Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Improved Adaptive Servotracking for a Class of Nonlinear Plants with Unmatched Uncertainties ⋆

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Abstract: The paper addresses the problem of adaptive tracking of multi-sinusoidal reference signal for the class of nonlinear systems with unknown unmatched parameters. It is assumed that the frequencies, amplitudes and phases of the reference harmonics are *a priori* unknown. The problem solution uses linear parameterization of reference signal, modular backstepping design and special adaptation algorithm (identifier) with *memory regressor extension*. The algorithm has two important properties. First of all, it offers improved parametric convergence achieved by regressor recording over past period of time. Recording is provided by involving a linear SISO filter of sufficiently large relative degree into the structure of the algorithm. Second, the structure of the filter allows us to apply the adaptation algorithm for generating the high-order time derivatives of adjustable parameters used in virtual and actual controls.

Keywords: Adaptive backstepping, internal model principle, adaptive tracking, improved parametric convergence.

1. INTRODUCTION

In the field of adaptive control, one of the actual completely unresolved problems is the improvement of the closed-loop transient performance. To overcome this problem, a number of adaptation algorithms with improved parametric convergence was proposed.

More than fifty years ago it was showed by Lion (1967) that multiple filtering of a regression, linear in unknown parameters, yields adaptation algorithm whose rate of convergence can be increased arbitrarily, if regressor satisfies the persistency of excitation (PE) condition. In accordance with the terminology used in Ortega et al. (2020) this technique is called *dynamic regressor extension* (DRE). Ten years later in Kreisselmeier (1977) (see also Kreisselmeier and Joos (1982)) a simpler solution was proposed and later was called adaptation algorithm with integral cost function. In this solution instead of several filters, only one filter of the first order was applied to nonlinearly transformed regression model. It was proved that arbitrary rate of parametric convergence under the PE condition is ensured due to "memory" effect of this filter. In this paper we generalize the idea of Kreisselmeier and use a filter of a sufficiently large relative degree. For this generalization we will stick to the terminology *memory regressor extension* (MRE) recently proposed in Gerasimov et al. (2019b); Ortega et al. (2020).

The ideas of Lion and Kreisselmeier and the schemes with DRE and MRE were later developed: in identification-based backstepping approach (see Kalkkuhl et al. (2002)), which in turns is based on *multiple models* by Narendra and Balakrishnan (1994); in concurrent, combined and composite adaptive control (see Chen et al. (2010); Ciliz (2009); Cho et al. (2017); Kamalapurkar et al. (2017); Lavretsky (2009)); in direct adaptive control with improved tuning properties (see Kreisselmeier (1977); Gerasimov et al. (2018b,a) and survey of Ortega et al. (2020)); in the problems of improved adaptive disturbance compensation (see Gerasimov et al. (2017, 2018c)) and adaptive servotracking (see Gerasimov et al. (2019a)). However, in all the above papers except for the paper of Kalkkuhl et al. (2002); Gerasimov et al. (2020) the algorithms with improved parametric convergence were considered for the case of matched uncertainties. In Kalkkuhl et al. (2002) the authors use Kreisselmeier's estimator together with traditional backstepping procedure with *tuning functions*. The scheme is designed under restrictive PE condition (see Theorem 1).

The control problem for a certain class of nonlinear plants with unmatched linearly parameterized uncertainties was resolved in Kanellakopoulos et al. (1991), where the authors proposed *adaptive backstepping* procedure with step-by-step design of adjustable stabilizing functions. The procedure was later called adaptive backstepping with *overparameterization.* Later, in Krstić et al. (1992), Krstić et al. (1995) this method was simplified, and two modifications of iterative design procedures without overparameterization were presented and called *adaptive backstepping with tuning functions* and *adaptive backstepping with modular identifiers.* The latter approach implies independent step-by-step design of stabilizing controller that ensures strong

⋆ This work is supported by the Russian Science Foundation grant (project 17-79-20341).

input-to-state stability (ISS) property, and design of an adaptation algorithm (identifier) that drives the tracking error to zero. The strong ISS property is achieved due to nonlinear *damping terms* in stabilizing functions derived at each step of the backstepping procedure. In Krstić et al. (1995) the ISS controller was combined with the identifiers designed on the basis of two different approaches called *passivity-based* approach and *swapping-based* one. In both cases gradient-like adaptation algorithms were used.

In this paper we solve the problem of servotracking based on modular backstepping design with a special type of MRE identifier. This identifier generates the high-order time derivatives (HOTD) of the adjustable parameters which are used directly in backstepping procedure for design of virtual and actual controls. An alternative backstepping design procedure with the use of the HOTD of adjustable parameters was proposed by Nikiforov and Voronov (2001) where Morse's high-order tuner of gradient type (see Morse (1992)) was applied. The dual problem of adaptive compensation of external multi-sinusoidal disturbances has recently been reported in Gerasimov et al. (2020).

The main contributions of the paper consist in the following.

1. A new adaptive servotracking controller is designed by applying the adaptive modular backstepping procedure involving the HOTD of the adjustable parameters. The controller provides complete compensation for unmatched uncertainties of both nonlinear plant and multi-sinusoidal reference signal and drives the tracking error to zero with all the closed-loop signals bounded.

2. Special modification of the adaptation algorithm with MRE is constructed at the final step of the design procedure and permits:

— to generate the HOTD of the adjustable parameters of virtual and actual control laws;

— to accelerate the tuning of control law and, hence improve transient performance of the closed-loop system.

The remainder of the paper is organized as follows. In the second section the problem of adaptive tracking is formulated. In the third section the procedure of reference parameterization is presented. In the forth section by means of backstepping procedure the actual adjustable controller is designed and the closed-loop error model is obtained. In the fifth section based on the error model the adaptation algorithm with improved parametric convergence is presented and analyzed. In the sixth section simulation results are demonstrated and discussed.

Notations: $s := d/dt$ is the derivative operator, $O_{i\times j}$ is zero $i \times j$ matrix, I_i is $i \times i$ identity matrix, $||\cdot||$ is the 2-norm of a matrix or a vector, $adj\{\cdot\}$ is the adjoint matrix, $\otimes$ is the Kroneker product operator.

2. PROBLEM STATEMENT

We consider uncertain SISO nonlinear plant presentable in the *parametric state feedback* (PSF) form

$$\dot{x}_i = x_{i+1} + \phi_i^\top(x_1, x_2, \cdots, x_i)\theta, \quad i = \overline{1, n-1}, \tag{1}$$

$$\dot{x}_n = \phi_n^\top(x)\theta + \alpha(x) + \beta(x)u, \tag{2}$$

$$y = x_1, \tag{3}$$

where $u \in \mathbb{R}$ is the control variable, $y \in \mathbb{R}$ is the regulated variable, $x_j,\ j = \overline{1, n}$ are the elements of the state vector $x \in \mathbb{R}^n$, $\theta \in \mathbb{R}^q$ is the constant vector of unknown parameters, $\alpha(x) \in \mathbb{R}$, $\beta(x) \in \mathbb{R}$ and $\phi_i \in \mathbb{R}^q$ are the smooth functions [1]. Function $\beta(x)$ is bounded away from zero by some constant $b_{min} > 0$, i.e. $\beta(x) \geq b_{min}$.

The control objective considered is divided into two parts. The first part is to design a state-feedback control providing the boundedness of all the closed-loop signals and driving the tracking error ε to zero in accordance with equality

$$\lim_{t\to\infty} \|\varepsilon\| = \lim_{t\to\infty} \|g - y\| = 0, \tag{4}$$

in which g is the reference signal.

The second part of the objective is to accelerate controller tuning by applying an adaptation algorithm with improved parametric convergence instead of the standard gradient-based adaptation algorithm.

We make the following assumptions.

(A.1) State x is measurable.

(A.2) Reference g is modelled as the output of the autonomous exosystem

$$\dot{\eta} = \Gamma\eta, \quad \eta(0), \tag{5}$$
$$g = h^\top\eta$$

with the unmeasurable vector $\eta \in \mathbb{R}^m$, $\Gamma \in \mathbb{R}^{m\times m}$ is the unknown matrix with simple eigenvalues on the imaginary axis, $h \in \mathbb{R}^m$ is the unknown vector.

(A.3) Pair (Γ, h) is observable, order m is known.

Assumption (A.1) is standard for the problems of control of PSF systems (see Krstić et al. (1995)). Assumptions (A.2) and (A.3) mean that model (5) generates multi-sinusoidal biased signals with *a priori* unknown frequencies, amplitudes and phases, however known upper bound of the number of harmonics (sinusoids) .

The startpoint of problem solution consists of in linear parameterization of the reference signal and its time derivative.

3. REFERENCE SIGNAL PARAMETERIZATION

We represent signal g in the form of linear regression (see Nikiforov (1997)):

$$g = \tau_0^\top\xi + \tau_0^\top\sigma, \tag{6}$$

where $\tau_0 \in \mathbb{R}^m$ is the vector of unknown parameters dependent on the elements of matrices Γ and h, σ is the exponentially decaying vector satisfying equation

$$\dot{\sigma} = G\sigma, \quad \sigma(0), \tag{7}$$

$\xi \in \mathbb{R}^m$ is the state of the filter

$$\dot{\xi} = G\xi + lg, \quad \xi(0) \tag{8}$$

[1] For the sake of simplicity of paper presentation the arguments of functions ϕ_i and the argument t will be omitted except when necessary.

with arbitrary preselected Hurwitz matrix $G \in \mathbb{R}^{m\times m}$ and the vector $l \in \mathbb{R}^m$ chosen so that the pair (G, l) is controllable.

It worth noting that vector ξ is measurable, since the filter (8) involves the measurable input g and contains known matrices G and l.

Replacing (6) in (8) we obtain the model

$$\dot{\xi} = (G + l\tau_0^\top)\xi + l\tau_0^\top\sigma. \tag{9}$$

As it will be shown in the backstepping procedure, instead of parameterization (6) we will need parameterization of the first time derivative $\dot{g}$ obtained using (6), (7) and (9):

$$\dot{g} = \tau_1^\top\xi + \tau_1^\top\sigma, \tag{10}$$

where $\tau_1 = (G + l\tau_0^\top)^\top\tau_0$ is the vector of unknown parameters.

Remark 1. In the sequel, the exponentially decaying term σ is to be taken into account, since this term excites nonlinear system and therefore affects its stability.

4. TRACKING CONTROLLER AND ERROR MODEL DESIGN

Applying the modular design technique with *swapping identifiers* (see Krstić et al. (1995)) we propose new procedure with the following distinguishing features: 1) the procedure involves nonlinear damping terms related to the unknown plant and reference model parameters θ, τ_0 and τ_1 and, as well as to the exponentially decaying term σ; 2) the virtual and the actual control laws explicitly depend on the HOTD of adjustable parameters generated by proposed adaptation algorithm; 3) design of adaptation algorithm is based on the *nonlinear swapping* lemma (see Appendix F of Krstić et al. (1995)) and generalization of Kreisselmeier's (see Kreisselmeier (1977)) adaptation algorithm (the algorithm with MRE).

Taking into account parameterizations (9) and (10), the first and the next ith steps ($i = \overline{2,n}$) of the backstepping procedure resulting in virtual ($i < n$) and actual ($i = n$) controls are the following:

$$\begin{aligned} &U_1 = (k_1 + F_1)z_1 - \phi_1^\top\hat{\theta} + \xi^\top\hat{\tau}_1, \\ &U_i = z_{i-1} + (k_i + F_i)z_i + \\ &\sum_{l=1}^{i-1}\left[\frac{\partial U_{i-1}}{\partial x_l}x_{l+1} + \frac{\partial U_{i-1}}{\partial\hat{\theta}^{(l-1)}}\hat{\theta}^{(l)} + \frac{\partial U_{i-1}}{\partial\hat{\tau}_0^{(l-2)}}\hat{\tau}_0^{(l-1)} + \right. \\ &\left.\frac{\partial U_{i-1}}{\partial\hat{\tau}_1^{(l-1)}}\hat{\tau}_1^{(l)}\right] + \frac{\partial U_{i-1}}{\partial\xi}G\xi + \omega_{i0}^\top\hat{\theta} + \omega_{i1}\xi^\top\hat{\tau}_0 + \omega_{i2}\xi^\top\hat{\tau}_1, \end{aligned} \tag{11}$$

where $z_1 = \varepsilon = g - x_1$, $z_i = U_{i-1} - x_i$ are the state errors, $k_i > 0$ are the constants,

$$\begin{aligned} F_1 &= \kappa_1(||\xi||^2 + 1) + g_1||\phi_1||^2, \\ F_i &= \kappa_i(\omega_{i1}^2 + \omega_{i2}^2)(||\xi||^2 + 1) + g_i||\omega_{i0}||^2 \end{aligned} \tag{12}$$

are the damping terms with positive constants κ_i and g_i,

$$\omega_{i0} = -\phi_i + \sum_{j=1}^{i-1}\frac{\partial U_{i-1}}{\partial x_j}\phi_j, \tag{13}$$

$$\omega_{i1} = \frac{\partial U_{i-1}}{\partial\xi}l, \quad \omega_{i2} = \frac{\partial U_{i-1}}{\partial g}, \tag{14}$$

$\hat{\theta}$, $\hat{\tau}_0$ and $\hat{\tau}_1$ are the estimates of θ, τ_0 and τ_1, respectively.

Finally, in the last nth step the *actual control* is derived:

$$u = \frac{1}{\beta}\left(-\alpha + U_n\right). \tag{15}$$

Note that all the indexed variables in formulas (11) are assumed zero, if the index is below one.

Lemma 1. The control law (11)-(15) being applied to the plant (1)-(3) yields the closed-loop error model

$$\begin{aligned} \dot{z} = &A_z(z, x, \bar{\vartheta})z + W(z, x, \bar{\vartheta})\tilde{\vartheta} + \\ &Q_1(z, x, \bar{\vartheta})\tau_0^\top\sigma + Q_2(z, x, \bar{\vartheta})\tau_1^\top\sigma, \end{aligned} \tag{16}$$

where $z = col(z_1, z_2, ..., z_n)$ is the vector of state errors,

$$A_z = \begin{bmatrix} -k_1 - F_1 & 1 & \cdots & 0 \\ -1 & -k_2 - F_2 & \cdots & 0 \\ 0 & -1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & -k_n - F_n \end{bmatrix},$$

$$W = [\Phi, \xi^\top \otimes Q_1, \xi^\top \otimes Q_2]$$

is the $n \times (q + 2m)$ matrix with

$$\begin{aligned} \Phi &= col(-\phi_1^\top, \omega_{20}^\top, \omega_{30}^\top, \cdots, \omega_{n0}^\top), \\ Q_1 &= col(0, \omega_{21}, \omega_{31}, \cdots, \omega_{n1}), \\ Q_2 &= col(1, \omega_{22}, \omega_{32}, \cdots, \omega_{n2}), \end{aligned}$$

$\tilde{\vartheta} = \vartheta - \hat{\vartheta}$ is the aggregated vector of parametric error,

$$\vartheta = col(\theta, \tau_0, \tau_1) \in \mathbb{R}^{q+2m}$$

is the aggregated vector of unknown parameters,

$$\hat{\vartheta} = col(\hat{\theta}, \hat{\tau}_0, \hat{\tau}_1)$$

is the aggregated vector of adjustable parameters, $\bar{\vartheta}$ is the vector including all the adjustable parameters and their time derivatives up to a required order.

The lemma is proved by calculating the time derivatives $\dot{z}_i = \dot{U}_{i-1} - \dot{x}_i$ in a view of (1)-(3), control (11)-(15), parameterization of the reference time derivative (10) and model (9).

One of the key properties of control law (11)-(15) is that, due to the damping terms F_i included, it ensures the ISS property of the closed-loop system (16), in which $\tilde{\vartheta}$ and σ are considered as the inputs.

Lemma 2. If $\tilde{\vartheta}$ is bounded, then in the closed-loop system (16) vector z is bounded and exponentially converges to a compact residual set with zero origin.

The lemma can be proved by selection of Lyapunov function candidate

$$V_z = \frac{1}{2}||z||^2 + \int_t^\infty \|\sigma(\tau)\|^2 d\tau$$

and calculation of its time derivative in a view of (16) and (7).

Lemma 2 is similar to Lemma 5.8 of Krstić et al. (1995) in *mutatis mutandis.*

5. DESIGN OF ADAPTATION ALGORITHM

Following Krstić et al. (1995) and Gerasimov et al. (2020) we represent model (16) in the form of linear regression.

Lemma 3. Together with the closed-loop model (16) consider filters

$$\dot{\zeta} = A_z\zeta + W\hat{\vartheta}, \tag{17}$$

$$\dot{\overline{W}} = A_z\overline{W} + W \tag{18}$$

and define the *augmented error*

$$\bar{z} = z + \zeta. \tag{19}$$

Then $\bar{z}$ can be represented as

$$\bar{z} = \overline{W}\vartheta + \epsilon, \tag{20}$$

where ϵ exponentially decays.

Proof. By taking time derivative of the signal $\epsilon = z + \zeta - \overline{W}\vartheta$ derived from (20) and (19) in view of (16), (17) and (18) we have:

$$\dot{\epsilon} = A_z\epsilon + Q_1{\tau_0}^\top\sigma + Q_2{\tau_1}^\top\sigma. \tag{21}$$

The proof is completed by selecting Lyapunov function

$$V = \frac{1}{2}||\epsilon||^2 + \int_t^\infty ||\sigma(\tau)||^2 d\tau$$

and calculating its time derivative with the use of (21) and the structures of matrices A_z, Q_1 and Q_2. □

It is worth noting that due to the special structures of matrices A_z, Q_1 and Q_2 the filter state $\overline{W}$ in (18) is bounded regardless of the boundedness of W (see Lemma 6.1 of Krstić et al. (1995)).

Now we need to design an adaptation algorithm that will simultaneously allow us to: 1) generate the HOTD of the adjustable parameters up to a required order; 2) accelerate transients of adjustable parameters and improve parametric convergence.

To this end, we introduce the SISO transfer function

$$\mathcal{H}(s) = \frac{1}{d(s)},$$

where $d(s) = s^{n-2} + d_{n-3}s^{n-3} + ... + d_0$ is the Hurwitz polynomial of degree $(n-2)$ with preselected constant coefficients. Then, we modify the regression (20) multiplying it by $\overline{W}$ and applying operator $\mathcal{H}(s)$:

$$Z = \Omega\vartheta + \bar{\epsilon}, \tag{22}$$

where $Z = \mathcal{H}(s)[\overline{W}^T\bar{z}]$ is the vector output of new regression, $\Omega = \mathcal{H}(s)[\overline{W}^\top\overline{W}]$ is the matrix regressor, $\bar{\epsilon} = \mathcal{H}(s)[\overline{W}^\top\epsilon]$. Note that Ω is the positive semi-definite matrix, and the term $\bar{\epsilon}$ exponentially decays due to the boundedness of $\overline{W}$ and exponentially decaying term ϵ. Then, based on regression (22) we form the adaptation algorithm

$$\dot{\hat{\vartheta}} = \gamma\left(Z - \Omega\hat{\vartheta}\right). \tag{23}$$

In view of (20) it can be shown that in this case the parametric error model is given by

$$\dot{\tilde{\vartheta}} = -\gamma\Omega\tilde{\vartheta} - \gamma\bar{\epsilon}. \tag{24}$$

Since the relative degree of the transfer function $\mathcal{H}(s)$ is $(n-2)$, the HOTD of Z and Ω are implementable, and the adaptation algorithm (23) can be used for calculation of the HOTD of adjustable parameters $\hat{\vartheta}$ up to the $(n-1)$th order. Indeed, direct differentiation of (23) gives:

$$\hat{\vartheta}^{(k+1)} = \gamma\left(Z^{(k)} - \sum_{j=0}^{k} C_j^k\Omega^{(k-j)}\hat{\vartheta}^{(j)}\right), \tag{25}$$

where C_j^k are the binomial coefficients, $k = \overline{1, n-2}$.

Adaptation algorithm (23) allows one to derive several alternative expressions for calculation of the HOTD of the adjustable parameters. In particular, instead of direct calculation of the HOTD of $\hat{\vartheta}$ in accordance with (25) (that requires substantial computational efforts) the HOTD can be generated by an adaptation algorithm presented in the closed-loop form (for more detail see Gerasimov and Nikiforov (2020)):

$$d(s)[\dot{\hat{\vartheta}} + \gamma\Omega\hat{\vartheta}] = \gamma\overline{W}^T\bar{z}. \tag{26}$$

Remark 2. On the one hand, we are free to use unnormalized adaptation algorithm (23) (or (26)) due to the boundedness of Ω (since $\overline{W}$ is bounded). At the same time, since the control law (11) – (15) provides the ISS property for the closed-loop system, we can choose different types of normalized algorithms regardless of the fact that the plant is nonlinear (see Krstić et al. (1995)).

Now, we are in position to present our main result. Before this, we define $\lambda(t) \geq 0$ as the minimum eigenvalue of Ω.

Proposition 1. The control law (11) – (15) together with the adaptation algorithm (23) being applied to the plant (1)-(3) provides the following properties in the closed-loop system:

- i boundedness of all the signals and asymptotic convergence of the tracking error $\varepsilon = g - y = z_1$ to zero;
- ii if $\lambda(t) \notin L_1$, $||\tilde{\vartheta}||$ converges to zero asymptotically; the rate of convergence can be increased by increasing adaptation gain γ.
- iii if $\lambda(t) \geq \lambda_0 > 0$ ($\lambda_0 = const$), $||\tilde{\vartheta}||$ approaches zero exponentially fast with the rate that can be increased by increasing gain γ.

Proof. The proof of the first property is followed from the proof of Theorem 6.3 in Krstić et al. (1995) and uses results of stability proof presented by Kreisselmeier and Joos (1982). First, since Ω is positive semi-definite and bounded, while $\bar{\epsilon}$ exponentially decays, we can conclude from (24) that $\tilde{\vartheta}, \dot{\hat{\vartheta}} \in L_\infty$, $||\overline{W}\tilde{\vartheta}|| \to 0$ and $||\dot{\hat{\vartheta}}|| \to 0$ as $t \to \infty$ (see Kreisselmeier and Joos (1982)). It follows from Lemma 2 that since $\dot{\hat{\vartheta}} \in \infty$, $z \in L_\infty$. As a result, all the closed-loop signals are bounded. Since $||\dot{\hat{\vartheta}}|| \to 0$ and $||\overline{W}\tilde{\vartheta}|| \to 0$ as $t \to \infty$, we have that $z \to \overline{W}\tilde{\vartheta}$ (see *nonlinear swapping lemma* in Krstić et al. (1995)) and therefore $z \to 0$ and $\varepsilon \to 0$ as $t \to \infty$.

Properties (ii) and (iii) are proved by selecting Lyapunov function $V = ||\tilde{\vartheta}||^2/2$ and calculating its time derivative in view of (24) and Rayleigh quotient:

$$\dot{V} = -\gamma\vartheta^\top\Omega\tilde{\vartheta} + \gamma\vartheta^\top\bar{\epsilon} \leq -2\gamma\lambda V + \gamma||\tilde{\vartheta}||||\bar{\epsilon}||.$$

It follows from the last inequality that

$$||\tilde{\vartheta}(t)||^2 \leq 2e^{-2\gamma\int_0^t \lambda(r)dr}V(0) +$$
$$2\int_0^t \gamma e^{-2\gamma\int_{r_1}^t \lambda(r_2)dr_2}||\tilde{\vartheta}(r_1)||||\bar{\epsilon}(r_1)||dr_1.$$

Since $||\tilde{\vartheta}|| \in L_\infty$, $||\bar{\epsilon}||$ exponentially decays and $\lambda(t) \geq 0 \ \forall t \geq 0$, $||\tilde{\vartheta}(t)||$ approaches zero asymptotically, while time tends to infinity, if $\lambda(t) \notin L_1$. If additionally $\lambda(t) \geq \lambda_0$, $||\tilde{\vartheta}(t)||$ decays exponentially. In both cases the rate of convergence of the norm $||\tilde{\vartheta}(t)||$ to zero can be increased by increasing gain γ. This completes the proof. $\square$

It is seen form Proposition 1 that the adaptation algorithm proposed improves transient performance (in comparison with standard modular backstepping design procedure with swapping-based gradient algorithms of adaptation) in two directions: 1) property (ii) relaxes the PE condition in the sense that it provides asymptotic (not exponential) convergence of parametric errors to zero under some weak condition, even if $\overline{W} \notin PE$; 2) properties (ii) and (iii) allow us to accelerate tuning of adaptive controller and to improve transient performance of the closed-loop system by increasing adaptation gain γ. Of course, the latter can be achieved in theory after some period of time only due to the exponentially decaying term $\bar{\epsilon}$ with dynamics simultaneously depending on the roots of polynomial $d(s)$, the properties of matrix A_z and the eigenvalues of matrix G.

6. SIMULATION

Consider the third order plant

$$\begin{aligned} \dot{x}_1 &= x_2 + \phi_1^\top(x_1)\theta, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= u, \\ y &= x_1 \end{aligned}$$

with unknown vector $\theta = col(4,5)$, nonlinear function $\phi_1(x_1) = col(x_1^2, \sin(x_1))$. Initial conditions of the plant are $x(0) = col(1,0,0)$.

Reference signal to be tracked is given by $g = 10\sin(\omega t)$. The amplitudes, phases and frequencies of this signal are unknown.

To demonstrate the adaptive properties of the system we change the reference frequency ω from 2 [rad/sec] to 5 [rad/sec] at 100 [sec] as shown by Fig.1c.

Passing through three steps of the backstepping procedure we obtain the control law (15) that contains:

- virtual control laws U_1, U_2, U_3 defined by (11) and functions $\alpha = 0$, $\beta = 1$ (i.e. $u = U_3$).
- $\hat{\vartheta}$, $\dot{\hat{\vartheta}}$, $\ddot{\hat{\vartheta}}$ are generated by the adaptation algorithm (26) with polynomial $d(s) = s + 1$ and zero initial conditions.
- functions ω_{i0}, ω_{i1} and ω_{i2} defined by (13), (14) for $i = 1,2,3$.
- parameters $k_i = 1$ for $i = 1,2,3$.
- vector ξ is calculated using (8) with matrices
$$G = \begin{bmatrix} 0 & 1 \\ -100 & -50 \end{bmatrix}, \ l = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
and zero initial conditions.
- damping terms (12) with coefficients $\kappa_i = 10^{-3}$, $g_i = 10^{-6}$ for $i = 1,2,3$.

Evolutions of the tracking errors in the adaptive systems closed by control law (15) with adaptation algorithm (26) for $\gamma = 20$ (plot a)) and for $\gamma = 5000$ (plot b)) are presented in Fig.1.It is seen from the results that, despite the frequency change (see Fig1.c), the tracking error approaches zero. At the same time, comparing transients of ε shown in Fig.1(plot a)) and Fig.1(plot b)) we observe increase of the rate of parameters convergence caused by increasing γ.

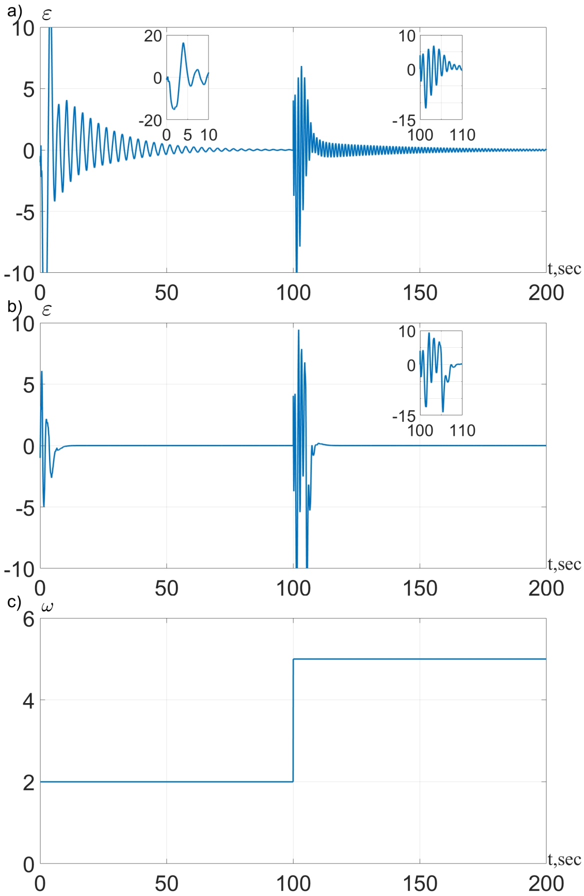

Fig. 1. Simulation results: evolutions of tracking errors ε in the closed-loop adaptive systems with a) $\gamma = 20$ and b) $\gamma = 5000$; c) change of reference signal frequency ω

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