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Dynamic output feedback sliding mode control for non-minimum phase systems with application to an inverted pendulum ⋆

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Abstract: In this paper, a class of nonlinear systems is considered, where the nominal system representation is allowed to be non-minimum phase. A sliding surface is proposed which is a function of the measured system output and an estimated state. A linear coordinate transformation is introduced so that the stability analysis of the reduced order sliding mode dynamics can be conveniently performed. A robust output feedback sliding mode control (OFSMC) is then designed to drive the system states to the sliding surface in finite time and maintain a sliding motion thereafter. A simulation example is used to demonstrate the effectiveness of the proposed method and the method is successfully applied to an inverted pendulum.

Keywords: Sliding mode technique; Non-minimum phase; Dynamical output feedback; Regular form; Inverted pendulum.

1. INTRODUCTION

Physical systems are frequently affected by external disturbances and modeling uncertainties, which can have a great impact on system performance. Sliding mode variable structure control has been widely considered as an effective method to tackle this problem due to its strong robustness properties and straightforward implementation. Much of the early work assumed that all system state variables are measurable (see e.g. Utkin (1977)). However, in actual engineering systems, only output information may be available. This has motivated the current emphasis on the study of OFSMC.

Many OFSMC algorithms have been proposed for robust stabilization of uncertain systems. A static output feedback variable structure control has been investigated for linear systems without disturbances in El-Khazali and Decarlo (1995). The case of matched disturbances and OFSMC has been considered by appealing to a particular canonical form in Edwards and Spurgeon (1998). However unmatched disturbances also act on many systems. Modeling errors may also be unmatched. In sliding mode control, unmatched disturbances directly affect the dynamics of the system when sliding. Output feedback sliding mode schemes in the presence of unmatched linear disturbances are considered in Zak and Hui (1993) and Choi (2002). The disturbances experienced by physical systems may be nonlinear and classes of more general unmatched disturbances have been considered in the design of OFSMC in Yan et al. (2004a) and Liu et al. (2019). All the literature above requires that the considered system is minimum phase which limits application.

More recently, the study of OFSMC for non-minimum phase systems has received attention. The tracking of specific signals has been considered in Spurgeon and Lu (1997), Shkolnikov and Shtessel (2002). In Yan et al. (2004b), a class of nonlinear systems has been stabilized based on a dynamic OFSMC, where the nominal system may be non-minimum phase. The results have been extended to time-delay systems (see e.g. Yan et al. (2009) and Yan et al. (2010)), and interconnected systems (see e.g. Yan et al. (2006)). Although the case of more general unmatched disturbances with nonlinear bounds has been considered in the literature above, an equivalent control method has been used to analyze the stability of the corresponding sliding mode dynamics, which makes the analysis somewhat complicated.

In this paper, a dynamic OFSMC strategy is proposed for a class of nonlinear non-minimum phase systems. Under the conditions that the considered system can be

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transformed to the regular form (see e.g. Edwards and Spurgeon (1998)), a new linear coordinate transformation is presented so that the stability analysis of the reduced order sliding motion dynamics can be clearly performed. In comparison with most of the existing OFSMC methods (see e.g. Gao et al. (2019) and Ji et al. (2019)), the nominal part of the nonlinear system considered in this paper is allowed to be non-minimum phase, which extends both the potential practical application and the theoretical contribution. It should be noted that the proposed new linear coordinate transformation greatly facilitates the stability analysis of the sliding motion dynamics. In light of this transformation, the dynamical OFSMC can accommodate unmatched uncertainty well and the stability analysis is more intuitive and straight forward compared with the results in Yan et al. (2004b), Yan et al. (2009) and Yan et al. (2010).

2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Consider a nonlinear system

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}u + \tilde{f}(\tilde{x},t) \\ y &= \tilde{C}\tilde{x} \end{aligned} \tag{1}$$

where $\tilde{x} \in R^n, y \in R^p, u \in R^m$ are the system state, output and input vectors respectively with $m \leq p < n$, and $\tilde{A}, \tilde{B}, \tilde{C}$ are constant matrices with appropriate dimensions, the function $\tilde{f}(\tilde{x},t)$ represents the modeling error and external disturbances. The following Assumptions are imposed on system (1).

Assumption 1. System (1) is controllable and observable. It follows from the observability of $(\tilde{A},\tilde{C})$ that there exists a matrix $\tilde{L} \in R^{n\times p}$ such that $\tilde{A} - \tilde{L}\tilde{C}$ is stable. For any symmetric matrix $Q_1 > 0$, the Lyapunov equation

$$(\tilde{A} - \tilde{L}\tilde{C})^T P_1 + P_1(\tilde{A} - \tilde{L}\tilde{C}) = -Q_1 \tag{2}$$

has a unique symmetric solution $P_1 > 0$.

Assumption 2. $\tilde{f}(\tilde{x},t)$ has a structural decomposition:

$$\tilde{f}(\tilde{x},t) = \tilde{E}\Delta\xi(\tilde{x},t) \tag{3}$$

where $\tilde{E}$ is a known constant matrix, and $\|\Delta\xi(\tilde{x},t)\| \leqslant \zeta(\tilde{x},t) = \eta(\tilde{x},t)\,\|\tilde{x}\|$ with $\zeta(\tilde{x},t)$ Lipschitz with respect to $\tilde{x}$ and where K_ζ represents the Lipschitz constant.

Assumption 3. There is a known matrix F such that

$$\tilde{E}^T P_1 = F\tilde{C} \tag{4}$$

holds, where $\tilde{E}$ is given by Assumption 2, and P_1 satisfies (2).

Assumption 4. $\tilde{B}$ and $\tilde{C}$ are both full rank, $rank(\tilde{C}\tilde{B}) = m$.

Under Assumption 4, it follows from Lemma 5.3 in Edwards and Spurgeon (1998) that there exists a nonsingular coordinate transformation $x = \tilde{T}\tilde{x}$ such that the system (1) can be described as:

$$\begin{aligned} \dot{x} &= Ax + Bu + f(x,t) \\ y &= Cx \end{aligned} \tag{5}$$

where $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \tilde{T}\tilde{A}\tilde{T}^{-1}$ with $A_{22} \in R^{m\times m}$, $B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} = \tilde{T}\tilde{B}$ with nonsingular matrix $B_2 \in R^{m\times m}$, $C = \tilde{C}\tilde{T}^{-1}$ and $f(x,t) = \tilde{T}\tilde{E}\Delta\xi(\tilde{T}^{-1}x,t)$.

It follows from Proposition 3.3 in Edwards and Spurgeon (1998) that the pair (A_{11}, A_{12}) is controllable. Thus there exists a matrix $K \in R^{m\times(n-m)}$ such that $A_{11} - A_{12}K$ is stable. It follows that for any symmetric matrix $Q_2 > 0$, the Lyapunov equation

$$(A_{11} - A_{12}K)^T P_2 + P_2(A_{11} - A_{12}K) = -Q_2 \tag{6}$$

has a unique symmetric solution $P_2 > 0$.

From (2) and the relationship between (1) and (5),

$$(\tilde{T}^{-1}A\tilde{T} - \tilde{L}C\tilde{T})^T P_1 + P_1(\tilde{T}^{-1}A\tilde{T} - \tilde{L}C\tilde{T}) = -Q_1 \tag{7}$$

Let $P_3 = \left(\tilde{T}^{-1}\right)^T P_1 \left(\tilde{T}^{-1}\right)$, $Q_3 = \left(\tilde{T}^{-1}\right)^T Q_1 \left(\tilde{T}^{-1}\right)$, $L = \tilde{T}\tilde{L}$, $E = \tilde{T}\tilde{E}$.

It follows from (7) and Assumption 3 that

$$(A - LC)^T P_3 + P_3\,(A - LC) = -Q_3 \tag{8}$$

with P_3, Q_3 are all symmetric positive matrices and

$$E^T P_3 = FC \tag{9}$$

From the above analysis, the following lemma can be obtained:

Lemma 1. The constrained Lyapunov equations (2) and (4) hold if and only if (8) and (9) hold.

From Lemma 1, Assumption 3 together with Assumption 1 represents a structural characteristic that is independent of the coordinate system.

The objective of this paper is to design a composite sliding surface formed by the system output and the estimated state for system (5) such that the reduced order sliding mode is asymptotically stable. For system (5), a dynamic output feedback control of the following form

$$\dot{\hat{x}} = \hat{x}\,(t,u,\hat{x},y) \tag{10}$$

$$u = u\,(t,\hat{x},y) \tag{11}$$

will be designed such that the associated closed-loop system formed by (5), (10) and (11) can be driven to the pre-designed sliding surface in finite time and a sliding motion maintained thereafter even in the presence of unmatched uncertainties.

3. SLIDING MOTION STABILITY ANALYSIS

3.1 Dynamic compensator design

Based on the analysis above, the following dynamical compensator is constructed for the system (5):

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) + \Phi(\hat{x},y,t) \tag{12}$$

where L satisfies (8) and

$$\Phi(\hat{x},y,t) = \begin{cases} E\dfrac{F\,(y - C\hat{x})}{\|F\,(y - C\hat{x})\|}\zeta(\tilde{T}^{-1}\hat{x},t), & F\,(y - C\hat{x}) \neq 0 \\ 0, & F\,(y - C\hat{x}) = 0 \end{cases} \tag{13}$$

where $E = \tilde{T}\tilde{E}$ with $\tilde{E}$ given in (3) and F satisfies (4).

It follows from (5) and (12) that

$$\dot{e} = (A - LC)\, e + f(x,t) - \Phi(\hat{x},y,t) \tag{14}$$

where $e = x - \hat{x}$. In the (x, e) coordinate system, system (5) and (14) can be described by

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} f \\ f - \Phi \end{bmatrix} \tag{15}$$
$$y = Cx$$

For the closed-loop analysis of (15) the following Lemma will be required.

Lemma 2. (see e.g. Yan et al. (2004b)) There exists a positive constant γ such that $\|e\| \leqslant \gamma$ if $\underline{\lambda}(Q_3) > 2K_\zeta \|FC\| \left\|\tilde{T}^{-1}\right\|$ where

$$\gamma \geqslant \sqrt{\frac{(e^T P_3 e)|_{t=t_0} \exp\{\alpha t_0\}}{\underline{\lambda}(P_3)}} \tag{16}$$

with $\alpha = \frac{\underline{\lambda}(Q_3) - 2K_\zeta \|FC\| \|\tilde{T}^{-1}\|}{2\bar{\lambda}(P_3)}$ and e is given by (14).

Here $\underline{\lambda}(\cdot)$ and $\bar{\lambda}(\cdot)$ denote the minimum and maximum eigenvalues respectively.

3.2 Sliding surface design and stability analysis of the sliding motion

In order to construct a stable sliding mode dynamics, a further transformation $z = Tx$ will be introduced, where $T = \begin{bmatrix} I_{n-m} & 0 \\ K & I_m \end{bmatrix}$ and K satisfies (6). In the new coordinates, system (5) can be described by:

$$\dot{z} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u + Tf\left(T^{-1}z, t\right) \tag{17}$$

where $z_1 \in R^{n-m}, z_2 \in R^m$, $\bar{A}_{11} = A_{11} - A_{12}K$. Choose the sliding function

$$\sigma(y, \hat{x}) = S_1 y + S_2 \hat{x} \tag{18}$$

where $S_1 \in R^{m \times p}$ is design parameters, and $S_2 = S - S_1 C$ with $S = [\, 0 \;\; I_m \,] T$.

Remark 1. It should be noted that $\bar{A}_{11}$ in (17) is stable. When compared with the equivalent control method used in Yan et al. (2004b), this simplifies the stability analysis of the sliding mode dynamics. An additional advantage is that the formulation provides a constructive design method to determine S.

From (18), it follows that

$$\begin{aligned} \sigma &= S_1 C x + S_2 (x - e) \\ &= Sx - S_2 e \\ &= ST^{-1} z - S_2 e \\ &= z_2 - S_2 e \end{aligned} \tag{19}$$

In the new coordinates the sliding surface is given by:

$$z_2 = S_2 e \tag{20}$$

When system (15) is restricted to the sliding surface (20), the sliding mode dynamics are described by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{e} \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} S_2 \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} z_1 \\ e \end{bmatrix} + \begin{bmatrix} f_1 \\ f - \Phi \end{bmatrix} \tag{21}$$

where f_1 is the first m components of $Tf\left(T^{-1}z, t\right)\big|_{z_2 = S_2 e}$. From Assumption 2 , it follows that

$$\left\|Tf(T^{-1}z,t)\right\| \leqslant \|TE\|\, \eta(\tilde{T}^{-1}T^{-1}z,t) \left\|\tilde{T}^{-1}T^{-1}\right\| \|z\| \tag{22}$$

Then, from the definition of f_1, (22) and the inequality

$$\|z\||_{z_2 = S_2 e} = \begin{bmatrix} z_1 \\ S_2 e \end{bmatrix} \leqslant \|z_1\| + \|S_2\| \|e\| \tag{23}$$

it can be seen that there exist functions χ_1 and χ_2 (dependent on η, T, E, $\tilde{T}$ and S_2) such that

$$\|f_1(z_1, e, t)\| \leqslant \chi_1 \|z_1\| + \chi_2 \|e\| \tag{24}$$

Now, consider the term $f(x,t) - \Phi(\hat{x}, y, t)$. From Assumption 2 and (13), it follows that:

(i) when $FCe = 0$, $\Phi(\hat{x}, y, t) = 0$ and thus

$$e^T P_3 (f(x,t) - \Phi(\hat{x},y,t)) = (FCe)^T \Delta\xi = 0 \tag{25}$$

(ii) when $FCe \neq 0$,

$$\begin{aligned} & e^T P_3 (f(x,t) - \Phi(\hat{x},t)) \\ &= (FCe)^T \Delta\xi(\tilde{T}^{-1}x, t) - \frac{(FCe)^T FCe}{\|FCe\|} \zeta(\tilde{T}^{-1}\hat{x}, t) \\ &\leqslant \|FCe\|\, \zeta(\tilde{T}^{-1}x, t) - \|FCe\|\, \zeta(\tilde{T}^{-1}\hat{x}, t) \\ &= K_\zeta \|FC\| \left\|\tilde{T}^{-1}\right\| \|e\|^2 \end{aligned} \tag{26}$$

Therefore from (i) and (ii) above, it follows that:

$$e^T P_3 (f(x,t) - \Phi(\hat{x},t)) \leqslant K_\zeta \|FC\| \left\|\tilde{T}^{-1}\right\| \|e\|^2 \tag{27}$$

Theorem 3. Suppose Assumptions 1-4 are satisfied. Then, the reduced order sliding mode dynamics (21) are asymptotically stable if the matrix M defined by

$$M = \begin{bmatrix} \underline{\lambda}(Q_2) - 2\chi_1 \|P_2\| & -\left\|P_2 \bar{A}_{12} S_2\right\| - \|P_2\| \chi_2 \\ -\left\|P_2 \bar{A}_{12} S_2\right\| - \|P_2\| \chi_2 & \underline{\lambda}(Q_3) - 2K_\zeta \|FC\| \left\|\tilde{T}^{-1}\right\| \end{bmatrix} \tag{28}$$

is positive definite.

Proof. For the system (21), consider the Lyapunov function candidate

$$V(z_1, e, t) = e^T P_3 e + {z_1}^T P_2 z_1 \tag{29}$$

The time derivative of V along the trajectories of the dynamic system (21) is given as

$$\begin{aligned} \dot{V} &= -e^T Q_3 e + 2e^T P_3 (f - \Phi) - {z_1}^T Q_2 z_1 \\ &\quad + 2{z_1}^T P_2 \bar{A}_{12} S_2 e + 2{z_1}^T P_2 f_1 \end{aligned} \tag{30}$$

Then from (24) and (27), it follows that

$$\begin{aligned} \dot{V} &\leqslant \left\{-\underline{\lambda}(Q_3) + 2K_\zeta \|FC\| \left\|\tilde{T}^{-1}\right\|\right\} \|e\|^2 \\ &\quad + \left\{-\underline{\lambda}(Q_2) + 2\chi_1 \|P_2\|\right\} \|z_1\|^2 \\ &\quad + 2\left\{\left\|P_2 \bar{A}_{12} S_2\right\| + \|P_2\| \chi_2\right\} \|e\| \|z_1\| \\ &= -[\, \|z_1\| \;\; \|e\| \,]\, M \begin{bmatrix} \|z_1\| \\ \|e\| \end{bmatrix} \end{aligned} \tag{31}$$

Hence, Theorem 3 follows since $M > 0$.

Remark 2. It should be pointed out that the stability analysis of the sliding mode dynamics (21) does not require that the considered system is minimum phase since the design of the sliding function (18) is based on the dynamic compensator (12) and the system output. Meanwhile as shown in (21), the unmatched uncertainty f_1 affects the sliding mode directly, but K in (6) and L in (8) can be designed by pole assignment to guarantee that the sliding motion is stable under the sufficient condition $M > 0$.

4. SLIDING MODE CONTROL DESIGN

Based on the estimated state given by (12) and the system output, the following sliding mode control is proposed

$$u = -(SB)^{-1}\left\{SA\hat{x} + S_2L\left(y - C\hat{x}\right) + \frac{\sigma}{\|\sigma\|}K\left(\hat{x}, y, t\right)\right\} \tag{32}$$

where $K\left(\hat{x}, y, t\right)$ is defined by

$$\begin{aligned} K\left(\hat{x}, y, t\right) = {} & K_\zeta \left\|S_1CE\right\| \left\|\tilde{T}^{-1}\right\| \|e\| + \gamma \left\|S_1CA\right\| \\ & + \left(\|S_1CE\| + \|S_2E\|\right)\zeta\left(\tilde{T}^{-1}\hat{x}, t\right) + \eta \end{aligned} \tag{33}$$

with η a positive constant.

Theorem 4. Suppose Assumptions 1-4 are satisfied. Then, the control (32) with $K(\cdot)$ defined in (33) will drive the system (15) to the sliding surface and maintain a sliding motion thereafter.

Proof. It follows from (15) and (18) that

$$\begin{aligned} \dot{\sigma}\left(y, \hat{x}\right) = {} & SBu + S_2\Phi(\hat{x}, y, t) + S_1Cf(x, t) \\ & + SA\hat{x} + \left(S_1CA + S_2LC\right)e \end{aligned} \tag{34}$$

By applying the control (32) to (34),

$$\begin{aligned} \sigma^T\dot{\sigma} \leqslant {} & -\|\sigma\|\left\{K\left(\hat{x}, y, t\right) - \|S_2\Phi(\hat{x}, y, t)\| \right. \\ & \left. - \|S_1Cf(x, t)\| - \|S_1CAe\|\right\} \end{aligned} \tag{35}$$

From Assumption 2,

$$\begin{aligned} \|S_1Cf(x, t)\| \leqslant {} & \|S_1CE\|\left(\zeta\left(\tilde{T}^{-1}x, t\right) - \zeta\left(\tilde{T}^{-1}\hat{x}, t\right)\right) \\ & + \|S_1CE\|\,\zeta\left(\tilde{T}^{-1}\hat{x}, t\right) \\ \leqslant {} & K_\zeta \|S_1CE\| \left\|\tilde{T}^{-1}\right\| \|e\| \\ & + \|S_1CE\|\,\zeta\left(\tilde{T}^{-1}\hat{x}, t\right) \end{aligned} \tag{36}$$

From (13) and Assumption 2,

$$\|S_2\Phi(\hat{x}, y, t)\| \leqslant \|S_2E\|\,\zeta\left(\tilde{T}^{-1}\hat{x}, t\right) \tag{37}$$

From Lemma 2,

$$\|S_1CAe\| \leqslant \gamma\,\|S_1CA\| \tag{38}$$

Based on the above analysis, the following inequality can be obtained:

$$\sigma^T\dot{\sigma} \leqslant -\eta\,\|\sigma\| \tag{39}$$

Hence, Theorem 4 follows since the so-called η reachability condition is satisfied.

5. SIMULATION AND EXPERIMENTAL RESULTS

The Process Modelling and Control Group at the China University of Petroleum (East China) has the inverted pendulum rig shown in Fig.1. This system is nonlinear with complex and non-minimum phase characteristics. The proposed algorithm will be validated by both simulation and experimental testing.

Through mechanism modeling and model identification, a linearized inverted pendulum model is obtained in the form (1) where the system states $\tilde{x} \in R^4$ represent the deviation of the slider's horizontal position (m), the slider's horizontal velocity (m/s), the pendulum angle (rad) and the pendulum angular velocity (rad/s) from their respective equilibrium points, $u \in R^1$ is the control input, representing the output torque of the motor drive (N), $\tilde{f}(\tilde{x}, t)$ represents the modeling error and external disturbances and $\tilde{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -14.19 & 0 & 2.91 & 0 \\ 0 & 0 & 0 & 1 \\ 285.68 & 0 & -18.96 & 0 \end{bmatrix}$, $\tilde{B} = \begin{bmatrix} 0 \\ -961.51 \\ 0 \\ 16060.57 \end{bmatrix}$, $\tilde{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$. This system has zeros at -5.45144 and 5.45144 and thus the nominal system is non-minimum phase. Suppose $\tilde{f}(\tilde{x}, t)$ satisfies

$$\tilde{f}(\tilde{x}, t) = \tilde{E}\Delta\xi(\tilde{x}, t) \tag{40}$$

where $\tilde{E} = 10^{-2} \times \left[\,8\ \ 0.54\ \ 0.02\ \ -3.41\,\right]^T$, $\|\Delta\xi(\tilde{x}, t)\| = \zeta(\tilde{x}, t) = 5 \times 10^{-4}\,\|\tilde{x}\|$ with $K_\zeta = 5 \times 10^{-4}$. The objective is to design a control so that the system states converge to zero asymptotically only using the system output information and the estimated state even in the presence of unmatched uncertainties and modelling errors.

The coordinate transformation $x = \tilde{T}\tilde{x}$ is given by

$$\tilde{T} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 16.70 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{41}$$

Then the system can be represented by

$$A = \begin{bmatrix} 0 & 1 & 0 & -16.70 \\ 29.72 & 0 & 48.70 & 0 \\ 0 & 0 & 0 & 1 \\ 2.91 & 0 & -14.19 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -961.51 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since $(\tilde{A}, \tilde{C})$ is observable, choose

$$\tilde{L} = \begin{bmatrix} 4 & 1 \\ -14.19 & 6 \\ 0 & 2.73 \\ 285.68 & -36.98 \end{bmatrix} \tag{42}$$

Then, $A - \tilde{T}\tilde{L}C$ is stable and for $Q_3 = 10^3 I_4$, the solution of Lyapunov equation (8) is

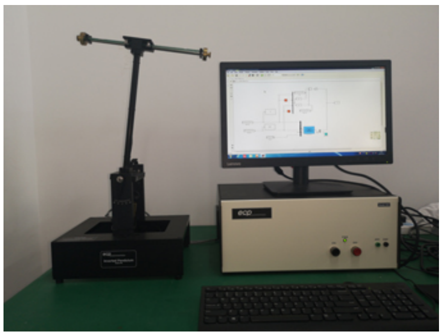

Fig. 1. Sliding self-balancing offset inverted pendulum rig produced by Educational Control Products

$$P_3 = \begin{bmatrix} 8122.20 & -500 & 0 & 4926.83 \\ -500 & 13908.57 & 0 & -144609.40 \\ 0 & 0 & 125 & 0 \\ 4926.83 & -144609.40 & 0 & 1512778.63 \end{bmatrix} \quad (43)$$

Since (A_{11}, A_{12}) is controllable with

$$A_{11} = \begin{bmatrix} 0 & 1 & 0 \\ 29.72 & 0 & 48.70 \\ 0 & 0 & 0 \end{bmatrix}, A_{12} = \begin{bmatrix} -16.70 \\ 0 \\ 1 \end{bmatrix} \quad (44)$$

Choose

$$K = [\,-0.41 \;\; -0.09 \;\; -0.88\,] \quad (45)$$

Then $A_{11} - A_{12}K$ is stable and for $Q_2 = 10^{-3} I_3$, the solution of Lyapunov equation (6) is

$$P_2 = 10^{-2} \times \begin{bmatrix} 0.81 & 0.16 & 1.87 \\ 0.16 & 0.04 & 0.38 \\ 1.87 & 0.38 & 11.28 \end{bmatrix} \quad (46)$$

The new coordinate transformation $z = Tx$ is given by

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -0.41 & -0.09 & -0.88 & 1 \end{bmatrix} \quad (47)$$

Choose $S = [\,-0.41 \;\; -0.09 \;\; -0.88 \;\; 1\,]$, $S_1 = [\,-0.02 \;\; -0.01\,]$, $S_2 = [\,-0.41 \;\; -0.09 \;\; -0.86 \;\; 1.01\,]$. Based on the above analysis, Assumptions 1-4 and Lemma 2 are all satisfied with $F = [\,10 \;\; 50\,]$. Meanwhile, by direct computation, it follows that $\chi_1 = 10^{-3}$, $\chi_2 = 1.4 \times 10^{-3}$ and

$$M = \begin{bmatrix} 0.0007668282825 & -0.3219264043330 \\ -0.3219264043330 & 999.1452458529587 \end{bmatrix}$$

Thus the conditions of Theorem 3 are satisfied. In the simulation, the initial condition is given by:

$$col(\tilde{x}, \hat{x}) = (-0.03, 0.01, 0.08, -0.02, 0.02, -0.01, -0.05, 0.01) \quad (48)$$

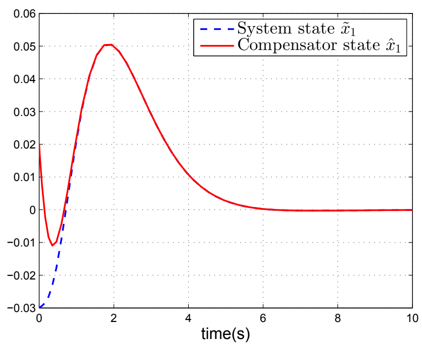

Fig. 2. The responses of $\tilde{x}_1$ and $\hat{x}_1$ in the simulation test

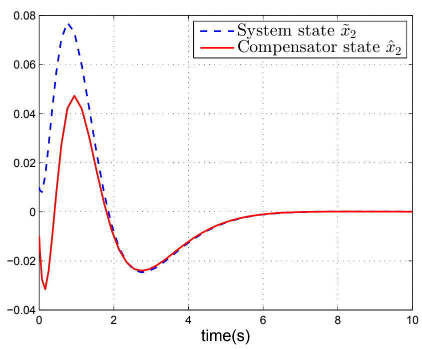

Fig. 3. The responses of $\tilde{x}_2$ and $\hat{x}_2$ in the simulation test

The simulation results in Figs.2-6 show the effectiveness of the designed control. The compensator can effectively observe the system states and the controlled system shows good robustness against unmatched disturbances.

The experimental results in Figs.7-10 show that the compensator states and system output are well-behaved, the control input is realistic and the sliding mode is reached. The proposed algorithm is found to be applicable to a nonlinear and non-minimum phase problem.

6. CONCLUSION

A dynamical OFSMC method has been proposed for a class of nonlinear non-minimum phase systems. A sliding mode control has been designed to ensure that the system states reach the designed sliding surface in finite time. Simulation and experimental test results are given to show the effectiveness of the proposed control scheme. Future

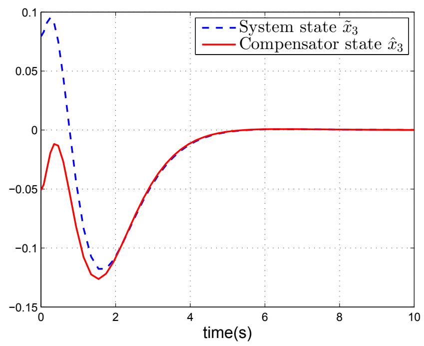

Fig. 4. The responses of $\tilde{x}_3$ and $\hat{x}_3$ in the simulation test

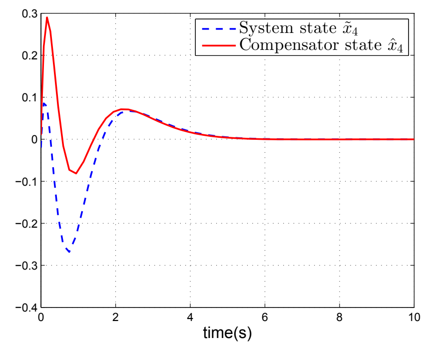

Fig. 5. The responses of $\tilde{x}_4$ and $\hat{x}_4$ in the simulation test

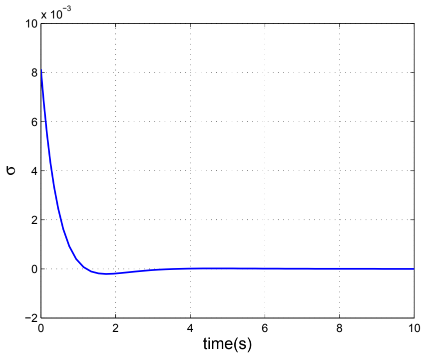

Fig. 6. The sliding function σ in the simulation test

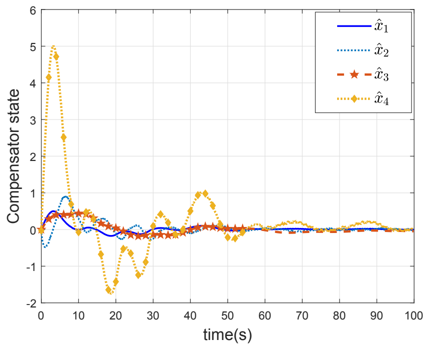

Fig. 7. The compensator states for the experimental tests

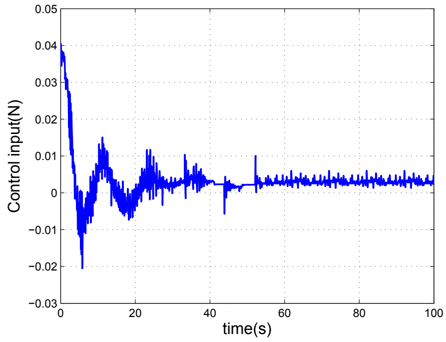

Fig. 8. The control input for the experimental tests

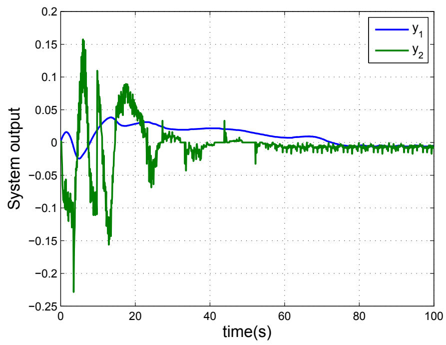

Fig. 9. The system output for the experimental tests

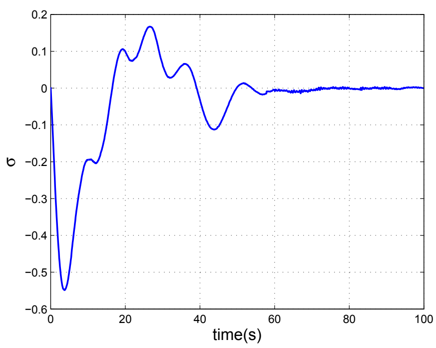

Fig. 10. The sliding function σ for the experimental tests

work will focus on the application of the proposed method to interconnected systems.

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