme is designed toward the time derivative of control inputs, and an auxiliary system is further introduced to obtain favorable performance even with input saturations.

The outline of this paper is organized as follows. In Section II, a novel tilting quadcopter and its dynamics is developed. In Section III, an ASOSMC is applied to obtain a fast convergence rate and high precision performance. In Section IV, simulation results are conducted to demonstrate the effectiveness and robustness of the proposed controller, followed by the conclusion in Section V.

2. DYNAMIC MODELING

In this section, a novel fully actuated tilting quadcopter is first proposed, which is able to decouple the translational and rotational movements by tilting the Push and the Servo parts as shown in Fig. 1. Moreover, unlike the tilting mechanisms in Segui-Gasco et al. (2014); Elfeky et al. (2016), which can only be within small ranges, the Push mechanisms proposed in this paper can be within $[-\frac{\pi}{2}, \frac{\pi}{2}]$ to get a more agile version.

2.1 Preliminaries

Before developing the dynamics, we set the following frames: the World frame R_W and the body frame R_B. The kinematic model of the vehicle can be described as,

$$\dot{\chi} = R\Omega, \tag{1}$$

where $\chi = [\phi, \theta, \psi, x, y, z]^{\mathrm{T}}$ is composed of the attitude and the position in the R_W; $\Omega = [p_b, q_b, r_b, v_{bx}, v_{by}, v_{bz}]^{\mathrm{T}}$ is the angular and linear velocities in the R_B. The matrix $R = \mathrm{diag}(R_1, R_2)$ is given by,

$$\begin{aligned} R_1 &= \begin{bmatrix} 1 & \mathrm{s}\phi\mathrm{t}\theta & \mathrm{c}\phi\mathrm{t}\theta \\ 0 & \mathrm{c}\phi & -\mathrm{s}\phi \\ 0 & \dfrac{\mathrm{s}\phi}{\mathrm{c}\theta} & \dfrac{\mathrm{c}\phi}{\mathrm{c}\theta} \end{bmatrix}, \\ R_2 &= \begin{bmatrix} \mathrm{c}\theta\mathrm{c}\psi & -\mathrm{c}\phi\mathrm{s}\psi+\mathrm{s}\phi\mathrm{s}\theta\mathrm{c}\psi & \mathrm{s}\phi\mathrm{s}\psi+\mathrm{c}\phi\mathrm{s}\theta\mathrm{c}\psi \\ \mathrm{c}\theta\mathrm{s}\psi & \mathrm{c}\phi\mathrm{c}\psi+\mathrm{s}\phi\mathrm{s}\theta\mathrm{s}\psi & -\mathrm{s}\phi\mathrm{c}\psi+\mathrm{c}\phi\mathrm{s}\theta\mathrm{s}\psi \\ -\mathrm{s}\theta & \mathrm{s}\phi\mathrm{c}\theta & \mathrm{c}\phi\mathrm{c}\theta \end{bmatrix}, \end{aligned} \tag{2}$$

where $\mathrm{c}\cdot = \cos(\cdot)$, $\mathrm{s}\cdot = \sin(\cdot)$, and $\mathrm{t}\cdot = \tan(\cdot)$.

Assumption 1. The attitude of the tilting quadcopter: roll ϕ, pitch θ, and yaw ψ are all bounded by $(-\frac{\pi}{2}, \frac{\pi}{2})$.

Assumption 2. There exist positive constants $\overline{r}_1$ and $\overline{r}_2$ such that $\|R\| \le \overline{r}_1$, $\|R^{-1}\| \le \overline{r}_1$, $\|\frac{d}{dt}(R)\| \le \overline{r}_2\|\dot{\chi}\|$, and $\|\frac{d}{dt}(R^{-1})\| \le \overline{r}_2\|\dot{\chi}\|$ hold.

2.2 The Dynamics of the Tilting Quadcopter

According to the Newton-Euler equations, the dynamic mode of a tilting quadcopter can be developed as,

$$J\dot{\Omega} + H(\Omega)\Omega + N(\Omega)\Omega + G_b + \Delta_\Omega = \tau_\Omega, \tag{3}$$

where $J = J_0 + \Delta J$ is the inertial matrix with the added mass m; $H(\Omega) = H_0(\Omega) + \Delta H$ is the Coriolis and centrifugal force matrix; $N(\Omega) = N_0(\Omega) + \Delta N$ is the nonlinear aerodynamic damping interaction; $G_b = G_{b0} + \Delta G_b$ is the gravity vector; Δ_Ω is the unknown but bounded external disturbance, and τ_Ω is the control input. J_0, $H_0(\Omega)$, $N_0(\Omega)$, and G_{b0} are the normal values of the corresponding terms. ΔJ, $\Delta C(\Omega)$, $\Delta N(\Omega)$, and ΔG_b are uncertain terms. More details can be found in Ji et al. (2019,). And we give the following assumptions:

Assumption 3. There exist positive constants b_i $(i = 1, \cdots, 5)$ such that the following inequalities hold.

$$\begin{aligned} &\|J\| \le b_1, \ \|H(\Omega)\| \le b_2\|\Omega\|, \\ &\|N(\Omega)\| \le b_3 + b_4\|\Omega\|, \ \|G_b\| \le b_5, \end{aligned} \tag{4}$$

where b_i $(i = 1, \cdots, 5)$ are unknown constants.

Assumption 4. The external disturbance and its time derivative: $\|\Delta_\Omega\| \le b_6$ and $\|\dot{\Delta}_\Omega\| \le b_7$, where b_6 and b_7 are unknown positive constants.

Substituting (1) into (3), we can have:

$$M(\chi)\ddot{\chi} + C(\Omega, \chi)\dot{\chi} + D(\Omega, \chi)\dot{\chi} + G_w + \Delta_\chi = \tau_\chi, \tag{5}$$

where $M(\chi) = M_0(\chi) + \Delta M$; $C(\Omega, \chi) = C_0(\Omega, \chi) + \Delta C$; $D(\Omega, \chi) = D_0(\Omega, \chi) + \Delta D$; $G_w = G_{w0} + \Delta G$; $\Delta_\chi = R^{-\mathrm{T}}\Delta_\Omega$, and $\tau_\chi = R^{-\mathrm{T}}\tau_\Omega$. The terms $M_0(\chi)$, $C_0(\Omega, \chi)$, $D_0(\Omega, \chi)$, and G_{w0} are their normal values as,

$$\begin{aligned} &M_0(\chi) = R^{-\mathrm{T}}J_0R^{-1}, \\ &C_0(\Omega, \chi) = R^{-\mathrm{T}}(H_0(\Omega) - J_0R^{-1}\dot{R})R^{-1} \\ &D_0(\Omega, \chi) = R^{-\mathrm{T}}N_0(\Omega)R^{-1}, \ G_{w0} = R^{-\mathrm{T}}G_{b0} \end{aligned} \tag{6}$$

And $\Delta M(\chi)$, $\Delta C(\Omega,\chi)$, $\Delta D(\Omega,\chi)$, and ΔG_w are uncertain terms due to the uncertainties. By integrating the overall uncertain terms and the external disturbance into a vector Δ, the dynamics (5) can be converted as:

$$M_0(\chi)\ddot{\chi} + C_0(\Omega,\chi)\dot{\chi} + D_0(\Omega,\chi)\dot{\chi} + G_{w0} + \Delta = \tau_\chi, \quad (7)$$

where $\Delta = \Delta M(\chi)\ddot{\chi} + \Delta C(\Omega,\chi)\dot{\chi} + \Delta D(\Omega,\chi)\dot{\chi} + \Delta G_w + \Delta_\chi$. According to (4), (5), Assumption 2, and 3, the following inequality holds:

$$\|\Delta\| \le a_1 + a_2\|\dot{\chi}\| + a_3\|\dot{\chi}\|^2, \quad (8)$$

where $a_i (i = 1, \cdots, 4)$ are unknown positive constants.

3. ADAPTIVE SECOND-ORDER SLIDING MODE CONTROL FOR A TILTING QUADCOPTER WITH INPUT CONSTRAINS

3.1 Preliminaries

Lemma 1. There exist positive constants: c, d, m, and real variables x_0 and y_0 such that Qian and Lin (2001):

$$|x_0|^c|y_0|^d \le \frac{c}{c+d} m|x_0|^{c+d} + \frac{d}{c+d} m^{-\frac{c}{d}}|y_0|^{c+d}. \quad (9)$$

Lemma 2. For $r_i \in R(i = 1, \cdots, n)$ and $0 < p \le 1$, the following inequality holds Huang et al. (2005):

$$(|r_1| + |r_2| + \cdots + |r_n|)^p \le |r_1|^p + |r_2|^p + \cdots + |r_n|^p \quad (10)$$

Lemma 3. Consider the Lyapunov function $V(\chi)$, which satisfies the following inequality Yu et al. (2005),

$$\dot{V}(\chi) \le -aV(\chi) - bV^\beta(\chi) + \varepsilon \quad (11)$$

where a, b, and ε are positive constants, $0 < \beta < 1$. Then the system state χ could converge to a small neighborhood around the origin in finite-time.

3.2 SOFNTSM

Let us review the second-order fast nonsingular terminal sliding mode (SOFNTSM) manifold, which is composed of the proportional-derivative sliding mode (PDSM) and the fast nonsingular terminal sliding mode (FNTSM) as:

$$s_1 = \dot{e} + k_1 e, \; s_2 = s_1 + k_2 s_1^{\beta_2} + k_3 \dot{s}_1^{\beta_1}, \quad (12)$$

where e represents the tracking error; $k_i (i = 1, 2, 3)$ is positive constants; $1 < \beta_1 < 2$, and $\beta_2 > \beta_1$. If the reaching phase control law makes the sliding mode manifold s_2 converge to zero, we then have,

$$s_1 + k_2 s_1^{\beta_2} + k_3 \dot{s}_1^{\beta_1} = 0 \quad (13)$$

According to Yang and Yang (2011), the sliding mode s_1 can be fast reached to zero in finite-time. When the sliding mode $s_1 = 0$, there is $\dot{e} + k_1 e = 0$, which means that the tracking errors converge to zero exponentially.

3.3 Adaptive Second-Order Sliding Mode Control with Input Saturations

As discussed above, we adopt PDSM manifold as,

$$s_1 = e_2 + K_1 e_1, \quad (14)$$

where $e_1 = \chi - \chi_d - \xi_1$ and $e_2 = \dot{\chi} - \dot{\chi}_d - \xi_2$ with χ_d being the desired tracking states and K_1 the positive definite matrix. ξ_1 and ξ_2 are auxiliary vectors designed later. The FNTSM manifold can be described as,

$$s_2 = s_1 + K_2 s_1^{\beta_2} + K_3 \dot{s}_1^{\beta_1}, \quad (15)$$

where K_2 and K_3 are positive definite matrices. Differentiating both sides of (15), we can have,

$$\dot{s}_2 = \dot{s}_1 + \beta_2 K_2 \mathrm{diag}(|s_1|^{\beta_2 - 1})\dot{s}_1 + \beta_1 K_3 \mathrm{diag}(|\dot{s}_1|^{\beta_1 - 1})\ddot{s}_1. \quad (16)$$

According to (7), we can further have,

$$\ddot{e}_1 = P_0(\Omega,\chi,\dot{\chi}) - M_0^{-1}\Delta - \ddot{\xi}_1, \quad (17)$$

where $P_0(\Omega,\chi,\dot{\chi}) = M_0^{-1}(\tau_\chi - C_0\dot{\chi} - D_0\dot{\chi} - G_{w0}) - \ddot{\chi}_d$. Similarly to e_2, we can get,

$$\ddot{e}_2 = M_0^{-1}\dot{\tau}_\chi + P(\Omega,\chi,\dot{\chi}) - (\frac{d}{dt}(M_0^{-1})\Delta + M_0^{-1}\dot{\Delta}) - \ddot{\xi}_2 \quad (18)$$

where $P(\Omega,\chi,\dot{\chi}) = \frac{d}{dt}(M_0^{-1})(\tau_\chi - C_0\dot{\chi} - D_0\dot{\chi} - G_{w0}) - M_0^{-1}\frac{d}{dt}(C_0\dot{\chi} + D_0\dot{\chi} + G_{w0}) - \frac{d}{dt}\ddot{\chi}_d$. Referring to (17) and (18), we can get,

$$\begin{aligned}\dot{s}_2 =& \dot{s}_1 + \beta_2 K_2 \mathrm{diag}(|s_1|^{\beta_2 - 1})\dot{s}_1 + \beta_1 K_3 \mathrm{diag}(|\dot{s}_1|^{\beta_1 - 1}) \cdot \\ & (\bar{\Delta} + M_0^{-1}\dot{\tau}_\chi + P + K_1 P_0 - \ddot{\xi}_2 - K_1\ddot{\xi}_1),\end{aligned} \quad (19)$$

where $\bar{\Delta} = -(\frac{d}{dt}(M_0^{-1}) + K_1 M_0^{-1})\Delta - M_0^{-1}\dot{\Delta}$. According to the above Assumptions and (8), the following inequality holds:

$$\|\bar{\Delta}\| \le \theta_1 + \theta_2\|\dot{\chi}\| + \theta_3\|\dot{\chi}\|^2 + \theta_4\|\dot{\chi}\|^3, \quad (20)$$

where $\theta_i (i = 1, \cdots, 4)$ are positive unknown constants and its proof is omitted here for brevity. Considering the practical applications, the control inputs are subject to saturation effect due to the physical limitations of actuators, and can be described as:

$$\dot{\tau}_{\chi_i} = \begin{cases} \dot{\tau}_i \ (\text{if } \tau_{\min} < \tau_{\chi i} < \tau_{\max}) \\ 0 \ (\text{others}) \end{cases} \quad (21)$$

where $\tau_{\min} < 0$ and $\tau_{\max} > 0$ are the minimum and maximum control force or torque generated by the tilting quadcopter and $\dot{\tau}_i$ will be designed later. To handle the input saturations (21), an auxiliary dynamic system is constructed as:

$$\begin{aligned}\dot{\xi}_1 &= -L_1\xi_1 + \xi_2 \\ \dot{\xi}_2 &= -L_2\xi_2 + \xi_3 \\ \dot{\xi}_3 &= -L_3\xi_3 + M_0^{-1}\Delta\dot{\tau},\end{aligned} \quad (22)$$

where $L_i (i = 1, 2, 3)$ are all positive matrices and $\Delta\dot{\tau} = \dot{\tau}_\chi - \dot{\tau}$ with $\dot{\tau}$ being the controller without input saturations. We then develop the reaching phase controller $\dot{\tau}$ including two parts: $\dot{\tau}_{eq}$ and $\dot{\tau}_{sw}$:

$$\dot{\tau} = \dot{\tau}_{eq} + \dot{\tau}_{sw} \quad (23)$$

with

$$\begin{aligned}\dot{\tau}_{eq} =& M_0(-P(\Omega,\chi,\dot{\chi}) - K_1 P_0(\Omega,\chi,\dot{\chi}) - \frac{1}{\beta_1}K_3^{-1}\dot{s}_1^{2-\beta_1} \\ & - \frac{\beta_2}{\beta_1}K_3^{-1}K_2\mathrm{diag}(|s_1|^{\beta_2-1})\dot{s}_1^{2-\beta_1} - L_2\dot{\xi}_2 - L_3\xi_3 \\ & + K_1(-L_1\dot{\xi}_1 + \dot{\xi}_2)),\end{aligned} \quad (24)$$

and

$$\begin{aligned}\dot{\tau}_{sw} =& -M_0[(\hat{\theta}_1 + \hat{\theta}_2\|\dot{\chi}\| + \hat{\theta}_3\|\dot{\chi}\|^2 + \hat{\theta}_4\|\dot{\chi}\|^3 + \rho)\mathrm{sgn}(s_2) \\ & + \frac{1}{\beta_1}K_3^{-1}K_4 s_2],\end{aligned} \quad (25)$$

where K_4 are positive definite matrix, ρ is positive constant, and $\hat{\theta}_i (i = 1 \cdots 4)$ are the estimated parameters of θ_i. The update laws for the estimated parameters θ_i can be described as,

$$\begin{aligned}\dot{\hat{\theta}}_1 &= \alpha_1(\beta_1\|s_2^{\mathrm{T}}K_3\mathrm{diag}(|\dot{s}_1|^{\beta_1-1})\| - \hat{\theta}_1)\\ \dot{\hat{\theta}}_2 &= \alpha_2(\beta_1\|s_2^{\mathrm{T}}K_3\mathrm{diag}(|\dot{s}_1|^{\beta_1-1})\|\|\dot{\chi}\| - \hat{\theta}_2)\\ \dot{\hat{\theta}}_3 &= \alpha_3(\beta_1\|s_2^{\mathrm{T}}K_3\mathrm{diag}(|\dot{s}_1|^{\beta_1-1})\|\|\dot{\chi}\|^2 - \hat{\theta}_3)\\ \dot{\hat{\theta}}_4 &= \alpha_4(\beta_1\|s_2^{\mathrm{T}}K_3\mathrm{diag}(|\dot{s}_1|^{\beta_1-1})\|\|\dot{\chi}\|^3 - \hat{\theta}_4),\end{aligned} \tag{26}$$

where $\alpha_i(i = 1, \cdots, 4)$ are positive constants. Then we can give the following theorem:

Theorem 4. Considering the closed-loop system (7) with parametric uncertainties and external disturbance, an adaptive SOSMC is developed based on the auxiliary dynamic system (22), the reaching phase controller (23), and adaptive laws (26). The PDSM s_1 and the FNTSM s_2 can fast converge to a small neighborhood around the equilibrium and the tracking error e is then guaranteed to converge to a small neighborhood around the origin exponentially.

Proof. We first select the following Lyapunov function candidate for the tilting quadcopter system as,

$$V_1 = \frac{1}{2}s_2^{\mathrm{T}}s_2 + \frac{1}{2}\sum_{i=1}^{3}\xi_i^{\mathrm{T}}\xi_i + \frac{1}{2}\sum_{i=1}^{4}\alpha_i^{-1}\tilde{\theta}_i^2, \tag{27}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i(i = 1, \cdots, 4)$ is the estimated error. The tiime derivative of (27) is,

$$\dot{V}_1 = s_2^{\mathrm{T}}\dot{s}_2 + \sum_{i=1}^{3}\xi_i^{\mathrm{T}}\dot{\xi}_i - \sum_{i=1}^{4}\alpha_i^{-1}\tilde{\theta}_i\dot{\hat{\theta}}_i. \tag{28}$$

Considering the input constrains, we let $\dot{\tau}_\chi = \dot{\tau} + \Delta\dot{\tau}$ with $\Delta\dot{\tau} = \dot{\tau}_\chi - \dot{\tau}$. In view of (19), (23), (24), and (25), we then have,

$$\begin{aligned}s_2^{\mathrm{T}}\dot{s}_2 =& s_2^{\mathrm{T}}\dot{s}_1 + \beta_2 s_2^{\mathrm{T}}K_2\mathrm{diag}(|s_1|^{\beta_2-1})\dot{s}_1\\ &+\beta_1 s_2^{\mathrm{T}}K_3\mathrm{diag}(|\dot{s}_1|^{\beta_1-1})(-P - K_1P_0\\ &-\frac{1}{\beta_1}K_3^{-1}\dot{s}_1^{2-\beta_1} - \frac{\beta_2}{\beta_1}K_3^{-1}K_2\mathrm{diag}(|s_1|^{\beta_2-1})\dot{s}_1^{2-\beta_1}\\ &-L_2\dot{\xi}_2 - L_3\xi_3 + K_1(-L_1\dot{\xi}_1 + \dot{\xi}_2) - (\hat{\theta}_1 + \hat{\theta}_2\|\dot{\chi}\|\\ &+\hat{\theta}_3\|\dot{\chi}\|^2 + \hat{\theta}_4\|\dot{\chi}\|^3 + \rho)\mathrm{sgn}(s_2) - \frac{1}{\beta_1}K_3^{-1}K_4s_2\\ &+M_0^{-1}\Delta\dot{\tau} + P + K_1P_0 + \bar{\Delta} + L_2\dot{\xi}_2 + L_3\xi_3\\ &-M_0^{-1}\Delta\dot{\tau} - K_1(-L_1\dot{\xi}_1 + \dot{\xi}_2))\\ =& \beta_1 s_2^{\mathrm{T}}K_3\mathrm{diag}(|\dot{s}_1|^{\beta_1-1})(\bar{\Delta} - (\hat{\theta}_1 + \hat{\theta}_2\|\dot{\chi}\| + \hat{\theta}_3\|\dot{\chi}\|^2\\ &+\hat{\theta}_4\|\dot{\chi}\|^3 + \rho)\mathrm{sgn}(s_2) - \frac{1}{\beta_1}K_3^{-1}K_4s_2)\end{aligned} \tag{29}$$

Substituting (29) into (28), we can get,

$$\begin{aligned}\dot{V}_1 \leq& \beta_1 s_2^{\mathrm{T}}K_3\mathrm{diag}(|\dot{s}_1|^{\beta_1-1})(\bar{\Delta} - (\hat{\theta}_1 + \hat{\theta}_2\|\dot{\chi}\| + \hat{\theta}_3\|\dot{\chi}\|^2\\ &+\hat{\theta}_4\|\dot{\chi}\|^3 + \rho)\mathrm{sgn}(s_2) - \frac{1}{\beta_1}K_3^{-1}K_4s_2) - (\lambda_{\min}(L_1)\\ &-\frac{1}{2})\xi_1^{\mathrm{T}}\xi_1 - (\lambda_{\min}(L_2) - 1)\xi_2^{\mathrm{T}}\xi_2 - (\lambda_{\min}(L_3)\\ &-1)\xi_3^{\mathrm{T}}\xi_3 + \frac{1}{2}\Delta\dot{\tau}^{\mathrm{T}}M_0^{-\mathrm{T}}M_0^{-1}\Delta\dot{\tau} - \sum_{i=1}^{4}\alpha_i^{-1}\tilde{\theta}_i\dot{\hat{\theta}}_i.\end{aligned} \tag{30}$$

By selecting parameters as: $\lambda_{\min}(L_1) > \frac{1}{2}$, $\lambda_{\min}(L_2) > 1$, and $\lambda_{\min}(L_3) > 1$, it can make the coefficients before variables positive. For the term $\xi_1^{\mathrm{T}}\xi_1$ using Lemma 1 property yields,

$$\begin{aligned}-\xi_1^{\mathrm{T}}\xi_1 =& -\xi_1^{\mathrm{T}}\xi_1 - \sqrt{2}(\xi_1^{\mathrm{T}}\xi_1)^{\frac{1}{2}} + \sqrt{2}(\xi_1^{\mathrm{T}}\xi_1)^{\frac{1}{2}}\\ \leq& -\frac{2-\sqrt{2}}{2}\xi_1^{\mathrm{T}}\xi_1 - \sqrt{2}(\xi_1^{\mathrm{T}}\xi_1)^{\frac{1}{2}} + \frac{\sqrt{2}}{2}\end{aligned} \tag{31}$$

$\xi_2^{\mathrm{T}}\xi_2$ and $\xi_2^{\mathrm{T}}\xi_2$ follow the same process. And we, thus, have,

$$\begin{aligned}\dot{V}_1 \leq& \beta_1 s_2^{\mathrm{T}}K_3\mathrm{diag}(|\dot{s}_1|^{\beta_1-1})(\bar{\Delta} - (\hat{\theta}_1 + \hat{\theta}_2\|\dot{\chi}\| + \hat{\theta}_3\|\dot{\chi}\|^2\\ &+\hat{\theta}_4\|\dot{\chi}\|^3 + \rho)\mathrm{sgn}(s_2) - \frac{1}{\beta_1}K_3^{-1}K_4s_2) - \frac{c_1}{2}\sum_{i=1}^{3}\xi_i^{\mathrm{T}}\xi_i\\ &-\frac{\sqrt{2}c_2}{2}\sum_{i=1}^{3}(\xi_i^{\mathrm{T}}\xi_i)^{\frac{1}{2}} + c_3 - \sum_{i=1}^{4}\alpha_i^{-1}\tilde{\theta}_i\dot{\hat{\theta}}_i,\end{aligned} \tag{32}$$

where $c_1 = \min((2-\sqrt{2})(\lambda_{\min}(L_1) - \frac{1}{2}), (2-\sqrt{2})(\lambda_{\min}(L_2) - 1), (2-\sqrt{2})(\lambda_{\min}(L_3) - 1))$, $c_2 = \min(2(\lambda_{\min}(L_1) - \frac{1}{2}), 2(\lambda_{\min}(L_2) - 1), 2(\lambda_{\min}(L_3) - 1))$, and $c_3 = \frac{\sqrt{2}}{2}(\lambda_{\min}(L_1) + \lambda_{\min}(L_2) + \lambda_{\min}(L_3) - \frac{5}{2}) + \frac{1}{2}\Delta\dot{\tau}^{\mathrm{T}}M_0^{-\mathrm{T}}M_0^{-1}\Delta\dot{\tau}$. Substitute (26) into (32), and we can have,

$$\begin{aligned}\dot{V}_1 \leq& \beta_1\|s_2^{\mathrm{T}}K_3\mathrm{diag}(|\dot{s}_1|^{\beta_1-1})\|(\|\bar{\Delta}\| - (\theta_1 + \theta_2\|\dot{\chi}\|\\ &+\theta_3\|\dot{\chi}\|^2 + \theta_4\|\dot{\chi}\|^3 + \rho)) - \frac{\sqrt{2}c_2}{2}\sum_{i=1}^{3}(\xi_i^{\mathrm{T}}\xi_i)^{\frac{1}{2}}\\ &-\lambda_{\min}(K_4\mathrm{diag}(|\dot{s}_1|^{\beta_1-1}))s_2^{\mathrm{T}}s_2 - \frac{c_1}{2}\sum_{i=1}^{3}\xi_i^{\mathrm{T}}\xi_i\\ &+c_3 + \sum_{i=1}^{4}\tilde{\theta}_i\hat{\theta}_i.\end{aligned} \tag{33}$$

For term $\tilde{\theta}_i\hat{\theta}_i$ using Young's inequality and Lemma 1, we can obtain,

$$\begin{aligned}\dot{V}_1 \leq& \beta_1\|K_3\mathrm{diag}(|\dot{s}_1|^{\beta_1-1})\|(\|\bar{\Delta}\| - (\theta_1 + \theta_2\|\dot{\chi}\| + \theta_3\|\dot{\chi}\|^2\\ &+\theta_4\|\dot{\chi}\|^3 + \rho))\|s_2\| - \lambda_{\min}(K_4\mathrm{diag}(|\dot{s}_1|^{\beta_1-1}))s_2^{\mathrm{T}}s_2\\ &-\frac{c_1}{2}\sum_{i=1}^{3}\xi_i^{\mathrm{T}}\xi_i - \frac{\sqrt{2}c_2}{2}\sum_{i=1}^{3}(\xi_i^{\mathrm{T}}\xi_i)^{\frac{1}{2}} - \frac{c_4}{2}\sum_{i=1}^{4}\tilde{\theta}_i^2\\ &-\frac{\sqrt{2}}{2}\sum_{i=1}^{4}|\tilde{\theta}_i| + c_5,\end{aligned} \tag{34}$$

where $c_4 = \frac{2-\sqrt{2}}{2}$ and $c_5 = c_3 + \sum_{i=1}^{4}(\frac{\sqrt{2}}{4} + \frac{1}{2}\theta_i^2)$. From (20), it can be known that $(\|\bar{\Delta}\| - (\theta_1 + \theta_2\|\dot{\chi}\| + \theta_3\|\dot{\chi}\|^2 + \theta_4\|\dot{\chi}\|^3 + \rho)) < 0$. Then applying Lemma 2 to (34), we can get,

$$\begin{aligned}\dot{V}_1 \leq& -\mu_1(\frac{1}{2}s_2^{\mathrm{T}}s_2 + \frac{1}{2}\sum_{i=1}^{3}\xi_i^{\mathrm{T}}\xi_i + \frac{1}{2}\sum_{i=1}^{4}\alpha_i^{-1}\tilde{\theta}_i^2)\\ &-\mu_2(\frac{\sqrt{2}}{2}(s_2^{\mathrm{T}}s_2)^{\frac{1}{2}} + \frac{\sqrt{2}}{2}\sum_{i=1}^{4}(\xi_i^{\mathrm{T}}\xi_i)^{\frac{1}{2}}\\ &+\frac{\sqrt{2}}{2}\sum_{i=1}^{4}|\tilde{\theta}_i|) + c_5\\ \leq& -\mu_1V_1 - \mu_2V_1^{\frac{1}{2}} + c_5,\end{aligned} \tag{35}$$

where $\mu_1 = \min(2\lambda_{\min}(K_4\mathrm{diag}(|\dot{s}_1|^{\beta_1-1})), c_1, c_4\alpha_i)$ and $\mu_2 = \min(\sqrt{2}\beta_1\|K_3\mathrm{diag}(|\dot{s}_1|^{\beta_1-1})\|(\|\bar{\Delta}\| - (\theta_1 + \theta_2\|\dot{\chi}\| + \theta_3\|\dot{\chi}\|^2 + \theta_4\|\dot{\chi}\|^3 + \rho)), c_2, \alpha_i^{\frac{1}{2}})$. According to Lemma 3, the sliding manifold s_2, auxiliary dynamic vectors ξ_i, and estimated errors $\tilde{\theta}_i$ are all ultimately bounded. And these values could converge to a small neighborhood $\sigma \leq \frac{c_5}{\mu_2(1-c)}$

around the equilibrium in finite-time with $0 < c < 1$. Let us rewrite (15) as,

$$s_1 + K_2 s_1^{\beta_2} + \dot{s}_1^{\beta_1}(K_3 - \frac{s_2}{\dot{s}_1^{\beta_1}}) = 0. \tag{36}$$

We first assume $\|\dot{s}_1\| > (\frac{\sigma}{K_3})^{\frac{1}{\beta_1}}$, which implies $K_3 - \frac{s_2}{\dot{s}_1^{\beta_1}} > 0$. As such, the function (36) could follow the same finite-time convergence property. In other words, $\|\dot{s}_1\| \leq (\frac{\sigma}{K_3})^{\frac{1}{\beta_1}}$ will hold in finite-time. Accordingly, we can further get the sliding mode s_1 could converge to the following region,

$$\|s_1\| \leq \|K_3 \dot{s}_1^{\beta_1}\| + \|s_2\| \leq 2\sigma \tag{37}$$

in finite-time as well. For the analysis of the PDSM manifold (14), follow the same process above. This completes the proof.

4. SIMULATION RESULTS

To demonstrate the effectiveness and robustness of the proposed ASOSMC, simulation is carried out in this section. The tilting quadcopter is commanded to track the desired trajectories to illustrate the superiority capability of the fully actuated vehicle over the conventional quadcopter. The desired trajectories are set as $\chi_d = [\cos(0.3t - 0.8), 0.6\cos(0.4t) + 0.2, \cos(0.45t + 0.4) - 0.3, 0.4\cos(0.4t) + 0.5, \cos(0.3t - 0.5), \cos(0.5t + 0.4) - 0.5]^{\mathrm{T}}$. The parameters of the dynamics are summarized in the following table. We further add 30% random un-

Table 1. Parameters of the tilting quadcopter

Parameters	Values	Parameters	Values
d_1, d_3, d_5	1.0	m	2.878 kg
d_2, d_4, d_6	1.0	I_x	0.04463 kgm^2
g_1, g_3, g_5	0.2	I_y	0.04463 kgm^2
g_2, g_4, g_6	0.3	I_z	0.04463 kgm^2

certainties to these parameters and external disturbance $\Delta_\Omega = [0.02\sin(0.2t), 0.01\cos(0.5t) + 0.01, 0.02\cos(0.3t - 0.2), 0.3\cos(0.1t), 0.1\cos(0.3t - 0.2) + 0.1, 0.3\sin(0.3t - 0.2)]^{\mathrm{T}}$. The parameters in ASOSMC scheme (23), adaptive law (26), and auxiliary system (22) are set as: $\beta_1 = 1.5$, $\beta_2 = 2.8$, $\rho = 0.1$, $K_1 = \mathrm{diag}(1.2, 1.2, 1.2, 1.3, 1.3, 1.3)$, $K_2 = \mathrm{diag}(0.7, 0.7, 0.7, 1, 1, 1)$, $K_3 = \mathrm{diag}(0.27, 0.27, 0.27, 0.9, 0.9, 0.9)$, $K_4 = \mathrm{diag}(75, 75, 75, 50, 50, 50)$, $L_1 = \mathrm{diag}(1, 1, 1, 1, 1, 1)$, $L_2 = \mathrm{diag}(1.5, 1.5, 1.5, 1.5, 1.5, 1.5)$, $L_3 = \mathrm{diag}(1.5, 1.5, 1.5, 1.5, 1.5, 1.5)$, $\tau_{\max} = [0.3, 0.3, 0.3, 5, 5, -10]^{\mathrm{T}}$, and $\tau_{\min} = [-0.3, -0.3, -0.3, -5, -5, -40]$. Then the simulation results are shown as follows.

The tilting quadcopter can well track the desired commands and the tracking errors converge to a small neighborhood around the equilibrium as shown in Fig. 2. It can be seen that the roll ϕ, x and y channels reach the saturated constrains at the beginning of the tracking as shown in Fig. 3. Between 15-20 seconds, the ϕ and ψ trigger the saturation constrains. All these controller are constrained within the predefined region, which means that the proposed scheme can deal with the physical constrains. Moreover, the control inputs are smooth and continuous, which alleviates the serious chattering effect dramatically.

It can further illustrate that the auxiliary system has an effect on the system once the inputs saturation is

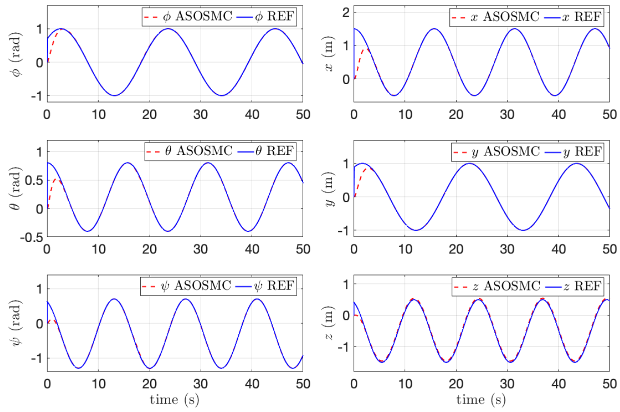

Fig. 2. The rotational and translational movements.

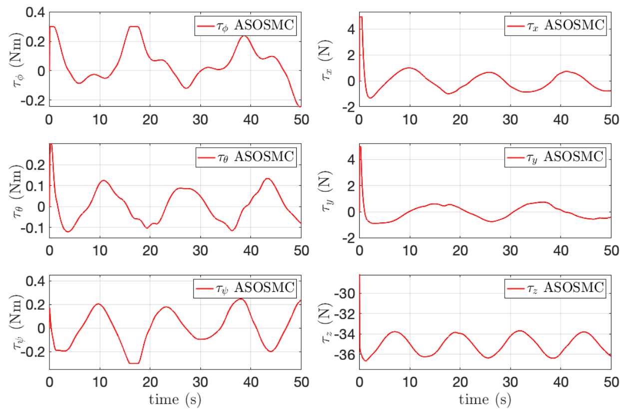

Fig. 3. The control inputs with input constrains.

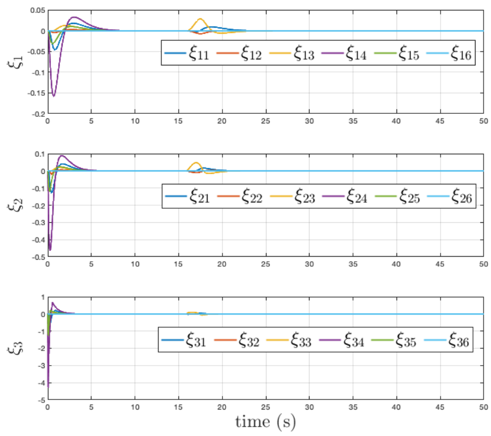

Fig. 4. The auxiliary variables.

triggered as shown in Fig. 4. And the auxiliary system finally converges to a small neighborhood around zero as derived in Section 3. The performance of the PDSM s_1 and the FNTSM manifold s_2 is shown in Fig. 5. The s_2 could fast converge to a small neighborhood around the origin in finite-time. And the PDSM s_1 can then fast converge to a small region around zero within a finite-time.

5. CONCLUSION

In this paper, we propose a novel tilting quadcopter featuring the capability of decoupling the translational and rotational movements. Considering uncertainties and input

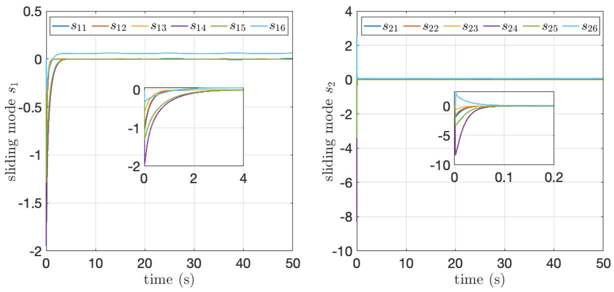

Fig. 5. The PDSM s_1 and the FNTSM s_2.

saturations, a new form of ASOSMC with an auxiliary system is proposed by combining the FNTSMC and the SOSMC to achieve fast convergence rate and high precision with alleviating chattering effect. It can be rigorously proved that the proposed ASOSMC can make the sliding mode manifolds fast converge to a small neighborhood around the origin in finite-time and tracking errors exponentially. Simulation results can further demonstrate the effectiveness and robustness of the proposed ASOSMC. In our future work, we will further consider the control allocation problem for this over-actuated system.

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