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Output sign-consensus of heterogenous multi-agent systems: an observer-based approach ⋆

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Abstract: This paper studies output sign-consensus of heterogeneous multi-agent systems over directed signed graphs. To remove the restriction that every follower knows the dynamics of the leader system, a distributed observer is maintained by each follower nodes, such that the state and dynamics of the leader node is estimated. Then a distributed state feedback control law is designed and analyzed. A simulation example is presented to illustrate the effectiveness of the proposed control law.

Keywords: antagonistic interaction; heterogeneous multi-agent system; sign-consensus; signed graph

1. INTRODUCTION

Over the past two decades, significant interests have been paid to distributed control of multi-agent systems, which has reached a relatively mature status (Ren and Beard, 2008; Lewis et al., 2014), with a primary focus on the consensus problem (Jadbabaie et al., 2003; Qin et al., 2017), assuming that all interactions between agents are collaborative. In other words, interactions/communication networks are modeled by nonnegative graphs, whose adjacency matrices are nonnegative (Berman and Plemmons, 1994).

However, in many practical scenarios, both cooperative and competitive interactions coexist within a group of agents, such as social network, biological network, and political campaign. Such interaction networks should be represented by signed graphs, where positive and negative edges stand for collaborative and antagonistic interactions, respectively. Obviously, adjacency matrix of a signed graph is a general matrix, instead of a nonnegative matrix. Thus, analysis of signed graph is much more challenging than that of nonnegative graph.

In the past several years, many researches have been conducted on distributed control of multi-agent systems over signed graphs, with a seminal work being Altafini (2013), which found a new collective behavior of multi-agent systems, called bipartite consensus. By bipartite consensus, it means that the whole group of agents will split into two subgroups, and agents within each subgroup reach conventional consensus, while consensus values of these two subgroups will have the same magnitude but different signs. This work was soon extended from single integrator dynamics to homogeneous and heterogeneous linear systems (Valcher and Misra, 2014; Zhang and Chen, 2017; Jiao et al., 2019), nonlinear systems (Yu et al., 2019), and to systems with measurement noise (Ma and Qin, 2016).

For bipartite consensus problems, it is always assumed that signed graphs should be structurally balanced (Altafini, 2013). However, this condition is very strong and can be easily violated by inappropriately adding an edge or changing the sign of a single edge. Therefore, investigating collective behaviors over structurally unbalanced signed graphs is important and interesting.

Very recently, Altafini and Lini (2015) studied opinion forming process over structurally unbalanced communication graphs. They found that when the communication graph is eventually positive, all agents will achieve an unanimous opinion, although some might be more convinced than others. In other words, the signs of all opinions will reach a consensus. Thus, we coined the term *sign-consensus* in Jiang et al. (2017), which extends Altafini and Lini (2015) to general homogeneous linear multi-agent systems. It is noticed that in practice almost all agent dynamics are inherently heterogeneous, which allows different agents to have different dimensions and/or different dynamics. For heterogeneous linear multi-agent systems, we designed distributed control law, which guarantees the output of each agent to achieve sign-consensus, i.e., output sign-consensus (Jiang and Zhang, 2020).

It is well known that the major advantage of multi-agent systems is its distributed feature, i.e., control law for each agent only use information of itself and its neighbors. However, most works of heterogeneous multi-agent sys-

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tems assume that all agent can access the information of dynamics of the exosystem (Wieland et al., 2011; Jiang and Zhang, 2020), which means there exist direct edges from the exosystem to all other agents. This is known as a kind of global information and not realistic. Therefore, how to design a fully distributed control law is of longstanding research interests in the control community. Motivated by the above-mentioned statements, we aim to remove this assumption and design a fully distributed control law for heterogeneous multi-agent systems, such that output sign-consensus can be achieved. Inspired by Cai et al. (2017), we propose an observer based control approach, where a distributed adaptive observer is maintained by each agent which can estimate the dynamics of the exosystem. With this observer, the whole control law can be designed in a fully distributed manner.

The rest of this paper is organized as follows. Section 2 introduces notations, preliminaries of signed graphs and problem formulation. In Section 3, an adaptive distributed observer is designed for each agent to estimate dynamics and states of the exosystem. In Section 4, a state feedback control law is proposed and analyzed. A simulation example is provided in Section 5 and Section 6 concludes the paper.

2. PRELIMINARIES

2.1 Notations

$\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{R}^n$ represent real space, positive real space and n-dimentional real space, respectively. The empty set is denoted as $\emptyset$. $\otimes$ denotes the Kronecker product. 1_n denotes an n dimensional vector, whose entries are all ones. A column vector $x \in \mathbb{R}^n$ is defined by $x = [x_1, x_2, \cdots, x_n]^T$ where x_i is the ith entry. A diagonal matrix $\Lambda \in \mathbb{R}^{n\times n}$ is denoted as $\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_n)$. Matrix $A \in \mathbb{R}^{n\times n}$ with entries defined as a_{ij} is denoted as $A = [a_{ij}]$. If the entries of matrix A are all positive, it is said to be positive, which is denoted as $A > 0$. A positive vector $v \in R^n$, whose entries are all positive, is denoted as $v > 0$. Eigenvalues of $A \in \mathbb{R}^{n\times n}$ are denoted by $\lambda_i(A)$, $i = 1, 2, \cdots, n$. Simultaneously, $Re(\lambda_i(A))$ denotes the real part of $\lambda_i(A)$. The identity matrix is denoted as $I_n \in \mathbb{R}^{n\times n}$. The spectral radius of A, denoted as $\rho(A)$, represents the smallest positive number such that $\rho(A) \geq |\lambda_i(A)|$, $\forall i = 1, 2, \cdots, n$. For $X_i \in \mathbb{R}^{m\times n}$, $col(X_1, X_2, \cdots, X_N) = [X_1^T, X_2^T, \cdots, X_N^T]^T$. For a column vector $x = col(x_1, x_2, \cdots, x_q) \in \mathbb{R}^{nq}$, where $x_i \in \mathbb{R}^n$, denote $M_n^q(x) = [x_1, x_2, \cdots, x_q] \in \mathbb{R}^{n\times q}$. For a matrix $X \in \mathbb{R}^{n\times m}$, its vector valued function is defined as $vec(X) = [X_1^T, X_2^T, \ldots, X_m^T]^T \in \mathbb{R}^{nm}$, where $X_i \in \mathbb{R}^n$ is the ith column of the matrix X. The entrywise sign function of $x = [x_1, x_2, \cdots, x_n]^T$ is defined as

$$sgn(x) = [sgn(x_1), sgn(x_2), \cdots, sgn(x_n)]^T$$

with

$$sgn(x_i) = \begin{cases} 1, & x_i>0 \\ 0, & x_i = 0, \qquad i = 1, 2, \ldots, n \\ -1, & x_i<0 \end{cases}$$

2.2 Signed graph

The communication or interaction of multi-agent systems can be naturally described as a graph, where each node or vertex represents an agent and each edge represents the communication/interaction between two agents. A directed graph is denoted as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ is the node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set. The topology of a graph can be completely depicted by its adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$, where a_{ij} represents the $(i, j)th$ entry and $a_{ij} \neq 0$ implies that node i can obtain information from node j, i.e., $(v_j, v_i) \in \mathcal{E}$, and v_j is thus called a neighboring agent of v_i; otherwise $a_{ij} = 0$. The set of all indices of neighboring agents of v_i can be denoted as $\mathcal{N}_i = \{j|(v_j, v_i) \in \mathcal{E}, j = 1, 2, \cdots, N\}$, called the neighboring set of node i. In this paper, only simple graph is considered, i.e., graph with no self-loops and no multiple edges. Particularly, for graph $\mathcal{G}(\mathcal{A})$, if all weights of edges are nonnegative, i.e. $a_{ij} \geq 0$, for $\forall i, j = 1, 2, \cdots, N$, it is called nonnegative graph; otherwise it is signed graph.

Definition 1. (Altafini and Lini, 2015)
A matrix $A \in \mathbb{R}^{n\times n}$ is said to have *strong Perron − Frobenius property* if the spectral radius $\rho(A)$ is a simple eigenvalue of A, and the associated eigenvector v_r is positive.

Definition 2. (Altafini and Lini, 2015)
A matrix $A \in \mathbb{R}^{n\times n}$ is said to be *eventually positive*, which can be denoted as $A \overset{\vee}{>} 0$, if a positive integer k_0 exists and is such that $A^k > 0$ for all $k \in \mathbb{N}$ with $k \geq k_0$.

Proposition 1. (Altafini and Lini, 2015)
For a matrix $A \in \mathbb{R}^{n\times n}$, the following statements are equivalent:

(i). A and A^T have *strong Perron-Frobenius* property;

(ii). $A \overset{\vee}{>} 0$;

(iii). $A^T \overset{\vee}{>} 0$.

2.3 Problem formulation

Consider the heterogeneous linear multi-agent system with N agents:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i \\ y_i &= C_i x_i, \qquad i = 1, 2, \cdots, N \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, $y_i \in \mathbb{R}^p$ are the state, input and output, respectively; and (A_i, B_i) is assumed to be stabilizable.

The exosystem, i.e., the leader node, has the following dynamics:

$$\begin{aligned} \dot{\eta} &= S_0 \eta \\ w &= R\eta \end{aligned} \tag{2}$$

where $\eta \in \mathbb{R}^q$ and $w \in \mathbb{R}^p$ are the state and output of the leader node. Matrix S_0 is assumed to have no eigenvalues on the left half plane (Huang, 2004).

The objective of this paper is to design a fully distributed controller by integrating an adaptive observer which can estimation the leader's signals and dynamics, such that output sign-consensus of all follower nodes is achieved.

Definition 3. (Jiang et al., 2018)
Let $y_i = [y_{i,1}, y_{i,2}, \cdots, y_{i,p}]^T$ and $y^* = [y_1^*, y_2^*, \cdots, y_p^*]^T$, $i = 1, 2, \cdots, N$. System (1) is said to achieve output sign-consensus if

$$\lim_{t\to\infty}(sgn(y_{i,l}(t)) - sgn(y_l^*(t))) = 0, \quad \forall l \in L_1$$
$$\lim_{t\to\infty}(y_{i,l}(t) - y_l^*(t)) = 0, \quad \forall l \in L_2$$

for all $i = 1, 2, \ldots, N$, where $L_1 = \{l | \lim_{t\to\infty} y_l^*(t) \neq 0, l = 1, 2, \cdots, p\}$, $L_2 = \{l | \lim_{t\to\infty} y_l^*(t) = 0, l = 1, 2, \cdots, p\}$ and $L_1 \cap L_2 = \emptyset$, $L_1 \cup L_2 = \{1, 2, \cdots, p\}$.

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be the communication graph of the follower nodes, where $\mathcal{V} = \{1, 2, \cdots, N\}$ and $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$. Denote the augmented graph, considering both communication among follower nodes and leader node, as $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$, where $\bar{\mathcal{V}} = \{0, 1, 2, \cdots, N\}$, and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. For the solvability of this problem, the following two assumptions are needed.

Assumption 1. The graph $\bar{\mathcal{G}}$ contains a spanning tree with the leader node, labeled 0, being the root.

Assumption 2. The adjacency matrix $\mathcal{A}$ of graph $\mathcal{G}$ is eventually positive.

3. DESIGN OF ADAPTIVE DISTRIBUTED OBSERVER

Since the dynamics of the leader is not available for all the followers in reality, an adaptive distributed observer is naturally required in order to estimate the leader's dynamics by each follower.

Assume that only the direct neighboring nodes of the leader node 0 can get the information of S_0. Denote the weight of the edge from leader node 0 to node i as g_{i0}, such that $g_{i0} > 0$ if node i is a direct neighbor of the leader; otherwise, $g_{i0} = 0$. Define a matrix $\mathcal{C} = \Sigma - \mathcal{A}$, where $\Sigma = diag(\sigma_1, \sigma_2, \cdots, \sigma_N)$ and $\sigma_i > 0$ for $i = 1, 2, \cdots, N$. Denote S_i to be the estimate of S_0 by node i.

From Assumption 2 and Proposition 1, we can conclude that the adjacency matrix $\mathcal{A}$ has *strong Perron − Frobenius property*, which means that there exists a positive vector $v_r = [v_{r_1}, v_{r_2}, \cdots, v_{r_N}]^T > 0$ such that $\mathcal{A}v_r = \rho(\mathcal{A})v_r$.

Then the adaptive observer can be designed as:

$$\dot{S}_i = \gamma_1 \left(-\sigma_i S_i + \sum_{j=1}^{N} a_{ij} S_j + g_{i0}(v_{r_i} S_0 - S_i) \right) \quad (3)$$

$$\dot{\zeta}_i = \frac{1}{v_{r_i}} S_i \zeta_i + \gamma_2 \left(-\sigma_i \zeta_i + \sum_{j=1}^{N} a_{ij} \zeta_j + g_{i0}(v_{r_i}\eta - \zeta_i) \right) \quad (4)$$

Lemma 1. Consider system(2) and adaptive observer (3) and (4). Let $\sigma_i = \rho(\mathcal{A}), \forall i = 1, 2, \cdots, N$; $\gamma_1 > 0$; $\gamma_2 > 0$ and is sufficiently large. Then for any initial condition $S_i(0)$ and $\zeta_i(0)$, we have:
(i). $\lim_{t\to\infty}(S_i(t) - v_{r_i} S_0) = 0, \quad \forall i \in \{1, 2, \cdots, N\}$;
(ii). $\lim_{t\to\infty}(\zeta_i(t) - v_{r_i}\eta) = 0, \quad \forall i \in \{1, 2, \cdots, N\}$.

Proof. Define $\tilde{S}_i = S_i - v_{r_i} S_0$ and $\varepsilon_i = \zeta_i - v_{r_i}\eta$.
Part(i). Since $\mathcal{A}v_r = \rho(\mathcal{A})v_r$ and $\sigma_i = \rho(\mathcal{A})$, it is trivial to show that $g_{i0}v_{r_i} = g_{i0}v_{ri} + \sigma_i v_{r_i} - \sum_{j=1}^{N} a_{ij}v_{r_j}$. Then from (3) we have

$$\begin{aligned}\dot{\tilde{S}}_i &= \gamma_1 \left(-\sigma_i S_i + \sum_{j=1}^{N} a_{ij} S_j + g_{i0}(v_{r_i} S_0 - S_i) \right) \\ &= \gamma_1 \left(-(\sigma_i + g_{i0}) S_i + \sum_{j=1}^{N} a_{ij} S_j \right) \\ &\quad + \gamma_1 \left((g_{i0} v_{r_i} + \sigma_i v_{r_i} - \sum_{j=1}^{N} a_{ij} v_{r_j}) S_0 \right)\end{aligned}$$

Denote $G = diag(g_{10}, g_{20}, \cdots, g_{N0})$, $S_c = col(S_1, \cdots, S_N)$, and $\tilde{S} = col(\tilde{S}_1, \tilde{S}_2, \cdots, \tilde{S}_N)$. We have

$$\begin{aligned}\dot{\tilde{S}} &= \dot{S}_c \\ &= -\gamma_1 \left((\mathcal{C} + G) \otimes I_q \right) (S_c - (v_r \otimes S_0)) \\ &= -\gamma_1 \left((\mathcal{C} + G) \otimes I_q \right) \tilde{S} \end{aligned} \quad (5)$$

Since assumptions 1 and 2 hold, it can be shown that $Re(\lambda_i(\mathcal{C}+G)) > 0, \forall i = 1, 2, \cdots, N$. Then $-\gamma_1(\mathcal{C}+G)\otimes I_q$ is *Hurwitz* for any $\gamma_1 > 0$. Therefore all $\tilde{S}_i$ will vanish exponentially as $t \to \infty$.

Part (ii). Similar to proof in Part (i), we have

$$\begin{aligned}\dot{\varepsilon}_i &= \frac{1}{v_{r_i}} S_i \zeta_i - S_0 \zeta_i + S_0 \zeta_i - v_{r_i} S_0 \eta \\ &\quad + \gamma_2 \left(-\sigma_i \zeta_i + \sum_{j=1}^{N} a_{ij} \zeta_j + g_{i0}(v_{r_i}\eta - \zeta_i) \right) \\ &= \frac{1}{v_{r_i}} \tilde{S}_i \zeta_i + S_0 \varepsilon_i \\ &\quad + \gamma_2 \left(-\sigma_i \zeta_i + \sum_{j=1}^{N} a_{ij} \zeta_j + g_{i0}(v_{r_i}\eta - \zeta_i) \right) \\ &= \frac{1}{v_{r_i}} \tilde{S}_i \varepsilon_i + \tilde{S}_i \eta + S_0 \varepsilon_i \\ &\quad + \gamma_2 \left(-\sigma_i \zeta_i + \sum_{j=1}^{N} a_{ij} \zeta_j + g_{i0}(v_{r_i}\eta - \zeta_i) \right)\end{aligned}$$

Let $\tilde{S}_d = diag(\tilde{S}_1, \cdots, \tilde{S}_N)$, $V_r = diag\left(\frac{1}{v_{r_1}}, \frac{1}{v_{r_2}}, \cdots, \frac{1}{v_{r_N}}\right)$, and $\varepsilon = col(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_N)$. Then we obtain

$$\begin{aligned}\dot{\varepsilon} &= (I_N \otimes S_0 - \gamma_2(\mathcal{C} + G) \otimes I_q)\, \varepsilon \\ &\quad + (V_r \otimes I_q)\tilde{S}_d \varepsilon + \tilde{S}_d (1_N \otimes \eta)\end{aligned} \quad (6)$$

For any

$$\gamma_2 > \frac{maxRe\,(\lambda_i(S_0))}{minRe\,(\lambda_i(\mathcal{C} + G))},$$

the matrix $(I_N \otimes S_0 - \gamma_2(\mathcal{C} + G) \otimes I_q)$ would be *Hurwitz*. Consider the system

$$\dot{x} = (I_N \otimes S_0 - \gamma_2(\mathcal{C} + G) \otimes I_q)\, x + (V_r \otimes I_q)\tilde{S}_d x \quad (7)$$

Using the same technique as in Example 9.6 of Khalil (2002), we can show that the origin of system (7) is globally exponentially stable. Then system(6) is input-to-state stable with $\tilde{S}_d(1_N \otimes \eta)$ as the input. Since $\tilde{S}_d$ and hence $\tilde{S}_d(1_N \otimes \eta)$ vanishes exponentially, it follows that $\lim_{t\to\infty} \varepsilon = 0$. Therefore, $\lim_{t\to\infty} \varepsilon_i = 0$ for all i. ■

4. CONTROL LAW DESIGN AND ANALYSIS

In this section we will design a state feedback controller to achieve output sign-consensus. Two lemmas are first presented.

Lemma 2. (Cai et al., 2017) Let $Q \in \mathbb{R}^{m\times n}, b \in \mathbb{R}^m$ and $rank(Q) = rank(Q, b) = k$ where $m, n, k \in \mathbb{R}^+$. Let $\mathcal{Q}(t) \in \mathbb{R}^{m\times n}$ such that $\lim_{t\to\infty}(\mathcal{Q}(t) - Q) = 0$. Then for any initial condition and sufficiently large $\mu > 0$, the state of system

$$\dot{x} = -\mu \mathcal{Q}(t)^T(\mathcal{Q}(t)x - b)$$

will tend to x^* exponentially, where $Qx^* = b$.

Lemma 3. Let $b_i = vec\left(\begin{bmatrix} 0_{n_i\times q} \\ -R \end{bmatrix}\right)$ and $P_i(t) = S_i^T \otimes \begin{bmatrix} I_{n_i} & 0 \\ 0 & 0 \end{bmatrix} - I_q \otimes \begin{bmatrix} v_{r_i}A_i & B_i \\ C_i & 0 \end{bmatrix}$. Each function

$$\dot{\psi}_i = -\gamma_3 P_i(t)^T(P_i(t)\psi_i - b_i),\ \forall i = 1, 2, \cdots, N \quad (8)$$

with γ_3 positive and sufficiently large, has a unique bounded solution, if the following regulator equations are solvable

$$\begin{aligned} v_{r_i}\Pi_i S_0 &= v_{r_i}A_i\Pi_i + B_i\Gamma_i \\ C_i\Pi_i &= R. \end{aligned} \quad (9)$$

And further let

$$\Psi_i(t) = M^q_{(n_i+m_i)}(\psi_i(t)) = \begin{bmatrix} \Psi_{1i}(t) \\ \Psi_{2i}(t) \end{bmatrix}.$$

Then for some solution (Π_i, Γ_i) of equation(9), we have $\lim_{t\to\infty}\left(\Psi_i(t) - \begin{bmatrix} \Pi_i \\ \Gamma_i \end{bmatrix}\right) = 0$.

Proof. Rewrite (9) as

$$v_{r_i}\begin{bmatrix} I_{n_i} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \Pi_i \\ \Gamma_i \end{bmatrix} S_0 - \begin{bmatrix} v_{r_i}A_i & B_i \\ C_i & 0 \end{bmatrix}\begin{bmatrix} \Pi_i \\ \Gamma_i \end{bmatrix} = \begin{bmatrix} 0_{n_i\times q} \\ -R \end{bmatrix}$$

which can be further put into the form

$$P_i\chi_i = b_i$$

where $P_i = v_{r_i}S_0^T \otimes \begin{bmatrix} I_{n_i} & 0 \\ 0 & 0 \end{bmatrix} - I_q \otimes \begin{bmatrix} v_{r_i}A_i & B_i \\ C_i & 0 \end{bmatrix}$ and $\chi_i = vec(\begin{bmatrix} \Pi_i \\ \Gamma_i \end{bmatrix})$. From Lemma 1, $\lim_{t\to\infty}(P_i(t) - P_i) = 0$. Thus from Lemma 2, $\lim_{t\to\infty}(\psi_i(t) - \chi_i) = 0$. By noting that

$$\Psi_i(t) - \begin{bmatrix} \Pi_i \\ \Gamma_i \end{bmatrix} = M^q_{(n_i+m_i)}(\psi_i(t) - \chi_i)$$

this completes the proof. ■

Then design the following state feedback controller

$$u_i = K_i x_i + \mathcal{K}_{\zeta_i}(t)\zeta_i \quad (10)$$

$$\mathcal{K}_{\zeta_i}(t) = \frac{1}{v_{r_i}}\left(\Psi_{2i}(t) - v_{r_i}K_i\Psi_{1i}(t)\right) \quad (11)$$

where K_i is a constant matrix such that $A_i + B_iK_i$ is *Hurwitz* and $\mathcal{K}_{\zeta_i}(t)$ is determined by solving (8).

Theorem 1. Consider system(1) and (2) with its control law (3), (4),(8),(10) and (11). Suppose assumptions 1 and 2 hold, and regulator equations (9) are solvable. Then if $\gamma_1, \gamma_2, \gamma_3 \in \mathbb{R}^+$ and γ_2, γ_3 are sufficiently large, output sign-consensus can be achieved.

Proof. Let $\tilde{x}_i = x_i - v_{r_i}\Pi_i\eta$, $\tilde{u}_i = u_i - \Gamma_i\eta$, $e_i = y_i - v_{r_i}w$, $K_{\zeta_i} = \frac{1}{v_{r_i}}(\Gamma_i - v_{r_i}K_i\Pi_i)$ and $\tilde{\mathcal{K}}_{\zeta_i}(t) = \mathcal{K}_{\zeta_i}(t) - K_{\zeta_i}$.

For each $i \in \{1, 2, \cdots, N\}$, by utilizing (9) we obtain that

$$\begin{aligned} \dot{\tilde{x}}_i &= \dot{x}_i - v_{r_i}\Pi_i\dot{\eta} \\ &= A_ix_i + B_iu_i - v_{r_i}\Pi_iS_0\eta \\ &= A_i(\tilde{x}_i + v_{r_i}\Pi_i\eta) + B_i(\tilde{u}_i + \Gamma_i\eta) - v_{r_i}\Pi_iS_0\eta \\ &= A_i\tilde{x}_i + B_i\tilde{u}_i \end{aligned} \quad (12)$$

$$\begin{aligned} e_i &= y_i - v_{r_i}w \\ &= C_ix_i - v_{r_i}R\eta \\ &= C_i\tilde{x}_i \end{aligned} \quad (13)$$

$$\begin{aligned} \tilde{u}_i &= K_ix_i + \mathcal{K}_{\zeta_i}(t)\zeta_i - \Gamma_i\eta \\ &= K_i(\tilde{x}_i + v_{r_i}\Pi_i\eta) + \mathcal{K}_{\zeta_i}(t)(\varepsilon_i + v_{r_i}\eta) - \Gamma_i\eta \\ &= K_i\tilde{x}_i + \mathcal{K}_{\zeta_i}(t)\varepsilon_i + (\Gamma_i - v_{r_i}K_{\zeta_i})\eta + v_{r_i}\mathcal{K}_{\zeta_i}(t)\eta - \Gamma_i\eta \\ &= K_i\tilde{x}_i + \mathcal{K}_{\zeta_i}(t)\varepsilon_i + v_{r_i}(\mathcal{K}_{\zeta_i}(t) - K_{\zeta_i})\eta \\ &= K_i\tilde{x}_i + \mathcal{K}_{\zeta_i}(t)\varepsilon_i + v_{r_i}\tilde{\mathcal{K}}_{\zeta_i}(t)\eta \end{aligned} \quad (14)$$

Substituting (14) into (12) leads to

$$\begin{aligned} \dot{\tilde{x}}_i &= (A_i + B_iK_i)\tilde{x}_i \\ &\quad + B_i\mathcal{K}_{\zeta_i}(t)\varepsilon_i + v_{r_i}B_i\tilde{\mathcal{K}}_{\zeta_i}(t)\eta \end{aligned} \quad (15)$$

When $\gamma_1 > 0$ and γ_2 sufficiently large, by Lemma 1, we know $\tilde{S}_i$ vanishes exponentially and $\varepsilon_i \to 0$ as $t \to \infty$. Since

$$v_{r_i}\tilde{\mathcal{K}}_{\zeta_i}(t) = (\Psi_{2i}(t) - \Gamma_i) - v_{r_i}K_i(\Psi_{1i}(t) - \Pi_i)$$

then $\lim_{t\to\infty}\tilde{\mathcal{K}}_{\zeta_i}(t) = 0$ if γ_3 is large enough (see Lemma 3). Thus $v_{r_i}B_i\tilde{\mathcal{K}}_{\zeta_i}(t)\eta$ tends to zero exponentially. Similar to the proof of Lemma 1, one can show that system (15) is input-to-state stable with the vanishing input term $\left(B_i\mathcal{K}_{\zeta_i}(t)\varepsilon_i + v_{r_i}B_i\tilde{\mathcal{K}}_{\zeta_i}(t)\eta\right)$. Therefore for any initial condition $\tilde{x}_i(0)$, $\lim_{t\to\infty}\tilde{x}_i = 0$. Then $\lim_{t\to\infty} e_i = 0$, which implies $\lim_{t\to\infty}(y_i - v_{r_i}R\eta) = 0$. This means that output sign-consensus is achieved. ■

Remark. Although the selection of γ_2 and γ_3 depends on agents' dynamics and graph topology $\mathcal{G}$, we need not compute them in practice, but to guarantee that they are positive and sufficiently large.

5. SIMULATION EXAMPLE

Consider a group of 6 followers nodes (1) with dynamics

$$A_i = \begin{bmatrix} -0.5 & -1 \\ 2 & 0 \end{bmatrix}, \forall i \in \{1, 2, 3\}$$

$$A_i = \begin{bmatrix} 0.5 & -1 \\ 2 & 0 \end{bmatrix}, \forall i \in \{4, 5, 6\}$$

$$B_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ C_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \forall i \in \{1, \cdots, 6\}$$

Obviously (A_i, B_i) is stabilizable.

Consider an exosystem (2) with the following dynamics

$$S_0 = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The communication graph $\mathcal{G}$ of follower nodes is adopted from Jiang et al. (2018) and is shown in Fig. 1. Suppose only node 1 has a direct communication with the leader, i.e., $g_{10} = 1$ and $g_{i0} = 0$ for $i = 2, 3, \cdots, 6$. We can easily show that assumptions 1 and 2 are satisfied.

Consider system(1) and (2) with control law (3), (4),(8),(10) and (11). Let $\gamma_1 = 5$, $\gamma_2 = 20$ and $\gamma_3 = 10$. The output trajectories of all agents are shown in Figs. 2 and 3. Output sign-consensus is achieved.

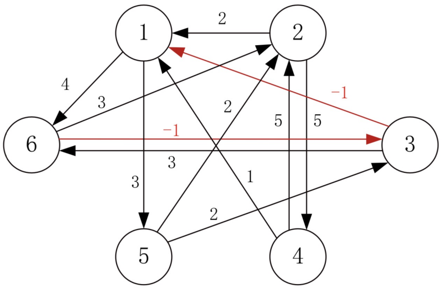

Fig. 1. Communication graph $\mathcal{G}$

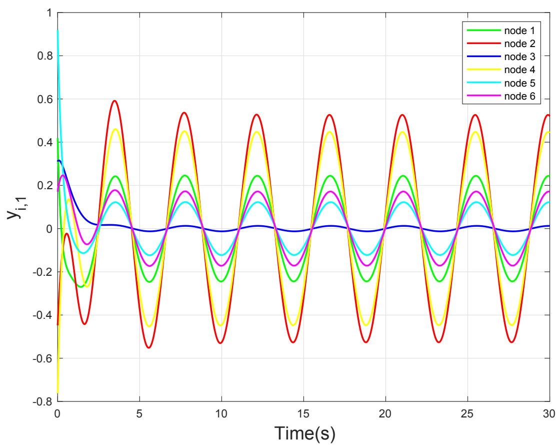

Fig. 2. Output trajectories of $y_{i,1}$ by state feedback control

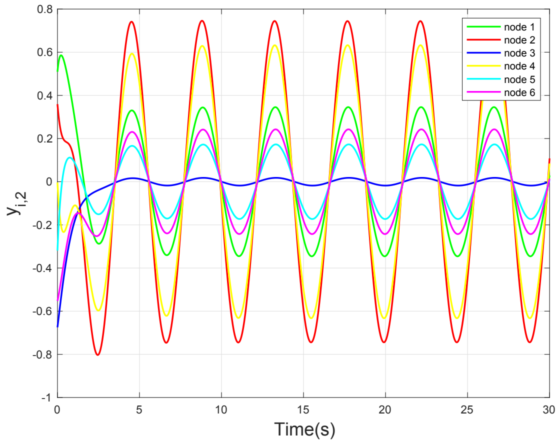

Fig. 3. Output trajectories of $y_{i,2}$ by state feedback control

6. CONCLUSION

This paper considered output sign-consensus of heterogeneous multi-agent systems. The objective is to design a distributed control law such that the sign of outputs of all follower nodes can reach consensus, i.e., output sign-consensus. Compared with our recent work (Jiang and Zhang, 2020), this paper removes the restriction that each follower nodes should know the dynamics of the leader node by designing an adaptive distributed observer. A state feedback control law was designed. This paper only considered a fixed graph topology. Output sign-consensus problem over time varying graph topology is worthy of further investigation.

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