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Dissipativity and Stability Recovery by Fault Hiding ⋆

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Abstract: This paper addresses the problem of dissipativity-based fault tolerant control (FTC) based on fault hiding approach. In particular, a static reconfiguration block (RB) is used for reconfiguration of faulty systems. Such block performs a loop transformation by inserting series, feedback, and feedforward gains to a system including plant, sensor or actuator faults. The proposed approach consists in recovering dissipativity and passivity conditions of a previously dissipative system, ensuring that the reconfigured system has the same supply function of the nominal system. Numerical examples illustrate how such approach can be used to recover the asymptotic stability by fault hiding even for nonlinear systems. Furthermore, LMI-based conditions for designing the proposed RB are provided for stability recovery for linear systems.

Keywords: Fault tolerant control, Reconfiguration Block, Fault hiding, Dissipativity, Passivity

1. INTRODUCTION

The dissipativity and passivity theory (Bao and Lee, 2007) provides powerful tools for dynamic systems analysis based on the intuitive energy balance concept, and eases the investigation of the input-output stability problem (Kottenstette et al., 2014; Khalil, 2000) as well as the analysis of interconnected systems. For this reason, the dissipativity/passivity framework is used to design control systems for several classes of systems, e.g., switched systems (Wu et al., 2013) and networked systems (Hirche et al., 2009), but there are few results for fault tolerant control (FTC).

FTC techniques are grouped into the active and passive techniques. Passive FTC (PFTC) considers the fault as uncertainty and designs controllers for achieving the control goals even with the fault occurrence. Active FTC (AFTC) modifies the controller or the control loop after a fault detection in order to mitigate the fault effects. In (Yang et al., 2008), the concept of global dissipativity and passivity is proposed to quantify the fault tolerance of dynamical systems and decide if it is necessary to design an FTC law for each fault mode. Dissipativity theory is also used to obtain PFTC systems in some recent works (Sakthivel et al., 2017; Selvaraj et al., 2017).

Among the AFTC techniques, the fault hiding is extensively employed for systems with sensor and actuator faults. Fault hiding consists in inserting a reconfiguration block (RB) between the faulty plant and the controller to correct the sensor measurements sent to the controller and translate the control signals provided by the controller that does not receive the information about fault occurrence. Fault hiding does not require controller redesign, *i.e.*, it allows using the same controller designed for the nominal (fault-free) system during the fault occurrence. The RBs can be virtual sensors, used for sensor faults, or virtual actuators, used for actuator faults. Most of fault hiding applications are concerned with linear systems (Steffen, 2005), but there are also applications for nonlinear systems represented by linear parameter varying (Rotondo et al., 2018), Hammerstein-Wiener (Richter, 2011), and Takagi-Sugeno fuzzy models (Bessa et al., 2020).

In this work, a static RB is used to mitigate the fault effects and ensure the stability of the reconfigured system by means of passivity and dissipativity recovery for faulty (linear or nonlinear) systems. Compared with the previously published fault hiding approaches in the literature, the proposed dissipativity-based approach and static RB are able to mitigate both sensor and actuator faults simultaneously by means of the same RB and to ensure the recovering of exactly the same dissipativity properties as the healthy scenario, implying the recovering of stability and robustness properties. Furthermore, it is able to handle

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even nonlinear systems without polytopic representations unlike the previous nonlinear approaches (Bessa et al., 2020; Rotondo et al., 2018; Richter, 2011). In summary, the main contributions of this work are:

- to define the passivity and (Q,S,R)-dissipativity recovery by fault hiding;
- to establish the relation between passivity and (Q,S,R)-dissipativity recovery by fault hiding and the asymptotic stability recovery for nonlinear systems;
- to provide conditions for stability recovery after sensor, actuator, and plant faults.

The remaining of this paper is organized as follows: Section 2 discusses general aspects about dissipativity, and states the problem of stability and dissipativity recovery by fault hiding. Sections 3 and 4 describe the main contributions of this work, i.e., new conditions for asymptotic stabilization of faulty systems by means of passivity and dissipativity recovery with RBs. Section 5 presents an application example. Finally, Section 6 draws the main conclusions.

Notation. For a matrix M, $M \succ (\prec)\ 0$ means that M is a positive (negative) definite matrix; $M^\top$ is its transpose; the identity matrix of dimension n is denoted by I_n and the null matrix of order $n \times m$ by $0_{n\times m}$; $\mathrm{He}\{M\}$ denotes $\mathrm{He}\{M\} = M + M^\top$; in a symmetric block matrix, '$\star$' is the term deduced by symmetry. The notation $(\Sigma_1, \ldots, \Sigma_n)$ denotes the system obtained through the interconnection of the subsystems Σ_1, Σ_2, ..., and Σ_n.

2. PRELIMINARIES

2.1 Dissipativity and Stability

Dissipativity is a useful concept for dynamic system analysis that allows to investigate their stability by means of the energy balance, *i.e.*, the difference between the stored and supplied energy. A system is said to be dissipative if its stored energy, represented by the temporal derivative of a positive semidefinite continuously differentiable storage function, is always less than or equal to the supplied energy, represented by a supply rate function.

Different supply functions can be used for dissipativity analysis. An important case is the (Q,S,R)-dissipativity that assumes the following supply rate function

$$\mathcal{S}\left(u(t), y(t)\right) = y(t)^\top Q y(t) + 2u(t)^\top S y(t) + u(t)^\top R u(t), \qquad (1)$$

where $u(t)$ and $y(t)$ are the input and output signals of the system and $Q = Q^\top$, S, and $R = R^\top$ are the parameters of the supply rate function.

The (Q,S,R)-dissipativity eases the asymptotic stability analysis, since that a (Q,S,R)-dissipative system is asymptotically stable if $Q \prec 0$ (Hill and Moylan, 1976).

Passivity is a special case of dissipativity, *i.e.*, a system is said to be passive if it is dissipative w.r.t. the supply rate $\mathcal{S}(u(t), y(t)) = u(t)^\top y(t)$ and it is said strictly passive if it is dissipative with respect to the supply rate $\mathcal{S}\left(u(t), y(t)\right) = u(t)^\top y(t) + \psi\left(x(t)\right)$ for some positive definite function ψ.

Dissipativity theory provides an effective tool for analysis of interconnected systems. Lemma 1 summarizes some results on dissipativity and stability of a feedback interconnection between two systems (Khalil, 2000).

Lemma 1. Consider two systems interconnected by feedback, $\Sigma_a : (y_a, x_a) = \Omega_a(x_a(0), u_a)$ and $\Sigma_b : (y_b, x_b) = \Omega_b(x_b(0), u_b)$, such that $(\Sigma_a, \Sigma_b) : (\bar{y}(t), \bar{x}(t)) = \bar{\Omega}\left(\bar{x}(0), w(t)\right)$, where $\bar{x} = \begin{bmatrix} x_a(t)^\top & x_b(t)^\top \end{bmatrix}^\top$, $\bar{w} = \begin{bmatrix} w_a(t)^\top & w_b(t)^\top \end{bmatrix}^\top$, $\bar{y} = \begin{bmatrix} y_a(t)^\top & y_b(t)^\top \end{bmatrix}^\top$, $u_a(t) = w_a(t) - y_b(t)$, and $u_b(t) = w_b(t) + y_a(t)$.

- If Σ_a is strictly passive and Ω_a is a passive memoryless function, then the origin of (Σ_a, Σ_b) is asymptotically stable with $w_a(t) = w_b(t) = 0$.
- If Σ_a and Σ_b are strictly passive, then the origin of (Σ_a, Σ_b) is asymptotically stable with $w_a(t) = w_b(t) = 0$.
- If Σ_a and Σ_b are (Q_a,S_a,R_a)- and (Q_b,S_b,R_b)-dissipative, respectively, then (Σ_a, Σ_b) is ($\bar{Q}$,$\bar{S}$,$\bar{R}$)-dissipative with
$$\bar{Q} = \begin{bmatrix} Q_a + R_b & S_b - S_a^\top \\ S_b^\top - S_a & Q_b + R_a \end{bmatrix}, \bar{S} = \begin{bmatrix} S_a & \frac{1}{2}\mathrm{He}\{R_b\} \\ -\frac{1}{2}\mathrm{He}\{R_a\} & S_b \end{bmatrix},$$
$$\bar{R} = \begin{bmatrix} R_a & 0 \\ 0 & R_b \end{bmatrix}.$$
Furthermore, for $w_a(t) = w_b(t) = 0$, if $\bar{Q} \prec 0$ then (Σ_a, Σ_b) is asymptotically stable.

For the sake of simplicity the time indication is omitted of the signals used in the remaining of this paper, e.g., $x(t)$ will be simply represented by x.

2.2 Problem statement

Consider the system Σ_P subject to faults whose faulty model is Σ_{P_f} and is interconnected with a controller Σ_C. The fault hiding approach consists in inserting an RB Σ_R between the faulty plant and the controller, as depicted in Fig. 1, to recover the nominal performance or stability.

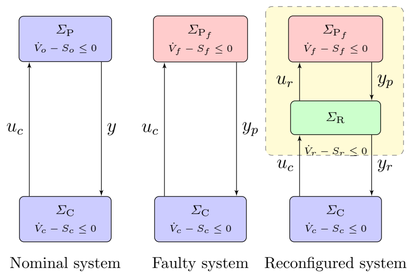


Fig. 1. Control reconfiguration by fault hiding.

In this paper, it is adopted the following RB Σ_R proposed in (Bessa et al., 2020)

$$\Sigma_R : \begin{cases} y_r = R_1 y_p + R_2 u_p \\ u_r = R_3 y_p + R_4 u_p \end{cases} \qquad (2)$$

allowing (Σ_{P_f}, Σ_R) to recover the dissipativity properties and asymptotic stability of $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$.

The closed loop system (Σ_P, Σ_C) depicted in Fig. 2 is modified by the RB Σ_R described in (2) such that the reconfigured closed loop is depicted in Fig. 3.

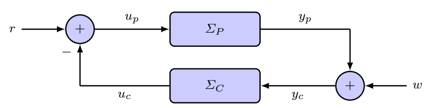


Fig. 2. Nominal closed loop system (Σ_P, Σ_C).

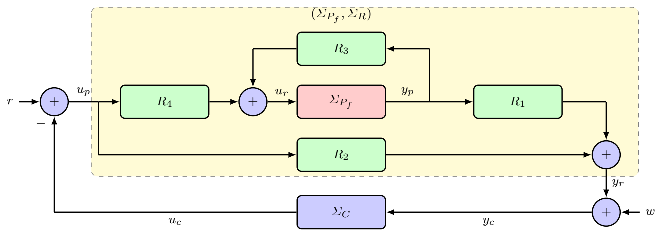


Fig. 3. Equivalent reconfigured loop of $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$.

This paper addresses the problem of dissipativity recovery by means of the RB described in (2), and the consequent stability recovery. The following definitions 1 and 2 describe, respectively, the problems of stability and dissipativity recovery by fault hiding respectively.

Definition 1. **Stability recovery by fault hiding** Let Σ_P and Σ_{P_f} be the nominal and the faulty models, respectively, with dynamics described as follows:

$$\Sigma_P : \begin{cases} \dot{x} = f(x, u_p) \\ y_p = h(x, u_p) \end{cases} \tag{3}$$

$$\Sigma_{P_f} : \begin{cases} \dot{x} = f_f(x, u_r) \\ y_p = h_f(x, u_r) \end{cases} \tag{4}$$

both interconnected by feedback to controller Σ_C. Assume that the origin of (Σ_P, Σ_C) is asymptotically stable. Σ_{P_f} is stable by fault hiding if there exists an RB Σ_R such that the origin of $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is also asymptotically stable.

Definition 2. **Dissipativity recovery by fault hiding** Let Σ_P and Σ_{P_f} be the nominal and the faulty models with dynamics described, respectively, by (3) and (4). Assume that Σ_P is dissipative with respect to a supply rate function $\mathcal{S}(t)$ with a positive semidefinite continuously differentiable storage function $\mathcal{V}(x)$. Σ_{P_f} is dissipative by fault hiding if there exists an RB Σ_R such that (Σ_{P_f}, Σ_R) is also dissipative with respect to $\mathcal{S}(t)$.

Therefore, Sections 3 and 4 provide conditions for passivity and (Q,S,R)-dissipativity recovery based on the obtaining of storage and supply rate functions for the nominal case. Afterward, it is designed an RB that is able to ensure that the fault system is dissipative with respect to the same storage and supply rate functions.

3. PASSIVITY RECOVERY BY FAULT HIDING

The following lemma provides sufficient conditions for asymptotic stability of faulty systems by means of passivity recovery by fault hiding.

Lemma 2. Let Σ_P and Σ_{P_f} be the nominal and the faulty models, respectively, with dynamics described as in (3) and (4) interconnected, as depicted in Fig. 2, to a strictly passive output feedback controller Σ_C. Assume that Σ_P is strictly passive and that there exists a positive definite continuously differentiable $\mathcal{V}(\cdot)$ such that $\dot{\mathcal{V}}(x) < u_p^\top y_p$. If there exist R_1, R_2, R_3, R_4, such that the reconfigured plant (Σ_{P_f}, Σ_R), as depicted in Fig. 3, is also strictly passive and $\dot{\mathcal{V}}(x) < u_p^\top y_r$ for the same $\mathcal{V}(\cdot)$, then Σ_{P_f} is stable by fault hiding with Σ_R described in (2).

Proof. Since Σ_P and Σ_C are strictly passive, Lemma 1 ensures that the origin of feedback interconnection (Σ_P, Σ_C) is asymptoticaly stable for $w = 0$ and $r = 0$, and, according to the Lyapunov converse Theorem (Khalil, 2000), there exist a smooth Lyapunov function $\mathcal{V}(x)$ and a continuous positive definite function $W_1(x)$, such that $\frac{\partial \mathcal{V}}{\partial x} f(x, 0) \leq \mathcal{W}_1(x)$. In addition, by assuming that $\mathcal{W}_1(x) = u_p^\top y_p - \psi_1(x)$, for some positive definite $\psi(x)$, it is possible to show that Σ_P is strictly passive with the storage function $\mathcal{V}(x)$. Assume that the same $\mathcal{V}(x)$ is taken as storage function for (Σ_{P_f}, Σ_R). If there exists any Σ_R, such that $\frac{\partial \mathcal{V}}{\partial x} f_f(x, u_p) \leq \mathcal{W}_2(x)$, then Σ_{P_f} is dissipative by fault hiding with respect to $\mathcal{S} = \mathcal{W}_2(x) = u_p^\top y_r - \psi_2(x)$, according to Lemma 2, and (Σ_{P_f}, Σ_R) is strictly passive. Finally, (Σ_{P_f}, Σ_R) and Σ_C are interconnected strictly passive as depicted in Fig. 3, and Lemma 1 ensures that the unforced origin of $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is asymptotically stable, therefore Σ_{P_f} is stable by fault hiding according to Definition 1. □

Example 1. (Adapted from (Khalil, 2000)) Let Σ_P and Σ_{P_f} be, respectively, the nominal and faulty model for the same system described as follows

$$\Sigma_P : \begin{cases} a\dot{x} = -x + u \\ y = h(x) \end{cases}, \quad \Sigma_{P_f} : \begin{cases} a\dot{x} = -x + f \cdot u \\ y = h(x) \end{cases}$$

such that $h(\cdot) \in (0, \infty)$ and $0 < f < 1$. Consider that the origin of (Σ_P, Σ_C) is asymptotically stable with the strictly passive output feedback controller Σ_C connected as depicted in Fig. 2.

Considering $\mathcal{V}(x) = \int_0^x h(\sigma) d\sigma$, it is possible to show that $\dot{\mathcal{V}}(x) < u_c y$, thus, Σ_P is passive.

Using the same storage function $\mathcal{V}(x) = \int_0^x h(\sigma) d\sigma$ for the configuration (Σ_{P_f}, Σ_R), it follows:

$$\begin{aligned} \dot{\mathcal{V}}(x) - u_c y_r = \; & h(x)[-x + f(R_3 h(x) + R_4 u_c)] \\ & -(R_1 h(x) + R_2 u_c) u_c < 0 \end{aligned}$$

Thus, if $R_1 = f \cdot R_4$, $R_2 > 0$, and $R_3 < 0$, then $\dot{\mathcal{V}}(x) < u_c y_r$ implies that (Σ_{P_f}, Σ_R) is passive. Given that Σ_C is strictly passive, the origin of $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is stable, *i.e..*, Σ_{P_f} is asymptotically stable by fault hiding with Σ_R such that $R_1 = f \cdot R_4$, $R_2 > 0$, and $R_3 < 0$.

3.1 Passivity recovery for linear systems

The next theorem tackles the particular case of passivity recovery for linear systems.

Theorem 1. Let Σ_P and Σ_{P_f} be the nominal and faulty models, respectively, with dynamics described as follows

$$\Sigma_P : \begin{cases} \dot{x} = Ax + Bu_p \\ y_p = Cx \end{cases} \tag{5}$$

$$\Sigma_{P_f} : \begin{cases} \dot{x} = A_f x + B_f u_r \\ y_r = C_f x \end{cases} \tag{6}$$

for the same plant interconnected to a strictly passive output feedback controller Σ_C, as depicted in Fig. 2. Assume that there exists $P = P^\top \succ 0$, such that $A^\top P + PA \prec 0$ and $B^\top P = C$. The faulty system Σ_{P_f} is stable by fault hiding with Σ_R described in (2) if there exist R_1, R_2, R_3, and R_4 that satisfy, with the same P, the following inequality:

$$\begin{bmatrix} \mathrm{He}\left\{P\left(A_f + B_f R_3 C\right)\right\} & PB_f R_4 - C_f^\top R_1^\top \\ \star & -\mathrm{He}\left\{R_2\right\} \end{bmatrix} \prec 0. \tag{7}$$

Proof. According to KYP lemma (Bao and Lee, 2007), if there exists $P = P^\top \succ 0$, such that $A^\top P + PA \prec 0$ and

$B^\top P = C$, then Σ_P is strictly passive. Given that Σ_C is also strictly passive, then Lemma 1 ensures that (Σ_P, Σ_C) is asymptotically stable, since it is an interconnection of strictly passive systems.

Lemma 2 indicates that the same storage function, with the same matrix P, can be used to ensure the passivity of the reconfigured plant (Σ_P, Σ_R). The reconfigured system (Σ_{P_f}, Σ_R) is described as follows, with $A_R = A_f + B_f R_3 C_f$, $B_R = B_f R_4$, $C_R = R_1 C_f$, and $D_R = R_2$:

$$(\Sigma_{P_f}, \Sigma_R) : \begin{cases} \dot{x} = A_R x + B_R u_c \\ y_r = C_R x + D_R u_c \end{cases} . \tag{8}$$

Thus, according to KYP Lemma (Bao and Lee, 2007), (Σ_{P_f}, Σ_R) is strictly passive if (7) is satisfied for some R_1, R_2, R_3, and R_4. In this case, the closed-loop reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is asymptotically stable according to Lemma 1, since Σ_C is also a strictly passive system. Therefore, according to Lemma 2, Σ_{P_f} is stable by fault hiding with Σ_R defined as (2). □

Remark 1. If Σ_P is an SISO strictly passive system, Σ_C is a static output feedback controller such that $u_c = k y_c$ and $k > 0$, then the result of Theorem 1 is also valid, since it is a passive memoryless function.

Example 2. Let Σ_P be a heat exchange system model (Bao and Lee, 2007), controlled by a static output feedback controller $\Sigma_C : u_c = y_r$, and described as in (5) with

$$A = \begin{bmatrix} -690.87 & 279.17 \\ 69.254 & -375.29 \end{bmatrix}, \quad B = \begin{bmatrix} 411.7 & 0 \\ 0 & 306.03 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

According to KYP lemma, Σ_P is strictly passive for the storage function $\mathcal{V}(x) = x^T P x$ with

$$P = \begin{bmatrix} 0.0008 & 0.0003 \\ 0.0003 & 0.0017 \end{bmatrix}. \tag{9}$$

Consider that an actuator fault occurs such that $\Sigma^1_{P_f}$ is described as in (6) with

$$A^1_f = A, \quad B^1_f = \begin{bmatrix} 0 & 0 \\ 0 & 275.4270 \end{bmatrix}, \quad C^1_f = C.$$

The following RB Σ^1_R is obtained based on Theorem 1 by solving the LMI (7) using (9) and the LMILAB

$$R^1_1 = \begin{bmatrix} 0.7179 & 0.3951 \\ 0.1019 & 1.3910 \end{bmatrix}, \quad R^1_2 = \begin{bmatrix} 0.9715 & 0 \\ 0 & 0.9715 \end{bmatrix},$$

$$R^1_3 = \begin{bmatrix} 0 & 0 \\ -0.1327 & -0.8374 \end{bmatrix}, \quad R^1_4 = \begin{bmatrix} 1.9431 & -1.0235 \\ 1.0235 & 2.1976 \end{bmatrix}.$$

Consider now that a plant fault occurs such that $\Sigma^2_{P_f}$ is described as in (6) with

$$A^2_f = \begin{bmatrix} -461.7000 & 50.0000 \\ 69.2540 & -375.2900 \end{bmatrix}, \quad B^2_f = B, \quad C^2_f = C.$$

In this case, an RB Σ^2_R is obtained based on Theorem 1 by means of LMILAB. The computed gains of Σ^2_R are described as follows:

$$R^2_1 = \begin{bmatrix} 41.8884 & -7.0504 \\ 17.3065 & 57.8189 \end{bmatrix}, \quad R^2_2 = \begin{bmatrix} 38.3822 & 0 \\ 0 & 38.3822 \end{bmatrix},$$

$$R^2_3 = \begin{bmatrix} -127.0821 & 45.0768 \\ 24.1794 & -86.7559 \end{bmatrix}, \quad R^2_4 = \begin{bmatrix} 87.1790 & 34.6017 \\ -32.6280 & 85.8490 \end{bmatrix}.$$

Consider a third scenario with sensor fault occurrence such that $\Sigma^3_{P_f}$ is described as in (6) with

$$A^3_f = A, \quad B^3_f = B, \quad C^3_f = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

In this case, according to Theorem 1, an RB Σ^3_R is obtained with the following gains

$$R^3_1 = \begin{bmatrix} 1.0669 & -0.0476 \\ 0.0476 & 1.8886 \end{bmatrix}, \quad R^3_2 = \begin{bmatrix} 0.9443 & 0 \\ 0 & 0.9443 \end{bmatrix},$$

$$R^3_3 = \begin{bmatrix} -1.5051 & 0 \\ 0.4646 & 0 \end{bmatrix}, \quad R^3_4 = \begin{bmatrix} 2.0816 & -0.2607 \\ -0.0764 & 1.2709 \end{bmatrix}.$$

Finally, suppose that the three faults above described occurs simultaneously, i.e., $\Sigma^4_{P_f}$ is described as (6) with

$$A^4_f = A^2_f, \quad B^4_f = B^1_f, \quad C^4_f = C^3_f.$$

In this case, according to Theorem 1, an RB Σ^4_R is obtained by means of LMILAB with the following gains

$$R^4_1 = \begin{bmatrix} 0.4968 & -0.0986 \\ 0.0986 & 1.4511 \end{bmatrix}, \quad R^4_2 = \begin{bmatrix} 0.7255 & 0 \\ 0 & 0.7255 \end{bmatrix},$$

$$R^3_3 = \begin{bmatrix} 0 & 0 \\ -0.0560 & 0 \end{bmatrix}, \quad R^3_4 = \begin{bmatrix} 1.4511 & -0.1907 \\ 0.1907 & 0.9579 \end{bmatrix}.$$

4. DISSIPATIVITY RECOVERY BY FAULT HIDING

The next lemma provides conditions for asymptotically stabilization with RBs after a fault occurrence by means of dissipativity recovery by fault hiding.

Lemma 3. Let Σ_P and Σ_{P_f} be nominal and faulty models, respectively, with dynamics described by (3) and (4) for the same plant interconnected, as depicted in Fig. 2, to an output feedback controller Σ_C. Assume that Σ_P and Σ_C are respectively (Q,S,R)-dissipative with the storage function $\mathcal{V}(\cdot)$ and (Q_c,S_c,R_c)-dissipative such that the following inequality is satisfied

$$\begin{bmatrix} Q + R_c & S_c - S^\top \\ S_c^\top - S & Q_c + R \end{bmatrix} \prec 0. \tag{10}$$

If there exist R_1, R_2, R_3, R_4, such that (Σ_{P_f}, Σ_R) is also (Q,S,R)-dissipative with the same storage function $\mathcal{V}(\cdot)$, then Σ_{P_f} is stable by fault hiding with Σ_R described in (2).

Proof. Assuming that Σ_P is (Q,S,R)-dissipative, Σ_C is (Q_c,S_c,R_c)-dissipative and (10) is satisfied. Then, the unforced origin of (Σ_P, Σ_C) is asymptotically stable according to Lemma 1, and according to the Lyapunov converse Theorem (Khalil, 2000), there exists a smooth Lyapunov function $\mathcal{V}(x)$ such that $\frac{\partial \mathcal{V}}{\partial x} f(x, 0) < 0$. Assume that the same $\mathcal{V}(x)$ is taken as storage function for (Σ_{P_f}, Σ_R). If there exists any Σ_R, such that (Σ_{P_f}, Σ_R) is also (Q,S,R)-dissipative, then (10) is still satisfied and therefore Σ_{P_f} is stable by fault hiding according to Definition 1. □

Example 3. Consider the nonlinear system with the following nominal (Σ_P) and fault (Σ_{P_f}) models:

$$\Sigma_P : \begin{cases} \dot{x} = -4x^3 - 4u_p x \\ y_p = x^2 \end{cases}, \quad \Sigma_{P_f} : \begin{cases} \dot{x} = 2x^3 - 4u_r x \\ y_p = x^2 \end{cases}$$

interconnected as depicted in Fig. 2 to an output feedback controller described as follows

$$\Sigma_C : \begin{cases} \dot{x}_c = -x_c + y_r \\ u_c = \frac{1}{2}x_c \end{cases} \tag{11}$$

Adopting $\mathcal{V}(x) = \frac{1}{2}x^2$ as the storage function, Σ_P is (Q,S,R)-dissipative, *i.e.*, $\dot{\mathcal{V}}(x) \leq \mathcal{S}(y_p, u_p)$ for the supply function $\mathcal{S}(y_p, u_p)$ defined in (1), if the following inequality is satisfied for some Q, S, and R

$$\dot{\mathcal{V}}(x) - \mathcal{S}(y_p, u_p) = -(Q+4)x^4 - (2S+4)u_p x^2 - Ru_p^2 \leq 0 \tag{12}$$

Notice that (12) is satisfied with $Q = -4$, $S = -2$, and $R = 0$. Similarly, adopting $\mathcal{V}_c(x_c) = \frac{1}{2}x_c^2$, Σ_C is (Q_c,S_c,R_c)-dissipative if

$$\dot{\mathcal{V}}_c(x_c) - \mathcal{S}(u_c, y_r) = -\left(\frac{Q_c}{4} + 1\right)x_c^2 - (S_c - 1)\, y_r x_c - R_c y_r^2 \leq 0$$

is satisfied for some Q_c, S_c, and R_c, implying that $Q_c = -4$, $S_c = 1$, and $R_c = 0$. Note that Q, S, R, Q_c, S_c, and R_c satisfy (10), then the nominal system is asymptotically stable. According to Lemma 3, Σ_{P_f} is stable by fault hiding if (Σ_{P_f}, Σ_R), as depicted in Fig. 3, is also (Q,S,R)-dissipative. The model of (Σ_{P_f}, Σ_R) is described as follows

$$(\Sigma_{P_f}, \Sigma_R) : \begin{cases} \dot{x} = (2 - 4R_3)\, x^3 - 4R_4 u_p x \\ y_r = R_1 x^2 + R_2 u_p \end{cases} \tag{13}$$

(Σ_{P_f}, Σ_R) is (Q,S,R)-dissipative with the same storage and supply functions used for Σ_P if there exist R_1, R_2, R_3, and R_4 that satisfy $\dot{\mathcal{V}}(x) - \mathcal{S}(y_p, u_p) \leq 0$, or equivalently

$$-\left(4R_3 - 4R_1^2 - 2\right)x^4 - (4R_4 - 8R_1R_2 - 4)\, u_p x^2 + \left(4R_2 + 4R_2^2\right)u_p^2 \leq 0 \tag{14}$$

Therefore, any Σ_R satisfying (14) recovers the asymptotic stability of Σ_{P_f}, e.g., $R_1 = \frac{1}{4}$, $R_2 = -\frac{1}{2}$, $R_3 = \frac{3}{2}$, and $R_4 = \frac{3}{4}$.

Remark 2. It is worth noticing that one of the main advantages of the dissipativity- and passivity-based fault hiding approaches proposed in this work is their applicability for any nonlinear system since their storage and supply rate functions are known, as illustrated in Example 3. Indeed, such requirement is also the main drawback of the proposed approaches, because obtaining such functions is still a challenging task for nonlinear systems, although there are some recent developments on data-driven estimation of dissipative properties (Romer et al., 2019; Lei et al., 2016; Tang and Daoutidis, 2019) that can effectively enable the use of the proposed approach for any nonlinear system.

Theorem 2. Let Σ_P and Σ_{P_f} be the nominal and faulty models, respectively, with dynamics described in (3) and (6) interconnected (as depicted in Fig. 2) to an output feedback (Q_c,S_c,R_c)-dissipative controller Σ_C. Assume that there exist $P = P^\top \succ 0$, $Q = Q^\top \prec 0$, S, and $R = R^\top$ that satisfy the inequalities (10) and

$$\begin{bmatrix} \mathrm{He}\{PA\} - C^\top QC & PB - C^\top S^\top \\ \star & -R \end{bmatrix} \preceq 0. \tag{15}$$

For given P, Q, S and R satisfying (15), if there exist R_1, R_2, R_3, and R_4 that satisfy

$$\begin{bmatrix} Q^{-1} & R_1 C_f & R_2 \\ \star & \mathrm{He}\{PA_R\} & PB_f R_4 - C_f^\top R_1^\top S^\top \\ \star & \star & -R \end{bmatrix} \prec 0 \tag{16}$$

with $A_R = A_f + B_f R_3 C_f$, then Σ_{P_f} is stable by fault hiding with Σ_R described in (2).

Proof. According to (Kottenstette et al., 2014), Σ_P is (Q,S,R)-dissipative with the storage function $\mathcal{V}(x) = x^\top Px$ if and only if there exist $P = P^\top \succ 0$, $Q = Q^\top \prec 0$, S, and $R = R^\top$ that satisfy (15). According to Lemma 1, assuming that Σ_C is (Q_c,S_c,R_c)-dissipative, the unforced origin of nominal closed-loop system (Σ_P, Σ_C), interconnected as depicted in Fig. 2, is asymptotically stable if (10) is satisfied.

According to (Kottenstette et al., 2014), the reconfigured system (Σ_P, Σ_R), interconnected as depicted in Fig. 3 and described as (8), is (Q,S,R)-dissipative if and only if the following inequality is satisfied

$$\begin{bmatrix} \mathrm{He}\{PA_R\} - C_R^\top QC_R & PB_R - C_R^\top S^\top - C_R^\top QD_R \\ \star & -R - \mathrm{He}\left\{D^\top S\right\} - D_R^\top QD_R \end{bmatrix} \preceq 0. \tag{17}$$

According to Schur complement lemma, if (16) is satisfied, given that $Q \prec 0$, then the following inequality is also satisfied, considering $L \triangleq \begin{bmatrix} R_1 C_f & R_2 \end{bmatrix}$:

$$\begin{bmatrix} A_R^\top P + PA_R & PB_f R_4 - C_f^\top R_1^\top S^\top \\ \star & -R - \mathrm{He}\left\{D^\top S\right\} \end{bmatrix} - L^\top QL \prec 0$$

that is equivalent to (17), *i.e.*, (16) is sufficient to (Σ_{P_f}, Σ_R) be (Q,S,R)-dissipative. Finally, if (10) and (16) are satisfied, then the unforced origin of $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is asymptotically stable by fault hiding, and Σ_{P_f} is stable by fault hiding with Σ_R described in (2). □

5. APPLICATION EXAMPLES

Consider the aircraft yaw control described in (Lunze and Richter, 2008), where the simplified and linearized model as (5) is considered for the yaw angle ψ with $x = \begin{bmatrix} x_R & x_T & \dot{\psi} \end{bmatrix}^\top$, $u = \begin{bmatrix} u_R & u_T \end{bmatrix}^\top$,

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0.27 & 0.13 & -10^{-3} \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^\top$$

where x_R and x_T are the states of the approximated first order dynamics of rudder and turbine actuation systems respectively, and u_R and u_T are, respectively, the rudder deflection angle and the differential thrust.

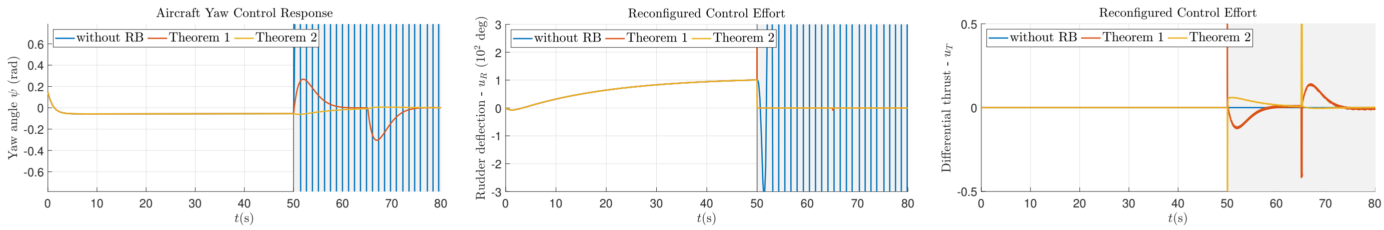
The yaw dynamics is controlled by an output feedback controller Σ_C interconnected as depicted in Fig. 2

$$\Sigma_C : \begin{cases} \dot{x}_c = -0.05x_c + 0.05y_r \\ u_p = \begin{bmatrix} 200 \\ 0 \end{bmatrix} x_c \end{cases}$$

Consider that such system is subject to an actuator fault, such that its matrix B_f is described as follows $B_f = B\mathrm{diag}\{0, 1\}$. Two RBs are designed for this fault scenario: Σ_R^p is designed for passivity recovery based on Theorem 1; and Σ_R^d is designed for dissipativity recovery based on Theorem 2. The obtained gains are the following

$$R_1^p = \begin{bmatrix} 0.6202 & -0.0556 \\ 0.0556 & 2.2842 \end{bmatrix}, \quad R_2^p = \begin{bmatrix} 1.4630 & 0.4650 \\ -0.3470 & 1.5252 \end{bmatrix},$$

$$R_3^p = \begin{bmatrix} 0 & 0 \\ -0.4187 & 0 \end{bmatrix}, \quad R_4^p = \begin{bmatrix} 2.2842 & -0.1227 \\ 0.1227 & 0.8143 \end{bmatrix},$$



(a) Output response. (b) Rudder actuator signals. (c) Differential thrust signals.

Fig. 4. Aircraft yaw control results.

$$R_1^d = \begin{bmatrix} 0.5003 & 0 \\ 0 & 5.7735 \times 10^8 \end{bmatrix}, \quad R_2^d = \begin{bmatrix} 0.4477 & 0.0049 \\ 0 & 0 \end{bmatrix},$$
$$R_3^d = \begin{bmatrix} 0 & 0 \\ -1.0675 & 0 \end{bmatrix}, \quad R_4^d = \begin{bmatrix} 0 & 0 \\ -0.0494 & -0.0028 \end{bmatrix}.$$

The results of the actuator fault simulation comparing the reconfigured systems with Σ_R^p, Σ_R^d, and without RB are depicted in Fig. 4 with initial condition $\begin{bmatrix} -1.25 & 1 & 0.15 \end{bmatrix}^\top$, and with the fault occurring at $t = 50$ s. An impulse disturbance is inserted at the output at $t = 65$s.

Fig. 4a depicts the output signals indicating that the aircraft yaw becomes unstable without reconfiguration, but both, Σ_R^p and Σ_R^d, recovers the stability after the fault occurrence. Figs. 4b and 4c depict the control efforts during the simulation of the rudder actuator u_R and differential thrust u_T, respectively. Since the controller Σ_C initially does not use the differential thrust, when the fault occurs the u_R tends to increase and becomes unstable when there is no RB. However, Figs. 4b and 4c indicate that the RBs perform a control reallocation using the other actuator to compensate for the fault and recover the aircraft stability.

6. CONCLUSION

This paper presents novel fault hiding conditions based on dissipativity theory. For this purpose, a static RB is used by combining feedback, feedforward and series gains to mitigate the fault effects. Such block can be used for either actuator, sensor or plant faults, even for nonlinear systems. The proposed approach consists in recovering the supply function and consequently the passivity or dissipativity, after a fault occurrence. LMI-based conditions are provided for stability recovery for linear systems by means of passivity recovery and dissipativity recovery using fault hiding. Numerical and application examples illustrate the versatility and effectiveness of the proposed approaches.

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