

Navigation and Source Localization based on Single Pseudo-Ranges with an Unknown Multiplicative Factor[★]

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Abstract: This paper presents a novel estimation solution for the problems of navigation and source localization based on pseudo-range measurements to a single pinger. In particular, the distance measurements are assumed to be corrupted by an unknown multiplicative factor, which is explicitly taken into consideration in the design. First, the equivalence between the problems of navigation and source localization is established, as well as cooperative navigation of two vehicles in tandem. Then, an augmented system is derived and its observability is carefully studied. The analysis is constructive, in the sense that the means to design an observer for the new system dynamics with globally exponentially stable error dynamics are readily available, resorting to linear systems theory. Moreover, the new augmented system is shown to be equivalent to the original one. Finally, simulations results are presented and discussed to assess the performance of the proposed solution in the presence of sensor noise.

Keywords: Pseudo-ranges, navigation, single ranges, Kalman filter, nonlinear estimation.

1. INTRODUCTION

Navigation systems are of the utmost importance in the development of autonomous vehicles, since they provide the required state for its successful operation. Moreover, for geo-referencing purposes, the position of the vehicle is usually required. The global positioning system (GPS) gives, in an easy and convenient way, the position of autonomous vehicles. However, in underwater applications, the GPS is not available and other solutions must be employed. Generally speaking, the most common classes of underwater positioning systems are long baseline (LBL), short baseline (SBL), and ultra-short baseline acoustic positioning systems. In these, a set of transponders (or pingers) are disposed in such a way that the position of another transponder (or pinger) can be computed. To do so, the travel time of the acoustic signals emitted by the positioning system is computed and the speed of propagation of the sound in water is also required. For the latter, a sound speed profile is typically obtained prior to the deployment of the vehicles.

More recently, the problems of navigation and source localization based on distance measurements to a single source have attracted the attention of the research community in recent years since the deployment is considerable easier. Roughly speaking, in the latter, an agent has access to distance measurements to a source that is fixed in an unknown position, as well as some other measurements

about the state of the agent, and aims to estimate the position of the source. In the former, a vehicle has access to distance measurements to a known pinger, as well as some other measurements about its own relative movement between consecutive distance measurements, and aims to estimate its inertial position.

One of the earlier contributions in the field can be found in Larsen (2000), where an algorithm is proposed that essentially builds a synthetic long baseline, and hence standard trilateration techniques can be applied. The vehicle measures the range to a single transponder and, between sampling instants, a high performance dead-reckoning system is used to compute the motion of the vehicle. A discrete-time Kalman filter is applied to a linearized model of the system. The linearization of a nonlinear system is considered again in Gadre and Stilwell (2005). Local observability results are obtained and an extended Kalman filter (EKF) is implemented to estimate the state, with no guarantees of global asymptotic stability. The same problem has been studied in Casey et al. (2007) and Lee et al. (2007), where EKFs have been extensively used as the estimation solution. In Jouffroy and Reger (2006), the observability analysis of the problem of single transponder navigation was carried out resorting to an algebraic approach and algebraic observers were also proposed. In Webster et al. (2009), preliminary experimental results with single beacon acoustic navigation were presented, where the EKF is employed as the state estimator. Observability questions and robustness issues of single range navigation are addressed in Indiveri et al. (2016). The problem of source localization has been addressed in Fidan et al. (2009), where the authors propose a localization algorithm based on the range to the source (more specifically

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its square) and the inertial position of the agent, which provides the necessary self-awareness of the agent motion. Global exponential stability (GES) is achieved under a persistent excitation condition.

In previous work Batista et al. (2011), the problems of single range navigation and source localization were addressed, where a novel solution was proposed with globally exponentially stable error dynamics. More recently, the problem of estimation of the sound speed propagation was addressed in a long baseline configuration Batista (2015). The main contribution of this paper is to bring both concepts together. In particular, the problems of navigation and source localization based on single pseudo-ranges are addressed. The range measurements are assumed to be known only up to an unknown multiplicative factor, accounting for an unknown speed of propagation of the signals in water. From a theoretical point of view, this is a very demanding framework, since one aims to estimating the state with scaled distance measurements, whereas in Batista et al. (2011) the distance measurements were assumed to be known exactly. Additionally, the problem of tandem navigation considering pseudo-range measurements is also considered. In fact, all three problems are shown to be equivalent.

To successfully tackle the problem, new system states and outputs are proposed and it is shown that the new system dynamics can be seen as linear in the state for observability analysis and observer design purposes. Its observability is carefully studied and the Kalman filter provides the estimation solution, with globally exponentially stable error dynamics. The transformation from the new states to the original system states is also addressed and it is shown that the convergence properties are kept. This is, to the best of the author's knowledge, the first solution to this problem, with GES error dynamics.

The paper is organized as follows. The different problem frameworks that are addressed herein are introduced in Section 2, where an equivalent system is also derived that encompasses all the frameworks. The design of the estimation solution is detailed in Section 3. Simulation results are presented in Section 4 and Section 5 summarizes the main results of the paper.

1.1 Notation

Throughout the paper, the symbol $\mathbf{0}_{n \times m}$ denotes a $n \times m$ matrix of zeros and \mathbf{I}_n an identity matrix. When the dimensions are omitted, they can be inferred from the context. For $\mathbf{x} \in \mathbb{R}^3$ and $\mathbf{y} \in \mathbb{R}^3$, $\mathbf{x} \cdot \mathbf{y}$ represents the inner product.

2. PROBLEM FORMULATION

2.1 Single vehicle navigation

Let $\{I\}$ denote a local inertial coordinate reference frame and consider a vehicle moving in a scenario where a fixed single pinger, or transponder, is installed. Let $\mathbf{p}(k) \in \mathbb{R}^3$ denote the inertial position of the vehicle at the time instant t_k and $\mathbf{s} \in \mathbb{R}^3$ denote the inertial position of the pinger. The discrete-time dynamics of the vehicle can be written as

$$\mathbf{p}(k+1) = \mathbf{p}(k) + \mathbf{u}_v(k), \quad (1)$$

where $\mathbf{u}_v(k)$ corresponds to the displacement of the vehicle from time instant t_k to time instant t_{k+1} .

In practice, the displacement of the vehicle can be obtained from sensors installed on-board. For instance, an Inertial Navigation System (INS) provides the displacement between consecutive sampling times through integration of inertial sensors. As another example, in underwater scenarios, a Doppler Velocity Log (DVL) with bottom-lock measures the inertial velocity of the vehicle expressed in the body-frame of the vehicle. With an Attitude and Heading Reference System (AHRS), the DVL measurements can be rotated to the inertial frame and then integrated, which gives the displacement of the vehicle.

In order to obtain the distance to the pinger, or transponder, the speed of propagation of the emitted signals is required. In this paper, this quantity is assumed constant but unknown. As such, the range measurements, which are measured periodically, are only available up to a scaling factor. Thus, instead of range measurements, the vehicle has access to pseudo-range measurements, as given by

$$r(k+1) = v_s(k+1) \|\mathbf{p}(k+1) - \mathbf{s}\|, \quad (2)$$

where $v_s(k)$ corresponds to a dimensionless scaling factor that accounts for the unknown speed of propagation of the signals in the medium. In short, a nominal speed of propagation is assumed by the range sensor, which does not necessarily correspond to the actual speed of propagation, which is assumed unknown. The scaling factor $v_s(k)$ accounts for that relation. This scaling factor is assumed constant, hence

$$v_s(k+1) = v_s(k). \quad (3)$$

This scenario is similar to the one addressed in Batista (2015), only now there is just one pinger in the scenario, as opposed to the long baseline configuration composed of several pingers that is considered in Batista (2015).

The navigation problem for a single vehicle based on pseudo-range measurements to a single pinger that is here considered is that of designing an estimation solution, with globally exponentially stable error dynamics, for the nonlinear system

$$\begin{cases} \mathbf{p}(k+1) = \mathbf{p}(k) + \mathbf{u}_v(k) \\ v_s(k+1) = v_s(k) \\ r(k+1) = v_s(k+1) \|\mathbf{p}(k+1) - \mathbf{s}\| \end{cases}, \quad (4)$$

where \mathbf{s} is known and both $\mathbf{u}_v(k)$ and $r(k)$ are measured.

2.2 Cooperative navigation in tandem

Consider a scenario where two vehicles operate in tandem, where one of the vehicle serves as support vehicle for the other. As a practical example, consider an autonomous surface craft (ASC) operating as a communication relay and navigation support ship for an autonomous underwater vehicle (AUV). In this scenario, the pinger is installed on the surface vehicle, which also sends, through communications, its position to the AUV.

Let $\{I\}$ denote a local inertial coordinate reference frame and denote by $\mathbf{p}(k) \in \mathbb{R}^3$ and $\mathbf{p}_s(k)$ the positions of the AUV and the ASC, respectively, at time instant t_k . In this case, the AUV kinematics are given by (1), whereas the pseudo-range measurements are given by

$$r(k+1) = v_s(k+1) \|\mathbf{p}(k+1) - \mathbf{p}_s(k+1)\|,$$

where $v_s(k)$ is again the scaling factor that accounts for the unknown speed of propagation of the emitted signals, satisfying (3).

The tandem navigation problem based on pseudo-range measurements considered in this paper is that of designing an estimation solution, with globally exponentially stable error dynamics, for the nonlinear system

$$\begin{cases} \mathbf{p}(k+1) = \mathbf{p}(k) + \mathbf{u}_v(k) \\ v_s(k+1) = v_s(k) \\ r(k+1) = v_s(k+1) \|\mathbf{p}(k+1) - \mathbf{p}_s(k+1)\| \end{cases}, \quad (5)$$

where $\mathbf{u}_v(k)$ and $r(k)$ are measured and $\mathbf{p}_s(k)$ is assumed available through communications. This problem is similar to the one addressed in Viegas et al. (2014) but instead of distance measurements, pseudo-range measurements are considered herein.

2.3 Source localization

Consider a scenario where a fixed source periodically emits, from an unknown position, a signal that is received by an autonomous vehicle, whose goal is to determine the position of the source. Since the speed of propagation of the signal is unknown, the vehicle can only determine pseudo-range distances to the source, as given by (2), where \mathbf{s} is the position of the source and $\mathbf{p}(k)$ is the position of the vehicle, both described in a local inertial coordinate reference frame $\{I\}$.

The source localization problem addressed in this paper based on single pseudo-range measurements is that of designing an estimation solution, with globally exponentially stable error dynamics, for the nonlinear system

$$\begin{cases} \mathbf{s}(k+1) = \mathbf{s}(k) \\ v_s(k+1) = v_s(k) \\ r(k+1) = v_s(k+1) \|\mathbf{p}(k+1) - \mathbf{s}(k+1)\| \end{cases}, \quad (6)$$

where $\mathbf{p}(k)$ and $r(k)$ are measured.

2.4 Problem equivalence

In this section, it is shown that the problems previously described are equivalent to the design of an estimator with globally exponentially stable error dynamics for the nonlinear system

$$\begin{cases} \mathbf{x}_1(k+1) = \mathbf{x}_1(k) + \mathbf{u}(k) \\ x_2(k+1) = x_2(k) \\ r(k+1) = x_2(k+1) \|\mathbf{x}_1(k+1)\| \end{cases}, \quad (7)$$

where $\mathbf{x}_1(k) \in \mathbb{R}^3$ and $x_2 \in \mathbb{R}$ are the system states, $\mathbf{u}(k) \in \mathbb{R}^3$ is the system input, and $r(k)$ is the system output.

Assumption 1. The pseudo-range measurements are positive, i.e., $r(k) > 0$ for all k .

Remark 1. The technical condition stated in Assumption 1 is a mild one, which is always verified in practice.

Single vehicle navigation Consider the system dynamics (4) and define the state transformation

$$\begin{cases} \mathbf{x}_1(k) := \mathbf{p}(k) - \mathbf{s} \\ x_2(k) := v_s(k) \end{cases}. \quad (8)$$

Then, it is a matter of straightforward computations to show that $\mathbf{x}_1(k)$ and $\mathbf{x}_2(k)$ satisfy the system dynamics (7), with $\mathbf{u}(k) = \mathbf{u}_v(k)$.

Notice that the state transformation (8) is invertible and always well-defined. Moreover, \mathbf{s} is available. Hence, if an estimator is designed for (7), estimates for the original states in (4) are readily available.

Cooperative navigation in tandem Consider the system dynamics (5) and define the state transformation

$$\begin{cases} \mathbf{x}_1(k) := \mathbf{p}(k) - \mathbf{p}_s(k) \\ x_2(k) := v_s(k) \end{cases}. \quad (9)$$

Again, it is a matter of straightforward computations to show that $\mathbf{x}_1(k)$ and $\mathbf{x}_2(k)$ satisfy the system dynamics (7), with $\mathbf{u}(k) = \mathbf{u}_v(k) - [\mathbf{p}_s(k+1) - \mathbf{p}_s(k)]$. Also, the state transformation (9) is invertible and always well-defined. Hence, if an estimator is designed for (7), estimates for the original states in (5) are readily available. However, while in the previous case the quantity involved in the state transformation, \mathbf{s} , was constant and known, in this case $\mathbf{p}_s(k)$ is required. Even though $\mathbf{p}_s(k)$ is available, it might be measured and thus subject to noise. An alternative observer design for the navigation problem consists in designing an observer (or filter) for (7) and then apply the inverse state transformation to the observer dynamics, yielding an observer directly for (5), and thus avoiding the direct injection of noise through the inversion of the state transformation.

Source localization Consider the system dynamics (6) and define the state transformation

$$\begin{cases} \mathbf{x}_1(k) := \mathbf{p}(k) - \mathbf{s}(k) \\ x_2(k) := v_s(k) \end{cases}. \quad (10)$$

Again, it is a matter of straightforward computations to show that $\mathbf{x}_1(k)$ and $\mathbf{x}_2(k)$ satisfy the system dynamics (7), with $\mathbf{u}(k) = \mathbf{p}(k+1) - \mathbf{p}(k)$. As before, the state transformation (10) is invertible and always well-defined. Moreover, the design of an estimator follows similarly.

3. FILTER DESIGN

3.1 State augmentation

Define a new system state as

$$\begin{cases} \mathbf{z}_1(k) := x_2^2(k) \mathbf{x}_1(k) \\ z_2(k) := x_2^2(k) \\ z_3(k) := r(k) \end{cases}. \quad (11)$$

The evolutions of $\mathbf{z}_1(k)$ and $z_2(k)$ are trivially obtained from (7), as given by

$$\begin{cases} \mathbf{z}_1(k+1) := \mathbf{z}_1(k) + z_2(k) \mathbf{u}(k) \\ z_2(k+1) := x_2(k) \end{cases}. \quad (12)$$

In order to derive the equation that describes the evolution of $z_3(k)$, notice that, by definition, $r^2(k) = x_2^2(k) \|\mathbf{x}_1(k)\|^2$ for the discrete time instant k , which gives

$$r^2(k+1) = x_2^2(k+1) \|\mathbf{x}_1(k+1)\|^2 \quad (13)$$

for the discrete time instant $k+1$. Substituting (7) in (13) gives

$$\begin{aligned} r^2(k+1) &= x_2^2(k) \|\mathbf{x}_1(k)\|^2 + 2x_2^2(k) \mathbf{u}(k) \cdot \mathbf{x}_1(k) \\ &\quad + x_2^2(k) \|\mathbf{u}(k)\|^2, \end{aligned}$$

which using the new state definition (11) and rearranging can be rewritten as

$$r^2(k+1) = 2\mathbf{u}(k) \cdot \mathbf{z}_1(k) + \|\mathbf{u}(k)\|^2 z_2(k) + r^2(k)$$

or, equivalently,

$$r(k+1) = \frac{2\mathbf{u}(k)}{r(k+1)} \cdot \mathbf{z}_1(k) + \frac{\|\mathbf{u}(k)\|^2}{r(k+1)} z_2(k) + \frac{r(k)}{r(k+1)} r(k). \quad (14)$$

Using the new state definition (11) selectively in (14) readily gives

$$z_3(k+1) = 2 \frac{\mathbf{u}(k)}{r(k+1)} \cdot \mathbf{z}_1(k) + \frac{\|\mathbf{u}(k)\|^2}{r(k+1)} z_2(k) + \frac{r(k)}{r(k+1)} z_3(k). \quad (15)$$

Now, define the state vector

$$\mathbf{z}(k) := \begin{bmatrix} z_1(k) \\ z_2(k) \\ z_3(k) \end{bmatrix}.$$

Using (12) and (15), and noticing that the new system state $z_3(k)$ is measured, allows to write the system dynamics

$$\begin{cases} \mathbf{z}(k+1) = \mathbf{A}(k)\mathbf{z}(k) \\ y(k+1) = \mathbf{C}\mathbf{z}(k+1) \end{cases}, \quad (16)$$

with

$$\mathbf{A}(k) := \begin{bmatrix} \mathbf{I}_3 & \mathbf{u}(k) & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & 1 & 0 \\ 2 \frac{\mathbf{u}^T(k)}{r(k+1)} & \frac{\|\mathbf{u}(k)\|^2}{r(k+1)} & \frac{r(k)}{r(k+1)} \end{bmatrix} \in \mathbb{R}^{5 \times 5},$$

and $\mathbf{C} = [\mathbf{0}_{1 \times 3} \ 0 \ 1] \in \mathbb{R}^{1 \times 5}$.

Notice that, in the system dynamics (16), the original nonlinear output $r(k+1) = x_2(k+1) \|\mathbf{x}_1(k+1)\|$ was replaced by $r(k+1) = z_3(k+1)$. This choice allows to derive a system that is linear in the state.

Remark 2. Notice that the system (16) is well defined under Assumption 1.

3.2 Observability analysis

The system (16) can be regarded as a discrete-time linear time-varying system for observer design purposes, since it is linear in the state, even though the system matrix $\mathbf{A}(k)$ depends on the pseudo-range and input measurements. This is possible because for observer (or filter) design purposes, both $r(k)$ and $\mathbf{u}(k)$ are available and, hence, they can be simply considered as functions of time. This idea was first pursued by the authors in (Batista et al., 2011, Lemma 1) for continuous systems, whose application is equivalent for the discrete-time case, as shown in (Batista, 2015, Lemma 1).

The following result addresses the observability of the discrete-time system (16).

Theorem 1. Suppose that, for some time instant $k_a \geq 0$,

$$\mathbf{L}(k_a) := \begin{bmatrix} \mathbf{L}_0(k_a) \\ \vdots \\ \mathbf{L}_3(k_a) \end{bmatrix} \in \mathbb{R}^{4 \times 4}$$

is full rank, i.e.,

$$\text{rank}(\mathbf{L}(k_a)) = 4, \quad (17)$$

with

$$\mathbf{L}_i(k_a) := \begin{bmatrix} 2 \sum_{j=0}^i \mathbf{u}(k_a + j) \\ \left\| \sum_{j=0}^i \mathbf{u}(k_a + j) \right\|^2 \end{bmatrix}^T \in \mathbb{R}^{1 \times 4}.$$

Then, the discrete-time system (16) is observable on the time interval $[k_a, k_a + 5]$, $k_a = 0, 1, 2, \dots$, in the sense that the initial state $\mathbf{z}(k_a)$ is uniquely determined by the input $\{\mathbf{u}(k) : k = k_a, \dots, k_a + 4\}$ and the output $\{y(k) : k = k_a, \dots, k_a + 4\}$.

Proof. The proof resorts to (Batista, 2015, Lemma 1) and it reduces to show that the observability matrix $\mathcal{O}(k_a, k_a + 5)$ associated with the pair $(\mathbf{A}(k), \mathbf{C})$ on $[k_a, k_a + 5]$, $k \geq 0$, has rank equal to the number of states of the system. It is omitted due to space limitations. \square

Finally, it is important to stress that, in the definition of the augmented system (16), the original nonlinear output equation $r(k) = x_2(k) \|\mathbf{x}_1(k)\|$ was discarded and artificial states were defined. As such, it is still necessary to relate this new augmented system (16) to the original nonlinear system (7). The following theorem addresses this issue and provides the means to design a state observer or filter for (16).

Theorem 2. Suppose that (17) holds. Then:

- i) the nonlinear system (7) is observable on any interval $[k_a, k_a + 5]$, $k_a = 0, 1, 2, \dots$, in the sense that the initial state $(\mathbf{x}_1(k_a), x_2(k_a))$ is uniquely determined by the input $\{\mathbf{u}(k) : k = k_a, \dots, k_a + 4\}$ and the output $\{r(k) : k = k_a, \dots, k_a + 4\}$; and
- ii) the initial condition of the augmented system (16) matches that of (7), i.e.,

$$\begin{cases} z_1(k_a) = x_2^2(k_a) \|\mathbf{x}_1(k_a)\| \\ z_2(k_a) = x_2^2(k_a) \\ z_3(k_a) = x_2(k_a) \|\mathbf{x}_1(k_a)\| \end{cases}.$$

Proof. The proof follows by comparison of the outputs of both systems along the time interval as a function of the initial conditions, which allows to conclude that they match. This is omitted due to space limitations. Notice that, using Theorem 1, the initial condition of (16) is uniquely determined. Hence, due to the correspondence between the two systems, it follows that the initial condition of (7) is also uniquely determined, thus completing the proof. \square

3.3 Estimation solution

Kalman filter The means to design an observer for (7) are readily provided by Theorem 2. Indeed, since the augmented system (16) is equivalent to (7), an observer (filter) for (16) is also an observer (filter) for (7). Furthermore, since the system (16) is linear in the state, for observer design purposes, a simple Kalman filter can be applied, yielding globally exponentially stable error dynamics if the system is shown to be uniformly completely observable (Jazwinski 1970). In this paper, the pair $(\mathbf{A}(k), \mathbf{C})$ was shown to be observable. The proof of uniform complete observability follows similar steps considering uniform

bounds in time. It is omitted due to space limitations. An alternative to the Kalman filter could be the design of a Luenberger observer as detailed in (Rugh, 1995, Theorem 29.2), which would allow to choose the convergence rate.

Estimates of $\mathbf{x}_1(k)$ and $x_2(k)$ Notice that, with the design of an observer (or filter) for the augmented system (16), one obtains estimates for $\mathbf{z}_1(k)$, $z_2(k)$, and $z_3(k)$, when one aims to estimate $\mathbf{x}_1(k)$ and $x_2(k)$. Nevertheless, the latter follow trivially, as it will be seen. First, some assumptions are introduced.

Assumption 2. The unknown factor $x_2(k)$ satisfies

$$V_m \leq x_2(t) \leq V_M,$$

with $V_m, V_M > 0$.

Assumption 3. The state $\mathbf{x}_1(k)$ is norm-bounded.

Considering estimates $\hat{x}_2(t)$ with globally exponentially stable error dynamics, the estimate of the speed of propagation of the signals in the medium can be obtained from

$$\hat{x}_2(t) = \begin{cases} V_m, & \hat{z}_2(t) < V_m^2 \\ \sqrt{\hat{z}_2(t)}, & V_m^2 \leq \hat{z}_2(t) \leq V_M^2 \\ V_M, & \hat{z}_2(t) > V_M^2 \end{cases}, \quad (18)$$

whose error also converges exponentially fast to zero for all initial conditions under Assumption 2. Estimates for the position then follow from

$$\hat{\mathbf{x}}_1(k) = \frac{\hat{\mathbf{z}}_1(t)}{\hat{x}_2^2(t)} \quad (19)$$

and it is possible to show that, under Assumptions 2 and 3, these also converge exponentially fast to zero for all initial conditions, see (Batista, 2015, Proposition 1).

4. SIMULATION RESULTS

In order to assess the performance of the proposed solution, numerical simulations are presented. These are preliminary results and extensive Monte Carlo simulations will be carried out in the future, prior to experimental validation, as well as comparison with the extended Kalman filter, which does not offer globally exponentially stable error dynamics.

A simple navigation setup framework is considered, without loss of generality, as it was seen in Section 2.4 that all three problems that were introduced in Section 2 are equivalent. In particular, the pinger is assumed to be placed at the origin of the inertial reference frame, i.e., $\mathbf{s} = \mathbf{0}$. In terms of measurements, the sampling rate is set to $T = 1$ s, which means that each discrete time instant occurs every second.

The initial position of the vehicle is $\mathbf{p}(0) = [20 \ 0 \ 0]^T$ m and the trajectory described by the vehicle is depicted in Fig. 1. This was obtained considering as input

$$\mathbf{u}_v(k) = \begin{bmatrix} \cos\left(\frac{2\pi k}{30}\right) \\ \cos\left(\frac{2\pi k}{20} + \frac{\pi}{6}\right) \\ 2 \cos\left(\frac{2\pi k}{45} + \frac{\pi}{9}\right) \end{bmatrix} \text{ m},$$

such that the observability condition is verified. The term that accounts for the unknown speed of propagation of

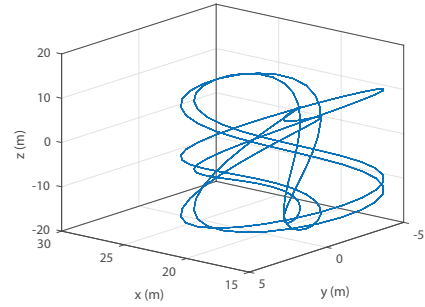


Fig. 1. Trajectory described by the vehicle

the signals in the medium was set to $v_s = 1.1$ (recall it is dimensionless).

In the simulations, sensor noise was considered for both the pseudo-range measurements and the input readings. In particular, zero-mean, additive uncorrelated white Gaussian noise was considered, with standard deviation of 5 cm for the pseudo-range measurements $r(k)$ and 1 cm for the input readings $\mathbf{u}(k)$. Recall that, for the navigation case, the input readings can be obtained through open-loop integration of inertial sensors, in this case over a period of 1 s. While small, the standard deviation that is considered here is realistic due to the small period of open-loop integration. Indeed, it is well known that INSs are very accurate for small periods of time. For specific examples, the Hydrins INS, or the Phins INS, both from iXblue, specify a 50% circular error probability (CEP) of 0.8 m, 3.2 m, and 20 m for 1, 2, and 5 minutes, respectively. This translates to a CEP 50 much lower than 1 cm for 1 second (about 2.22×10^{-4} m for 1 second).

To tune the Kalman filter, the state disturbance covariance matrix was set to $0.01^2 \mathbf{I}_5$ and the output noise covariance to 0.05^2 . All initial state estimates were set to zero but $\hat{z}_2(0) = 1$. In order to obtain estimates for the position and scale factor, the bounds in Assumption 2 were set to $V_m = 0.8$ and $V_M = 1.2$.

The initial convergence of the position and scale factor estimation errors is depicted in Figs. 2 and 3. For the sake of completeness, the initial convergence of the estimation error of the additional state $z_3(k)$ is also depicted in Fig. 4. As it is possible to see, the estimation error

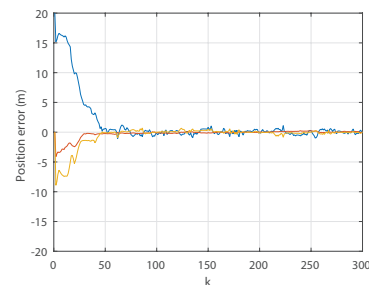


Fig. 2. Initial convergence of the position estimation error

converges very fast to a neighborhood of zero (due to the presence of sensor noise, otherwise it would converge to zero). Moreover, the effect of the bounds V_m and V_M used to obtain the estimates of $x_2(k)$ are clearly visible in its initial evolution, in Fig. 3. The detailed evolutions of the position and scale factor errors are depicted in Figs. 5 and

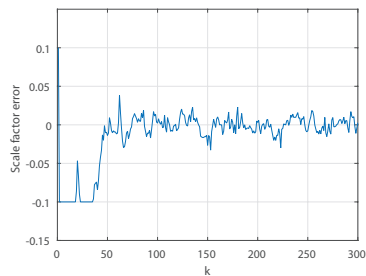


Fig. 3. Initial convergence of the scale factor estimation error

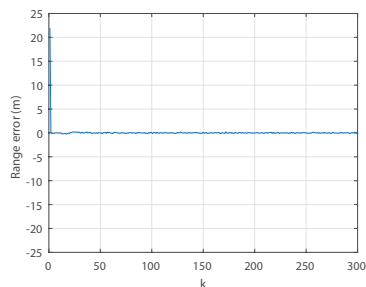


Fig. 4. Initial convergence of the estimation error of $z_3(k)$ 6, respectively. The most noticeable feature is that the position and scale factor errors remain, most of the time, below 0.5 m and 0.02 m/s, respectively.

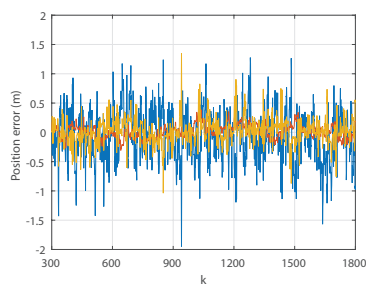


Fig. 5. Steady-state evolution of the position error

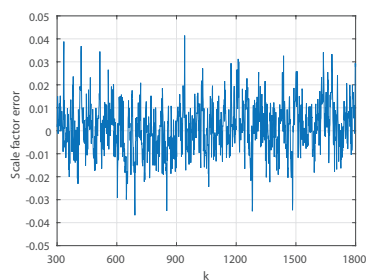


Fig. 6. Steady-state evolution of the scale factor error

5. CONCLUSIONS

This paper addressed the problems of autonomous vehicle navigation and source localization based on discrete-time pseudo-range measurements to a single beacon. In particular, the distance measurements to the single transponder (or pinger) are only assumed to be known up to a scaling factor. First, the problems were shown to be equivalent to the design of an observer for a simplified form of the system

dynamics. Then, a new augmented system was derived that can be seen as linear in the state for observer design purposes. Its observability was studied and the analysis, which is constructive, provides the means to design an observer for the original system dynamics, with globally exponentially stable error dynamics. Simulation results, including sensor noise, were presented, evidencing good filtering performance.

REFERENCES

- Batista, P. (2015). GES Long Baseline Navigation with Unknown Sound Velocity and Discrete-time Range Measurements. *IEEE Transactions on Control Systems Technology*, 23(1), 219–230.
- Batista, P., Silvestre, C., and Oliveira, P. (2011). Single Range Aided Navigation and Source Localization: observability and filter design. *Systems & Control Letters*, 60(8), 665–673.
- Casey, T., Guimond, B., and Hu, J. (2007). Underwater Vehicle Positioning Based on Time of Arrival Measurements from a Single Beacon. In *Proceedings of the MTS/IEEE Oceans 2007*, 1–8. Vancouver, BC, Canada.
- Fidan, B., Dandach, S., Dasgupta, S., and Anderson, B. (2009). A continuous time linear adaptive source localization algorithm robust to persistent drift. *Systems & Control Letters*, 58(1), 7–16.
- Gadre, A. and Stilwell, D. (2005). A complete solution to underwater navigation in the presence of unknown currents based on range measurements from a single location. In *Proceedings of the 2005 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 1420–1425. Edmonton AB, Canada.
- Indiveri, G., De Palma, D., and Parlangeli, G. (2016). Single range localization in 3-D: Observability and robustness issues. *IEEE Transactions on Control Systems Technology*, 24(5), 1853–1860.
- Jazwinski, A. (1970). *Stochastic Processes and Filtering Theory*. Academic Press, Inc.
- Jouffroy, J. and Reger, J. (2006). An algebraic perspective to single-transponder underwater navigation. In *Proceedings of the 2006 IEEE International Conference on Control Applications*, 1789–1794. Munich, Germany.
- Larsen, M. (2000). Synthetic long baseline navigation of underwater vehicles. In *Proc. of the 2000 MTS/IEEE Oceans*, volume 3, 2043–2050. Providence, RI, USA.
- Lee, P.M., Jun, B.H., Kim, K., Lee, J., Aoki, T., and Hyakudome, T. (2007). Simulation of an Inertial Acoustic Navigation System With Range Aiding for an Autonomous Underwater Vehicle. *IEEE Journal of Oceanic Engineering*, 32(2), 327–345.
- Rugh, W. (1995). *Linear system theory*. Prentice-Hall, Inc., 2nd edition.
- Viegas, D., Batista, P., Oliveira, P., and Silvestre, C. (2014). Position and Velocity Filters for ASC/I-AUV Tandems based on Single Range Measurements. *Journal of Intelligent & Robotic Systems*, 74(3-4), 745–768.
- Webster, S., Eustice, R., Singh, H., and Whitcomb, L. (2009). Preliminary deep water results in single-beacon one-way-travel-time acoustic navigation for underwater vehicles. In *Proceedings of the 2009 IEEE/RSJ International Conference on Intelligent Robots and Systems-IROS 2009*, 2053–2060. SaintLouis, MO, USA.