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Fault Prognostics of Rolling Bearings Using a Hybrid Approach ⋆

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Abstract: This paper presents a two-phase hybrid prognostics approach; in the first phase, the model's parameters are estimated using available training data in the least squares sense using the Levenberg-Marquardt algorithm. The second phase consists of using a particle filter to update the knowledge acquired so far and to predict future states of the system using in the Bayesian sense. The approach is used for an accelerated ball bearing data set, the PRONOSTIA platform, where a general fractional polynomial model is proposed as degradation model. The results of the Remaining Useful Life estimation are compared with another work in the literature, indicating its suitability and competitiveness for prognostics in this data set.

Keywords: Prognostics and health management, remaining useful life, particle filter, hybrid prognostics, accelerated ball bearings.

ACRONYMS

CBM	Condition-based Maintenance
CDF	Cumulative Distribution Function
FT	Fault Threshold
GD	Gradient Descent
GN	Gauss-Newton
HI	Health Index
LM	Levenberg-Marquardt
MAPE	Mean Absolute Percentage Error
PDF	Probability Density Function
PF	Particle Filter
PHM	Prognostics and Health Management
RA	Relative Accuracy
RMSE	Root Mean Squared Error
RUL	Remaining Useful Life

1. INTRODUCTION

In the last few decades, industry has been struggling to guarantee high reliability and safety for critical systems. These systems need maintenance due to their deterioration or aging to prevent unexpected failures and increase the reliability of such systems (Ma et al., 2019).

Maintenance policies are constantly evolving to become more and more cheap and reliable. The earliest policy consisted in unplanned actions in which the faulty component would be replaced after a breakdown. As industrial system becomes more complex, waiting for a breakdown could potentially increase the costs of maintenance, leading to the creation of preventive policies; a periodic time interval for maintenance would be set regardless the asset's health status. This kind of policy would also represent a major expense of industries since components would be replaced without necessity. To handle these situations, more efficient policies, such as Condition-based Maintenance (CBM), need to be developed (Jardine et al., 2006).

In the context of CBM, Prognostics and Health Management (PHM) enables the use of real monitoring data to create relevant health indicators and trends (Jouin et al., 2016). These indicators and trends aid in the system lifecycle support by reducing and eliminating inspections through incipient fault detection and prediction of impending faults (Chen et al., 2012). According to Lei et al. (2018), one major task in CBM is health prognostics which consists in predicting the Remaining Useful Life (RUL) of machinery. Prognostics is composed of four processes: data acquisition, Health Index (HI) construction, health stage division and RUL prediction.

The RUL prediction problem has been tackled by different authors using different tools. One way to distinguish between these methods is given in Liao and Kottig (2014),

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where three categories are presented, namely, experience-based, data-driven and physics-based models. Experience-based approaches usually take into account the historical data and knowledge from experts to create degradation models (Xu et al., 2014). Nevertheless, in practice, most degradation phenomena are nonlinear, stochastic and non-stationary, thus, it can be difficult to reuse models created from experience data (Tobon-Mejia et al., 2012).

Physics model-based approaches rely on mathematical models derived from physics of component to assess its current and future health condition (Cubillo et al., 2016). A widely used model is the Paris-Erdogan model for crack propagation, since the physical model of bearing degeneration is too complex (Liu et al., 2018). For state propagation, a particle filter with a modified crack growth model, based on Paris model, is used to predict the RUL of bearings in Liu et al. (2018). For the same component, Li et al. (2015) proposed an improved exponential model based on Paris law for RUL prediction. However, their restrict application is the main drawback of these approaches, since it is difficult to model the physics of damage in complex systems (Lei et al., 2018).

Data driven approaches includes, but are not limited to, statistical and artificial intelligence models. Statistical models generally predict RUL by fitting available observations into empirical models to be presented as a Probability Density Function (PDF) conditioned on these available observations (Si et al., 2011). In addition, artificial intelligence approaches are able to deal with complex systems by modeling degradation patterns disregarding any physical models of the assessed system (Peng et al., 2010).

Statistical models such as stochastic processes are also used for RUL prediction. A Wiener process is used in (Li et al., 2019) to predict the RUL of a turbofan engine considering variability between units. In prognostics, it is common to use monotonic degradation indices; thus, a Gamma process would be a better choice (Tsui et al., 2015). In (Le Son et al., 2016) a non-homogenic Gamma process is used to model degradation and RUL prediction. However, a single empirical model may not be enough when degradation path includes different fault modes. To address this, an Interacting Multiple Model associated with Takagi-Sugeno fuzzy systems is proposed in Cosme et al. (2019). For a similar reason, an interacting diagnostic hybrid bond graph associated with a particle filter is used for RUL prediction of electrical circuits in (Yu et al., 2015).

In this work, a two-phase hybrid prognostics approach is presented and applied to a real prognostics data set; it consists in training and updating a general fractional polynomial model through the Bayesian framework of Particle Filter (PF). In summary, the main contributions of this work are listed as follows:

- to define a new generic fractional polynomial model to estimate degradation indices given data is available;
- to incorporate new measurements for updating the proposed model by using a PF with initial conditions optimized by means of the Levenberg-Marquardt algorithm, enabling the l-steps ahead prediction of the degradation index;
- to apply the proposed approach for prognostics in a real data set of accelerated rolling bearings called PRONOSTIA [1] .

The remainder of this paper is organized as follows. Section 3 illustrates the PF framework for Bayesian state estimation. Section 2 introduces the proposed model for state prediction and shows how the RUL is predicted using the PF. In Section 4, the PRONOSTIA data set is detailed and the prognostics results using the proposed model are compared with another model's results. Finally, Section 5 concludes the present work.

2. DEGRADATION MODEL

To deal with nonlinear HIs, allowing their propagation several steps ahead, the following generic fractional polynomial model is proposed

$$\hat{x}_k(\boldsymbol{\theta}) = \theta_1 + \theta_2 k^{\theta_3} + \theta_4 x_{k-1}^{\theta_5}. \tag{1}$$

Since the used HI provides high trendability, i.e., it has a strong correlation with the time index, adding the time step k into the degradation model (1) is reasonable.

Considering that the degradation model at a given time step k is given by a nonlinear function of n_ψ past states with known parameters

$$\hat{x}_k = g(x_{k-1}, \ldots, x_{k-n_\psi}; \boldsymbol{\theta}), \tag{2}$$

where $\boldsymbol{\theta} \in \mathbb{R}^{n_\theta}$ is the estimated parameter vector from a data set $\boldsymbol{X} \in \mathbb{R}^{n_d}$ and $g\colon \mathbb{R}^{n_\psi} \mapsto \mathbb{R}$. The parameter vector is estimated by minimizing the sum of weighted squares of the residuals generated by the candidate fit function (2), as shown below

$$\begin{aligned}\chi^2(\boldsymbol{\theta}) &= \sum_{k=n_\psi+1}^{n_d} \left[\frac{x_k - g(x_{k-1}, \ldots, x_{k-n_\psi}; \boldsymbol{\theta}))}{\sigma_k}\right]^2 \\ &= \sum_{k=n_\psi+1}^{n_d} \left[\frac{x_k - \hat{x}_k}{\sigma_k}\right]^2 \\ &= (\boldsymbol{X} - \widehat{\boldsymbol{X}})^\top \boldsymbol{W} (\boldsymbol{X} - \widehat{\boldsymbol{X}}) = \boldsymbol{R}^\top \boldsymbol{W} \boldsymbol{R},\end{aligned} \tag{3}$$

where σ_k is the measurement error for measurement x_k and $\boldsymbol{R}$ is the vector of residuals. Typically, the weighting matrix $\boldsymbol{W}$ is diagonal with $w_{kk} = 1/(\sigma_k)^2$ (Gavin, 2019). Since (2) is nonlinear in the parameters $\boldsymbol{\theta}$, (3) must be minimized iteratively, with the goal of finding a perturbation $\boldsymbol{h}$ to the parameters $\boldsymbol{\theta}$ that reduces the error.

The Levenberg-Marquardt algorithm is chosen as optimization algorithm because it combines the features of the Gauss-Newton (GN) and Gradient Descent (GD) methods while avoiding their most serious limitations (Marquardt, 1963). The parameter increments ($\boldsymbol{h}_{\text{LM}}$) are computed adaptively by weighting between GN and GD updates through the solution of

$$\left[(\boldsymbol{J}^\top \boldsymbol{W} \boldsymbol{J}) + \xi\, \boldsymbol{I}\right] \boldsymbol{h}_{\text{LM}} = \boldsymbol{J}^\top \boldsymbol{W} \boldsymbol{R}, \tag{4}$$

where $\boldsymbol{J} = \partial\hat{\boldsymbol{x}}/\partial\boldsymbol{\theta}$ is the Jacobian matrix, ξ is a damping parameter to balance between GN and GD updates. The damping parameter starts large, favoring the GD increments; as the solution improves, ξ decreases, leading the

[1] Provided for the PHM IEEE 2012 Prognostic Data Challenge.

solution to a local minimum through GN. Furthermore, ξ increases whenever there is a worse approximation, that is

$$\chi^2(\boldsymbol{\theta} + \boldsymbol{h}_{\text{LM}}) > \chi^2(\boldsymbol{\theta}). \tag{5}$$

In order to use the PF for state estimation in the new unit, the initial distributions of the tracked parameters must be provided. Once the initial set of parameters $\boldsymbol{\theta}$ is estimated through Levenberg-Marquardt, its covariance matrix can be used as their initial distribution as

$$\text{cov}[\hat{\boldsymbol{\theta}}] = \frac{(\boldsymbol{J}^\top \boldsymbol{J})^{-1} \boldsymbol{R}^\top \boldsymbol{R}}{n_d - n_\theta + 1}. \tag{6}$$

3. NONLINEAR BAYESIAN STATE ESTIMATION

An important part of prognostics is the identification of model's parameters that affects its behavior. In cases where this model is linear with Gaussian noise, Kalman Filters yields the exact PDF of the parameters. However, nonlinear models with non-Gaussian noise requires another approach, such as the PF, that uses samples to construct the posterior distribution of the model's parameters (An et al., 2013).

Considering a system described by the following state space model:

$$\boldsymbol{x}_k = \boldsymbol{f}_k(\boldsymbol{x}_{k-1}, \boldsymbol{\omega}_k), \tag{7}$$

$$\boldsymbol{z}_k = \boldsymbol{h}_k(\boldsymbol{x}_k, \boldsymbol{\nu}_k), \tag{8}$$

where $\boldsymbol{x}_k \in \mathbb{R}^{n_x}$ is the system's state at time index k, $\boldsymbol{z}_k \in \mathbb{R}^{n_z}$ is the system's measurement at time index k, $\boldsymbol{f}_k\colon \mathbb{R}^{n_x} \times \mathbb{R}^{n_\omega} \mapsto \mathbb{R}^{n_x}$ is the state transition function, $\boldsymbol{h}_k\colon \mathbb{R}^{n_x} \times \mathbb{R}^{n_\nu} \mapsto \mathbb{R}^{n_z}$ is the measurement function, $\boldsymbol{\omega}_k \in \mathbb{R}^{n_\omega}$ is an independent identically distributed (i.i.d.) state noise vector of a known distribution, and $\boldsymbol{\nu}_k \in \mathbb{R}^{n_\nu}$ is an i.i.d. measurement noise vector of a known distribution.

The Bayesian technique aims at estimating the system's state $\boldsymbol{x}_k$, given the set of available measurements $\boldsymbol{z}_{1:k}$ through the steps of prediction and update. The prior information is assumed to be known and $p(\boldsymbol{x}_0|\boldsymbol{z}_0) \equiv p(\boldsymbol{x}_0)$. In the prediction step, the objective is to find the prior PDF of the state $p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1})$, using the Chapman-Kolmogorov equation, in the following way:

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1}) = \int p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})\, p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k-1})\, d\boldsymbol{x}_{k-1}, \tag{9}$$

where $p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ is defined as (7). At time instant k, a new measurement $\boldsymbol{z}_k$ is done; then, the second step updates the prior distribution using via Bayes' rule:

$$\begin{aligned} p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k}) &= \frac{p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1})\, p(\boldsymbol{z}_k|\boldsymbol{x}_k)}{p(\boldsymbol{z}_k|\boldsymbol{z}_{1:k-1})} \\ &= \frac{p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1})\, p(\boldsymbol{z}_k|\boldsymbol{x}_k)}{\int p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1})\, p(\boldsymbol{z}_k|\boldsymbol{x}_k)\, d\boldsymbol{x}_k}. \end{aligned} \tag{10}$$

To compute the normalizing constant in (10), the evaluation of complex high-dimensional integrals is required (Zio and Peloni, 2011). The PF overcomes it by representing the required posterior distribution as a set of N_s random samples associated with weights $\{\boldsymbol{x}^i_{0:k}, w^i_k\}_{i=1}^{N_s}$ through Monte Carlo simulations. As $N_s \to \infty$, the true posterior density is estimated by (Arulampalam et al., 2009):

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k}) \approx \sum_{i=1}^{N_s} w^i_k\, \delta(\boldsymbol{x}_k - \boldsymbol{x}^i_k). \tag{11}$$

The particles' weights can degenerate over time, i.e., all but one particle will have negligible weights. One approach to overcome this problem is to re-sample the weights. The strategy adopted in this paper is the inverse Cumulative Distribution Function (CDF) method, in which the CDF of the likelihood function $p(\boldsymbol{z}_k|\boldsymbol{x}_k)$ and the particles are randomly chosen according to a uniform distribution. This CDF will have N_s values and the particle is chosen as $\inf\{i \,|\, 1 \le i \le N_s, \text{CDF}_i \ge u_i\}$, where $u_i \sim \mathcal{U}(0,1)$. This procedure is repeated N_s times to generate the set of re-sampled particles.

3.1 RUL computation procedure

The l-step posterior distribution, using the state transition function (7), is computed by

$$p(\boldsymbol{x}_{k+l}|\boldsymbol{z}_{1:k}) = \int \cdots \int \prod_{j=k+1}^{k+l} p(\boldsymbol{x}_j|\boldsymbol{x}_{j-1})\, p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k}) \prod_{j=k}^{k+l-1} d\boldsymbol{x}_j. \tag{12}$$

These integrals are difficult to be evaluated and require significant computational effort. However, in a PF, (12) can be approximated iteratively by using the law of total probabilities

$$\hat{p}(\boldsymbol{x}_{k+n}|\hat{\boldsymbol{x}}_{1:k+n-1}) \approx \sum_{i=1}^{N_s} w^i_{k+n-1}\, \hat{p}(\boldsymbol{x}_{k+n}|\hat{\boldsymbol{x}}^i_{k+n-1}). \tag{13}$$

A simple approach to compute (13) consists of not updating the particles' weights, i.e., $w^i_{k+n} = w^i_{k+n-1}$ for $n > 0$. This approach considers that the error generated by not adjusting the weights at each time step is negligible (Orchard and Vachtsevanos, 2009); the state value estimate associated with each particle is propagated as

$$\hat{\boldsymbol{x}}^i_{k+p} = \mathbb{E}\left[\boldsymbol{f}_k(\hat{\boldsymbol{x}}^i_{k+p-1}, \boldsymbol{\omega}_{k+p})\right], \tag{14}$$

where $\mathbb{E}[\cdot]$ denotes the expected value. The output of the PF for prognostics is the RUL distribution, which is computed by taking the point where each particle crosses the Fault Threshold (FT), computed as

$$\hat{p}(\text{RUL} \le p|\boldsymbol{z}_{1:k}) = \hat{p}(x_{k+p} \ge \eta|\boldsymbol{z}_{1:k}), \tag{15}$$

where η is the FT. In this framework, the RUL is a random variable whose distribution is approximated by particles evolved through the successive application of the model update equation (7) for each particle.

Using the proposed fractional model (1), the state transition function and the measurement function are defined, respectively, as

$$f_k(x_{k-1}) = \theta_1 + \theta_2 k^{\theta_3} + \theta_4 x_{k-1}^{\theta_5}, \tag{16}$$

$$h_k(x_k, \omega_k) = x_k + \omega_k, \tag{17}$$

where the state transition noise ν_k is omitted because this uncertainty is modeled by the covariance matrix of the parameters $\boldsymbol{\theta}$. Assuming a Gaussian distribution for ω_k, the likelihood of the measurement can be expressed as

$$L(z_k|x^i_k, \boldsymbol{\theta}^i_k, \sigma^i_k) = \frac{1}{\sqrt{2\pi}\sigma^i_k} \exp\left(-\frac{1}{2}\left(\frac{z_k - f_k(x^i_{k-1})}{\sigma^i_k}\right)^2\right). \tag{18}$$

4. APPLICATION AND RESULTS

To test the proposed model, data from the PRONOSTIA platform are used. According to Nectoux et al. (2012), there are two main reasons for using data-driven techniques to tackle PRONOSTIA: the first one is that nothing is known about the degradation nature and origin; the other reason regards to the existence of a mismatch between the experiments the theoretical framework such as L_{10} life, ball pass frequency of inner ring, ball pass frequency of outer ring, etc.

4.1 PRONOSTIA data set

The PRONOSTIA database consists of accelerated degradation of ball bearings under different operating conditions along their whole operational life (Nectoux et al., 2012). The test bed, shown in Fig. 1, has three parts: (i) rotating part, composed by an asynchronous motor that allows the bearing to rotate through a system of gearing and different couplings; (ii) degradation generation part, composed by a pneumatic jack applying a radial force that reduces the bearing's life duration; (iii) measurements part to obtain instantaneous measurements from the radial force applied on the bearing, the rotation speed of the shaft handling the bearing and the torque inflicted to the bearing. The training and test data sets under different operation conditions are summarized in Table 1.

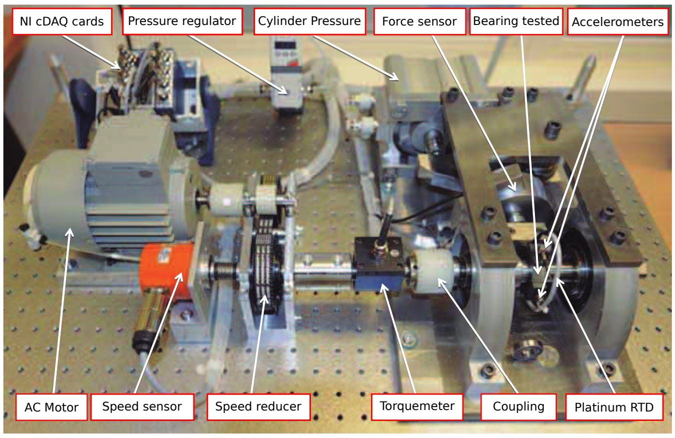

Fig. 1. Overview of PRONOSTIA test bed.

Table 1. PRONOSTIA training and test data sets under different operation conditions.

Operating conditions	Training sets	Test sets
1800rpm / 4000N	Be_{1-1} and Be_{1-2}	Be_{1-3} to Be_{1-7}
1650rpm / 4200N	Be_{2-1} and Be_{2-2}	Be_{2-3} to Be_{2-7}
1500rpm / 5000N	Be_{3-1} and Be_{3-2}	Be_{3-3}

4.2 Degradation model

The first step towards a data-driven approach in prognostics is extracting candidate features for HI (or degradation index) creation. In this paper, these features are obtained from vibration sensors as described in Javed et al. (2015), in which there are three sequential steps to build an index that improves trendability and monotonicity:

(1) application of discrete wavelet transform followed be the use of a mixture of trigonometric and statistics functions for feature extraction;
(2) some smoothing technique is used for denoising;
(3) finally, cumulative sum is computed to build a monotonic degradation process.

For the first item, using fourth order Daubechies at the fourth level of decomposition along with the standard deviation of the arctangent trigonometric function yielded the best results in terms of trendability and monotonicity. In the second item, an exponential moving average with window size of 12 samples is used as smoothing technique.

4.3 Results

The initial distributions of the tracked parameters must be provided; once the initial set of parameters $\boldsymbol{\theta}$ is estimated through Levenberg-Marquardt, its covariance matrix can be used as their initial distribution as

$$\boldsymbol{\theta}_0 \sim \mathcal{N}\left(\hat{\boldsymbol{\theta}}, \mathrm{cov}[\hat{\boldsymbol{\theta}}]\right). \tag{19}$$

The measurement noise is modeled as

$$\omega_k \sim \mathcal{N}(0, \sigma_k), \tag{20}$$

$$\sigma_0 \sim \mathcal{U}(0.04, 0.06), \tag{21}$$

where $\mathcal{U}$ is a uniform distribution. For each bearing operating condition shown in Table 1, a set of initial parameters $\hat{\boldsymbol{\theta}}_0$ is obtained, as shown in Table 2.

Table 2. Initial parameters of PRONOSTIA degradation model.

Condition	$\hat{\theta}_0^1$	$\hat{\theta}_0^2$	$\hat{\theta}_0^3$	$\hat{\theta}_0^4$	$\hat{\theta}_0^5$
1	0.0098	0.3807	-0.6165	0.9984	1.0004
2	-0.0654	0.3671	-0.4166	1.0134	0.9964
3	-0.0160	0.3576	-0.5100	1.0022	0.9995

Three accuracy metrics are used to evaluate the performance of the proposed method at each time step k: Mean Absolute Percentage Error (MAPE), Root Mean Squared Error (RMSE) and Relative Accuracy (RA). Such indicators are computed as follows (Saxena et al., 2008)

$$\mathrm{MAPE}_k = \frac{100}{H} \sum_{n=k+1}^{k+H} \left| \frac{x_n - \hat{x}_n}{x_n} \right|, \tag{22}$$

$$\mathrm{RMSE}_k = \sqrt{\frac{1}{H} \sum_{n=k+1}^{k+H} (x_n - \hat{x}_n)^2}, \tag{23}$$

$$\mathrm{RA}_k = 1 - \frac{|r_k - \hat{r}_k|}{r_k}, \tag{24}$$

where H is the prognostic horizon, r_k is the true RUL at k and $\hat{r}_k$ is the expected value of the estimated RUL distribution at k. These metrics are well stabilised in the field of fault prognostics and allow a fair comparison between different methods.

Since there are differences in the bearings' lifetime for each operation condition, different start times and threshold are set for the prognostics task. For bearing conditions 1, 2 and 3, prognostics started at $\tau_1 = 100$, $\tau_2 = \tau_3 = 20$, respectively; the FT is set to $\eta_1 = 20$, $\eta_2 = 8$ and $\eta_3 = 10$, respectively. This choice allows comparisons with the IMMF (Cosme et al., 2019), for which the proposed method achieves competitive results. As shown in Table 3, the RMSE and MAPE are lower for almost all bearing conditions while the RA is greater; this means that the proposed model achieves results for RUL prediction closer to the ground truth. In IMMF, the nonlinear degradation model is achieved by a convex combination of a fixed number of linear models obtained from data, assuming that they approximate the true HI behavior. This assumption reduces the model's ability to explain observations outside its knowledge domain which is a possible reason for the better performance of the proposed model.

Table 3. RMSE, MAPE and RA computed at $\tau_1 = 100$, $\tau_2 = \tau_3 = 20$ for operation conditions 1, 2 and 3 respectively, compared with IMMF (Cosme et al., 2019), with best values in bold.

Bearing	Proposed model			IMMF		
	RMSE	MAPE	RA	RMSE	MAPE	RA
1–3	**0.43**	**2.88%**	0.96	0.90	3.85%	**0.97**
1–4	**1.73**	**8.63%**	0.82	2.15	9.73%	**0.91**
1–5	**0.32**	**1.60%**	**0.91**	1.47	6.41%	0.85
1–6	**0.77**	5.45%	**0.96**	0.97	**3.01%**	0.94
1–7	**0.35**	**2.69%**	**0.99**	0.52	2.77%	0.97
2–3	**0.72**	**10.20%**	0.75	4.72	20.17%	**0.84**
2–4	**0.10**	**1.74%**	**1.00**	1.86	12.56%	0.91
2–5	**1.59**	21.32%	0.42	4.98	**19.00%**	**0.53**
2–6	**0.31**	**4.65%**	**0.82**	2.93	24.91%	0.75
2–7	**0.11**	**1.87%**	**0.95**	0.40	6.05%	0.85
3–3	**0.41**	**6.37%**	**1.00**	0.55	7.66%	0.96

The estimated RUL is defined as the mean value of all particles, whose distributions are shown in Fig. 2 for bearings 1–3, 2–3 and 3–3.

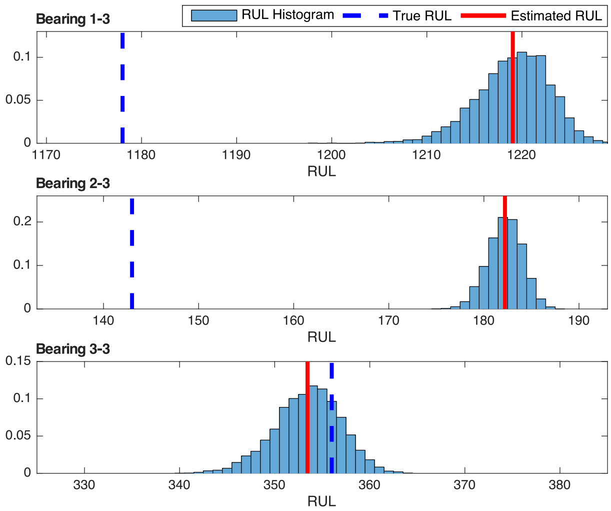

Fig. 2. RUL distribution approximated by a PF for bearings 1–3, 2–3 and 3–3.

To evaluate the prognostics tasks through time, the $\alpha - \lambda$ plot is used. The estimated RUL is compared with the true RUL at each time step λ until the fault occurrence. The desired result is that the predicted RUL fall within the region defined by α (Saxena et al., 2010). The successive computation of RUL for bearing condition 1–3 is depicted are Fig. 3, where the accuracy cone is computed for $\alpha = 0.2$; for each time instant in Fig. 3, the prognostics task is executed to estimate the RUL at that same instant. Another way to evaluate the RUL estimate through time is depicted in Fig. 4 where the ground truth is represented in the $x - y$ axis while the PDF shape estimated through PF is shown in $y - z$ axis for each time instant.

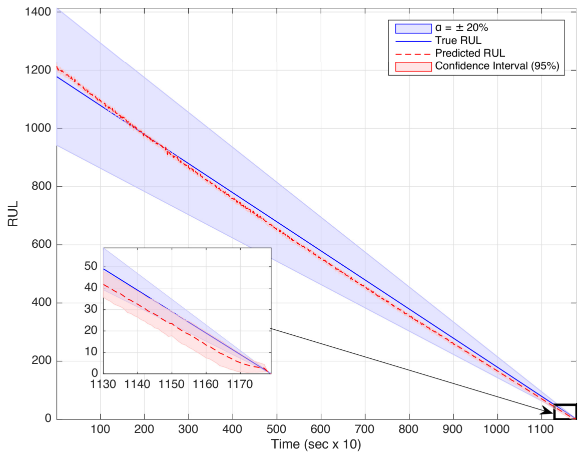

Fig. 3. Accuracy cone for true and predicted RUL of bearing condition 1–3. The southwest box shows the $\alpha - \lambda$ plot zoomed for the last 500 seconds prediction.

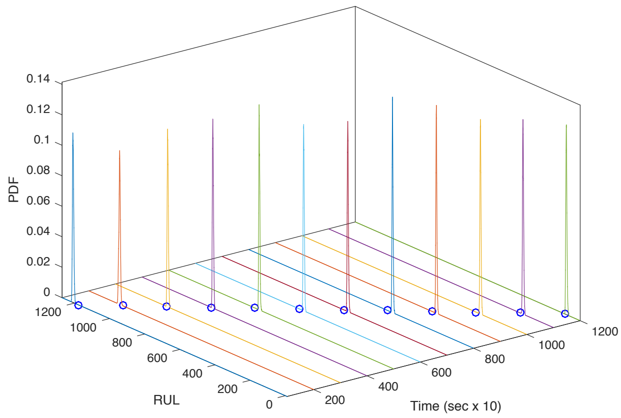

Fig. 4. RUL distribution computed at multiple time steps for bearing 1–3; the blue dot represents the true RUL.

5. CONCLUSION

In this work, a general fractional polynomial model is proposed to represent the degradation index introduced by Javed et al. (2015) for an accelerated ball bearing platform called PRONOSTIA. The nonlinear model's parameters are estimated in the least squares sense using Levenberg-Marquardt algorithm with the platform's provided training data. The prognostics is performed in the test units where the system's state and parameters are predicted and updated in the Bayesian sense through a PF. The RUL prediction results using the single fractional polynomial model are compared with the multiple model approach proposed by Cosme et al. (2019). The results indicate that proposed model can be used for prognostics when training data are available, and that it is competitive, in comparison with another technique in the literature, in terms of RMSE, MAPE and RA.

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