The Influence of Mode Change Penalties on the Comparison of Hybrid Drivetrain Topologies

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Abstract: Hybrid drivetrains are systems with complex behavior of which finding the optimal design is a problem with a large design space. To assess a design on efficiency over a driving cycle, a control strategy is needed. Introducing mode change and engine start penalties in the optimization of the control increases the accuracy of the results at the cost of increased computation time. Yet, due to the large design space of the design problem, computation time is critical. In this work, an extensive case study is presented to analyze the influence of penalizing mode changes and engine starts on the comparison of hybrid drivetrain topologies. Eight different drivetrain topologies are considered, including parallel, series-parallel, and multi-mode powersplit topologies. For these topologies, the control is optimized over two driving cycles using dynamic programming with and without penalties. The introduction of mode change and engine start penalties reduces the number of mode changes by a factor of three to five, and the number of engine starts by approximately a factor of three. Yet, the influence on the fuel consumption comparison between the topologies is small: the largest change in relative fuel consumption is 0.36 percentage points, with the average absolute change over both cycles being 0.15 percentage points. The computation time is increased by approximately a factor of 26 due to the introduction of the penalties. Therefore, in the context of the system level design of hybrid drivetrains, it can be argued that the additional computation time outweighs the minor increase in accuracy provided by mode change penalties.

Keywords: Hybrid electric vehicles, powertrain design, drivetrain topology, multi-mode, mode change, gear shift, powertrain architecture, dynamic programming

1. INTRODUCTION

To assess the efficiency of a hybrid drivetrain over a driving cycle, a control strategy is needed. In the context of the system-level design (SLD) of these systems, including topology optimization and component sizing, multiple criteria to the control strategy arise (Silvas et al. (2017)). The control strategy must be equally optimal for distinct component sizes and topologies to enable comparison of the designs. To ensure such comparability, optimal or near-optimal control is often applied in literature (Vinot et al. (2014); Zhang et al. (2015); Silvas et al. (2017); Goos et al. (2017)).

An additional criterion for the control strategy is that it must be as computationally efficient as possible, as the large design space of the SLD problem requires the evaluation of many designs. For this reason, many works can be found that propose computationally efficient control algorithms to solve the control of hybrid electric vehicles (HEVs) over a driving cycle (Ngo et al. (2012); Delprat and Hofman (2014); Zhang et al. (2015); Goos et al. (2017); Van Harselaar et al. (2019b)).

For the selection of the control algorithm and level of detail of the modeling, a trade-off exists between accuracy and computation time. One aspect of this dichotomy is the quantity of mode changes and internal combustion engine (ICE) starts. Using clutches and brakes, different topologies can enable different numbers of transmission modes. Especially considering topologies with very different numbers of transmission modes, it can be suspected that using a control algorithm that allows unlimited mode changing influences the topology comparison. However, for control strategies suited for automated topology comparison, penalizing mode changes significantly increases the computation time as the transmission mode must be added as a state for the control problem. This raises the question if it is necessary to penalize mode changes and ICE starts for a valid comparison of distinct hybrid drivetrain designs.

In this paper, the influence of penalizing mode changes and ICE starts on the comparison of hybrid drivetrain topologies is analyzed via an extensive case study. This study considers eight hybrid drivetrain topologies, ranging from parallel topologies with one electric machine (EM) to multi-mode topologies with two EMs, and ranging from a

Table 1. Overview of the transmission mode types with the directly controlled quantities, and the numbers of enabled modes for the eight example topologies.

	Controlled quantities		Number of mode type instances per topology							
Ψ: Mode Type	$x_{\mathrm{c},2}$	$x_{\mathrm{c},3}$	P2-8G	P3-5G	P2P3	TDT-RE	AHS	SEE	MME	L310
ICE only										
1: FG	-	-	-	-	3	-	-	-	-	-
EM only										
2: FG 1 EM	-	-	8	1	4	2	-	4	1	-
3: FG 2 EMs	$\tau_{\mathrm{em}_1}(x_{\mathrm{c},2})$	-	-	-	-	4	-	-	1	-
4: EVT 2 EMs	$\omega_{\mathrm{em}_1}(x_{\mathrm{c},2})$	-	-	-	-	-	-	-	-	-
Hybrid										
5: FG parallel 1 EM	$\tau_{\mathrm{ice}}(x_{\mathrm{c},2})$	-	8	5	6	2	1	4	1	-
6: FG parallel 2 EMs	$\tau_{\mathrm{em}_1}(x_{\mathrm{c},2})$	$\tau_{\mathrm{ice}}(x_{\mathrm{c},3})$	-	-	-	4	3	-	1	-
7: Series	$P_{\mathrm{ice}}(x_{\mathrm{c},2})$	-	-	-	-	2	-	-	-	-
8: EVT 1 EM	$\omega_{\mathrm{ice}}(x_{\mathrm{c},2})$	-	-	-	-	-	-	2	-	-
9: EVT 2 EMs	$\omega_{\mathrm{ice}}(x_{\mathrm{c},2})$	$\tau_{\mathrm{ice}}(x_{\mathrm{c},3})$	-	-	-	-	2	-	1	4
10: EVT 2 EMs τ-fix	$\omega_{\mathrm{ice}}(x_{\mathrm{c},2})$	$\omega_{\mathrm{em}_1}(x_{\mathrm{c},3})$	-	-	-	-	-	-	-	-
Charge										
11: FG 1 EM	$P_{\mathrm{ice}}(x_{\mathrm{c},2})$	-	1	-	1	-	1	2	-	1
12: FG 2 EMs	$P_{\mathrm{ice}}(x_{\mathrm{c},2})$	-	-	-	-	-	1	-	-	-
13: EVT 2 EMs	$P_{\mathrm{ice}}(x_{\mathrm{c},2})$	-	-	-	-	-	1	-	-	1
Neutral										
14: Neutral	-	-	1	-	1	1	1	1	-	1

topology with 5 transmission modes to a topology with 18 transmission modes. For all eight topologies, the control is optimized over two different driving cycles with and without these penalties. As power sources and ratios are not optimized, the presented simulation results can not be used for an actual comparison between the topologies, but solely serve to study the influence of penalizing mode changes and ICE starts. Here, this influence is studied in a systematic manner over a large and diverse set of topologies to obtain robust insights.

The remaining sections of this paper are organized as follows. Section 2 discusses the modeling and optimization, and explains how mode changes and ICE starts are penalized for arbitrary hybrid drivetrain topologies. In Section 3 the study itself is presented, addressing the method, the investigated topologies, the numerical penalty values, and the simulation results. This paper ends with a discussion in Section 4, and a conclusion in Section 5.

2. MODELING AND CONTROL OPTIMIZATION

2.1 Modeling

A backward facing model in the Matlab environment is used, which considers the longitudinal vehicle dynamics. To model the hybrid drivetrains, an automated modeling method is used that consists of a parameter determination and a generic transmission model (Van Harselaar et al. (2018, 2019a)). This method enables the automated modeling of all drivetrains with a maximum of one ICE and two EMs, consisting of gear pairs, planetary gear sets, clutches, brakes, and grounds. All transmission modes that are enabled by topologies with these components can be classified to one of 14 distinct mode types Ψ. Table 1 lists these mode types. (The *Controlled quantities* part of the table is explained in Section 2.2 and the *Number of mode type instances per topology* part is explained in Section 3.2.) The following terms are used to name the different mode types:

a) *Fixed gear (FG)* indicates that all rotational speeds are linearly dependent on each other, i.e. that there is no controllable degree of freedom (DOF) in rotational speed for a given speed at the wheels.
b) *Electric variable transmission (EVT)* indicates that there is at least one controllable DOF in rotational speed.
c) *Parallel* and *series* refer to the common classification of hybrid electric drivetrains (see e.g. Guzzella and Sciarretta (2005))
d) *τ-fix* indicates that all torques are linearly dependent on each other, i.e. that there is no DOF in torque for a given torque at the wheels.

The model does not include transmission losses and powertrain inertia.The ICE is modeled by a static lookup table for fuel mass flow as a function of its torque and rotational speed. EMs and power electronics are modeled by a static lookup table for the electric power as a function of EM torque and rotational speed.

2.2 Control Optimization Without Penalties

In this study, the control is optimized using dynamic programming (DP). An existing DP function for Matlab is used, and the implementation of the optimization without penalties is very similar to the example in the work that presents this Matlab DP function (Sundstrom and Guzzella (2009)). As the charge sustaining fuel consumption is minimized, the stage cost J^k only consists of the fuel mass:

$$J^k(\boldsymbol{x}_c^k) = \dot{m}_f(\Lambda(k), \boldsymbol{x}_c^k)\Delta t, \tag{1}$$

with stage k, fuel mass flow $\dot{m}_f$, driving cycle Λ, time step Δt, and control variables

$$\boldsymbol{x}_\mathrm{c} = \left[x_{\mathrm{c},1}\ \ x_{\mathrm{c},2}\ \ x_{\mathrm{c},3}\right]^T. \tag{2}$$

The first control variable $x_{\mathrm{c},1}$ is the discrete transmission mode, and the other two are continuous control variables. The continuous control variables are scaled variables between zero and one, and are discretized by the DP algo-

Table 2. Data of used power source models.

Description	Symbol	ICE	EM1	EM2	Unit
Maximum power	$P_{\max}$	85	100	20	kW
Maximum torque	$\tau_{\max}$	190	173	60	Nm
Minimum speed	$n_{\min}$	750	-16000	-9200	rpm
Maximum speed	$n_{\max}$	5500	16000	9200	rpm

rithm. Using scaled control variables is necessary as, e.g., a given topology with an ICE and one EM can enable a parallel hybrid mode ($\Psi = 5$) that requires the control of a torque and can also enable an EVT mode ($\Psi = 8$) that requires the control of a rotational speed. Thus, which quantities are directly controlled by $x_{\mathrm{c},2}$ and $x_{\mathrm{c},3}$ is dependent on the mode type. The controlled quantities are listed in Table 1. The only state of the control problem is the battery state of charge (SOC) ξ_{b}, with state function

$$\xi_{\mathrm{b}}^{k+1} = \xi_{\mathrm{b}}^{k} + \dot{\xi}_{\mathrm{b}}(\Lambda(k), \boldsymbol{x}_{\mathrm{c}}, \xi_{\mathrm{b}}^{k})\Delta t. \tag{3}$$

2.3 Penalizing Mode Changes and ICE Starts

To enable penalization of mode changes, the transmission mode is added as a state ξ_{m} to the control problem. This state can also be used to penalize ICE starts by additionally penalizing the transition from an electric mode to an ICE or hybrid mode. There are however two hybrid mode types that enable pure electric driving: series ($\Psi = 7$) and EVT mode $\Psi = 9$. Therefore, transmission modes of type $\Psi \in \{7, 9\}$ are doubled in the domain of $x_{\mathrm{c},1}$ and ξ_{m}, whereas one of the two allows electric operation and the other hybrid operation. As the value of $x_{\mathrm{c},1}$ is no longer directly the transmission mode, function $\Gamma(x_{\mathrm{c},1})$ is defined to return the corresponding transmission mode. Furthermore, function $\varepsilon(x_{\mathrm{c},1})$ is defined to return one for electric modes and zero for ICE and hybrid modes. To penalize mode changes and ICE starts, the state cost is defined as

$$J^k(\boldsymbol{x}_c^k, \xi_{\mathrm{m}}^k) = \dot{m}_f(\Lambda(k), \boldsymbol{x}_c^k)\Delta t + f_{\mathrm{mc}}(x_{\mathrm{c},1}^k, \xi_{\mathrm{m}}^k) + f_{\mathrm{is}}(x_{\mathrm{c},1}^k, \xi_{\mathrm{m}}^k), \tag{4}$$

with mode change penalty function f_{mc} and ICE start penalty function f_{is}

$$f_{\mathrm{mc}}(x_{\mathrm{c},1}^k, \xi_{\mathrm{m}}^k) = \begin{cases} p_{\mathrm{mc}}, & \text{if } \Gamma(x_{\mathrm{c},1}^k) \neq \Gamma(\xi_{\mathrm{m}}^k) \\ 0, & \text{if } \Gamma(x_{\mathrm{c},1}^k) = \Gamma(\xi_{\mathrm{m}}^k) \end{cases} \tag{5}$$

$$f_{\mathrm{is}}(x_{\mathrm{c},1}^k, \xi_{\mathrm{m}}^k) = \begin{cases} p_{\mathrm{is}}, & \text{if } \varepsilon(x_{\mathrm{c},1}^k) < \varepsilon(\xi_{\mathrm{m}}^k) \\ 0, & \text{if } \varepsilon(x_{\mathrm{c},1}^k) \geq \varepsilon(\xi_{\mathrm{m}}^k) \end{cases} \tag{6}$$

where p_{mc} and p_{is} are the numerical penalty values for a mode change and an ICE start, respectively. As both penalty values have unit [g], the cost function (4) also has unit [g]. The state function for the transmission mode state is

$$\xi_{\mathrm{m}}^{k+1} = x_{\mathrm{c},1}^k. \tag{7}$$

3. STUDY

3.1 Method

To study the influence of mode change and ICE start penalties on the comparison of hybrid drivetrain topologies, eight topologies are selected for which the control is optimized over two driving cycles with and without these penalties. For all topologies, fixed components are used which are not optimized. Data of the used power sources is listed in Table 2. The ratios of (planetary) gears are not optimized, yet, if neccesary, gears connecting the EMs are edited to match the speed ranges of the used EM models. The two used driving cycles are the worldwide harmonized light vehicles test cycle (WLTC) and the Artemis150 cycle (Urban + Rural + 150 km/h variant of Motorway) (DieselNet (2019)). All simulations are performed using the same vehicle parameters, corresponding to a C-segment car.

The authors would like to emphasize that the results presented below do not represent a valid topology comparison, as components and gear ratios are not varied nor optimized. Furthermore, for a valid topology comparison performance (e.g. acceleration, top speed, and towing) and functionality should be taken into account. The goal of this study is to analyze the influence of mode change penalties on the comparison of hybrid drivetrain topologies.

3.2 Investigated drivetrain topologies

Eight drivetrain topologies are selected, which enable a broad range of distinct mode types. Below, the eight topologies are briefly introduced. For all topologies, the stick diagrams are displayed in Fig. 1 and the number of enabled modes per mode type are listed in Table 1.

a) *P2-8G:* The first example topology is a parallel hybrid topology based on a conventional 8 speed gearbox with an EM connected to the input shaft of the gearbox, referred to as the P2-8G topology. Through the possibility to decouple the ICE, all gears are available as electric and as parallel hybrid modes. This type of drivetrain is mass-produced by a number of companies, see e.g. Harsch et al. (2019).
b) *P3-5G:* The second topology is based on a conventional 5 speed gearbox with an EM connected to the output shaft of the gearbox, referred to as the P3-5G topology. In contrast to the P2-8G topology, the EM can be used to reduce or eliminate torque interruption during a mode change.
c) *P2P3:* The P2P3 topology is a novel parallel hybrid concept that enables three different transmission ratios for the ICE, whereas these three ratios can also be used for the EM. Additionally the EM can be connected directly to the differential, combining the functionalities of a P2 and a P3 topology (Schleiffer et al. (2020)).
d) *TDT-RE:* The TDT-RE is a series-parallel hybrid drivetrain, consisting of a two drive transmission (TDT) with two EMs and an ICE as range extender. Both EMs have their own subtransmission, each with two gears. This topology is based on the DE-REX concept (Viehmann and Rinderknecht (2019)).
e) *AHS:* The fifth topology is the AHS, also called two-mode hybrid system (Grewe et al. (2007)). Additionally to two EVT modes ($\Psi = 9$), four fixed gear parallel hybrid modes are enabled.
f) *SEE:* The single EM example (SEE) topology is derived from the AHS by removing one EM and adding an additional clutch to enable decoupling of the ICE. This example topology is introduced by

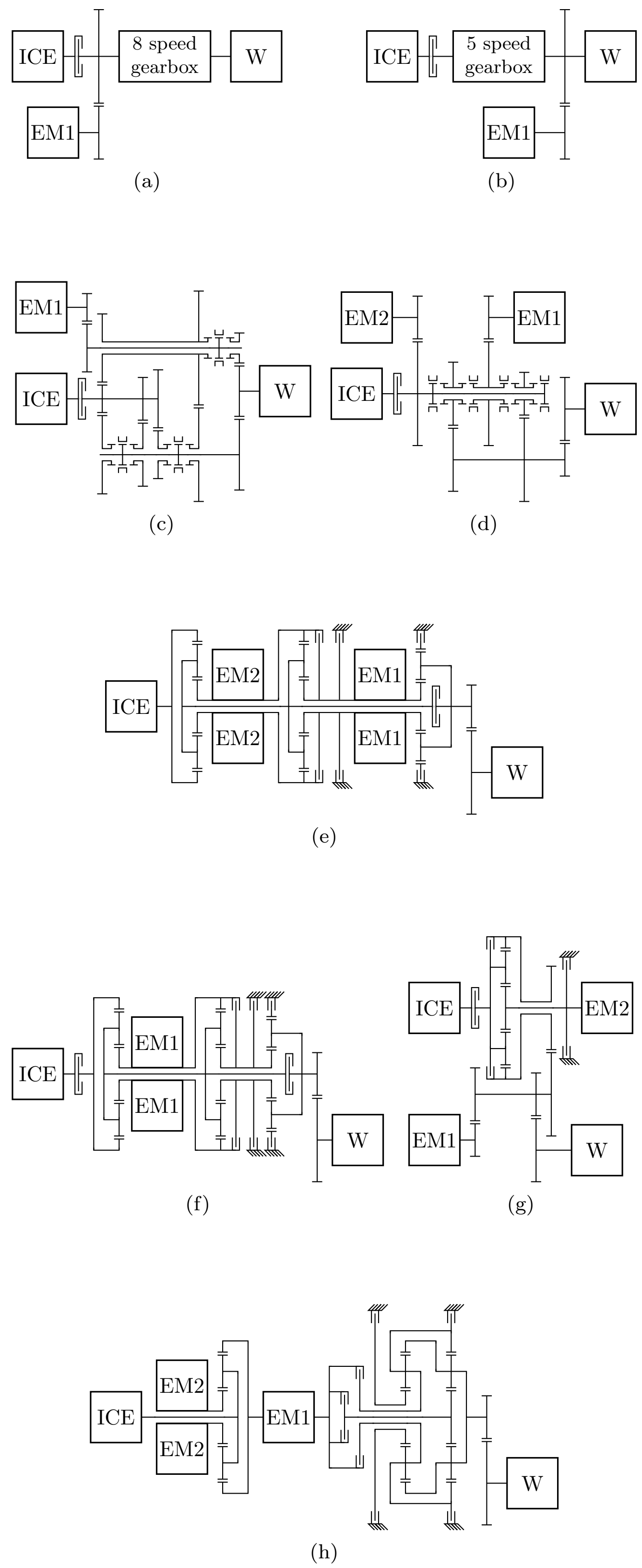

Fig. 1. Stick diagrams of the drivetrain topologies. (a) P2-8G, (b) P3-5G, (c) P2P3, (d) TDT-RE, (e) AHS, (f) SEE, (g) MME, and (h) L310.

Van Harselaar et al. (2019a). Two EVT modes of mode type $\Psi = 8$ are enabled, as are fixed gear electric and parallel hybrid modes.

g) *MME:* The multi-mode example (MME) topology is derived from the Toyota Hybrid System (THS) (Fushiki (2016)) by adding two clutches and one brake. The MME is a multi-mode topology that enables two electric modes, one EVT mode, and two fixed gear parallel hybrid modes (Van Harselaar et al. (2018)).

h) *L310:* The eight and last topology is the L310 (Okuda et al. (2017)). This topology enables four EVT modes of mode type $\Psi = 9$. In contrast the other seven example topologies, no fixed gear modes are enabled.

For the P2-8G and P3-5G topologies, the stick diagrams are simplified using a gearbox block as the gearbox type is not relevant for the modes that are enabled.

3.3 Numerical penalty values

In Section 2 it is explained how mode changes are penalized. Numerical values for p_{mc} and p_{is} must however still be determined. In this study, for each p_{mc} and p_{is}, one value is determined and used for all topologies, as all topologies are simulated using the same power sources.

To determine p_{mc}, the number of mode changes over the WLTC is evaluated as a function of p_{mc} for the P2-8G, AHS, TDT-RE, and P3-5G topologies. For these four topologies the control is optimized using nine different values of p_{mc}, from 0.001 [g] to 0.5 [g]. The number of mode changes resulting from these simulations are displayed in Fig. 2a. For the simulated penalty range, a variation in fuel consumption of up to 3.5% is observed. As a comparison, Fig. 2b shows the number of mode changes as published by Robinette and Wehrwein (2015), where causal shift schedules are optimized for multiple conventional vehicles featuring an ICE and an automatic transmission (AT) with different numbers of gears and optimized ratios.

To ensure a significant influence, and force the number of mode changes into a realistic range, a mode change penalty

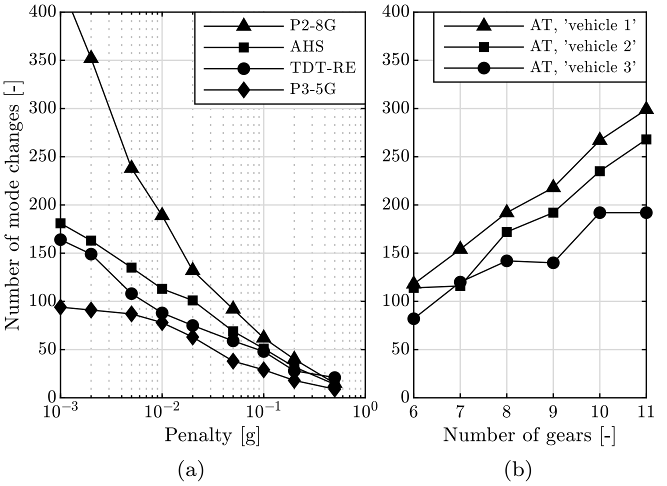

Fig. 2. Number of mode changes over WLTC. (a) Simulation results over numerical penalty value. (b) Results published by Robinette and Wehrwein (2015) over number of gears for conventional drivetrains.

Table 3. Fuel consumption increase per topology due to the penalties.

Topology	WLTC	Artemis150	Unit
P2-8G	0.71	0.71	%
P3-5G	0.52	0.47	%
P2P3	0.51	0.65	%
TDT-RE	0.64	0.60	%
AHS	0.50	0.58	%
SEE	0.45	0.72	%
MME	0.46	0.37	%
L310	0.53	0.66	%

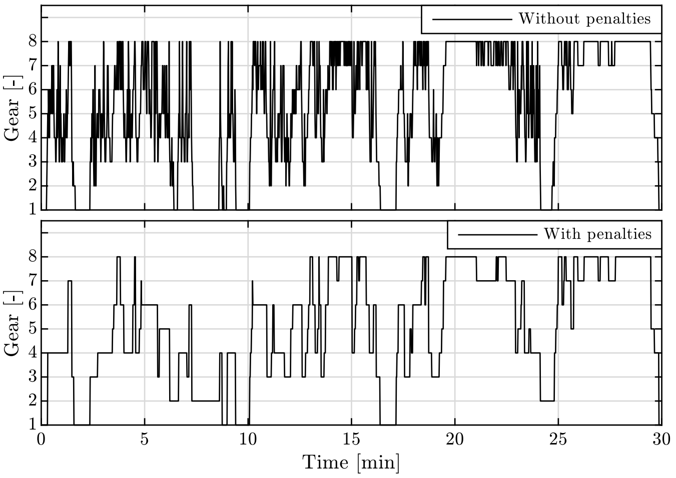

Fig. 3. Gear selection of P2-8G over WLTC with and without penalties.

of $p_{\mathrm{mc}} = 0.02$ [g] is selected. In a similar way, an ICE start penalty of $p_{\mathrm{is}} = 0.2$ [g] is selected to ensure a significant reduction of ICE starts.

3.4 Results

As shown in Fig. 2a, introducing penalties influences the number of mode changes significantly, especially for the P2-8G topology. For this topology, the 16 transmission modes that enable propulsion can be reduced to the eight gears of the gearbox by disregarding whether an hybrid or an electric mode is selected. This is done to visualize the influence of the penalties over the WLTC in Fig. 3. The total number of mode changes of the P2-8G over the WLTC is reduced from 667 to 132. The number of mode changes of all eight topologies over the WLTC and Artemis150 with and without penalties are displayed in the top of Fig. 4. The application of penalties reduces these numbers to below 150 and 300 for the WLTC and Artemis150, respectively, for all topologies. Depending on the topology, the application of penalties reduces the number of mode changes by a factor of three to five. Furthermore, for the simulations with penalties, the number of mode changes are more comparable across the eight topologies.

Fig. 4 also shows the number of ICE starts over both driving cycles. Both with and without penalties these numbers are relatively similar across the eight topologies. The application of penalties lowers the number of ICE starts by approximately a factor of three.

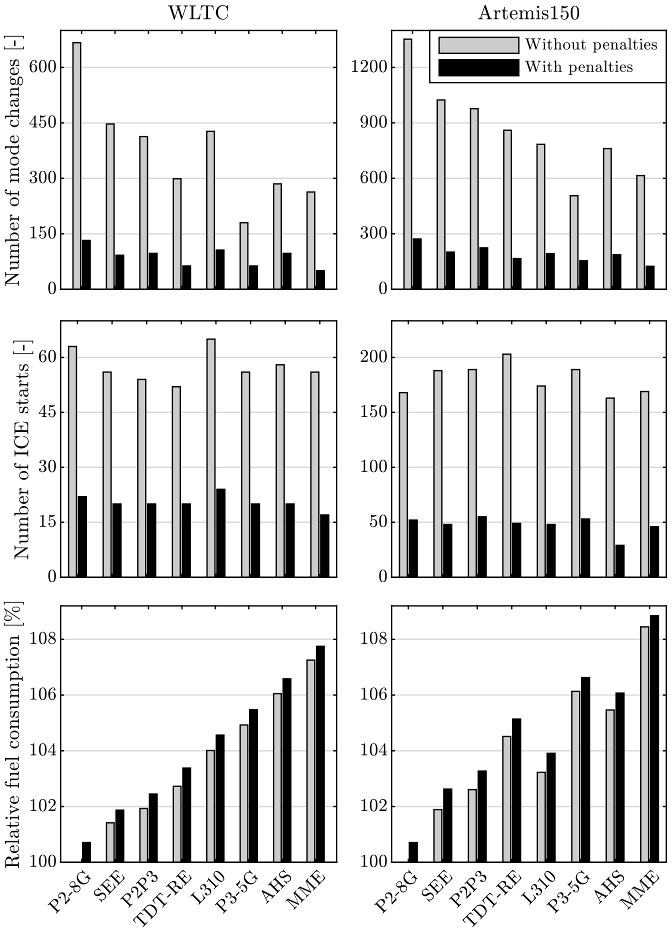

Fig. 4. Number of mode changes, number of ICE starts, and relative fuel consumption, with and without mode change and ICE start penalties.

For both driving cycles, without penalties, the P2-8G topology showed the lowest charge sustaining fuel consumption using the unoptimized components and ratios. To visualize the influence of the penalties, the fuel consumption of all topologies is determined relative to the fuel consumption of the P2-8G without penalties. With the introduction of mode change and ICE start penalties the fuel consumption increases and is also determined relative to the fuel consumption of the P2-8G without penalties. Thereby, the simulations with and without penalties have the same reference value, and the relative fuel consumption with and without penalties are plotted on the same axis in the bottom of Fig. 4. For all topologies, the increase in fuel consumption due to the introduction of mode change and ICE start penalties is small compared to the differences between the topologies. As the increase in fuel consumption is relatively small, this increase is also similar across the eight topologies. The fuel consumption increase per topology is listed in Table 3, and is between 0.45% and 0.71% for the WLTC. For the Artemis150, the fuel consumption increases between 0.40% and 0.74% due to the introduction of mode change and ICE start penalties. Taking the P2-8G as reference both with and without penalties, the largest change in relative fuel consumption is 0.36 percentage points and the average absolute change over both cycles is 0.15 percentage points.

4. DISCUSSION

In the introduction, it is mentioned that there is a trade-off between accuracy and computation time. As an indication; the computation time required to optimize the control for the P2-8G over the WLTC increases from approximately 40 seconds to approximately 18 minutes when mode change and ICE start penalties are introduced. For the TDT-RE, the computation time increases from approximately 54 minutes to approximately 22 hours. These numbers correspond to an increase in computation time by a factor of around 26. Therefore, it can be argued that the increase in accuracy provided by mode change penalties is not worth the additional computation time in the context of the SLD of hybrid drivetrains.

A disadvantage of the applied penalization method is that every mode change is equally penalized. Optimally, regular up-shifting during an acceleration phase would not be penalized and (high) frequent switching between two modes during constant driving would be heavily penalized. Furthermore, it would be desirable to penalize the skipping of a gear during acceleration, whereas the here implemented method essentially rewards the skipping of gears. More advanced penalization that aims for more realistic and comfortable behavior is, however, very challenging to implement for the automated assessment of arbitrary topologies, and might influence a design optimization in an unexpected way.

Although beyond the scope of this paper, the results in Fig. 4 show that the driving cycle does have a significant influence on the comparison of hybrid drivetrain topologies.

5. CONCLUSION

To study the influence of mode change penalties, for eight distinct hybrid drivetrain topologies the control is optimized over two different driving cycles, with and without penalties. To obtain robust insights, this study considers a large and diverse set of topologies in a systematic manner. These topologies not only enable very different numbers of transmission modes, but also differ greatly in their sets of enabled mode types. The introduction of mode change and ICE start penalties reduces the number of mode changes by a factor of three to five, and reduces the number of ICE starts by approximately a factor of three. Yet, the influence on the fuel consumption comparison between the topologies is small: the largest change in relative fuel consumption is 0.36 percentage points, with the average absolute change over both cycles being 0.15 percentage points. Therefore, in the context of the SLD of hybrid drivetrains, it can be argued that the additional computation time outweighs the minor increase in accuracy provided by mode change penalties.

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